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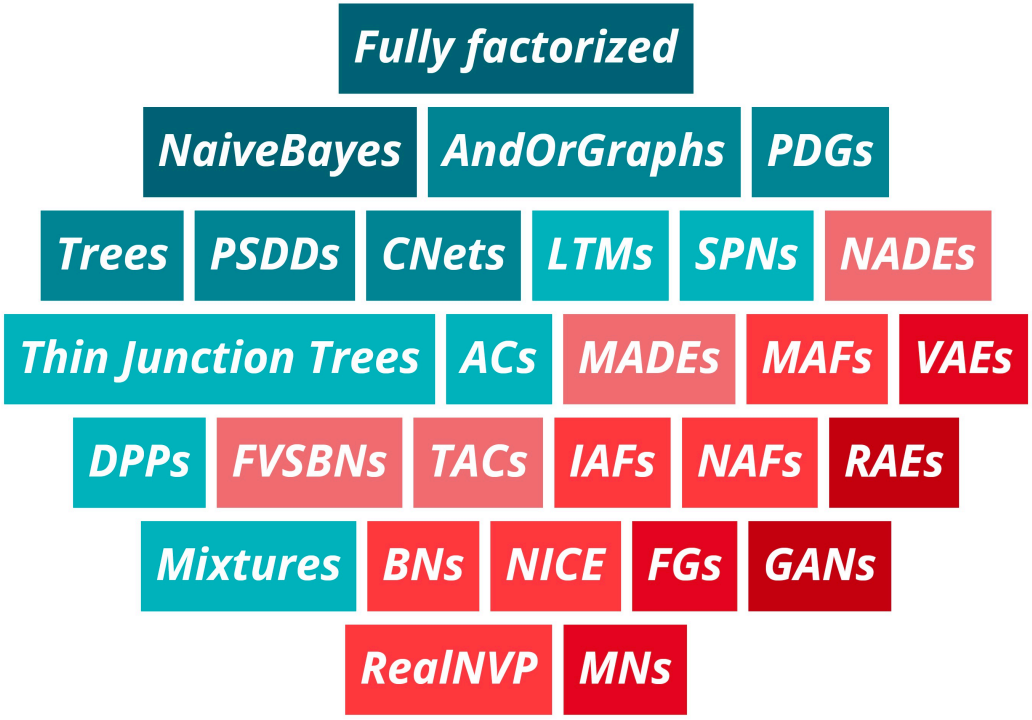
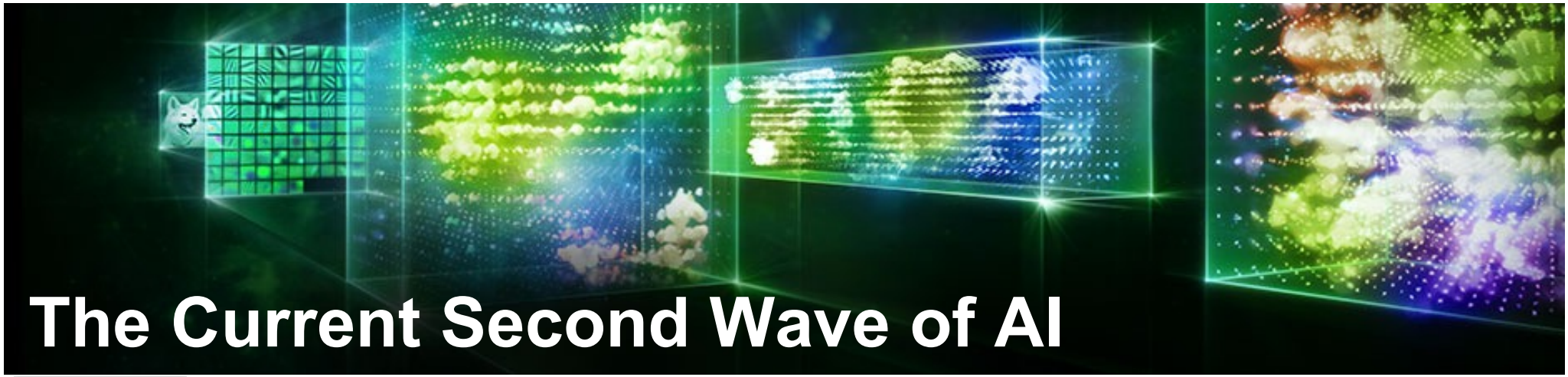
Tractable Probabilistic Models



Thanks to Pedro Domingos for making slides publically available
This is also based on joint work with Alejandro Molina, Antonio Vergari, Robert Peharz, Guy van den Broeck, Karl Stelzner and many others. Thanks!

tractability is a spectrum

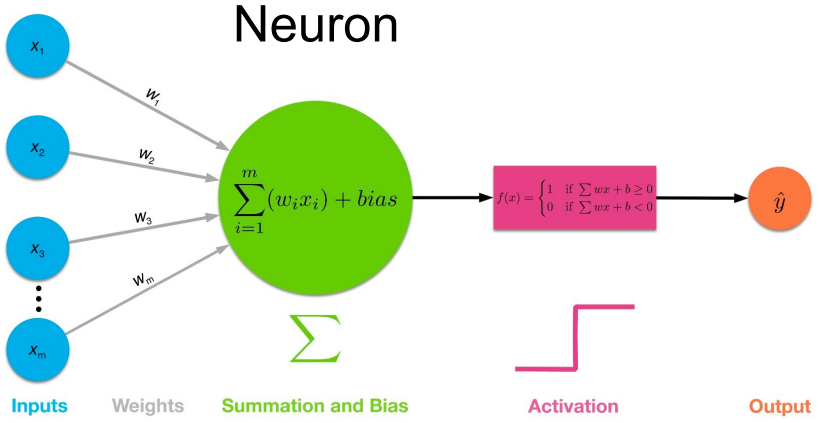


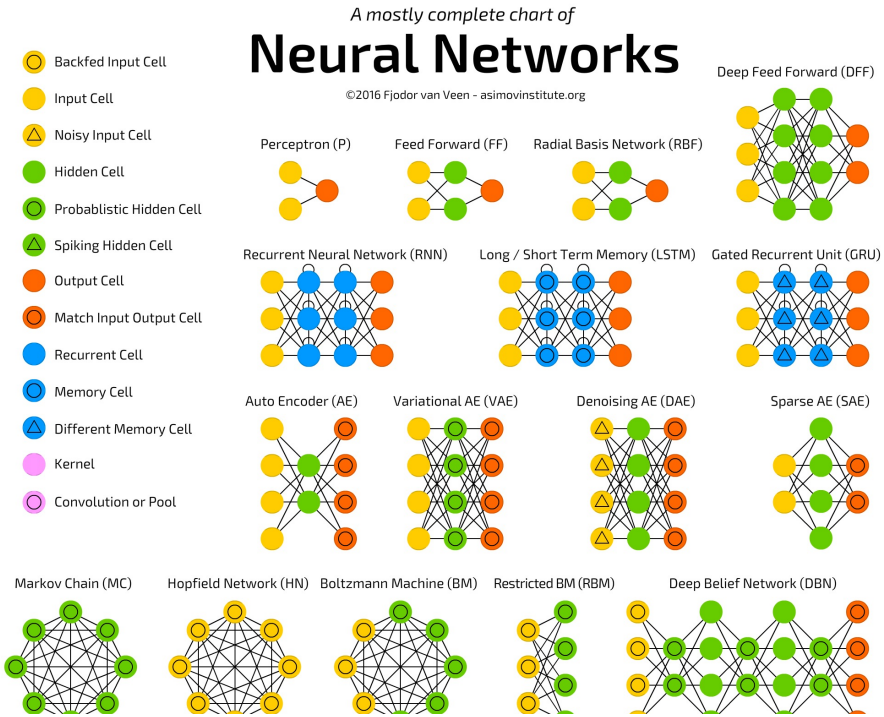
The Current Second Wave of AI

Deep Neural Networks

Potentially much more powerful than shallow architectures, represent computations

[LeCun, Bengio, Hinton Nature 521





Differentiable Programming

Deep Learning

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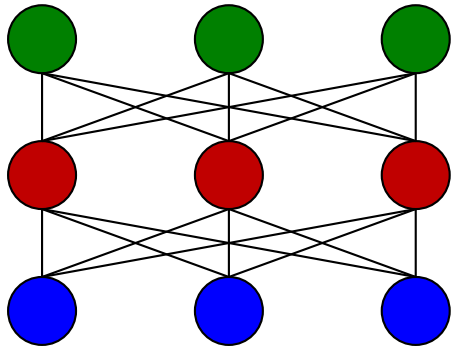
Stack many layers

E.g.: DBN [Hinton & Salakhutdinov,2006]

CDBN [Lee et al.,2009]

DBM [Salakhutdinov & Hinton,2010]



Potentially much more powerful than shallow architectures, represent computations [Bengio, 2009]

But …

- **Often no probabilistic semantics**
- **Learning requires extensive efforts**

DNNs often have no probabilistic semantics. They are not calibrated joint distributions.

$$P(Y|X) \neq P(Y,X)$$

MNIST

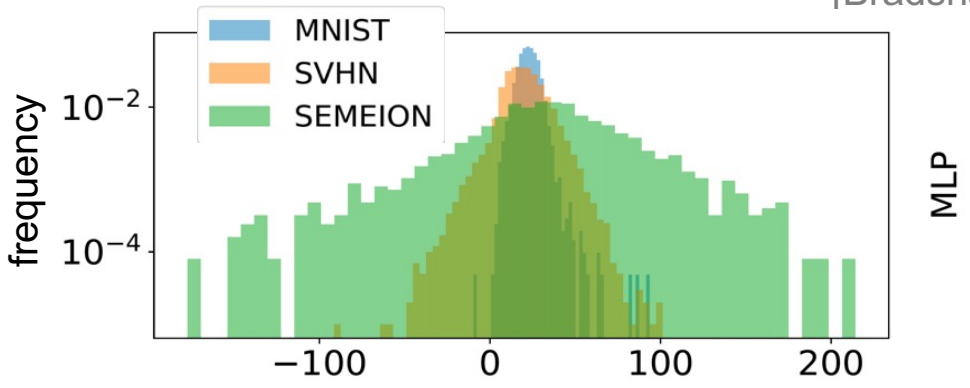
SVHN

SEMEION

Train & Evaluate

Transfer Testing

[Bradshaw et al. arXiv:1707.02476 2017]



Many DNNs cannot distinguish the datasets

[Peharz, Vergari, Molina, Stelzner, Trapp, Kersting, Ghahramani UAI 2019]

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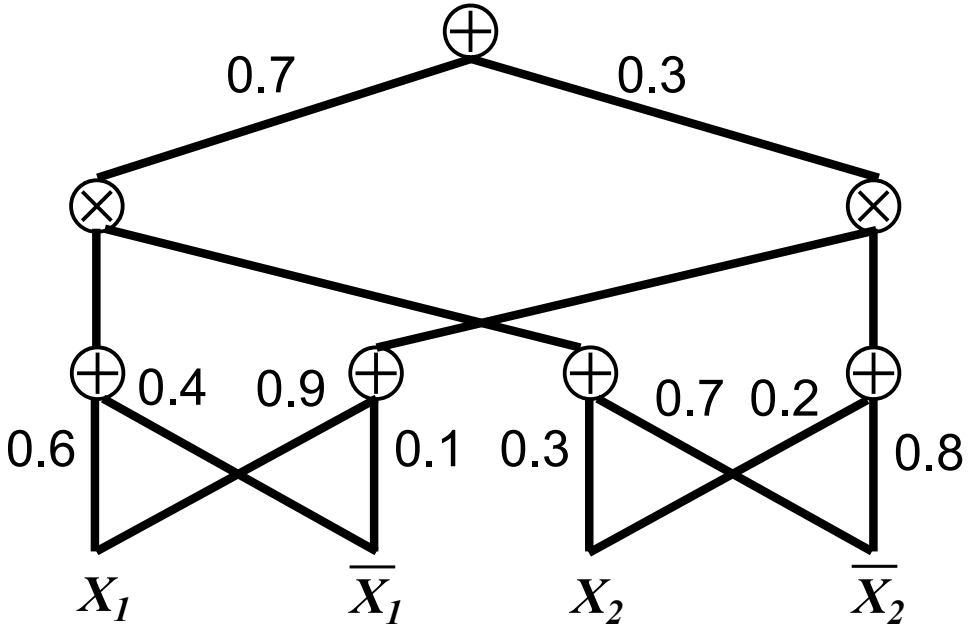
Probabilistic Circuits

a deep probabilistic learning framework (here sum-product networks)



