Maclaurin Series

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Made with LaTeX

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Outline

What is it?

- Approximation of a function with an infinite series
- Approximates near 0

Why?

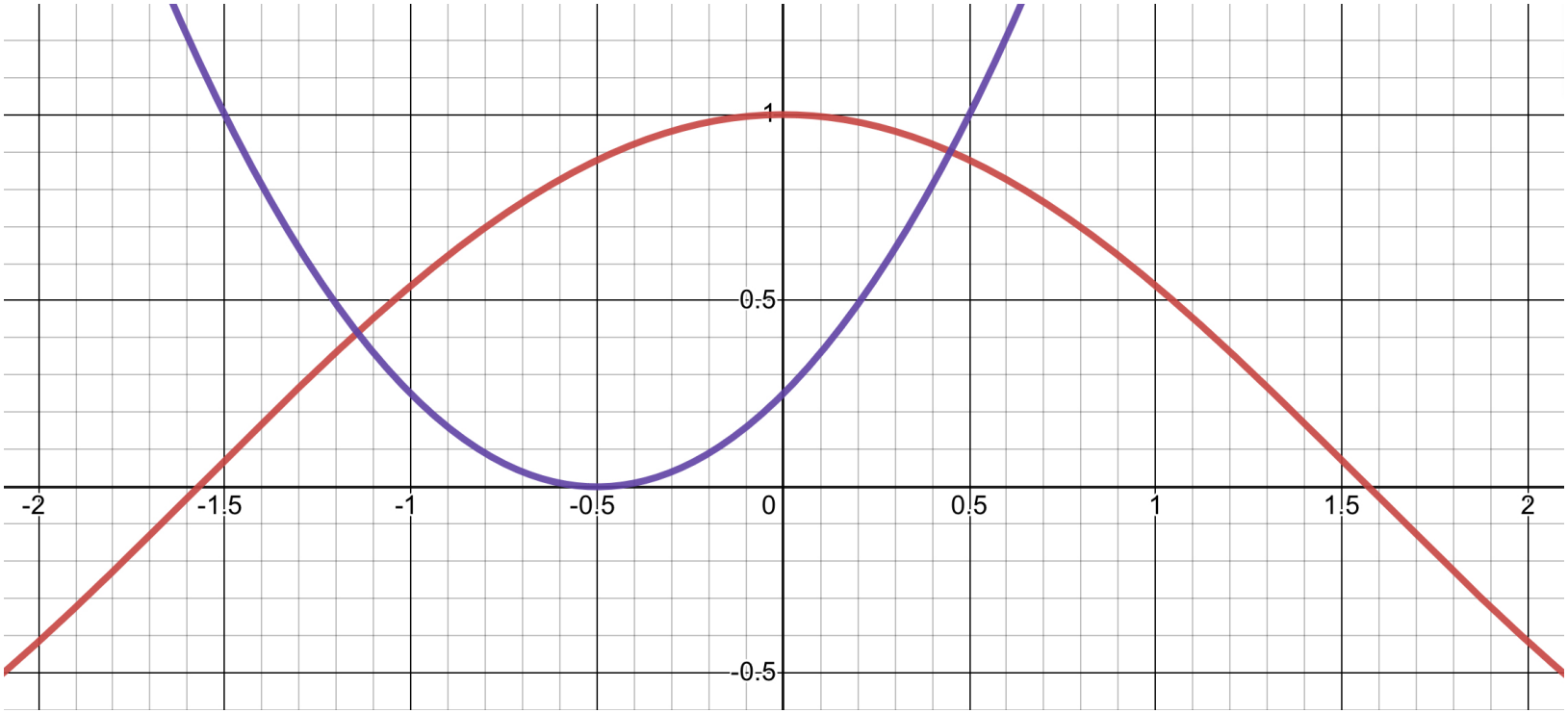
- To compute $\sin x$, $\cos x$, and e^x *fast*
- Calculators (your TI) use this technique
- To simplify equations/functions
- In simple pendulum, we *approximated* $\sin x$ with x

