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Three Uses for Springs in Legged Locomotion

Abstract

Running animals and robots can save energy and reduce unwanted heat production by bouncing along on springs, using the principle of the pogo stick. (The principal springs in animals are tendons.) They can make further energy savings by using return springs to halt the legs at the end of each foward or backward swing and start them swinging the other way. The forces at impact of feet with the ground can be moderated by compliant foot pads, which can improve road holding by preventing "chatter" (vibrations in which the foot repeatedly leaves and returns to the ground before settling). Animals use springs in all three ways but there seems to be a need for more use of springs in legged robots, especially in robots designed to run fast.

1. Introduction

This paper shows how springs can perform useful functions in robots that move on legs rather than on wheels. Mammals generally use springs in all the ways that I will describe, but few robots make much use of them: the robots described by Raibert (1986) are an exception.

Exchanges between different forms of mechanical energy will figure largely in the discussion that follows. I will distinguish between the external kinetic energy of an animal or vehicle, which is the energy associated with the movement of its center of mass, and the internal kinetic energy associated with movement of component parts relative to the center of mass (Alexander 1988). The first uses of springs that I will discuss relate to fluctuations of external kinetic energy.

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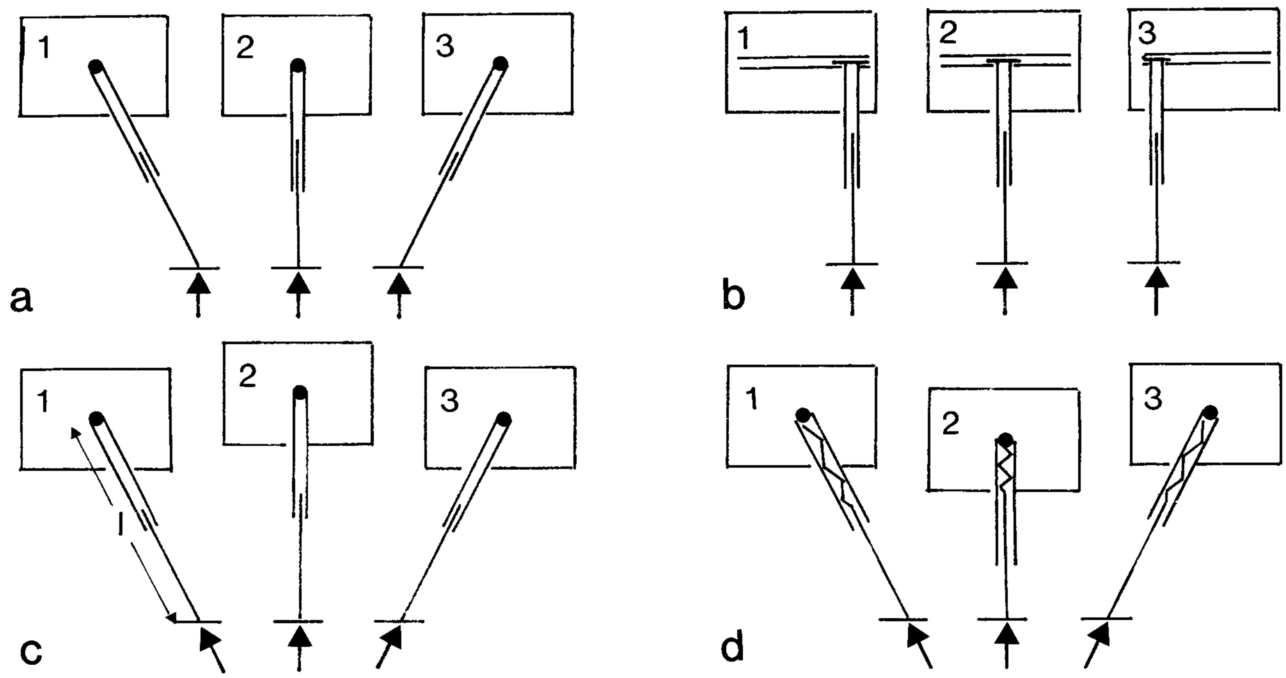
2. The Pogo Stick Principle

A wheeled vehicle moving at constant speed over level ground has constant kinetic energy and constant potential energy. The work needed to propel it is simply whatever is necessary to overcome friction between moving parts, and aerodynamic drag. This is not necessarily the case when wheels are replaced by legs. Paradoxically, it may even be desirable to make kinetic and potential energy fluctuate during each stride, so that the power needed to propel the vehicle can be reduced.

