Homework3

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10/9/2017

Due Tuesday, 10 October, 1:00 PM

50 points total.

$5^{n=day}$ points taken off for each day late.

This assigment includes turning in the first two assignments. All three should include knitr files (named `homework1.rmd`, `homework2.rmd`, `homework3.rmd`) along with valid PDF output files. Inside each file, clearly indicate which parts of your responses go with which problems (you may use the original homework document as a template). Add your name as `author` to the file's metadata section. Raw R code/output or word processor files are not acceptable.

Failure to properly name files or include author name may result in 5 points taken off.

Question 1

10 points

1. Use GitHub to turn in the first three homework assignments. Make sure the teacher (couthcommander) and TA (nstrayer) are collaborators. (5 points)
2. Commit each assignment individually. This means your repository should have at least three commits. (5 points)

Question 2

15 points

Write a simulation to calculate the power for the following study design. The study has two variables, treatment group and outcome. There are two treatment groups (0, 1) and they should be assigned randomly with equal probability. The outcome should be a random normal variable with a mean of 60 and standard deviation of 20. If a patient is in the treatment group, add 5 to the outcome. 5 is the true treatment effect. Create a linear model for the outcome by the treatment group, and extract the p-value (hint: see assigment1). Test if the p-value is less than or equal to the alpha level, which should be set to 0.05.

Repeat this procedure 1000 times. The power is calculated by finding the percentage of times the p-value is less than or equal to the alpha level. Use the `set.seed` command so that the professor can reproduce your results.

1. Find the power when the sample size is 100 patients. (10 points)
2. Find the power when the sample size is 1000 patients. (5 points)

```
set.seed(42)
x <- rbinom(1,100,.5); treatment0 <- c(rep(1,x), rep(0,100 - x)); treatment <- sample(treatment0)
outcome <- rnorm(100,60,20)
data <- data.frame(treatment, outcome)
data1 <- data$outcome[treatment == 1] + 5
data$outcome[treatment == 1] <- data1
mod <- lm(data[,2] ~ data[,1])
ls(summary(mod))
```

```
##  [1] "adj.r.squared" "aliased"       "call"          "coefficients"
##  [5] "cov.unscaled"  "df"            "fstatistic"    "r.squared"
##  [9] "residuals"     "sigma"         "terms"
```

```
summary(mod)$coefficients[2,4]
```

```
## [1] 0.003785009
```

Question 3

15 points

Obtain a copy of the football-values lecture. Save the `2017/proj_wr17.csv` file in your working directory. Read in the data set and remove the first two columns.