Computational graph (kind of TensorFlow graphs) that encodes how to compute probabilities

Inference is linear in size of network

Alternative Representation: Joint Distributions as Deep Networks

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X_1	X_2	$P(X)$
1	1	0.4
1	0	0.2
0	1	0.1
0	0	0.3

$$P(X) = 0.4 \cdot \mathrm{I}[X_1{=}1] \cdot \mathrm{I}[X_2{=}1] + 0.2 \cdot \mathrm{I}[X_1{=}1] \cdot \mathrm{I}[X_2{=}0] + 0.1 \cdot \mathrm{I}[X_1{=}0] \cdot \mathrm{I}[X_2{=}1] + 0.3 \cdot \mathrm{I}[X_1{=}0] \cdot \mathrm{I}[X_2{=}0]$$

Alternative Representation: Joint Distributions as (Deep) Networks

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X_1	X_2	$P(X)$
1	**1**	**0.4**
1	0	0.2
0	1	0.1
0	0	0.3

$$P(X) = \mathbf{0.4 \cdot I[X_1{=}1] \cdot I[X_2{=}1]} + 0.2 \cdot \mathrm{I}[X_1{=}1] \cdot \mathrm{I}[X_2{=}0] + 0.1 \cdot \mathrm{I}[X_1{=}0] \cdot \mathrm{I}[X_2{=}1] + 0.3 \cdot \mathrm{I}[X_1{=}0] \cdot \mathrm{I}[X_2{=}0]$$

Shorthand for Indicators

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X_1	X_2	$P(X)$
1	1	0.4
1	0	0.2
0	1	0.1
0	0	0.3

$$
\begin{aligned}
P(X) = 0.4 \cdot X_1 \cdot X_2 \\
+ 0.2 \cdot X_1 \cdot \overline{X}_2 \\
+ 0.1 \cdot \overline{X}_1 \cdot X_2 \\
+ 0.3 \cdot \overline{X}_1 \cdot \overline{X}_2
\end{aligned}
$$

Sum Out Variables

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X_1	X_2	$P(X)$
1	**1**	**0.4**
1	**0**	**0.2**
0	1	0.1
0	0	0.3

$$\boxed{e: X_1 = 1}$$

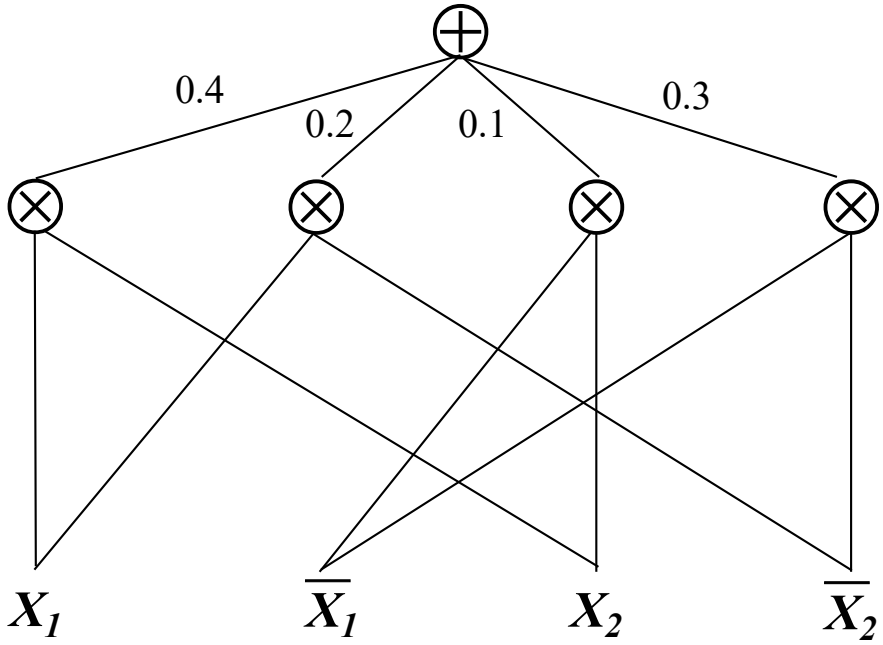
$$P(e) = \mathbf{0.4 \cdot X_1 \cdot X_2} + \mathbf{0.2 \cdot X_1 \cdot \overline{X}_2} + 0.1 \cdot \overline{X}_1 \cdot X_2 + 0.3 \cdot \overline{X}_1 \cdot \overline{X}_2$$

Set $X_1 = 1, \overline{X}_1 = 0, \boxed{X_2 = 1, \overline{X}_2 = 1}$

Easy: Set both indicators of X2 to 1

Idea: Deeper Network Representation of a Graphical Model that encodes how to compute probabilities

X_1	X_2	$P(X)$
1	1	0.4
1	0	0.2
0	1	0.1
0	0	0.3



Sum-Product Networks* (SPNs)

[Poon, Domingos UAI 2011]

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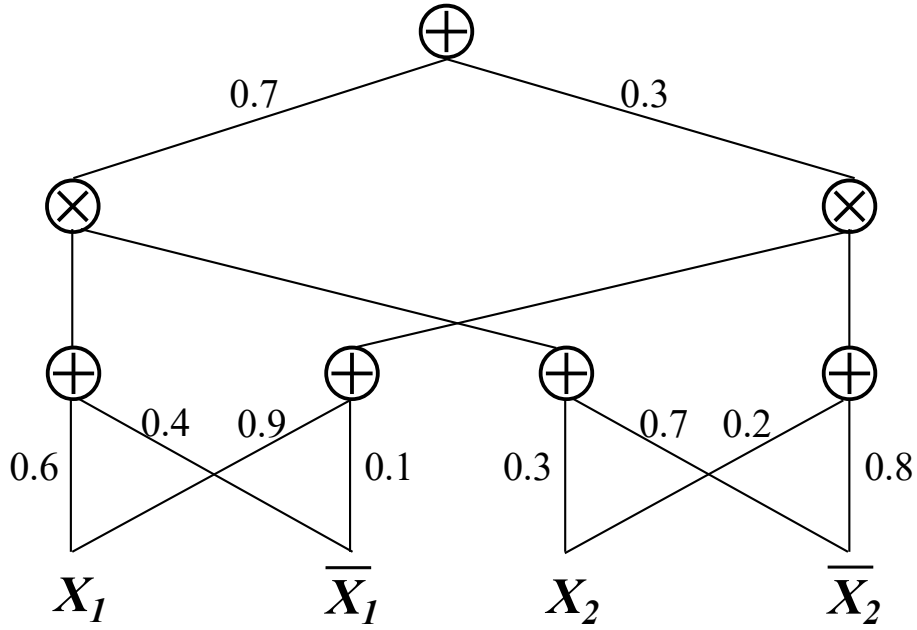
A SPN S is a rooted DAG where:
Nodes: Sum, product, input indicator
Weights on edges from sum to children



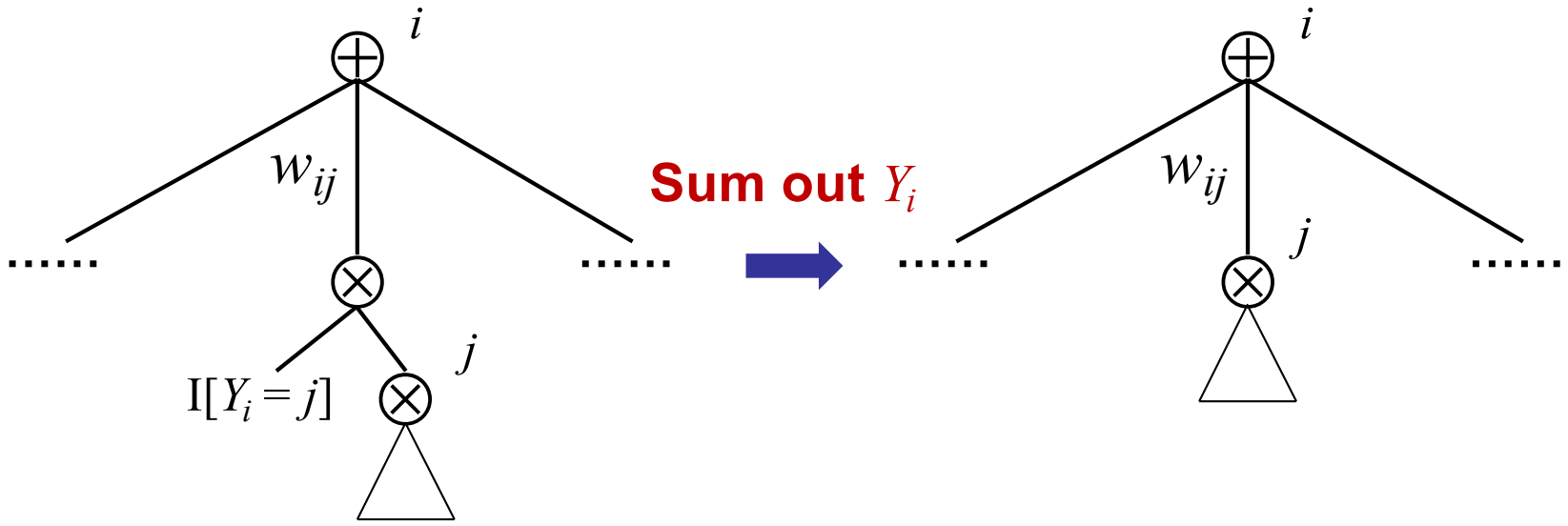
*SPNs are an instance of Arithmetic Circuits (ACs). ACs have been introduced into the AI literature more than15 years ago as a tractable representation of probability distributions

[Darwiche CACM 48(4):608-647 2001]

Semantics of Sums and Products

Product ~ Feature $\rightarrow$ Form feature hierarchy

Sum ~ Mixture (with hidden var. summed out)



Inference: Linear in Size of Network

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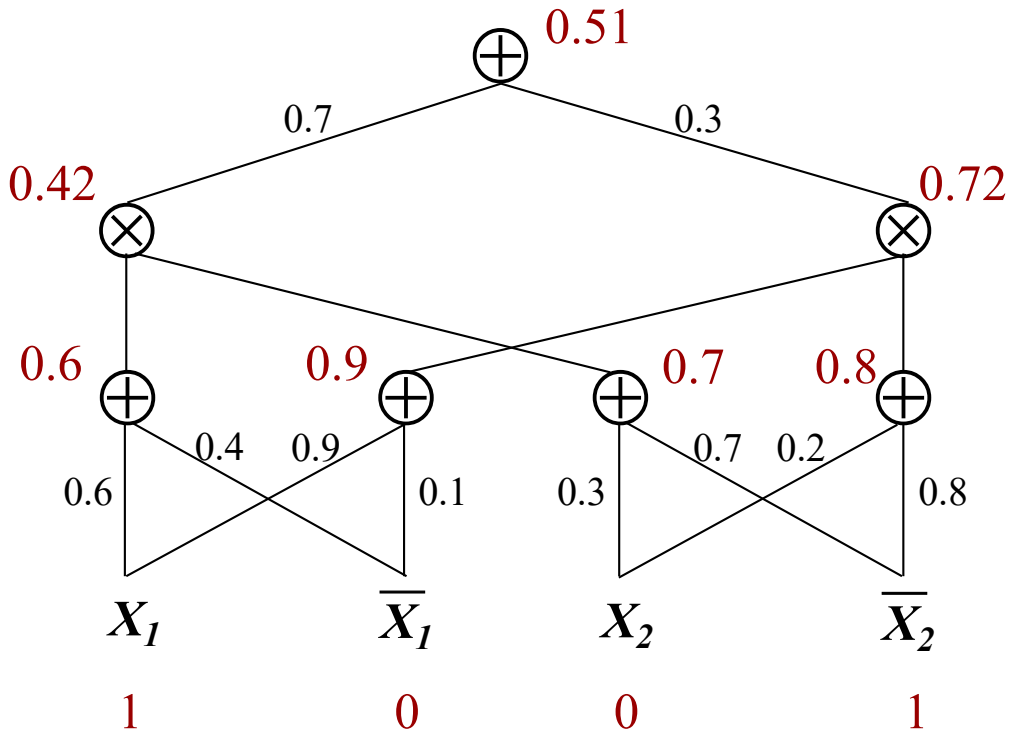
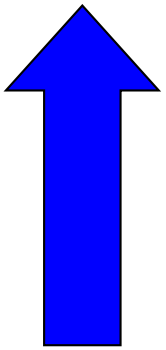
As long as weights sum to 1
at each sum node