Figure 1(A) shows a simple legged robot. The drawing shows only one leg, but the robot should be imagined as having several pairs, making it possible for equilibrium to be maintained throughout the stride. Its legs have joints that enable them to be pointed in any direction and lengthened or shortened as required. This particular design has a hinge joint at the hip (or a universal joint, if it is to be able to turn corners) and a sliding joint in the leg, but the sliding joint could be replaced by a hinge joint (a "knee"), as in animals and some robots. These joints are operated by actuators of some kind (in an animal, the actuators would be muscles).

Let the robot walk as shown in Figure 1(A), keeping the ground forces on the feet vertical and keeping its body constantly in equilibrium. The external kinetic energy and the potential energy remain constant. However, as the leg moves from position 1 to position 2, the hip actuator must do positive work: it swings the leg clockwise while exerting the clockwise torque that is needed to balance the counterclockwise moment of the ground force. At the same time the actuator of the sliding joint acts like a brake, pushing on the ground while allowing the leg to shorten; it does negative work. Between positions 2 and 3, the converse happens: the hip actuator does negative work, while the slider actuator does positive work. Throughout the step one or other of the actuators is doing work that

Fig. 1. Diagrams showing alternative patterns of movement for the legs of robots. Arrows show the direction of forces on the feet.

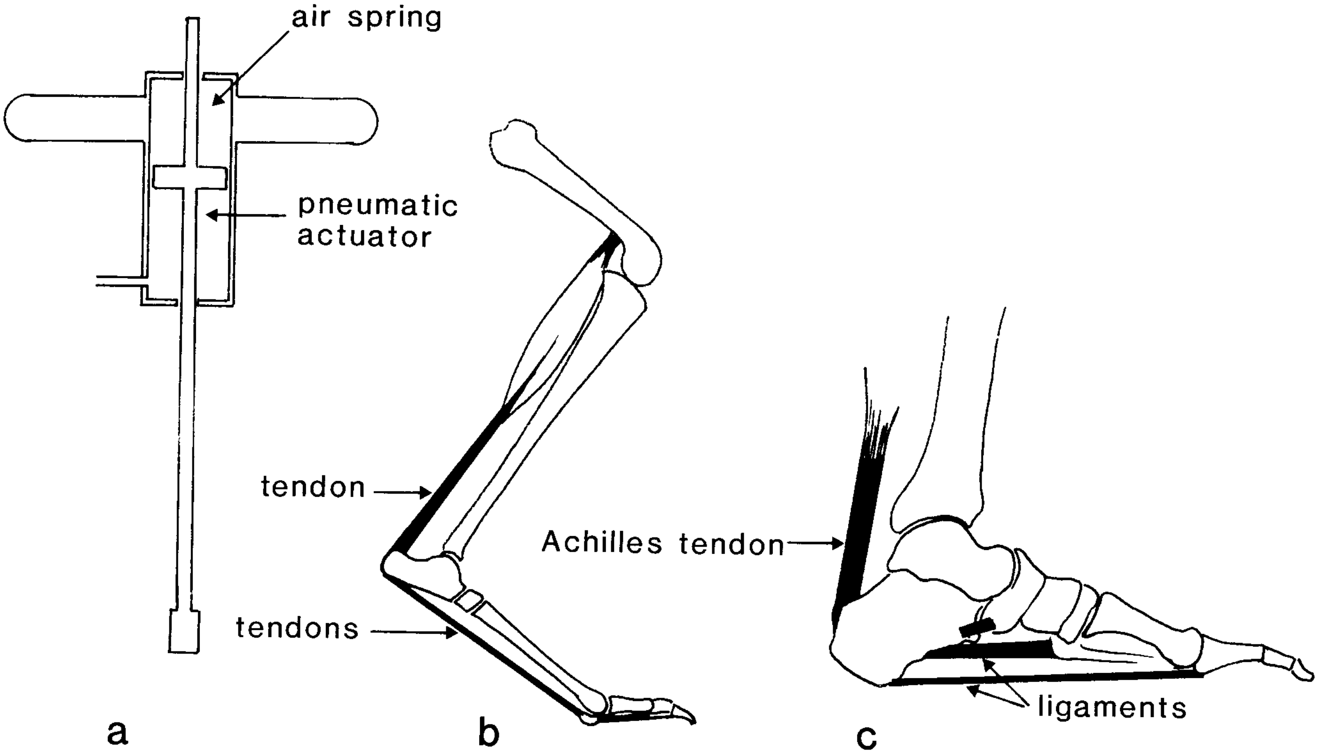


the other actuator degrades to heat, leaving the robot's mechanical energy unchanged. The same would happen if the slider were replaced by a hinged knee. This wastage of energy seems undesirable. Minimization of energy costs seems to have been an important consideration in the evolution of animals (Alexander 1988), and fuel consumption will become important in the design of legged robots when they come to be used commercially. The energy wastage may also be undesirable because the heat that is generated will have to be dissipated; this was a consideration in the design of the Adaptive Suspension Vehicle (Waldron et al. 1984).