Derivation

- Calculators can multiply, add, subtract, divide, and take powers of whole numbers *quickly*
- Let us use *polynomials*
- Polynomials are just multiplications, additions, and exponentiations

Derivation

Figure: The Function $\cos x$



- Approximate to two degrees
- Find real numbers for a, b, and c that approximate $\cos x$ the *best*

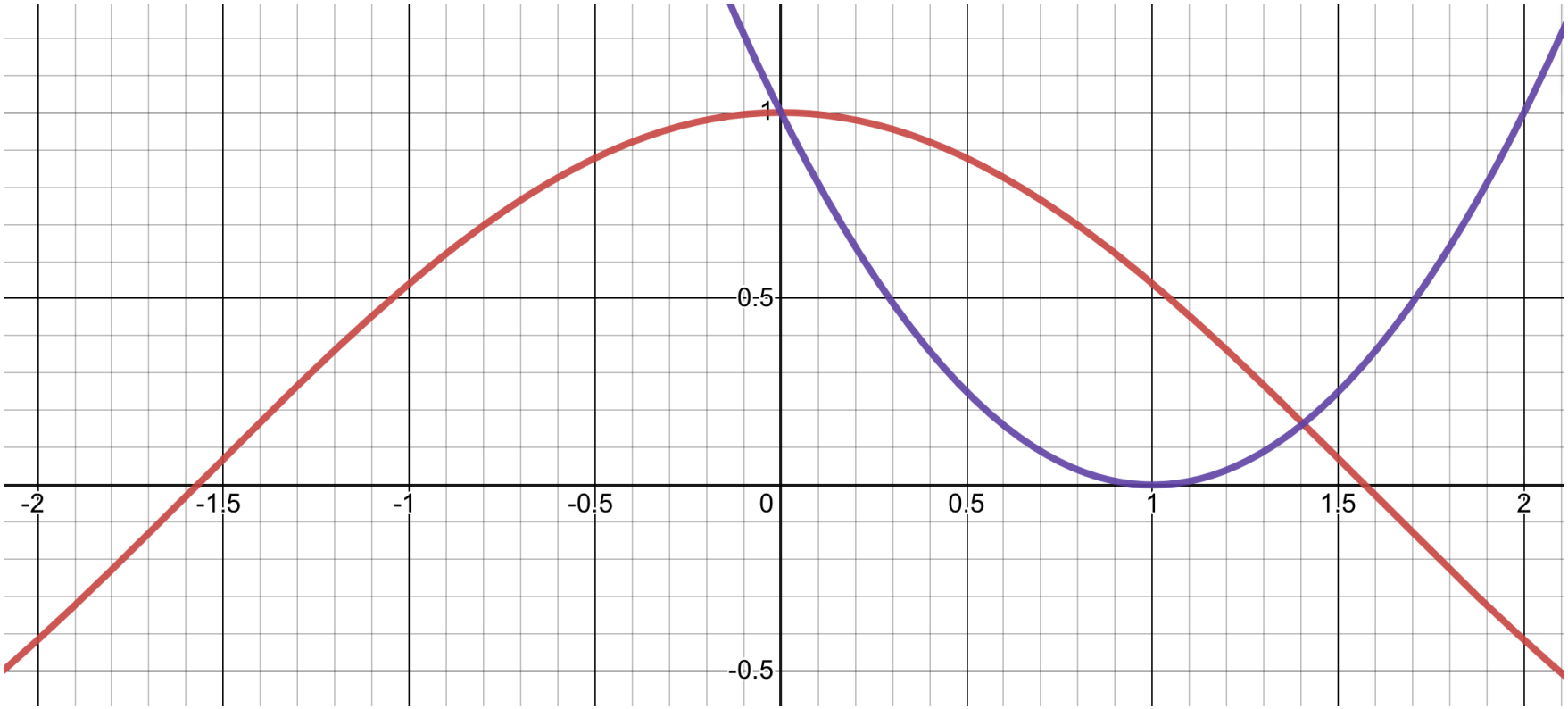
$$\cos x \approx a + bx + cx^2$$

Derivation

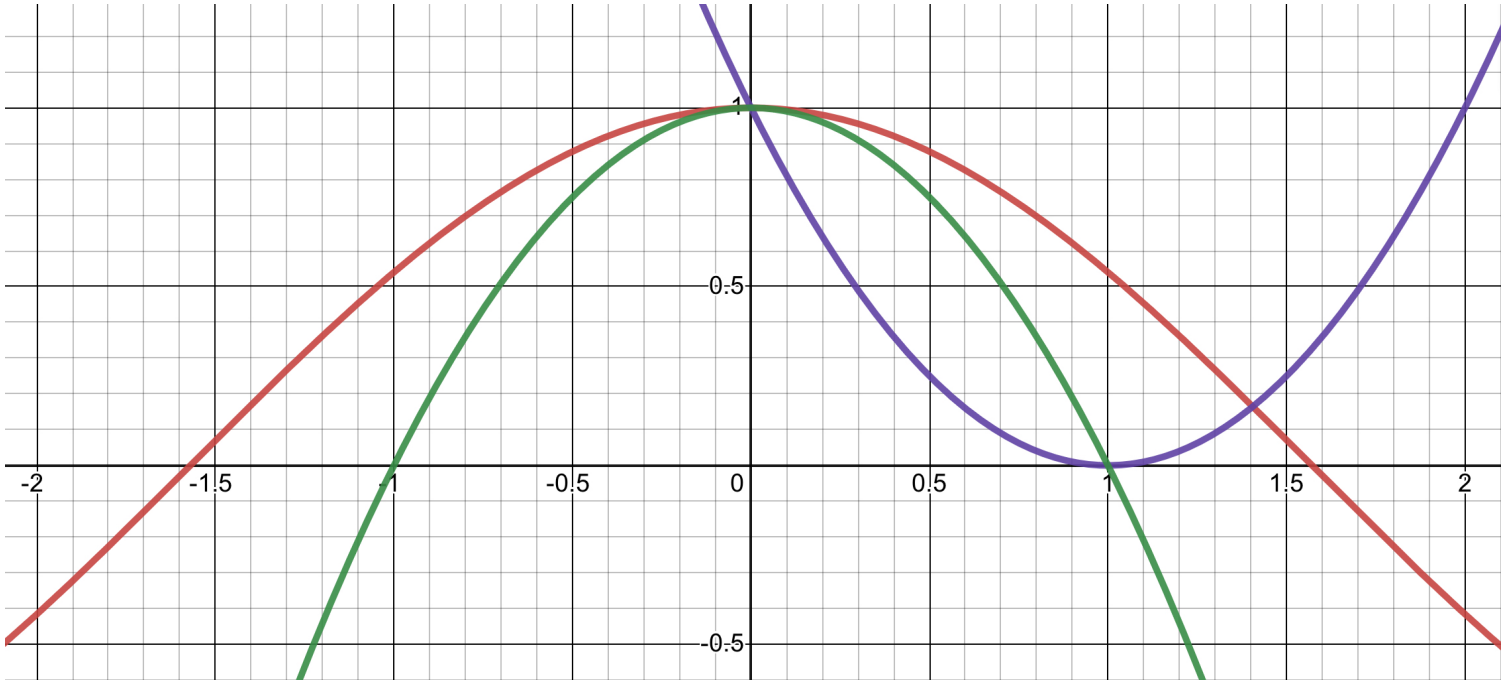
- We want to approximate *near* $x = 0$
- $\cos x = a + bx + cx^2$ at $x = 0$