$$P(X) = S(X)$$

X: $X_1 = 1, X_2 = 0$

X_1	1
$\overline{X}_1$	0
X_2	0
$\overline{X}_2$	1





Inference: Linear in Size of Network

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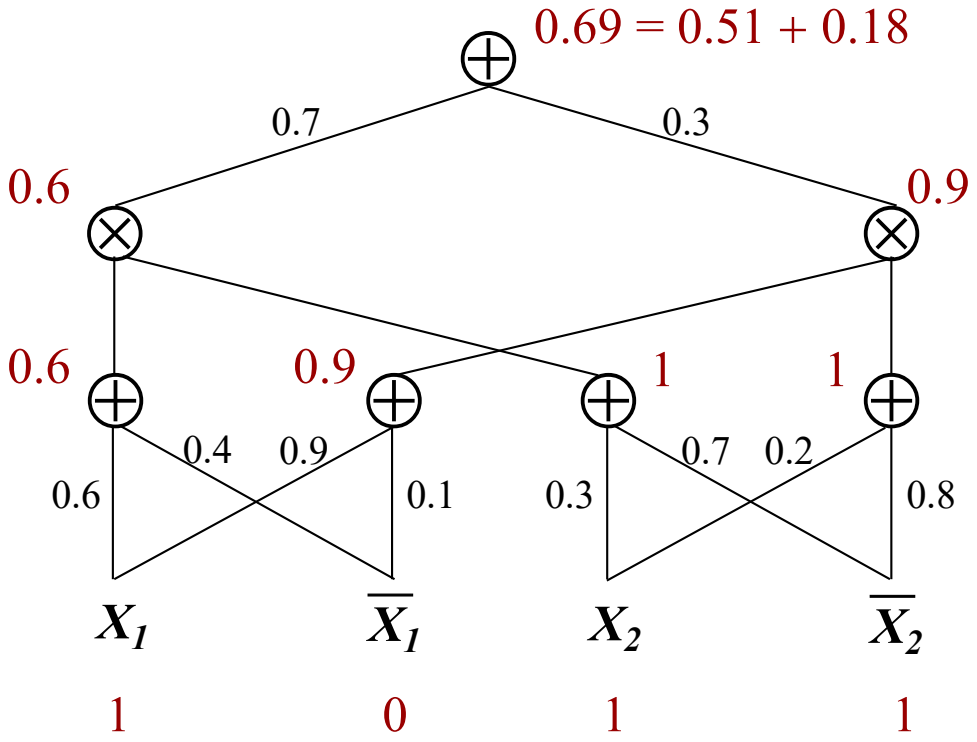
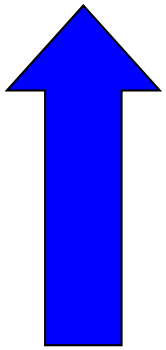
Marginal: $P(e) = S(e)$

e: $X_1 = 1$

X_1	1
$\overline{X}_1$	0
X_2	1
$\overline{X}_2$	1





Inference: Linear in Size of Network

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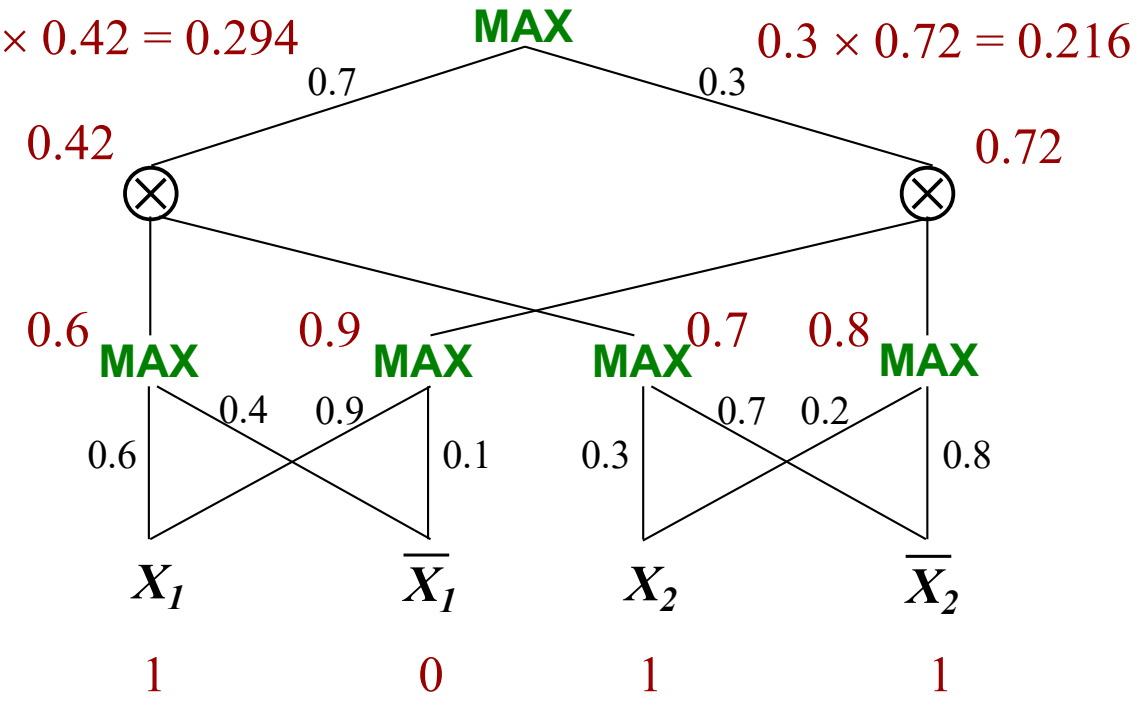
MAP: Replace sums with maxs

e: $X_1 = 1$

X_1	1
$\overline{X}_1$	0
X_2	1
$\overline{X}_2$	1



Inference: Linear in Size of Network

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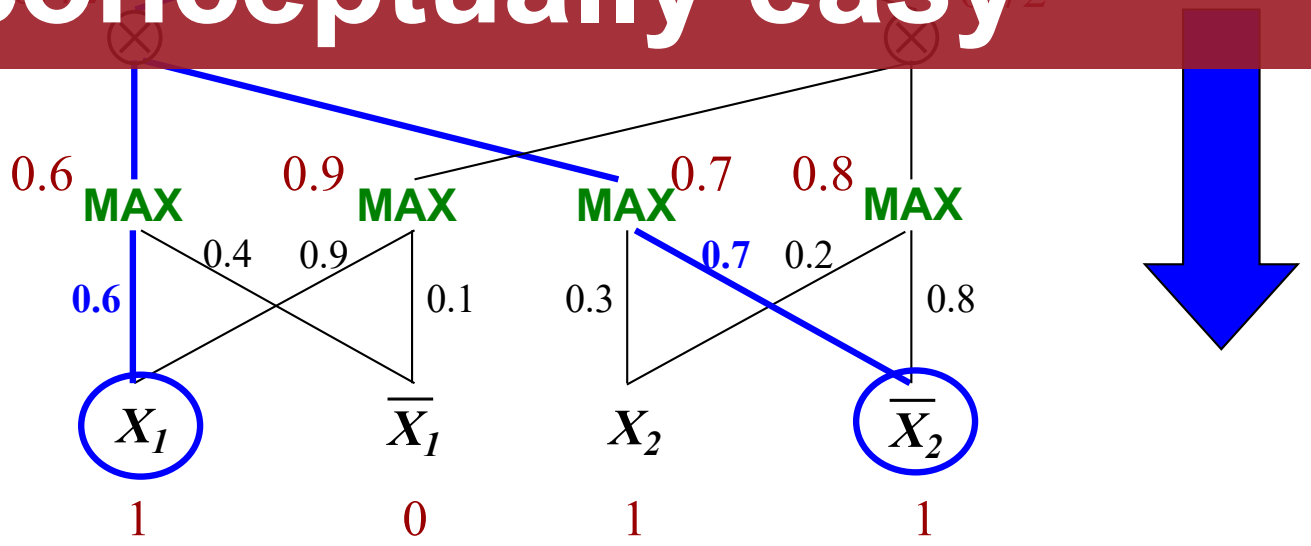
MAX: Pick child with highest value

MAP State: $X_1 = 1, X_2 = 0$

And also learning is conceptually easy

X_1	1
$\overline{X}_1$	0
X_2	1
$\overline{X}_2$	1



General Approach via Parameter Estimation assuming a fixed network (like in Deep Learning)

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- Start with a dense SPN
- Find the structure by (online) learning weights
  Zero weights signify absence of connections
- (Hard) EM beneficial to avoid gradient vanishing
  Each sum node is a mixture over children

In principle you can turn a given SPN into a TensorFlow computation graph and apply any known algorithm from there

Image Completion

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Main evaluation: Caltech-101 [Fei-Fei et al., 2004]

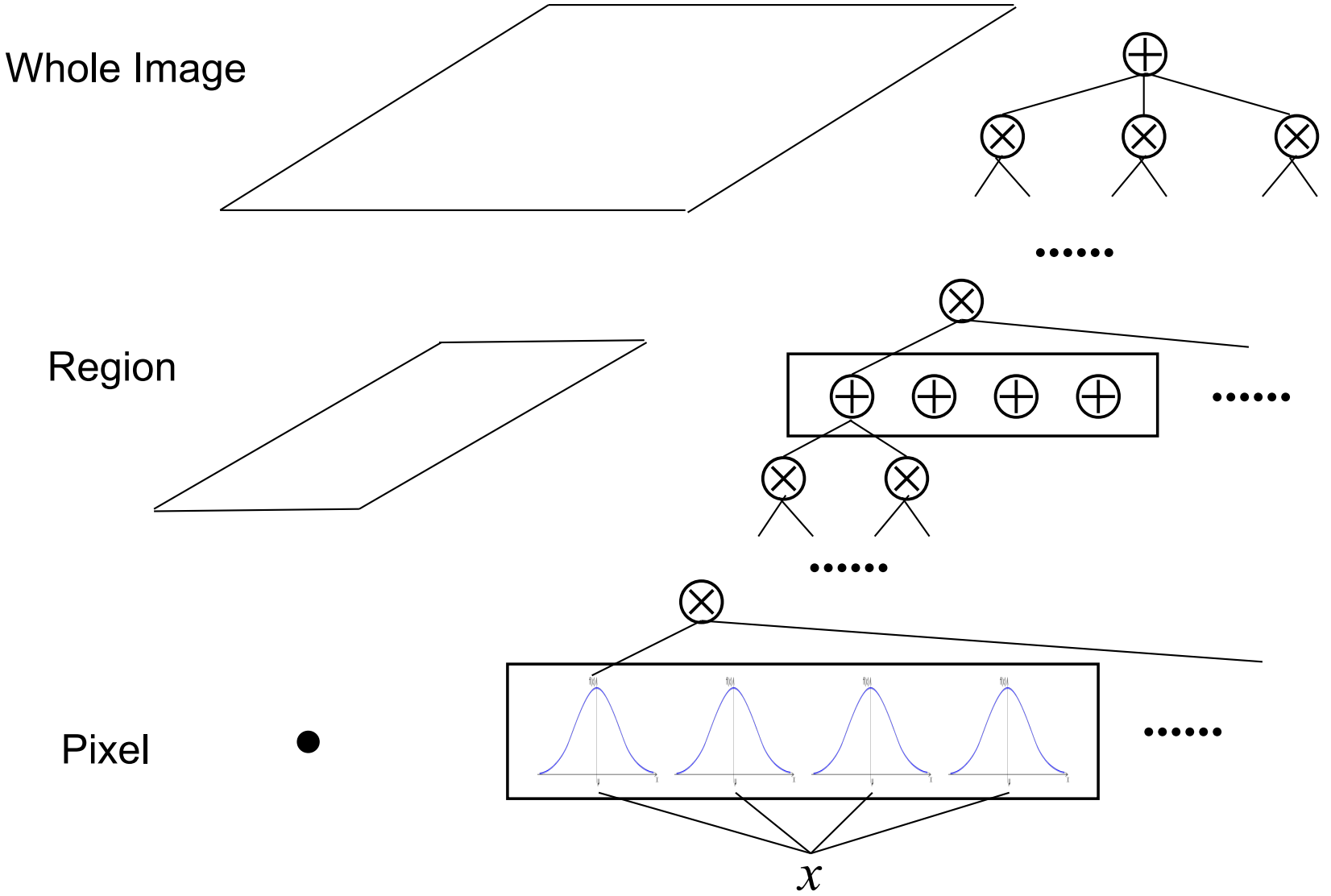
- 101 categories, e.g., faces, cars, elephants
- Each category: 30 – 800 images

Also, Olivetti [Samaria & Harter, 1994] (400 faces)