Figure 1(B) shows an alternative design that avoids the problem. The leg is constrained to be always vertical but can slide forward and back along a sliding joint with the body. It is extended to the appropriate length when it is set down, but is then held at constant length until the foot is lifted at the end of the step. Neither actuator is required to do negative work at any stage of the stride, apart from the very small quantity involved in lowering the foot onto the ground. They have to do only enough positive work to lift the foot and to overcome friction and drag. A robot constructed in this way could be almost as energy efficient as a wheeled vehicle. The same result could have been achieved in a different way by using pantograph legs like the ones that have been fitted to several robots (Waldron et al. 1984; Hirose 1984).

Figure 1(C) shows another alternative. The hinge is retained at the hip, but the leg is again held constant in length while the foot is on the ground. Consequently, the body rises and then falls in the course of each step. No torque is exerted about the hip while the foot is on the ground, so the ground force is aligned with the leg, sloping first one way and then the other, decelerating and re-accelerating the body. If there were no friction, no work would be done between position 1 and position 3, because the slider would not move and there would be no torque at the hinge. The changes in potential energy are equal and opposite to the changes in kinetic energy, as in a swinging pendulum. This is possible only at fairly low speeds. While the foot is on the ground, the body moves in an arc of a circle of radius equal to the length ℓ of the leg. If its speed is v, it has acceleration v^2/ℓ toward the foot, but unless the foot adheres to the ground, this cannot exceed the gravitational acceleration g. Thus the speed cannot exceed $\sqrt{g\ell}$.

Fig. 2. Legs with components that function like the springs of pogo sticks. (A). Air spring in the leg of Raibert's (1986) hopping robot. (B). Important tendons in the leg of a kangaroo. (C). Ligaments in the arch of the human foot, and the Achilles tendon.



The gait cannot be performed exactly as suggested because stage 3 of one step is stage 1 of the next, and the direction of the body's velocity would have to be changed instantaneously. Alexander (1980) analyzed similar gaits that did not involve infinite accelerations and showed that the positive and negative work requirements could be made very small if the foot were made to exert an appropriate pattern of force on the ground. However, at speeds above $\sqrt{g\ell}$, these walking gaits were less economical than running gaits, in which potential and kinetic energy fluctuate in phase with each other. The critical speed, $\sqrt{g\ell}$, is quite low by the standards of mammals: it is the speed of a slow trot, for mammals of all sizes (Alexander and Jayes 1983). However, it is fast by the standards of legged robots: 2.4 m/s for a vehicle with 0.6-m legs like Raibert's (1986) 3D hopper, and 4.2 m/s for one with 1.8-m legs like the ASV (Waldron et al. 1984). The 3D hopper has a maximum speed of 2.2 m/s, and the ASV is designed to reach 3.6 m/s. These are fast robots; most legged robots travel at speeds that are small fractions of $\sqrt{g\ell}$.

Figure 1(D) shows a fourth possible design, this time involving a spring. As in (C), the ground force is kept aligned with the leg, but leg length is not held constant while the foot is on the ground. The sliding joint has a spring that allows it to shorten as the ground force increases and lengthen as the ground force falls: the leg is constructed like a pogo stick. The body loses and regains kinetic and potential energy while the foot is on the ground, but this energy is stored temporarily as strain energy in the spring, and no work is required of the actuators (except to overcome energy losses in the spring and friction and drag). In this case, unlike cases (A) to (C), there may be stages of the stride when all the feet are off the ground. Raibert's (1986) one- and four-legged robots work like this (Fig. 2(A)). The pogo stick principle is also used by mammals, including humans and kangaroos as well as quadrupeds. In them, the principal springs are the tendons of leg muscles and ligaments in the foot (Fig. 2(B,C) (Alexander 1988). Tendons have good elastic properties: dynamic tests at a wide range of frequencies show that they return in their elastic recoil 93% of the work done stretching them (Ker 1981).