$$\cos 0 = a + b \cdot 0 + c \cdot 0^2$$

$$1 = a$$



Derivation

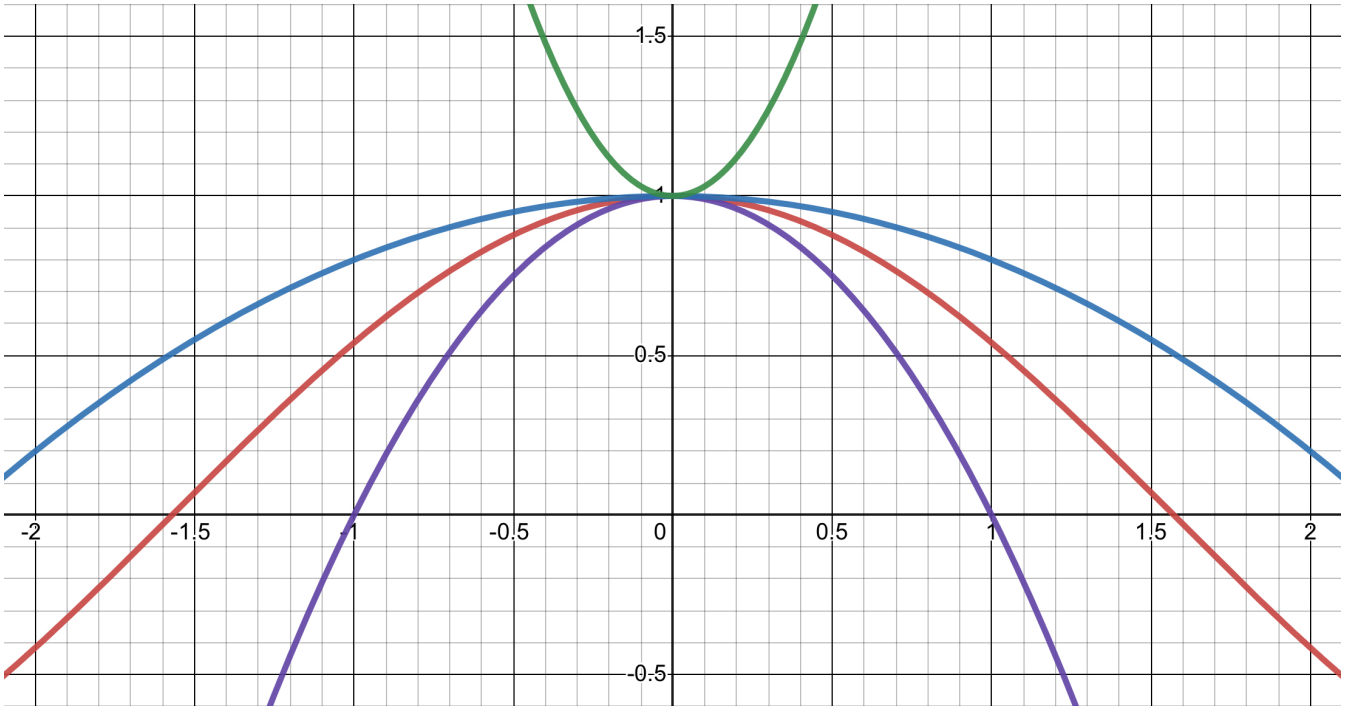


- The green function is better, but why?
- The rate of change is the same as $\cos x$ at $x = 0$
- Our approximation must have the same derivative at $x = 0$
- $\cos' x = -\sin x$ and $(a + bx + cx^2)' = b + 2cx$

$$-\sin 0 = 0 = b + 2c \cdot 0$$

$$b = 0$$

Derivation



- $\cos x$ *curves* downwards at $x = 0$
- So, the second derivative is negative
- Which means the rate of change is decreasing
- Same second derivative will ensure that they curve at the same rate

$$\cos'' x = -\cos x$$

$$(a + bx + cx^2)'' = 2c$$