Each category: Last third for test

Test images: Unseen objects

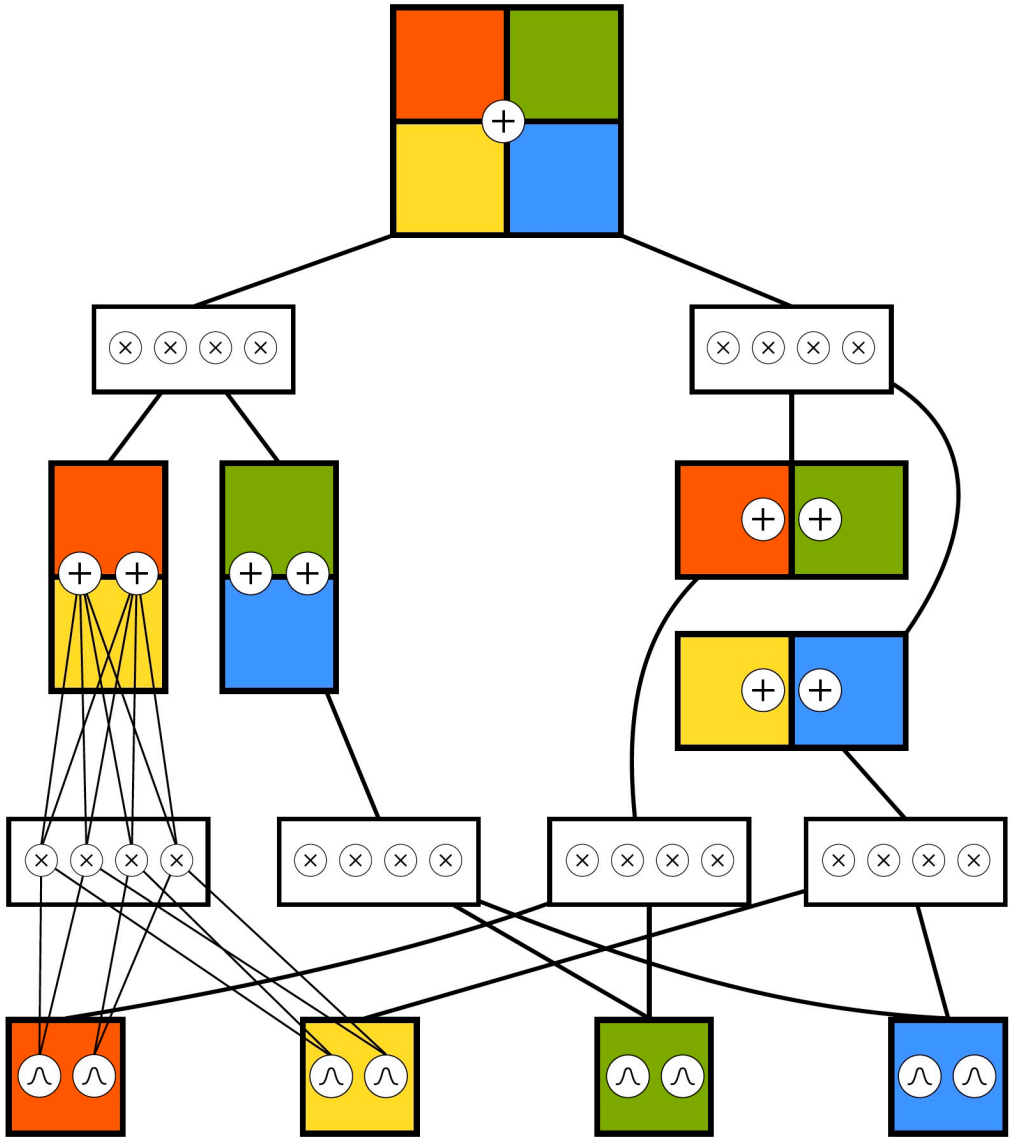
SPN Architecture



SPN Architecture

Whole Image

Regions



Caltech: Mean-Square Errors

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	LEFT	BOTTOM
SPN	**3551**	**3270**
DBM	9043	9792
DBN	4778	4492
PCA	4234	4465
Nearest Neighbor	4887	5505

SPN vs. DBM / DBN

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SPN is order of magnitude faster

	SPN	DBM / DBN
Learning	2-3 hours	Days
Inference	< 1 second	Minutes or hours

No elaborate preprocessing, tuning

Reduced errors by 30-60%

Learned up to 46 layers

Example Completions

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Original

SPN

DBM

DBN

PCA

Nearest Neighbor

Or we learn directly (Tree-)SPNs

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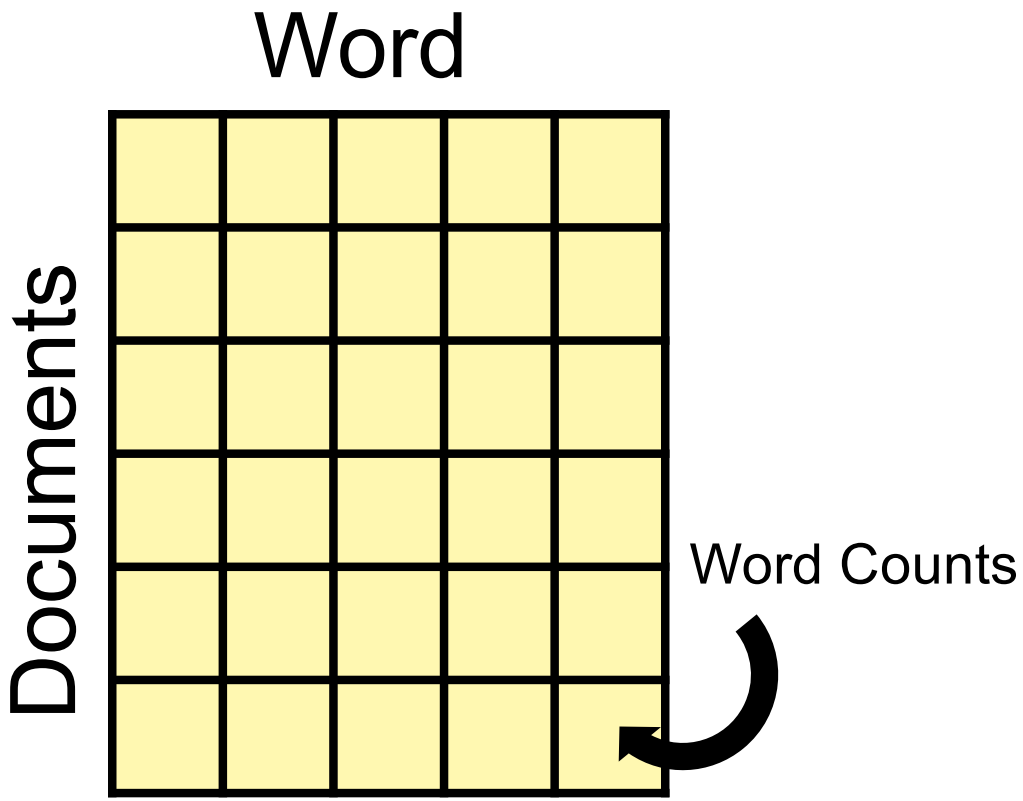
Testing independence of
Poisson random variables



[Poon, Domingos UAI'11; Molina, Natarajan, Kersting AAAI´17]

Or we learn directly (Tree-)SPNs

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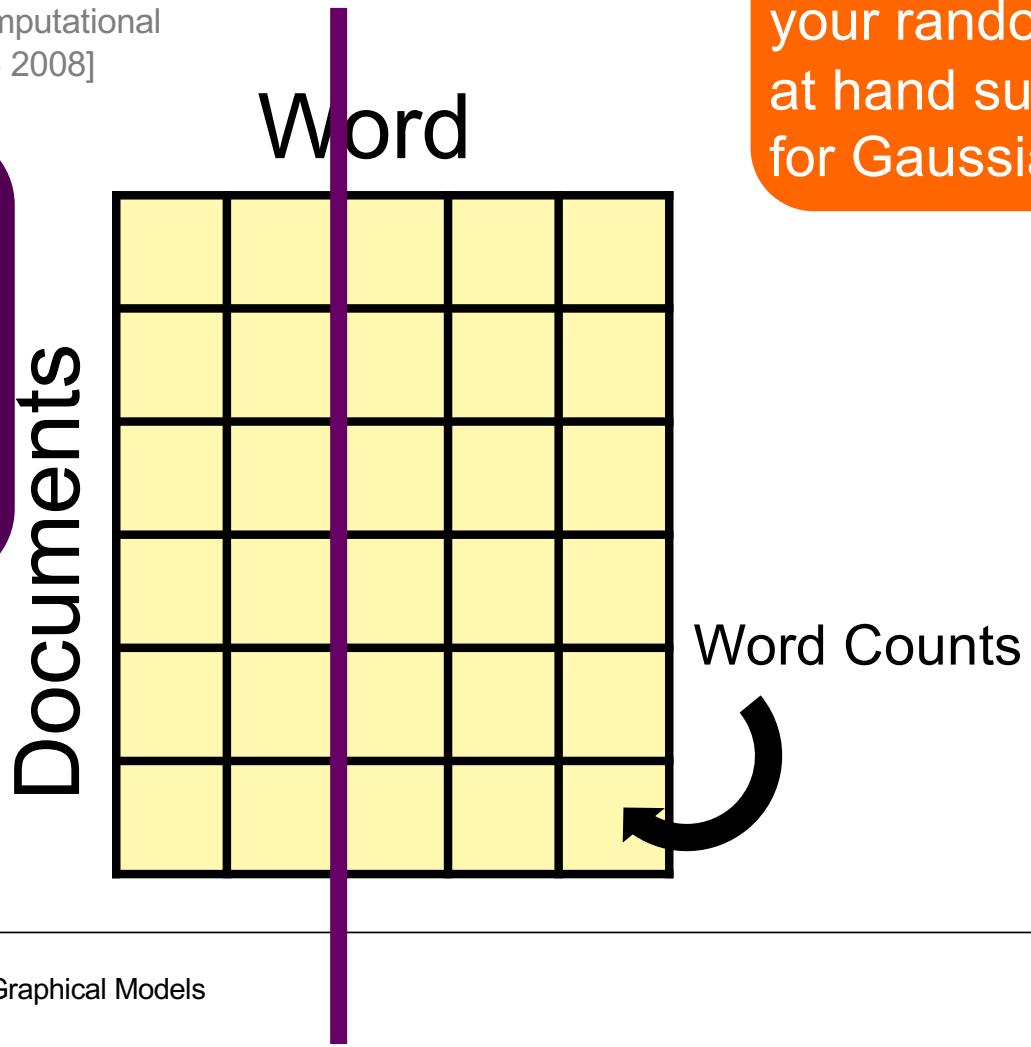
Testing independence of Poisson random variables

[Zeileis, Hothorn, Hornik Journal of Computational And Graphical Statistics 17(2):492–514 2008]

In general use the independency test for your random variables at hand such as g-test for Gaussians

Learn Poisson model trees for P(x|V-x) and P(y|V-y). Check whether X resp. Y is significant in P(y|V-x) resp. P(x|V-y)



[Poon, Domingos UAI'11; Molina, Natarajan, Kersting AAAI'17]