It should be possible to make pogo stick vehicles as energy efficient as wheeled ones. The bumpy ride that they give may be acceptable, as it apparently is to horse riders. There is no fundamental limit to speed like the $\sqrt{g\ell}$ limit for gaits that use the pendulum principle. [Alexander's (1980) argument that running was less economical than walking at speeds below $\sqrt{g\ell}$ envisaged legs without springs. The running action was like that of the pogo stick leg of Fig. 1(D), but the negative and positive work were done by a muscle that degraded mechanical energy to heat and then replaced it.] However, it may be desirable to adjust the stiffness of the spring to suit different speeds. If the leg is to be swung through the same angle while the foot is on the ground, higher speeds require it to remain on the ground for shorter times. A stiffer spring may be needed to achieve this, because the duration of ground contact is between half and one period of the vibration of the body mass on the leg spring, depending on the "Groucho number" (McMahon, Valiant, and Frederick 1987). That argument is not wholly convincing, because it treats the body as a mass bouncing vertically on a constantly vertical spring, but other, less general arguments lead to the same conclusion. Alexander and Vernon (1975) presented such an argument and treated a muscle and its tendon as an inelastic actuator in series with a spring. At low speeds the muscle would have to lengthen and then shorten to supplement the length changes of the tendon. At high speeds the muscle would have to shorten and then lengthen to partly counteract excessive length changes in the tendon. There would be only one speed at which the muscle would maintain constant length and the energy-saving potential of the pogo stick style of locomotion would be fully realized. Note that this argument assumes a leg that swings through the same angle, while on the ground, at all speeds. If the angle is allowed to change, a tendon of constant stiffness could allow the muscle to work isometrically at all speeds.

The short ground-contact times given by a spring stiff enough for fast running may be disadvantageous at low speeds, which require either a high stride frequency (if the Groucho number is kept low; McMahon, Valiant, and Frederick 1987) or high peak ground forces and a large amplitude of vertical movement (if the Groucho number is high). The higher the stride frequency, the more often work has to be done giving kinetic energy to the legs (see section 3). Large vertical movements may give an uncomfortable ride. If pogo stick robots are to be designed to perform well at a wide range of speeds, they will need springs of variable stiffness.

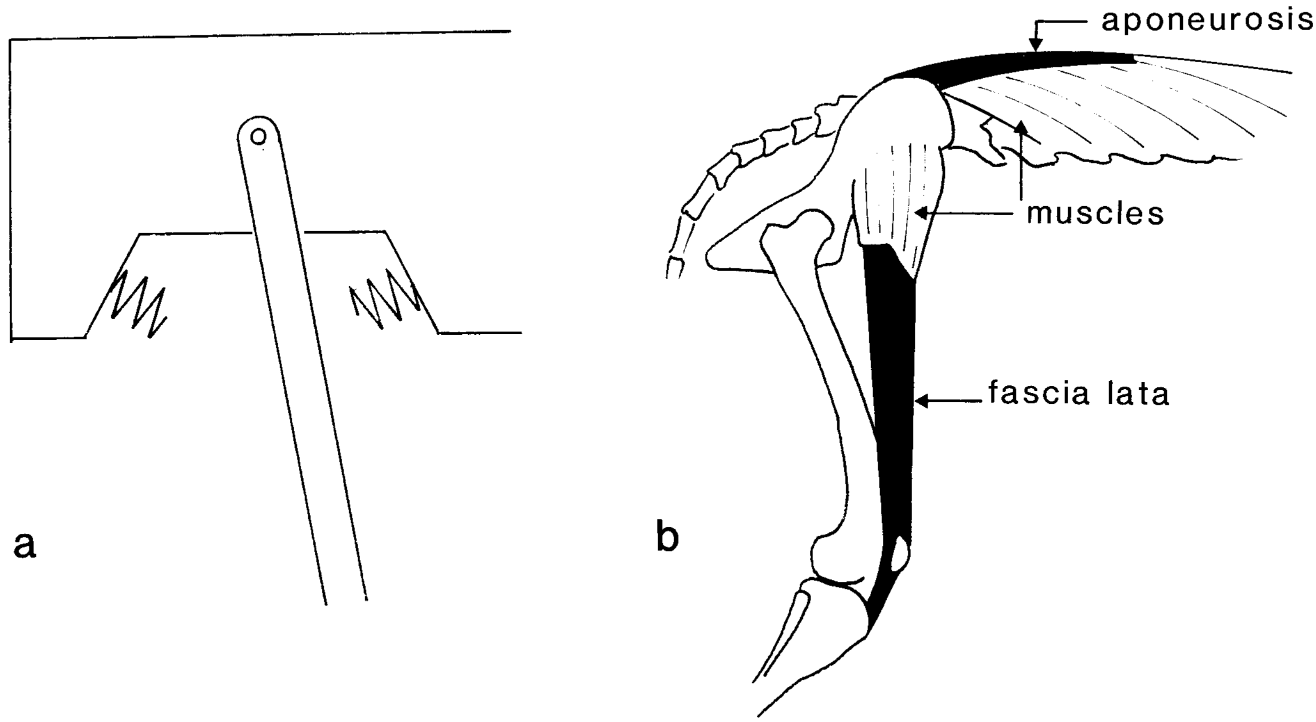
There is a fundamental design difference between running animals and Raibert's (1986) 3D hopper. In animals, the pogo stick springs (tendons) are connected in series with the actuators (muscles), but in the robot the spring works in parallel with the actuator. (Note that actuator and air spring exert forces on the same piston in Figure 2(A)).

