Velocity and Acceleration

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Velocity

- For a point P moving in frame A, the velocity of P in A is defined as:

$$ {}^{A}\vec{v}^{P} = \frac{{}^{A}d\vec{r}^{OP}}{dt} $$

where, O is a point fixed in A.

- And the acceleration of P in A :

$$ {}^{A}\vec{a}^{P} = \frac{{}^{A}d\,{}^{A}\vec{v}^{P}}{dt} $$

- Straightforward and simple in definition! If A is an inertial frame then the velocity and acceleration are inertial.

NOTE

- Velocities and Accelerations are always of points/particles, NEVER of Rigid Bodies
  - Though we may say ”velocity of an airplane” we either mean that the airplane has no angular velocity and thus all point do have the same velocity or we are implicitly referring to the velocity of a particular point on the aircraft
- Angular Velocities and Angular Accelerations are always of frames or rigid bodies, NEVER of points

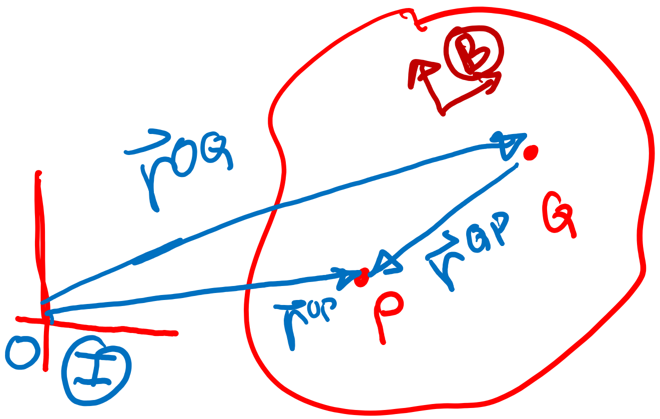
Special Case:
Two Points Fixed on a (Moving) Rigid Body

- For two points P and Q fixed in a rigid body B, the velocities and accelerations of the two points in A can be related

$$
{}^{A}\vec{v}^{P} = {}^{A}\vec{v}^{Q} + {}^{A}\vec{\omega}^{B} \times \vec{r}^{QP}
$$

$$
{}^{A}\vec{a}^{P} = {}^{A}\vec{a}^{Q} + \underbrace{{}^{A}\vec{\omega}^{B} \times \left({}^{A}\vec{\omega}^{B} \times \vec{r}^{QP}\right)} + {}^{A}\vec{\alpha}^{B} \times \vec{r}^{QP}
$$

- If you know velocity (or acceleration) of one point on a rigid body, you can find velocity (or acceleration) at any other point if you know the angular velocity (and angular acceleration) of the rigid body.
- The second term in the acceleration above should remind you of centripetal acceleration!



$$ {}^{I}\vec{v}^{Q} = \frac{{}^{I}d\,\vec{r}^{OQ}}{dt} $$

$$ {}^{I}\vec{v}^{P} = \frac{{}^{I}d\,\vec{r}^{OP}}{dt} $$

$$ \vec{r}^{OP} = \vec{r}^{OQ} + \vec{r}^{QP} $$

$$ {}^{I}\vec{v}^{P} = \frac{{}^{I}d\,\vec{r}^{OQ}}{dt} + \frac{{}^{I}d\,\vec{r}^{QP}}{dt} $$

$$ = {}^{I}\vec{v}^{Q} + \cancelto{0}{\frac{{}^{B}d\,\vec{r}^{QP}}{dt}} + {}^{I}\vec{\omega}^{B} \times \vec{r}^{QP} $$

$$ = {}^{I}\vec{v}^{Q} + {}^{I}\vec{\omega}^{B} \times \vec{r}^{QP} $$

$$ {}^{I}\vec{a}^{P} = \frac{{}^{I}d\,{}^{I}\vec{v}^{P}}{dt} = \frac{{}^{I}d\,{}^{I}\vec{v}^{Q}}{dt} + \frac{{}^{I}d\,{}^{I}\vec{\omega}^{B}}{dt} \times \vec{r}^{QP} + {}^{I}\vec{\omega}^{B} \times \frac{{}^{I}d\,\vec{r}^{QP}}{dt} $$

$$ = {}^{I}\vec{a}^{Q} + {}^{I}\vec{\alpha}^{B} \times \vec{r}^{QP} + {}^{I}\vec{\omega}^{B} \times \left({}^{I}\vec{\omega}^{B} \times \vec{r}^{QP}\right) $$

