

Orientation Angles



Dynamics: Orientation Angles

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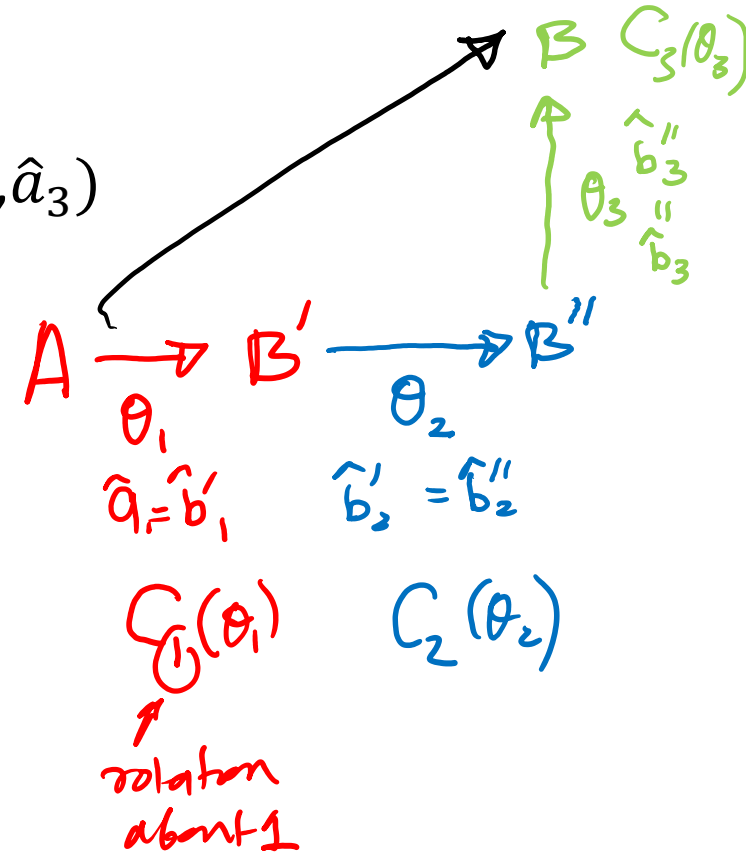
Three Angles

- Orientation of body B in reference frame A can be represented in terms of three angles ...
- Consider unit vectors: $(\hat{a}_1, \hat{a}_2, \hat{a}_3)$ and $(\hat{b}_1, \hat{b}_2, \hat{b}_3)$
- Starting with $(\hat{a}_1, \hat{a}_2, \hat{a}_3)$ and $(\hat{b}_1, \hat{b}_2, \hat{b}_3)$ aligned, one can rotate about three angles about three particular axes to get from $(\hat{a}_1, \hat{a}_2, \hat{a}_3)$ to $(\hat{b}_1, \hat{b}_2, \hat{b}_3)$
- These are three orientation angles
- See Appendix I and II in the book for all the possible angle combinations

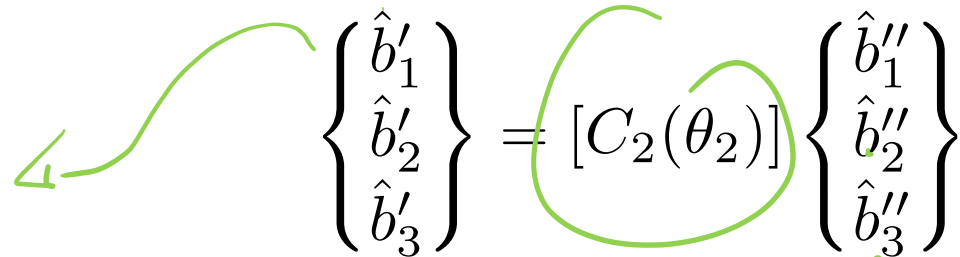
Example: Body 1-2-3

- Consider the body $(\hat{b}_1, \hat{b}_2, \hat{b}_3)$ is initially aligned with $(\hat{a}_1, \hat{a}_2, \hat{a}_3)$
- Consider a rotation of θ_1 about $\hat{b}'_1 = \hat{a}_1$
- This takes us to a new reference frame, say $(\hat{b}'_1, \hat{b}'_2, \hat{b}'_3)$

$$\begin{Bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \end{Bmatrix} = [C_1(\theta_1)] \begin{Bmatrix} \hat{b}'_1 \\ \hat{b}'_2 \\ \hat{b}'_3 \end{Bmatrix}$$