3. Return Springs

Fluctuations of external kinetic energy can be avoided in legged locomotion, as we have seen, but fluctuations of internal kinetic energy are an inevitable consequence of the reciprocating action that distinguishes legs from wheels. If the legs, swinging backward and forward, are started and stopped by inelastic actuators, the kinetic energy given to them at the beginning of each swing will be degraded to heat at the end. Energy will be saved if the legs can be halted and re-accelerated by springs.

A robot or animal running with velocity v has to accelerate each foot to velocity $-v$ (relative to the center of mass of the body) each time the foot is set down. The kinetic energy and thus the work involved are proportional to the square of the velocity. The fluctuations of internal kinetic energy involved in running by animals are smaller than the fluctuations of external kinetic energy at low speeds, but larger at higher speeds (Fedak, Heglund, and Taylor 1982). As speed increases, it becomes increasingly desirable to use springs to maintain the swinging of the legs, both for animals and for robots.The legged robots made so far have been rather slow in comparison with animals of similar size, and the energy cost of swinging their

Fig. 3. Diagrams showing springs that help to maintain the swinging of legs. (A). Return springs for a leg of a hypothetical robot. (B). Fascia lata and longissimus aponeurosis of a dog.



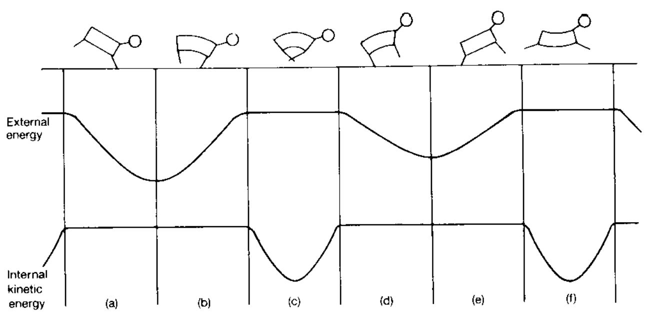
legs does not seem to be a very serious burden. For example, Raibert's (1986) Figure 3.8 shows that when his one-legged robot hopped at 1.6 m/s it gained and lost about 0.1 m/s in each stride, corresponding to 2.7 J external kinetic energy. The angular velocity of the leg in its backward swing was 2.8 rad/s and the associated internal kinetic energy only about 0.4 J. (Raibert's later robots are faster. I use this example here because the data needed for the calculation have been published.)

Figure 3(A) shows a simple design for a leg with springs to reverse its swing. It hits an elastic end-stop and bounces back at the end of each swing. The foot should not be decelerated (relative to the body) until it has left the ground, so the springs are arranged to contact the leg only while the foot is off the ground. I do not know of any robots that use this energy-saving principle, but it is incorporated in McGeer's (1989) theoretical model. In quadrupedal mammals, the fascia lata (Fig. 3(B)) may be a return spring for the hind leg (Bennett 1989). It is a broad sheet of collagen fibers with elastic properties like those of other tendons. It runs down the front of the thigh, connecting its muscle to the tibia (the principal bone of the lower leg); it may halt the backward swing of the leg and recoil to swing the leg forward. It seems to do this in quadrupedal mammals but not in humans, in which it runs down the side of the thigh instead of the front.