Or we learn directly (Tree-)SPNs

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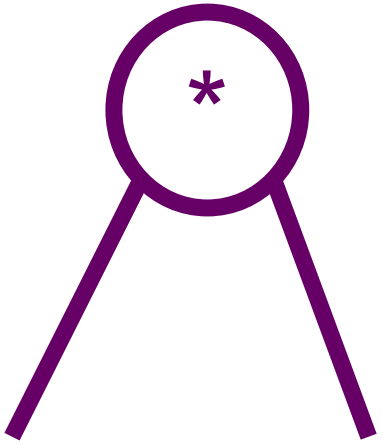
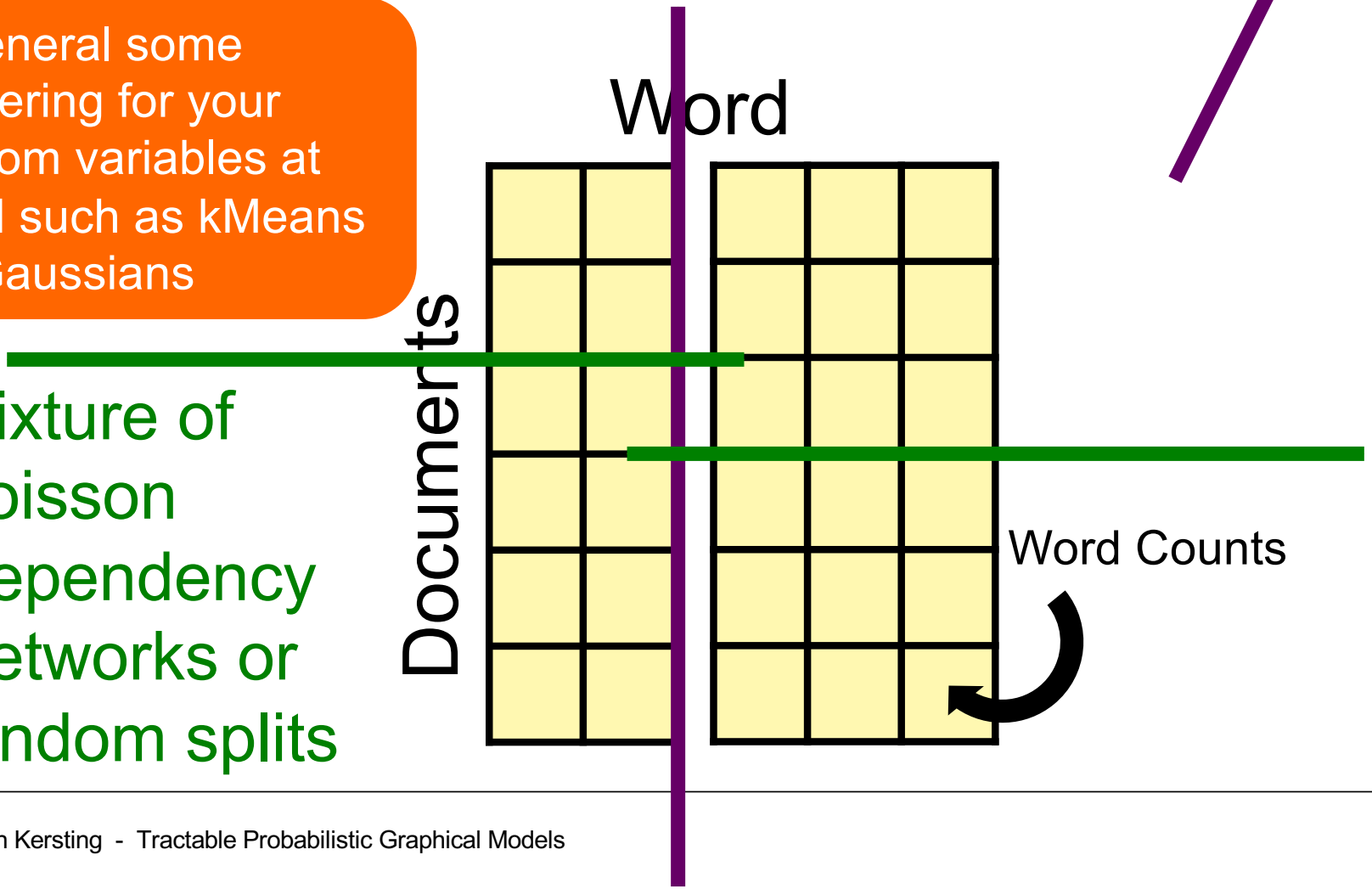
Testing independence of Poisson random variables



In general some clustering for your random variables at hand such as kMeans for Gaussians



Mixture of Poisson Dependency Networks or random splits

[Poon, Domingos UAI'11; Molina, Natarajan, Kersting AAAI`17]

Or we learn directly (Tree-)SPNs

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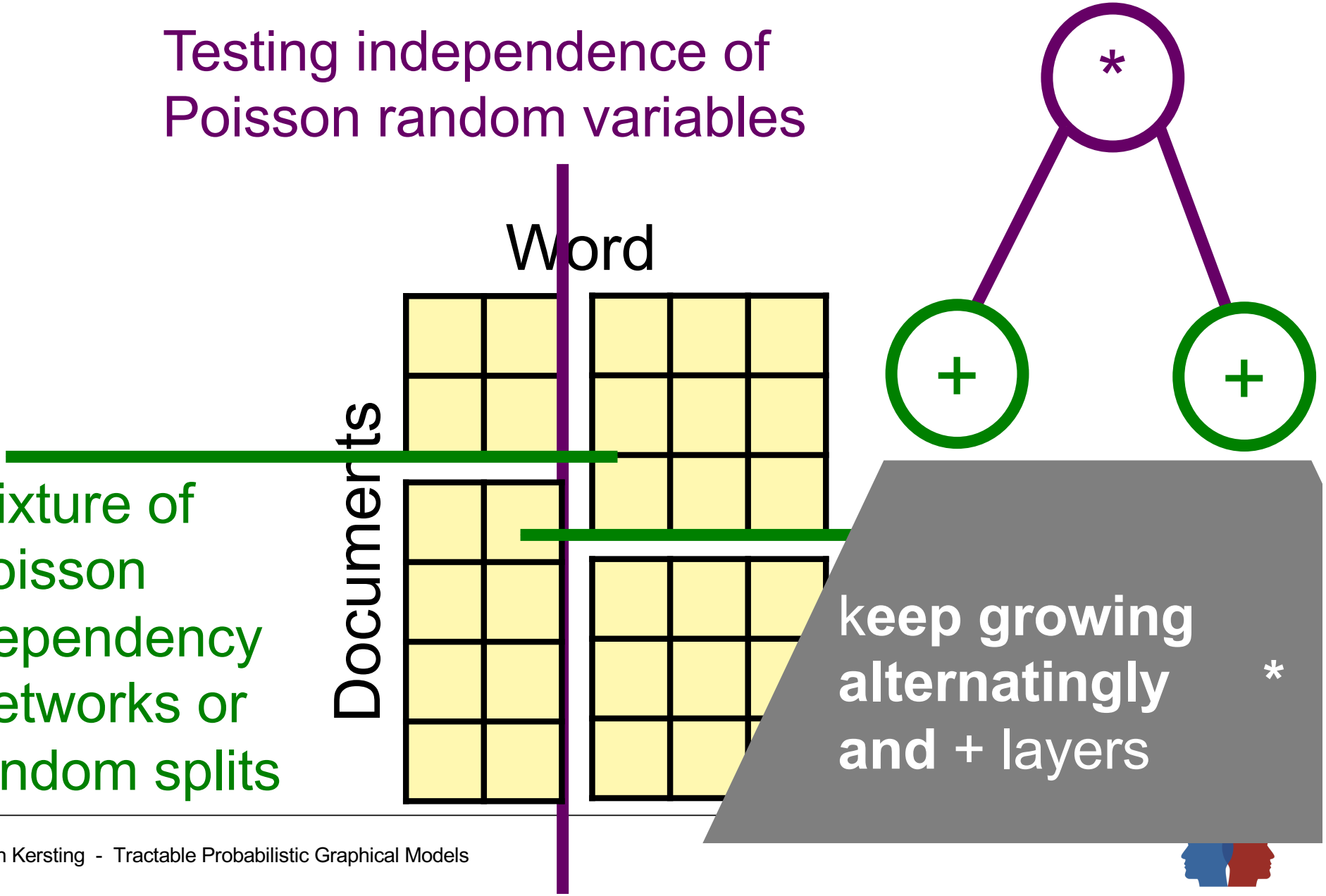
Testing independence of Poisson random variables



[Poon, Domingos UAI'11; Molina, Natarajan, Kersting AAAI'17; Vergari, Peharz, Di Mauro, Molina, Kersting, Esposito AAAI '18; Molina, Vergari, Di Mauro, Esposito, Natarajan, Kersting AAAI '18, Peharz et al. UAI 2019, Stelzner, Peharz, Kersting iCML 2019]

SPFlow: An Easy and Extensible Library for Sum-Product Networks

[Molina, Vergari, Stelzner, Peharz, Subramani, Poupart, Di Mauro, Kersting arXiv:1901.03704, 2019]

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CAML · MADESI · DFG · Federal Ministry of Education and Research

195 commits · 2 branches · 0 releases · 6 contrib...

Branch: master · New pull request · Create new file · Upload files · Find file · Clone or download

https://github.com/SPFlow/SPFlow

```
from spn.structure.leaves.parametric.Parametric import Categorical

from spn.structure.Base import Sum, Product

from spn.structure.base import assign_ids, rebuild_scopes_bottom_up


p0 = Product(children=[Categorical(p=[0.3, 0.7], scope=1), Categorical(p=[0.4, 0.6], scope=2)])
p1 = Product(children=[Categorical(p=[0.5, 0.5], scope=1), Categorical(p=[0.6, 0.4], scope=2)])
s1 = Sum(weights=[0.3, 0.7], children=[p0, p1])
p2 = Product(children=[Categorical(p=[0.2, 0.8], scope=0), s1])
p3 = Product(children=[Categorical(p=[0.2, 0.8], scope=0), Categorical(p=[0.3, 0.7], scope=1)])
p4 = Product(children=[p3, Categorical(p=[0.4, 0.6], scope=2)])
spn = Sum(weights=[0.4, 0.6], children=[p2, p4])

assign_ids(spn)
rebuild_scopes_bottom_up(spn)

return spn
```

Domain Specific Language, Inference, EM, and Model Selection as well as Compilation of SPNs into TF and PyTorch and also into flat, library-free code even suitable for running on devices: C/C++,GPU, FPGA

SPFlow, an open-source Python library providing a simple interface to inference, learning and manipulation routines for deep and tractable probabilistic models called Sum-Product Networks (SPNs). The library allows one to quickly create SPNs both from data and through a domain specific language (DSL). It efficiently implements several probabilistic inference routines like computing marginals, conditionals and (approximate) most probable explanations (MPEs) along with sampling

Inference on Devices

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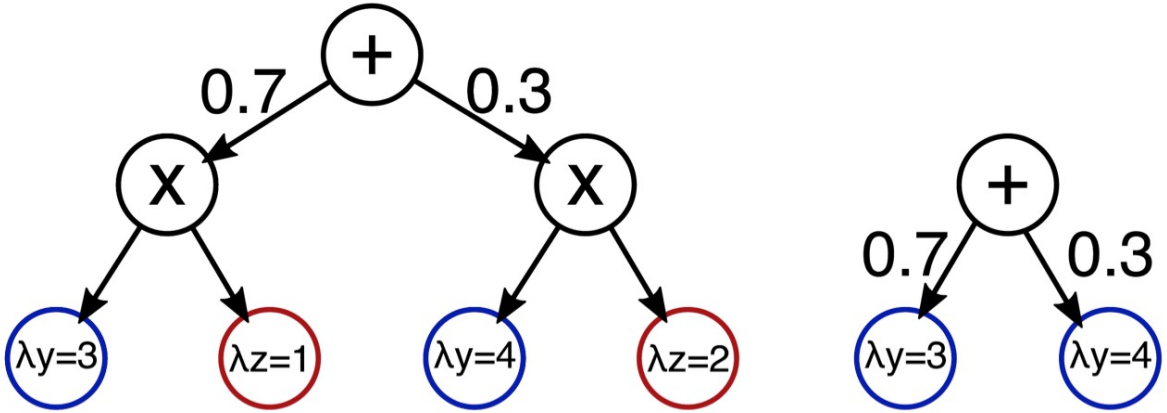
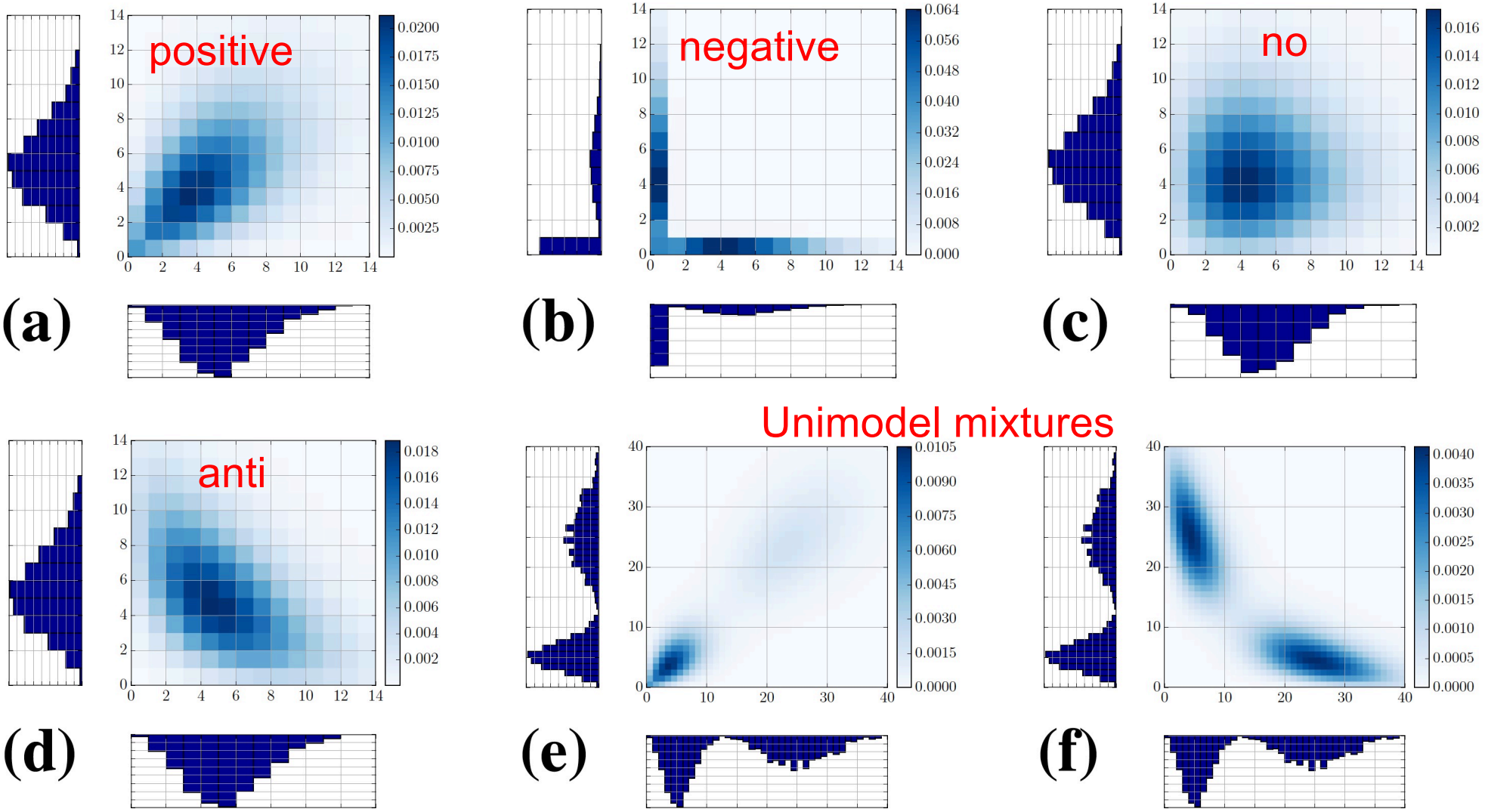