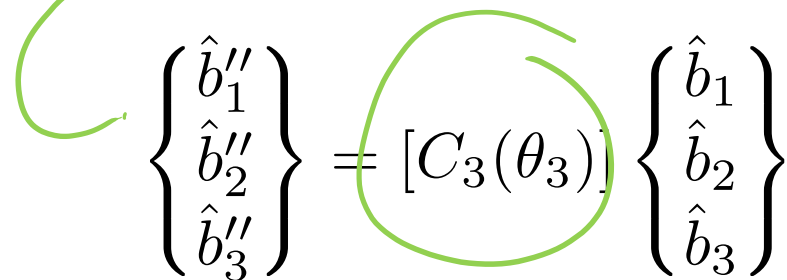


- Next, consider a rotation of θ_2 about $\hat{b}'_2 = \hat{b}''_2$
- This takes us to a new reference frame, say $(\hat{b}''_1, \hat{b}''_2, \hat{b}''_3)$



$$\begin{Bmatrix} \hat{b}'_1 \\ \hat{b}'_2 \\ \hat{b}'_3 \end{Bmatrix} = [C_2(\theta_2)] \begin{Bmatrix} \hat{b}''_1 \\ \hat{b}''_2 \\ \hat{b}''_3 \end{Bmatrix}$$

- Finally, consider a rotation of θ_3 about $\hat{b}''_3 = \hat{b}_3$
- This takes us to the body reference frame system $(\hat{b}_1, \hat{b}_2, \hat{b}_3)$



$$\begin{Bmatrix} \hat{b}''_1 \\ \hat{b}''_2 \\ \hat{b}''_3 \end{Bmatrix} = [C_3(\theta_3)] \begin{Bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \end{Bmatrix}$$

- The direction cosine matrix can be written as:

$$[{}^A C^B] = [C_1(\theta_1)] [C_2(\theta_2)] [C_3(\theta_3)] = \begin{bmatrix} \overset{\nearrow \cos(\theta_2)}{c_2 c_3} & -c_2 s_3 & s_2 \\ s_1 s_2 c_3 + s_3 c_1 & -s_1 s_2 s_3 + c_3 c_1 & -s_1 c_2 \\ -c_1 s_2 c_3 + s_3 s_1 & c_1 s_2 s_3 + c_3 s_1 & c_1 c_2 \end{bmatrix}$$

$$\begin{Bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \end{Bmatrix} = [{}^A C^B] \begin{Bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \end{Bmatrix}$$

- One can go from the angles to direction cosine matrix and one can go from the direction cosine matrix back to angles* (see section 10.3)

* except at singularities

Angular Velocity

- Use the addition theorem and the simple angular velocities to calculate the angular velocity vectors

$${}^A\vec{\omega}^B = \dot{\theta}_1 \hat{b}'_1 + \dot{\theta}_2 \hat{b}''_2 + \dot{\theta}_3 \hat{b}_3 \quad \rightarrow \text{in mixed basis}$$

$$= \omega_1 \hat{b}_1 + \omega_2 \hat{b}_2 + \omega_3 \hat{b}_3 \quad \rightarrow \text{in body frame}$$

- The measure numbers of the angular velocity vector can be represented in any reference frame, specifically we can find the measure numbers in the B reference frame

angular velocity components

$$\begin{Bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{Bmatrix} = \begin{bmatrix} c_2 c_3 & s_3 & 0 \\ -c_2 s_3 & c_3 & 0 \\ s_2 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{Bmatrix}$$

rate of change of Euler angles

Gimbal lock.

$\{ \dot{\theta} \} = [M]^T \{ \omega \}$

... these are the kinematical differential equations

... the above matrix is not always invertible – singularity!

Other Orientation Angle Representations

- Other axis sequence, e.g., Body 3-2-1 used in aircraft flight mechanics
 - The direction cosine matrix and kinematical differential equations for Body 3-2-1 were presented in Lecture 3

$$\vec{\omega} = p \hat{b}_1 + q \hat{b}_2 + r \hat{b}_3 \quad \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{bmatrix} 1 & 0 & -s_\theta \\ 0 & c_\phi & c_\theta s_\phi \\ 0 & -s_\phi & c_\theta c_\phi \end{bmatrix} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}$$

- Angles measured about the space fixed axes, e.g, Space 1-2-3
- 2-axis (not 2-angle) instead of 3-axis angles, e.g., Space 1-2-1
a student is going to figure it out
- See Appendix I and II for the direction cosine matrices and kinematical differential equations for 24 different orientation angle sequences