If springs are to play a major role in keeping legs swinging, the internal kinetic energies associated with the forward and backward swings must be about equal. Unless energy can be transferred from leg to leg, this implies that the forward swing of each leg (in which it is shortened to clear the ground, reducing its moment of inertia) should have a higher angular velocity than the backward swing. Within limits, the period of leg swinging could be matched to the desired stride frequency by using springs of appropriate stiffness: softer springs would take longer to halt and reaccelerate the leg.

When mammals run fast, they take long strides, setting down each foot for only a small fraction of the stride duration (Alexander and Jayes 1983). Each leg swings forward more slowly than it swings back, and the internal kinetic energy associated with the forward swing is much less than for the backward swing

Fig. 4. Diagrams of successive stages of a galloping stride, with schematic graphs showing energy fluctuations. This figure appeared in the Journal of Zoology, volume 207 (Alexander, Dimery, and Ker 1985) and is being reprinted with permission from the Zoological Society of London.



(Fedak, Heglund, and Taylor 1982). Consequently the leg movements cannot be wholly maintained by separate springs associated with individual legs. Galloping seems to be designed to overcome this difficulty by transferring kinetic energy from leg to leg.

Most mammals walk at low speeds and trot at intermediate speeds (Alexander and Jayes 1983). In these "symmetrical" gaits, the left and right leg of each pair move half a cycle out of phase with each other. In one step the left leg is swinging backward and the right forward, and in the next the other way round. The total internal kinetic energy is the same in both cases. It might be possible to devise some sort of elastic connection between the legs that could maintain their swinging, but no such device seems to have evolved. Instead, quadrupedal mammals change from symmetrical gaits to asymmetrical ones (the various kinds of gallop) for fast running at speeds above about $1.6\sqrt{g\ell}$ (Alexander and Jayes 1983). It is at these high speeds that fluctuations of internal kinetic energy are largest relative to the fluctuations of external kinetic energy.

Alexander, Dimery, and Ker (1985) showed how galloping might save energy. Figure 4 gives an impression of the movements and energy changes that occur during a galloping stride. For clarity, the two legs of each pair are shown moving in phase with each other, but it would have been more realistic to have shown them a little out of phase. In Figure 4, (a) and (b) show the fore feet on the ground, decelerating and re-accelerating the body. The external energy of the body (external kinetic plus potential energy) falls and then rises, while the pogo stick springs of the legs store strain energy and recoil. Similarly, in stages (d) and (e) external energy is stored briefly in the pogo stick springs of the hind legs.

At stage (c) of the stride, all the feet are off the ground, and the external energy of the body remains constant. The fore legs end their backward swing and start swinging forward, and the hind legs end their forward swing and start swinging backward; thus the internal kinetic energy falls and then rises. The back is bent at this stage of the stride, and a sheet of collagen fibers close under the skin of the back is tense, storing strain energy. This is the longissimus aponeurosis, which serves as the tendon of the principal extensor muscle of the back (Fig. 3(B)). Rough calculations suggest that a substantial proportion of the internal kinetic energy that is lost and regained during stage (c) may be stored as strain energy in the longissimus aponeurosis. Most of this energy is taken from the fore legs and returned to the hind.