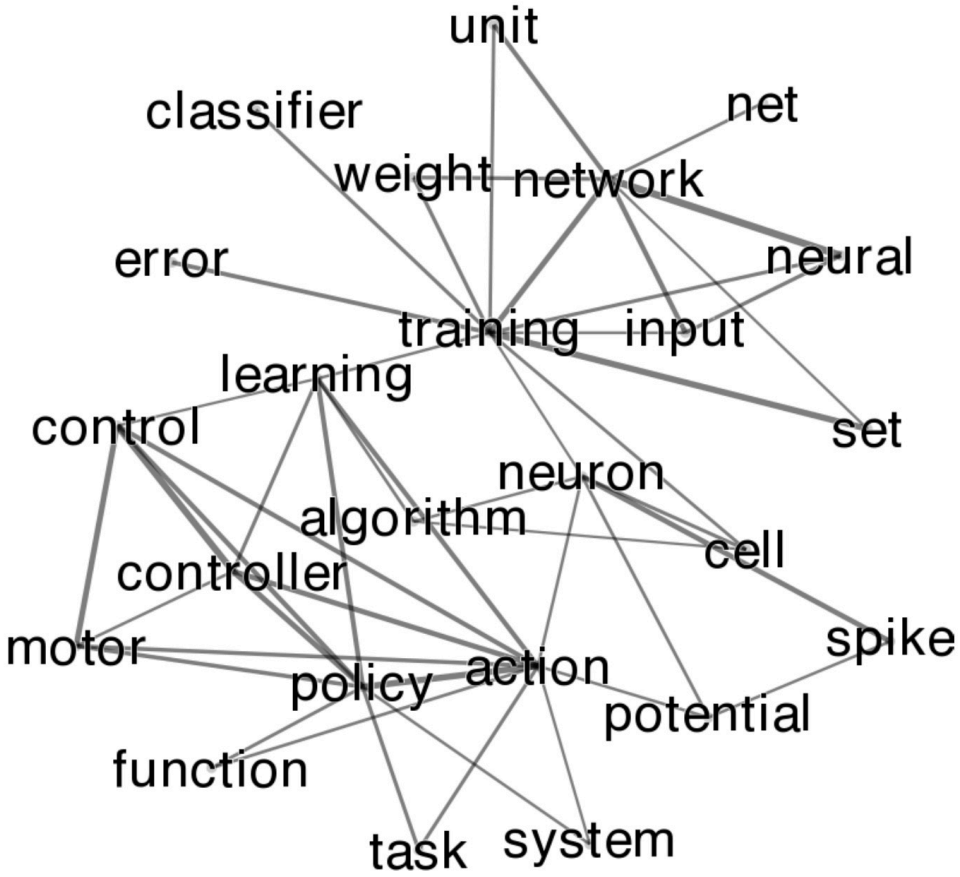
Ultimately, they may even be compiled into simple, flat, library-free code suitable for embedding in real-time applications and devices

SPNs encode polynomials over univariate distributions to build multivariate distributions. They can be fed into symbolic evaluation methods for inference

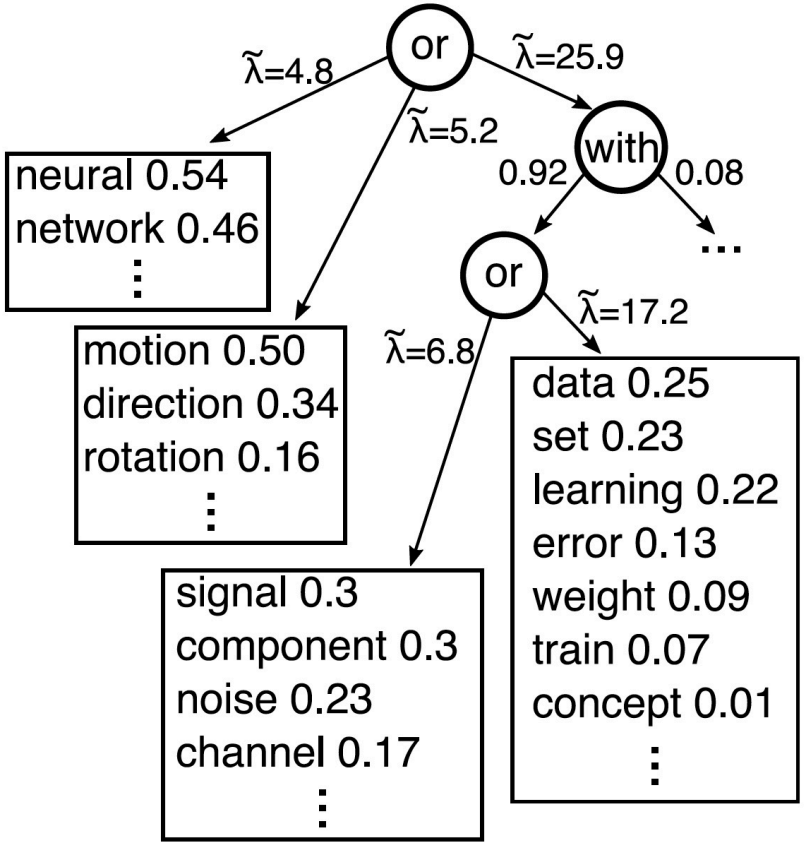
Flexible Dependencies even for challenging distributions, here multivariate Poisson distribution



Multiple views on you data due to efficient inference



Mutual Information
(NIPS corpus)



Poisson Multinomial SPN
= hierachical topic model

Poisson SPNs provide topics:
From Topic Models to Poisson Mean Fields

[Bishop, Fienberg, Holland. Discrete Multivariate Analysis: Theory and Practice. Springer, 2007]

Topic-word distributions of LDA can be viewed as a product of independent Poissons

$$\begin{aligned}
&\Pr_{\text{Poiss}}\left(\tilde{x}\,|\,\tilde{\lambda}\right)\Pr_{\text{Mult}}\left(\mathbf{x}\,|\,\theta=(\lambda_1,\cdots,\lambda_p)/\tilde{\lambda},\mathsf{N}=\tilde{x}\right)\\
&\quad=\frac{e^{-\tilde{\lambda}}}{\tilde{x}!}\tilde{\lambda}^{\tilde{x}}\frac{\tilde{x}!}{\prod_{s=1}^{p}x_s!}\prod_{s=1}^{p}\left(\frac{\lambda_s}{\tilde{\lambda}}\right)^{x_s}\\
&\quad=\frac{\tilde{x}!}{\tilde{x}!}\frac{e^{-\tilde{\lambda}}}{\prod_{s=1}^{p}x_s!}\prod_{s=1}^{p}\left(\frac{\tilde{\lambda}\lambda_s}{\tilde{\lambda}}\right)^{x_s}\\
&\quad=\Pr_{\text{Ind. Poiss}}(\mathbf{x}\,|\,\lambda_1,\cdots,\lambda_p)=\prod_{s=1}^{p}\frac{e^{-\lambda_s}}{x_s!}\lambda_s^{x_s}
\end{aligned}$$

Competitive Predictive Performance

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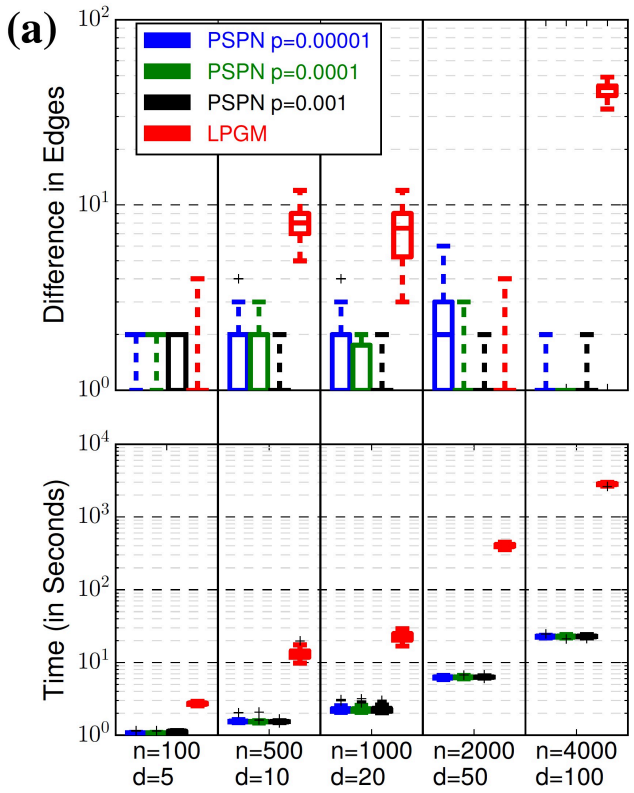
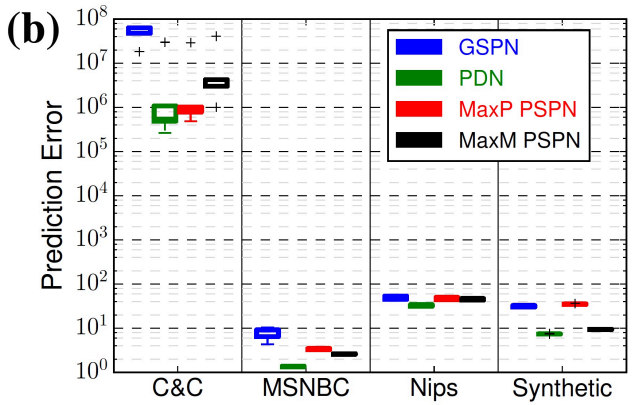
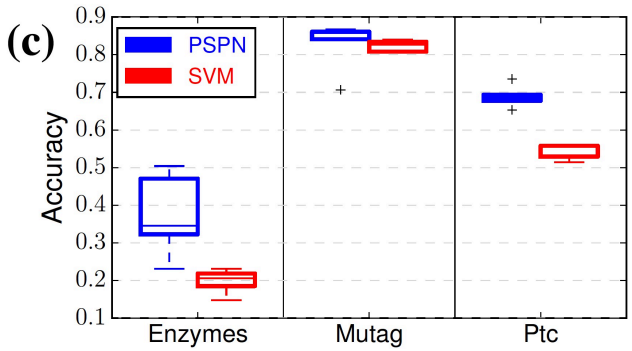
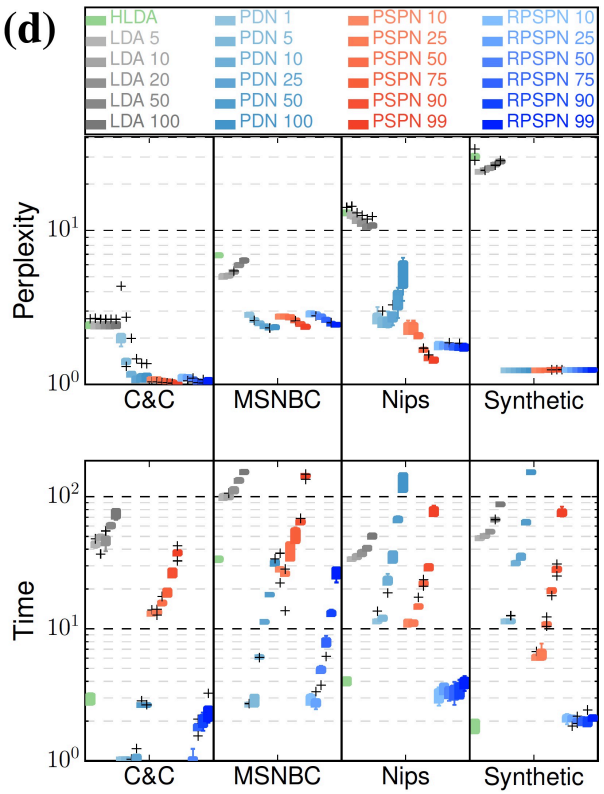
Prediction error