1. Show the correlation matrix of this data set. (3 points)

```
proj_wr17 <- read.csv("proj_wr17.csv", header = T, sep = ",")
summary(proj_wr17)
```

```
##            PlayerName       Team        rush_att          rush_yds
##  A.J. Green     :  1   BUF    : 11   Min.   : 0.0000   Min.   : -0.400
##  Aaron Burbridge:  1   NYJ    : 11   1st Qu.: 0.0000   1st Qu.:  0.000
##  Aaron Dobson   :  1   TEN    : 10   Median : 0.0000   Median :  0.100
##  Adam Humphries :  1   ARI    :  9   Mean   : 0.9388   Mean   :  5.359
##  Adam Thielen   :  1   CHI    :  9   3rd Qu.: 0.8000   3rd Qu.:  4.225
##  Albert Wilson  :  1   MIN    :  9   Max.   :25.3000   Max.   :163.100
##  (Other)        :236   (Other):183
##     rush_tds          rec_att          rec_yds          rec_tds
##  Min.   :0.00000   Min.   :  0.000   Min.   :   0.00   Min.   : 0.000
##  1st Qu.:0.00000   1st Qu.:  6.125   1st Qu.:  73.92   1st Qu.: 0.400
##  Median :0.00000   Median : 18.400   Median : 229.00   Median : 1.300
##  Mean   :0.02149   Mean   : 29.287   Mean   : 378.76   Mean   : 2.296
##  3rd Qu.:0.00000   3rd Qu.: 46.550   3rd Qu.: 600.48   3rd Qu.: 3.875
##  Max.   :1.10000   Max.   :111.200   Max.   :1512.00   Max.   :10.400
##
##     fumbles            fpts
##  Min.   :0.0000   Min.   :  0.00
##  1st Qu.:0.1000   1st Qu.: 10.03
##  Median :0.3000   Median : 30.10
##  Mean   :0.4058   Mean   : 51.53
##  3rd Qu.:0.6000   3rd Qu.: 83.15
##  Max.   :2.0000   Max.   :205.30
##
```

```
str(proj_wr17)
```

```
## 'data.frame':    242 obs. of  10 variables:
##  $ PlayerName: Factor w/ 242 levels "A.J. Green","Aaron Burbridge",..: 134 18 179 169 126 1 168 214 8
##  $ Team      : Factor w/ 32 levels "ARI","ATL","BAL",..: 2 27 23 30 12 7 22 14 28 9 ...
##  $ rush_att  : num  0.5 3.1 0.8 0 0 0 2 0 2.3 0 ...
##  $ rush_yds  : num  2.2 17.1 4.3 0 0 0 9.5 0 12.4 0 ...
##  $ rush_tds  : num  0 0 0 0 0 0 0 0 0.1 0 ...
##  $ rec_att   : num  100.6 111.2 96.4 90 90.7 ...
##  $ rec_yds   : num  1512 1441 1331 1301 1222 ...
##  $ rec_tds   : num  9.2 10.1 10 9.3 10.4 7.9 8.1 6.8 8.3 8.9 ...
```

```
##  $ fumbles   : num  0.8 1 1 0.6 1.1 0.5 0.6 0.3 0.5 1.2 ...
##  $ fpts      : num  205 205 192 185 182 ...
```

```
cor(proj_wr17[,3:10])
```

```
##           rush_att  rush_yds   rush_tds   rec_att    rec_yds    rec_tds
## rush_att 1.0000000 0.9852929 0.90483781 0.2319214 0.18035010 0.16743969
## rush_yds 0.9852929 1.0000000 0.92448344 0.2257124 0.17679740 0.16679516
## rush_tds 0.9048378 0.9244834 1.00000000 0.1362059 0.09557542 0.09869131
## rec_att  0.2319214 0.2257124 0.13620592 1.0000000 0.99074495 0.96555114
## rec_yds  0.1803501 0.1767974 0.09557542 0.9907449 1.00000000 0.97842956
## rec_tds  0.1674397 0.1667952 0.09869131 0.9655511 0.97842956 1.00000000
## fumbles  0.2498605 0.2625638 0.20256396 0.6696210 0.65373865 0.63586414
## fpts     0.2188717 0.2165635 0.13757275 0.9888153 0.99727772 0.98787533
##            fumbles      fpts
## rush_att 0.2498605 0.2188717
## rush_yds 0.2625638 0.2165635
## rush_tds 0.2025640 0.1375727
## rec_att  0.6696210 0.9888153
## rec_yds  0.6537386 0.9972777
## rec_tds  0.6358641 0.9878753
## fumbles  1.0000000 0.6482031
## fpts     0.6482031 1.0000000
```

```
library(MASS)
proj2 <- proj_wr17[,3:10]
(rho.proj2=cor(proj2))
```

```
##           rush_att  rush_yds   rush_tds   rec_att    rec_yds    rec_tds
## rush_att 1.0000000 0.9852929 0.90483781 0.2319214 0.18035010 0.16743969
## rush_yds 0.9852929 1.0000000 0.92448344 0.2257124 0.17679740 0.16679516
## rush_tds 0.9048378 0.9244834 1.00000000 0.1362059 0.09557542 0.09869131
## rec_att  0.2319214 0.2257124 0.13620592 1.0000000 0.99074495 0.96555114
## rec_yds  0.1803501 0.1767974 0.09557542 0.9907449 1.00000000 0.97842956
## rec_tds  0.1674397 0.1667952 0.09869131 0.9655511 0.97842956 1.00000000
## fumbles  0.2498605 0.2625638 0.20256396 0.6696210 0.65373865 0.63586414
## fpts     0.2188717 0.2165635 0.13757275 0.9888153 0.99727772 0.98787533
##            fumbles      fpts
## rush_att 0.2498605 0.2188717
## rush_yds 0.2625638 0.2165635
## rush_tds 0.2025640 0.1375727
## rec_att  0.6696210 0.9888153
## rec_yds  0.6537386 0.9972777
## rec_tds  0.6358641 0.9878753
## fumbles  1.0000000 0.6482031
## fpts     0.6482031 1.0000000
```