Special Case:
A $\boxed{\text{Point}}$ Moving on a (Moving) Rigid Body

- The velocity and acceleration of point P moving on rigid body B, while B is moving in reference frame A

$$
{}^{A}\vec{v}^{P} = {}^{A}\vec{v}^{\Phi} + {}^{B}\vec{v}^{P}
$$

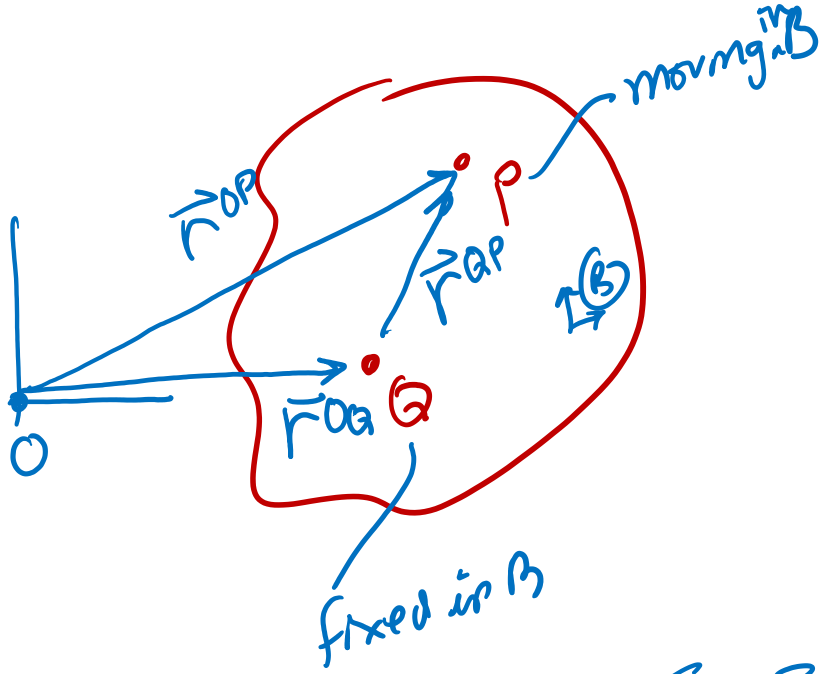
$$
{}^{A}\vec{a}^{P} = {}^{A}\vec{a}^{\Phi} + {}^{B}\vec{a}^{P} + \underbrace{2\,{}^{A}\vec{\omega}^{B} \times {}^{B}\vec{v}^{P}}
$$

3 Terms

$$
= {}^{A}\vec{a}^{Q} + {}^{A}\vec{\alpha}^{B} \times \vec{r}^{QP} + {}^{A}\vec{\omega}^{B} \times ({}^{A}\vec{\omega}^{B} \times \vec{r}^{QP}) + {}^{B}\vec{a}^{P} + 2\,{}^{A}\vec{\omega}^{B} \times {}^{B}\vec{v}^{P}
$$

5 terms

  - where, ϕ, at any instant is a point fixed in B that coincides with P
  - and, using the last slide, we know how to calculate the velocity and acceleration of any point fixed in B
- The last term in the acceleration should remind you of Coriolis's acceleration



$${}^{I}\vec{v}^{P} = \frac{{}^{I}d\,\vec{r}^{OP}}{dt}$$

fixed in I

$$= \frac{{}^{I}d\,\vec{r}^{OQ}}{dt} + \frac{{}^{I}d\,\vec{r}^{QP}}{dt}$$

$$= {}^{I}\vec{v}^{Q} + \underbrace{\frac{{}^{B}d\,\vec{r}^{QP}}{dt}}_{= {}^{B}\vec{v}^{P}} + {}^{I}\vec{\omega}^{B} \times \vec{r}^{QP}$$

fixed in B

whatever makes sense { 3 terms, 2 terms

$$\equiv {}^{I}\vec{v}^{\bar{P}} + {}^{B}\vec{v}^{P}$$

$\bar{P}$ is a point fixed in B where P is at this instant

Example