LDA







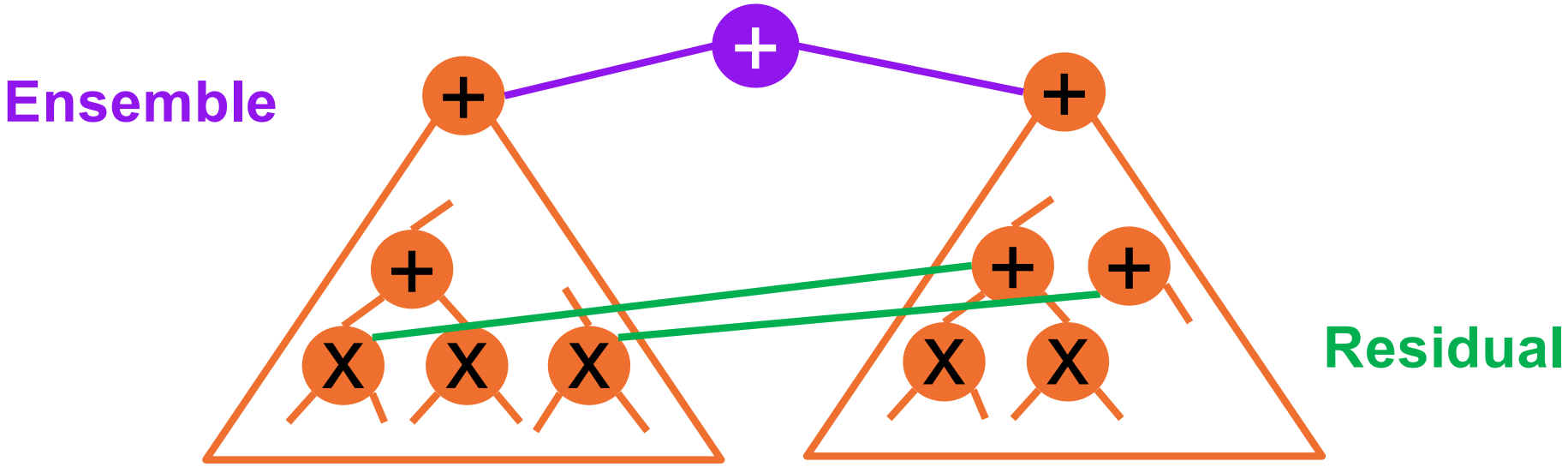


Network discovery

Deep Graph Kernels (Weisfeiler Lehman)

[Ventola, Stelzner, Molina, Kersting PGM 2020]

Residual Sum-Product Networks



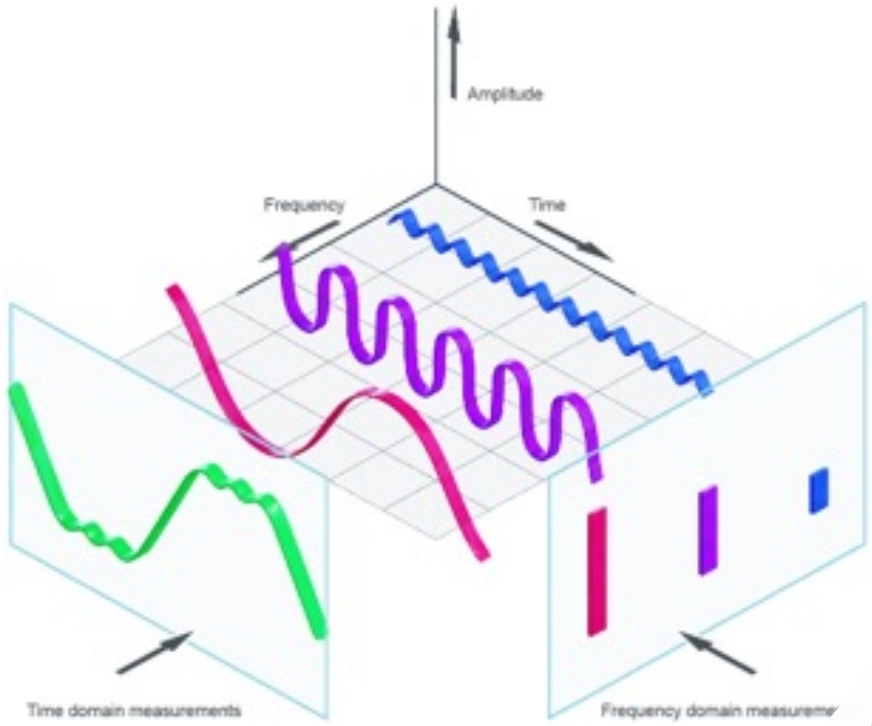
ResSPNs = Ensemble SPNs + Residual Links

	Test Accuracy			
	Baseline Singleton SPN Learners			Ensemble SPN
	Best ExtraSPN	LearnSPN	ID-SPN	ResSPN
NLTCS	-6.153	-6.11	**-6.02**	-6.040⇑
MSNBC	-6.433	-6.11	**-6.04**	-6.097⇑
Plants	-16.683	-12.98	**-12.54**	-13.908●
Audio	-42.639	-40.5	**-39.79**	-40.762●
Jester	-55.335	-53.48	**-52.86**	-53.956●
Netflix	-60.330	-57.33	**-56.36**	-57.867●

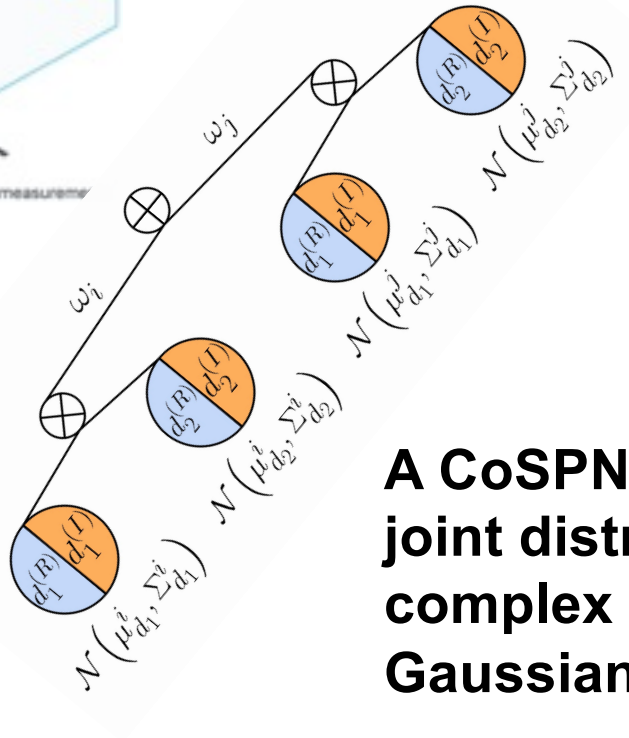
Whittle Networks

ICML 2021
38th International Conference on Machine Learning

[Yu, Ventola, Kersting ICML 2021]



Encode the Whittle likelihood for times series using a complex-valued SPN (CoSPN)



Whittle Likelihood:
The Fourier coefficients of stationary times series are independent complex-valued Gaussians with zero mean

A CoSPN models the joint distribution over complex (normal) Gaussians

Whittle Networks

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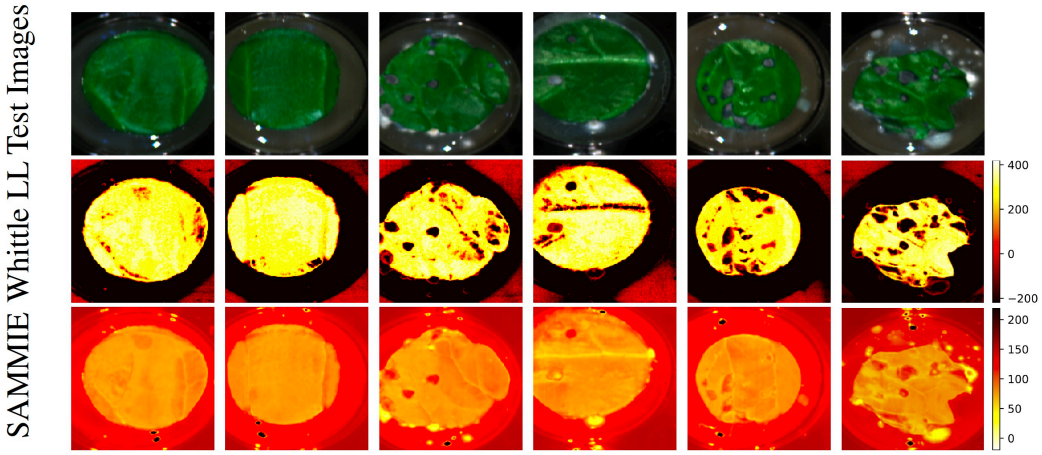
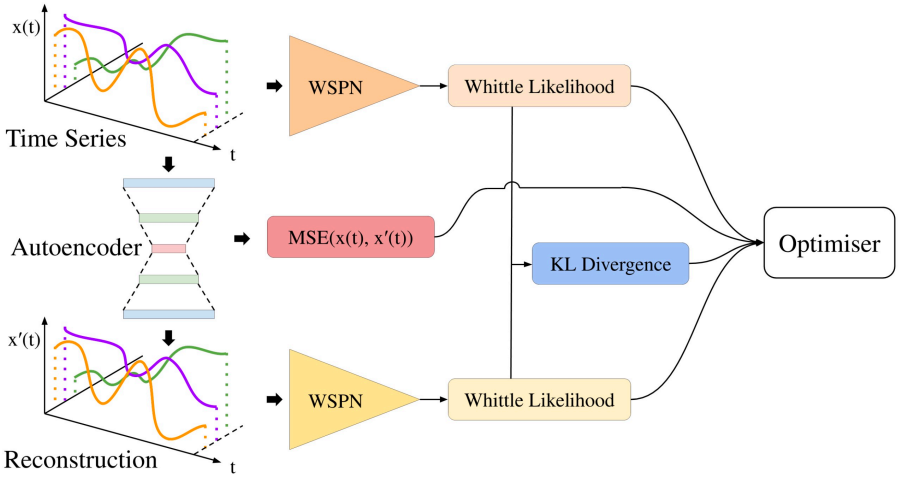
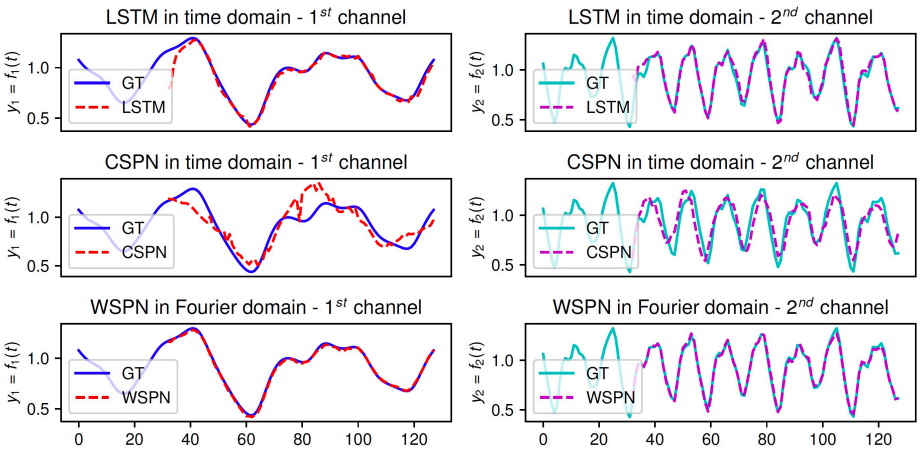
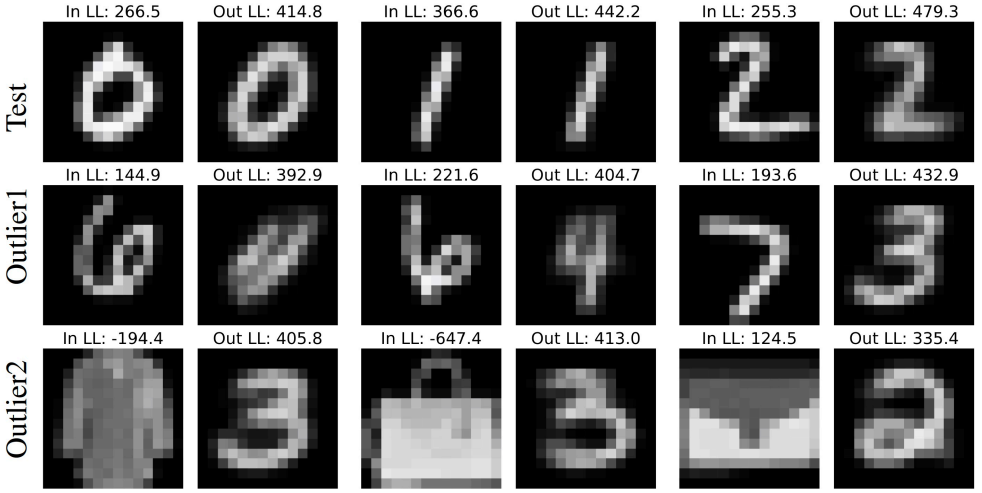
[Yu, Ventola, Kersting ICML 2021]