1. Generate a data set with 30 rows that has a similar correlation structure. Repeat the procedure 10,000 times and return the mean correlation matrix. (10 points)

```
(rho.proj2=cor(proj2))
```

```
##           rush_att  rush_yds   rush_tds   rec_att    rec_yds    rec_tds
## rush_att 1.0000000 0.9852929 0.90483781 0.2319214 0.18035010 0.16743969
## rush_yds 0.9852929 1.0000000 0.92448344 0.2257124 0.17679740 0.16679516
## rush_tds 0.9048378 0.9244834 1.00000000 0.1362059 0.09557542 0.09869131
## rec_att  0.2319214 0.2257124 0.13620592 1.0000000 0.99074495 0.96555114
```

```
## rec_yds  0.1803501 0.1767974 0.09557542 0.9907449 1.00000000 0.97842956
## rec_tds  0.1674397 0.1667952 0.09869131 0.9655511 0.97842956 1.00000000
## fumbles  0.2498605 0.2625638 0.20256396 0.6696210 0.65373865 0.63586414
## fpts     0.2188717 0.2165635 0.13757275 0.9888153 0.99727772 0.98787533
##            fumbles      fpts
## rush_att 0.2498605 0.2188717
## rush_yds 0.2625638 0.2165635
## rush_tds 0.2025640 0.1375727
## rec_att  0.6696210 0.9888153
## rec_yds  0.6537386 0.9972777
## rec_tds  0.6358641 0.9878753
## fumbles  1.0000000 0.6482031
## fpts     0.6482031 1.0000000
```

```
(vcov.proj2=var(proj2))
```

```
##             rush_att    rush_yds    rush_tds      rec_att      rec_yds
## rush_att   8.2762443   48.461529 0.263435753   18.0133788 1.846837e+02
## rush_yds  48.4615288  292.300941 1.599563801  104.1857169 1.075936e+03
## rush_tds   0.2634358    1.599564 0.010241761    0.3721525 3.442937e+00
## rec_att   18.0133788  104.185717 0.372152532  728.9124385 9.521274e+03
## rec_yds  184.6837001 1075.936191 3.442936799 9521.2743562 1.267042e+05
## rec_tds    1.1509910    6.813895 0.023865094   62.2887833 8.321890e+02
## fumbles    0.2909777    1.817170 0.008298412    7.3183341 9.419876e+01
## fpts      31.2469007  183.738930 0.690908405 1324.8099612 1.761621e+04
##               rec_tds      fumbles         fpts
## rush_att   1.15099105  0.290977676 3.124690e+01
## rush_yds   6.81389493  1.817169507 1.837389e+02
## rush_tds   0.02386509  0.008298412 6.909084e-01
## rec_att   62.28878331  7.318334076 1.324810e+03
## rec_yds  832.18902987 94.198760674 1.761621e+04
## rec_tds    5.70944343  0.615044752 1.171385e+02
## fumbles    0.61504475  0.163866808 1.302139e+01
## fpts     117.13852783 13.021388841 2.462643e+03
```

```
(means.proj2=colMeans(proj2))
```

```
##    rush_att    rush_yds    rush_tds     rec_att     rec_yds     rec_tds
##   0.9388430   5.3586777   0.0214876  29.2867769 378.7561983   2.2958678
##     fumbles        fpts
##   0.4057851  51.5252066
```

```
proj2.sim = mvrnorm(20, mu = means.proj2, Sigma = vcov.proj2)
proj2.sim = as.data.frame(proj2.sim)
(rho.sim=cor(proj2.sim))   ## Simulated correlation matrix
```

```
##             rush_att    rush_yds   rush_tds    rec_att    rec_yds
## rush_att  1.00000000  0.98241757  0.8548789 -0.3183961 -0.3412275
## rush_yds  0.98241757  1.00000000  0.8760203 -0.3613216 -0.3906897
## rush_tds  0.85487892  0.87602030  1.0000000 -0.3622526 -0.3651420
## rec_att  -0.31839609 -0.36132157 -0.3622526  1.0000000  0.9818659
## rec_yds  -0.34122749 -0.39068968 -0.3651420  0.9818659  1.0000000
## rec_tds  -0.29361925 -0.33467244 -0.2917701  0.9398650  0.9778538
## fumbles   0.07589688  0.09516521  0.0341450  0.4036428  0.4355268
## fpts     -0.29081404 -0.33858899 -0.3087052  0.9731680  0.9964131
##             rec_tds    fumbles       fpts
```

```