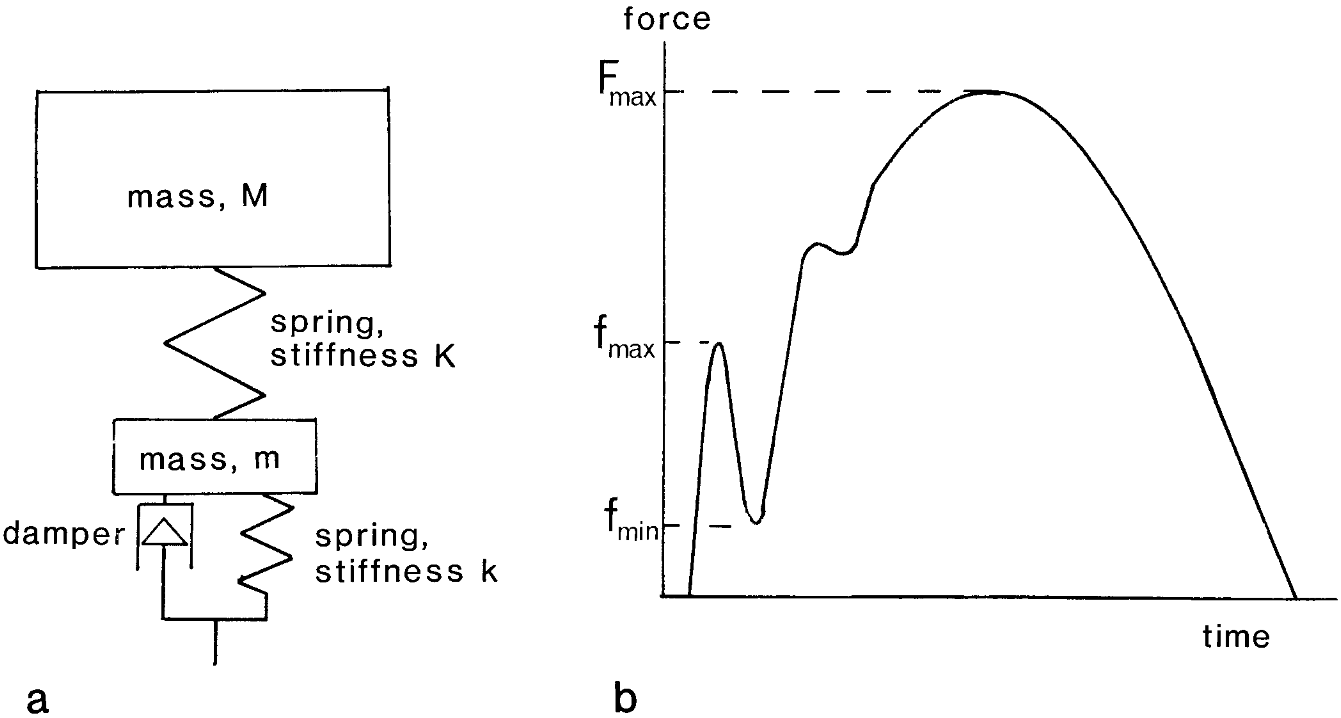
At stage (f) of the stride, the legs make the reverse change of direction: the fore legs start swinging backward, and the hind legs start swinging forward. The back is strongly extended. There seems to be scope for saving energy by storing strain energy in a spring somewhere in the belly, but no suitable spring has been identified. The fascia lata (Fig. 3(B)) stores strain energy at this stage, but it cannot by itself transfer kinetic energy from the hind legs to the fore legs.

4. Foot Pads

Feet should be made light, to keep fluctuations of internal kinetic energy small, but they must have some mass. A rigid foot landing on rigid ground would be decelerated very rapidly and would experience a large force. Some sort of padding is needed to avoid excessive impact forces. Most mammals (for example, dogs) have pads on their paws. People have fatty pads beneath the skin of the sole of the foot and use additional padding (in the form of shoes) for walking and running on hard surfaces. Similarly, Raibert's (1986) one-legged robot has a rubber foot pad.

Figure 5(A) is a simple model of a legged robot or animal. The body has mass M, and the foot has mass m. The leg has a pogo stick spring of stiffness K, and there is a pad of stiffness k under the foot. This pad has rate-independent damping, returning in its elastic recoil a constant fraction α of the work previously

Fig. 5. (A). Model of an animal or robot with a padded foot. (B). Schematic graph of force on the foot against time, following impact of the model with the ground.



done deforming it. Rate-independent rather than viscous damping has been postulated for mathematical convenience and because it is more characteristic of rubber-like polymers such as might be used to make a pad (Lazan 1968). Alexander, Bennett, and Ker (1986) found that the paw pads of mammals showed very nearly rate-independent damping.

Figure 5(B) shows the forces that act when the model's foot hits rigid ground. The foot (mass m) is rapidly decelerated, exerting a peak force f_{max}. A decaying oscillation follows as a result of vibration of mass m on spring k. This is superimposed on the slower rise of the ground force to the peak F_{max} caused by compression of spring K and deceleration of mass M. Force records like this are obtained when people and animals run across force plates (see, for example, Dickinson, Cook, and Leinhardt 1985; Bryant et al. 1987).

The following analysis of the model is taken from Alexander, Bennett, and Ker (1986). The mass m of the foot is presumably small compared to the mass M of the body. When force F_{max} acts on spring k, an almost equal force acts on spring K, but a pad of reasonable thickness will presumably deform much less than the leg as a whole, so stiffness k must be larger than stiffness K. The time from impact to force f_{max} is approximately one quarter of the period of vibration of mass m on spring k, or about $(\pi/2)(m/k)^{1/2}$. This is much shorter than the time from impact to force F_{max}, which (by a similar argument) is about $(\pi/2)(M/K)^{1/2}$. (These statements imply the assumption that forces f_{max} and F_{max} are large compared to the weights of the foot and body, respectively. The effect of stiffness k on the time to F_{max} has been ignored because k is much larger than K.)