## rush_att -0.2936193 0.07589688 -0.2908140
## rush_yds -0.3346724 0.09516521 -0.3385890
## rush_tds -0.2917701 0.03414500 -0.3087052
## rec_att   0.9398650 0.40364280  0.9731680
## rec_yds   0.9778538 0.43552685  0.9964131
## rec_tds   1.0000000 0.44839575  0.9897954
## fumbles   0.4483957 1.00000000  0.4400489
## fpts      0.9897954 0.44004889  1.0000000
```

```
rho.proj2
```

```
##           rush_att  rush_yds   rush_tds   rec_att    rec_yds    rec_tds
## rush_att 1.0000000 0.9852929 0.90483781 0.2319214 0.18035010 0.16743969
## rush_yds 0.9852929 1.0000000 0.92448344 0.2257124 0.17679740 0.16679516
## rush_tds 0.9048378 0.9244834 1.00000000 0.1362059 0.09557542 0.09869131
## rec_att  0.2319214 0.2257124 0.13620592 1.0000000 0.99074495 0.96555114
## rec_yds  0.1803501 0.1767974 0.09557542 0.9907449 1.00000000 0.97842956
## rec_tds  0.1674397 0.1667952 0.09869131 0.9655511 0.97842956 1.00000000
## fumbles  0.2498605 0.2625638 0.20256396 0.6696210 0.65373865 0.63586414
## fpts     0.2188717 0.2165635 0.13757275 0.9888153 0.99727772 0.98787533
##            fumbles      fpts
## rush_att 0.2498605 0.2188717
## rush_yds 0.2625638 0.2165635
## rush_tds 0.2025640 0.1375727
## rec_att  0.6696210 0.9888153
## rec_yds  0.6537386 0.9972777
## rec_tds  0.6358641 0.9878753
## fumbles  1.0000000 0.6482031
## fpts     0.6482031 1.0000000
```

```
keep.1=0
loops=10000

for (i in 1:loops) {
      proj2.sim = mvrnorm(30, mu = means.proj2, Sigma = vcov.proj2)
      keep.1=keep.1+cor(proj2.sim)/loops   # take average as it accumulates
}
rho.proj2 ; keep.1
```

```
##           rush_att  rush_yds   rush_tds   rec_att    rec_yds    rec_tds
## rush_att 1.0000000 0.9852929 0.90483781 0.2319214 0.18035010 0.16743969
## rush_yds 0.9852929 1.0000000 0.92448344 0.2257124 0.17679740 0.16679516
## rush_tds 0.9048378 0.9244834 1.00000000 0.1362059 0.09557542 0.09869131
## rec_att  0.2319214 0.2257124 0.13620592 1.0000000 0.99074495 0.96555114
## rec_yds  0.1803501 0.1767974 0.09557542 0.9907449 1.00000000 0.97842956
## rec_tds  0.1674397 0.1667952 0.09869131 0.9655511 0.97842956 1.00000000
## fumbles  0.2498605 0.2625638 0.20256396 0.6696210 0.65373865 0.63586414
## fpts     0.2188717 0.2165635 0.13757275 0.9888153 0.99727772 0.98787533
##            fumbles      fpts
## rush_att 0.2498605 0.2188717
## rush_yds 0.2625638 0.2165635
## rush_tds 0.2025640 0.1375727
## rec_att  0.6696210 0.9888153
## rec_yds  0.6537386 0.9972777
## rec_tds  0.6358641 0.9878753
## fumbles  1.0000000 0.6482031
```

```
## fpts     0.6482031 1.0000000
```

```
##           rush_att  rush_yds   rush_tds   rec_att    rec_yds    rec_tds
## rush_att 1.0000000 0.9847090 0.90156620 0.2271792 0.17635035 0.16398906
## rush_yds 0.9847090 1.0000000 0.92177511 0.2203726 0.17216643 0.16265174
## rush_tds 0.9015662 0.9217751 1.00000000 0.1322901 0.09240096 0.09596377
## rec_att  0.2271792 0.2203726 0.13229010 1.0000000 0.99037005 0.96413697
## rec_yds  0.1763503 0.1721664 0.09240096 0.9903701 1.00000000 0.97756279
## rec_tds  0.1639891 0.1626517 0.09596377 0.9641370 0.97756279 1.00000000
## fumbles  0.2446502 0.2567486 0.19693472 0.6629822 0.64697880 0.62821247
## fpts     0.2143455 0.2113787 0.13381033 0.9883280 0.99716412 0.98740604
##            fumbles      fpts
## rush_att 0.2446502 0.2143455
## rush_yds 0.2567486 0.2113787
## rush_tds 0.1969347 0.1338103
## rec_att  0.6629822 0.9883280
## rec_yds  0.6469788 0.9971641
## rec_tds  0.6282125 0.9874060
## fumbles  1.0000000 0.6411157
## fpts     0.6411157 1.0000000
```

1. Generate a data set with 30 rows that has the exact correlation structure as the original data set. (2 points)

Question 4

```
proj2.sim = mvrnorm(30, mu = means.proj2, Sigma = vcov.proj2, empirical=TRUE)
cor(proj2.sim) ; rho.proj2
```

```
##           rush_att  rush_yds   rush_tds   rec_att    rec_yds    rec_tds
## rush_att 1.0000000 0.9852929 0.90483781 0.2319214 0.18035010 0.16743969
## rush_yds 0.9852929 1.0000000 0.92448344 0.2257124 0.17679740 0.16679516
## rush_tds 0.9048378 0.9244834 1.00000000 0.1362059 0.09557542 0.09869131
## rec_att  0.2319214 0.2257124 0.13620592 1.0000000 0.99074495 0.96555114
## rec_yds  0.1803501 0.1767974 0.09557542 0.9907449 1.00000000 0.97842956
## rec_tds  0.1674397 0.1667952 0.09869131 0.9655511 0.97842956 1.00000000
## fumbles  0.2498605 0.2625638 0.20256396 0.6696210 0.65373865 0.63586414
## fpts     0.2188717 0.2165635 0.13757275 0.9888153 0.99727772 0.98787533
##            fumbles      fpts
## rush_att 0.2498605 0.2188717
## rush_yds 0.2625638 0.2165635
## rush_tds 0.2025640 0.1375727
## rec_att  0.6696210 0.9888153
## rec_yds  0.6537386 0.9972777
## rec_tds  0.6358641 0.9878753
## fumbles  1.0000000 0.6482031
## fpts     0.6482031 1.0000000
```

```
##           rush_att  rush_yds   rush_tds   rec_att    rec_yds    rec_tds
## rush_att 1.0000000 0.9852929 0.90483781 0.2319214 0.18035010 0.16743969
## rush_yds 0.9852929 1.0000000 0.92448344 0.2257124 0.17679740 0.16679516
## rush_tds 0.9048378 0.9244834 1.00000000 0.1362059 0.09557542 0.09869131
## rec_att  0.2319214 0.2257124 0.13620592 1.0000000 0.99074495 0.96555114
## rec_yds  0.1803501 0.1767974 0.09557542 0.9907449 1.00000000 0.97842956
```

```
## rec_tds  0.1674397 0.1667952 0.09869131 0.9655511 0.97842956 1.00000000
## fumbles  0.2498605 0.2625638 0.20256396 0.6696210 0.65373865 0.63586414
## fpts     0.2188717 0.2165635 0.13757275 0.9888153 0.99727772 0.98787533
##            fumbles      fpts
## rush_att 0.2498605 0.2188717
## rush_yds 0.2625638 0.2165635
## rush_tds 0.2025640 0.1375727
## rec_att  0.6696210 0.9888153
## rec_yds  0.6537386 0.9972777
## rec_tds  0.6358641 0.9878753
## fumbles  1.0000000 0.6482031
## fpts     0.6482031 1.0000000
```

```
proj2.sim
```

```
##         rush_att   rush_yds      rush_tds    rec_att     rec_yds
##  [1,]  1.5754772   4.211786  0.1164924419  56.240488  687.434316
##  [2,]  9.3304626  53.588733  0.3029475710  22.677015  257.740415
##  [3,]  2.7204414  14.972911  0.0565046918  60.603128  825.653708
##  [4,] -2.4855812 -15.327036 -0.1114110628  58.280142  878.254445
##  [5,] -1.3817706  -2.949417  0.0030519366  11.155197  217.073241
##  [6,]  2.2927557  11.417717  0.0209009819  27.988638  360.665485
##  [7,]  0.7496331   6.811580  0.0131855688  -1.386773  -33.763883
##  [8,]  5.7532612  34.637101  0.1406167028  55.469297  662.983764
##  [9,] -0.1873799  -4.334009 -0.0409710737  38.040269  522.666807
## [10,] -1.4484733  -9.185154 -0.1025871820  38.116966  542.372631
## [11,]  1.1879075   7.513546  0.0552502469   5.664459    2.412513
## [12,]  0.7559732   7.932634  0.0110898555  88.583539 1062.506221
## [13,] -3.1835083 -20.494988 -0.1884372158  35.215681  452.290408
## [14,]  3.1045217  19.716922  0.0968342281  34.329108  422.541467
## [15,] -0.1199922  -4.111912  0.0043089785   8.331937  109.819804
## [16,]  1.6515419   8.128833  0.0594465835  -5.887124  -80.678305
## [17,] -2.7482758 -13.184848 -0.0602950482  14.861982  207.463997
## [18,]  0.8836141  11.690034  0.1291046216  23.249231  370.759015
## [19,] -3.6768986 -23.272580 -0.0499905602   9.213644  134.459849
## [20,]  3.3857763  15.839956  0.0906118461  12.560325  118.786011
## [21,]  0.7735350   2.378393  0.0008144142  79.202319 1053.239926
## [22,] -3.7009028 -21.923688 -0.1502956703 -43.915077 -618.499582
## [23,] -0.5770420  -4.475083 -0.0481680562  13.959089  162.771614
## [24,]  1.9396164   9.197112 -0.0072139707  46.761530  560.526886
## [25,]  4.6080126  31.922578  0.1712669576  38.951519  429.747648
## [26,]  1.7353738  12.417677  0.0768851308   8.851291  145.530084
## [27,]  1.6675691  13.022827  0.0594968262  46.189295  646.507378
## [28,]  4.0642044  20.755432  0.0830488178  32.006253  449.393025
## [29,]  0.2470225  -1.709878 -0.0535535594  15.965843  219.568056
## [30,] -0.7515859  -4.426849 -0.0343069033  47.324092  592.459009
##           rec_tds     fumbles       fpts
##  [1,]  4.15458011  0.67514811  93.276382
##  [2,]  1.65380929  0.28484778  42.629327
##  [3,]  5.01949695  1.19855233 112.297807
##  [4,]  5.91501879  0.84740159 119.764013
##  [5,]  1.12915848  0.04127021  28.008765
##  [6,]  2.34417940  0.43586268  50.570003
##  [7,] -0.72031731  0.26982148  -7.057819
##  [8,]  3.72800892  0.53615613  91.556789
```

```
##  [9,]  2.63717343 -0.22112638  67.783792
## [10,]  3.14871216  0.14264036  71.278226
## [11,] -0.62288788  0.81897033  -3.977245
## [12,]  6.61284132  1.05194899 144.875941
## [13,]  2.26347443  0.33721702  55.182473
## [14,]  2.08111147  0.96517764  55.243514
## [15,]  0.52776506  0.27823741  13.442144
## [16,] -0.90595664  0.35155080 -13.308881
## [17,]  1.45276207  0.59719475  26.312626
## [18,]  2.55317534 -0.01798629  54.323920
## [19,]  1.40795112  0.02862812  19.088731
## [20,]  0.03042153 -0.13243537  14.728478
## [21,]  7.23427825  0.73958821 147.742142
## [22,] -3.56962082 -0.58623497 -85.285731
## [23,]  0.95390529  0.52779627  20.029373
## [24,]  3.43449687  0.54130942  76.532356
## [25,]  3.51329729  0.33626317  67.508597
## [26,] -0.12207362  0.26696110  14.929055
## [27,]  4.33830572  0.86104714  90.662040
## [28,]  4.11231341  0.58053570  70.831493
## [29,]  0.89473419 -0.08143834  27.148703
## [30,]  3.67591847  0.49864833  79.639185
```

10 points

Use LaTeXto create the following expressions.

1. Hint: `\Rightarrow` (4 points)

$$P(B) = \sum_j P(B|A_j)P(A_j)$$
$$\Rightarrow P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_j (B|A_j)P(A_j)}$$

1. Hint: `\zeta` (3 points)

$$\hat{f}(\zeta) = \int_{-\infty}^{\infty} f(x)e^{-2\pi ix\zeta}dx$$

1. Hint: `\partial` (3 points)

$$\mathbf{J} = \frac{df}{dx} = \left[\frac{\partial \mathbf{f}}{\partial x_1} \cdots \frac{\partial \mathbf{f}}{\partial x_n}\right] = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$