Immediately before impact, the body (mass M) has downward velocity V and the foot (mass m) has downward velocity v. The momentum $-MV$ of the body is removed in time $(\pi/2)(M/K)^{1/2}$, so the mean force required is $(2V/\pi)(MK)^{1/2}$. Similarly, the mean force required to remove the momentum of the foot is $(2v/\pi)(mk)^{1/2}$. This crude argument tells us that

$$f_{max}/F_{max} \approx (v/V)(mk/MK)^{1/2} \qquad (1)$$

To keep f_{max} small, the foot should have small mass and a soft pad (low m and k) and should be set down gently (low v).

There is another less obvious point that may be important. The first maximum in the ground force (f_{max}) is followed by a minimum (f_{min}; Fig. 5(B). If f_{min} reaches zero, the foot leaves the ground. With certain values of the masses and spring constants, the foot might leave the ground and return repeatedly before settling on the ground, a phenomenon that Alexander, Bennett, and Ker (1986) called "chattering." This would presumably be undesirable, because a chattering foot would be apt to shift its position on the ground. We will consider how chattering can be prevented.

Just before impact, the foot has kinetic energy $\frac{1}{2}mv^2$ and momentum $-mv$. If it were not connected to the body, it would rebound with kinetic energy $\frac{1}{2}\alpha mv^2$ and momentum $\alpha^{1/2}mv$, leaving the ground at time $\pi(m/k)^{1/2}$ after impact. If chatter is to be prevented, spring K must by then have delivered a downward impulse $\alpha^{1/2}mv$. This spring is compressed at an initial rate V, so it has developed by this time a force of almost $VK\pi(m/k)^{1/2}$ and delivered an impulse of almost $\frac{1}{2}VK(\pi^2 m/k)$. The condition for no chatter is that this should be more than $\alpha^{1/2}mv$, that is that

$$k/K < 5V/\alpha^{1/2}v \qquad (2)$$

The foot should have a soft pad (low k) and should be set down gently (low v). It may also help if the resilience α of the pad is low, but it will be argued later that this may be undesirable. Condition (2) was obtained by a crude argument but has been checked by numerical simulation and found to be remarkably accurate (Alexander, Bennett, and Ker 1986).

High impact forces and chattering are both prevented by soft foot pads and by setting the feet down gently. But which is more easily prevented? If chattering is just avoided, V/v equals $\alpha^{1/2}k/5K$ (condition 2). Putting this value in equation (1) gives

$$f_{max}/F_{max} \approx 5(mK/\alpha Mk)^{1/2} \qquad (3)$$

We already know that m is small compared to M and k is large compared to K, so it seems unlikely that f_{max}/F_{max} will be large. A foot that does not chatter is unlikely to suffer high impact forces.

If a linear spring in a pad of reasonable thickness is not to bottom out under load F_{max}, its spring constant k must be reasonably high, but condition (2) shows that low values of k may be needed to prevent chatter. It seems that a foot pad should ideally have nonlinear elastic properties: it should be soft under low loads and become stiffer as load increases. Pads made of rubber-like polymers behave like this in compression, as also do the paw pads of mammals (Alexander, Bennett, and Ker 1986).

It seems inevitable that the initial kinetic energy of the foot ($\frac{1}{2}mv^2$) will be lost in the damped vibration following impact. It may nevertheless be desirable to make the resilience α of the foot pad as large as possible, so that as much as possible of the strain energy stored in the pad under the load F_{max} is recovered. In dynamic compressive tests over a wide range of frequencies, most of the paw pads tested by Alexander, Bennett, and Ker (1986) gave resiliences between 0.6 and 0.8.

5. Conclusion

This paper has identified three uses for springs in legged locomotion. Pogo stick-like springs and return springs can save energy, reducing fuel consumption and unwanted heat production. Foot pads can moderate the force at impact of the foot on the ground and improve road holding by preventing chatter. Raibert's (1986) robots make exciting use of pogo stick-like springs, but roboticists do not seem thus far to have used return springs to reverse the swing of legs, or to have given much thought to foot pads. Running animals use springs for all three functions.

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