Random sum-product networks

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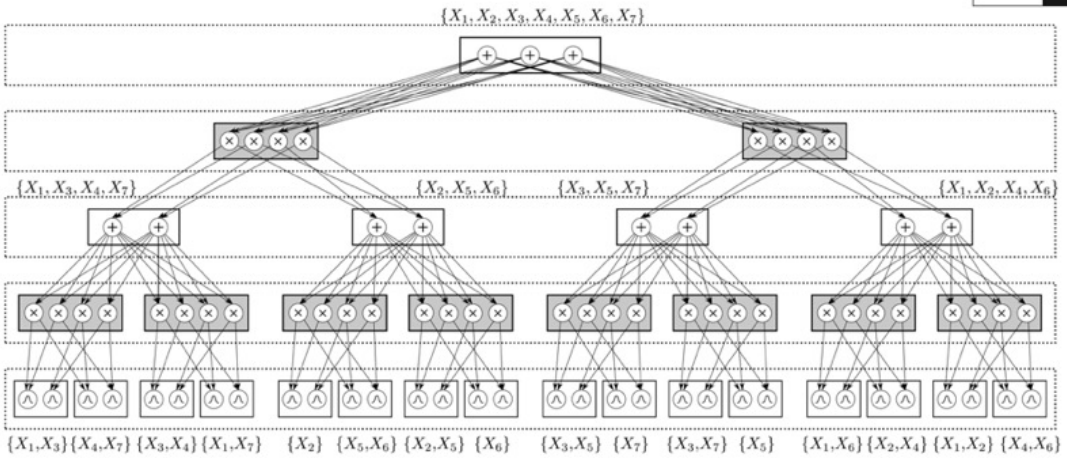
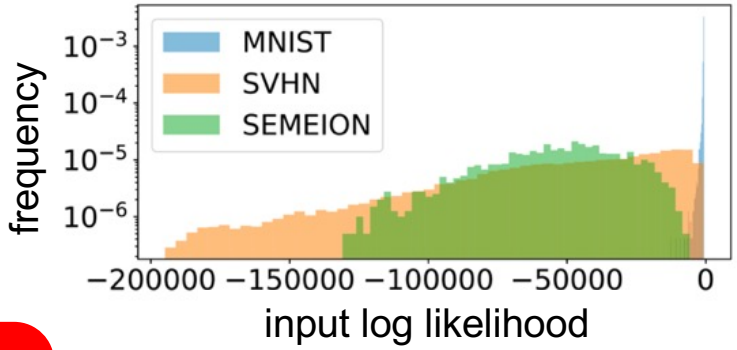
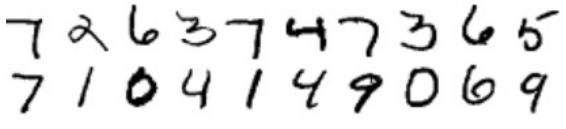
Conference on Uncertainty in Artificial Intelligence
Tel Aviv, Israel
July 22 - 25, 2019
uai2019



Similar to Random Forests, build a random SPN structure. This can be done in an informed way or completely at random

		RAT-SPN	MLP	vMLP
Accuracy	MNIST	98.19 (8.5M)	98.32 (2.64M)	98.09 (5.28M)
	F-MNIST	89.52 (0.65M)	90.81 (9.28M)	89.81 (1.07M)
	20-NG	47.8 (0.37M)	49.05 (0.31M)	48.81 (0.16M)
Cross-Entropy	MNIST	0.0852 (17M)	0.0874 (0.82M)	0.0974 (0.22M)
	F-MNIST	0.3525 (0.65M)	0.2965 (0.82M)	0.325 (0.29M)
	20-NG	1.6954 (1.63M)	1.6180 (0.22M)	1.6263 (0.22M)





outliers

prototypes

outliers

prototypes

SPNs can have similar predictive performances as (simple) DNNs

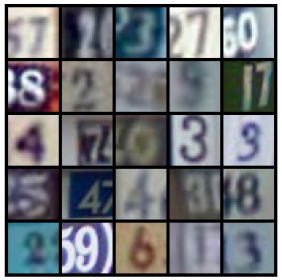
SPNs can distinguish the datasets

SPNs know when they do not know by design

Einsum Networks

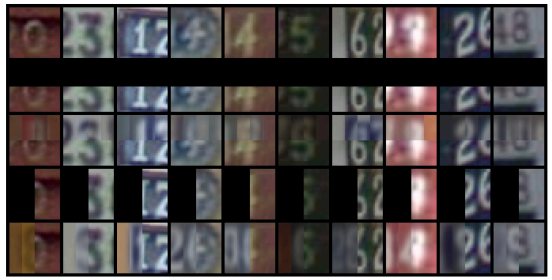
[Peharz, Lang, Vergari, Stelzner, Molina, Trapp, Van den Broeck, Kersting, Ghahramani ICML 2020]

ICML | 2020
Thirty-seventh International Conference on Machine Learning

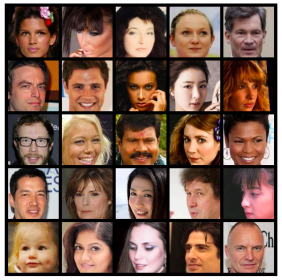


(a) Real SVHN images.

(b) EiNet SVHN samples.

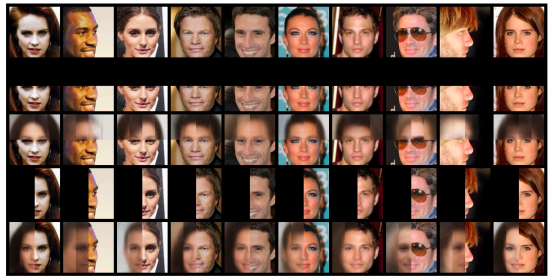


(c) Real images (top), covered images, and EiNet reconstructions



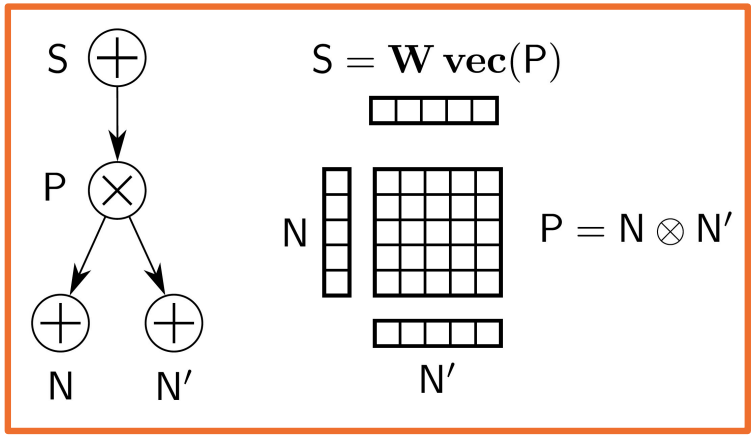
(d) Real Celeba samples.

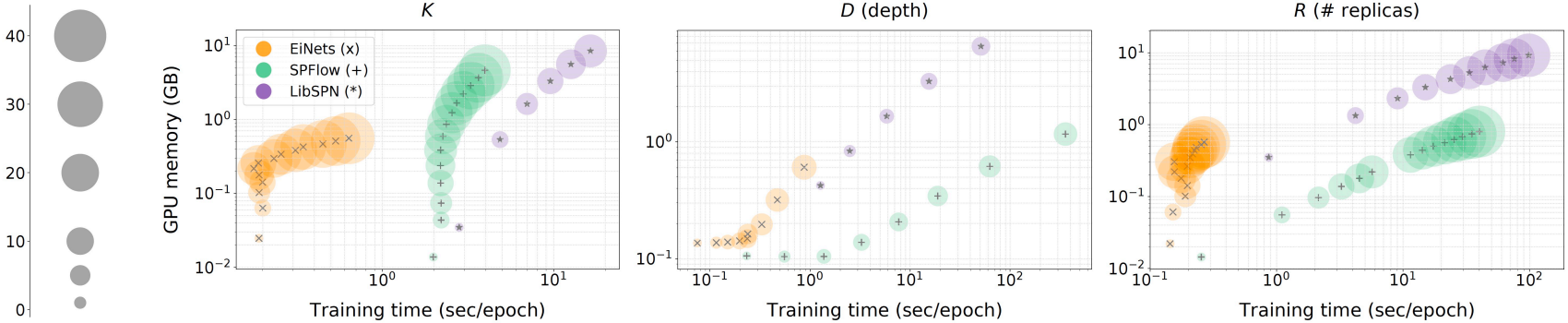
(e) EiNet Celeba samples.



(f) Real images (top), covered images, and EiNet reconstructions

Compilation of PCs into efficient to evaluate computational graphs





Unsupervised scene understanding

[Stelzner, Peharz, Kersting ICML 2019, Best Paper Award at TPM@ICML2019] https://github.com/stelzner/supair

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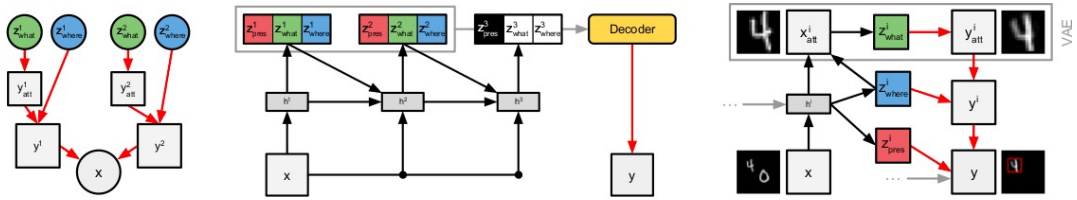
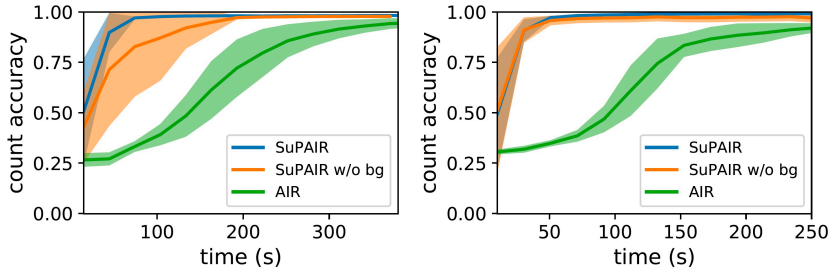
ICML | 2019
Thirty-sixth International Conference on Machine Learning

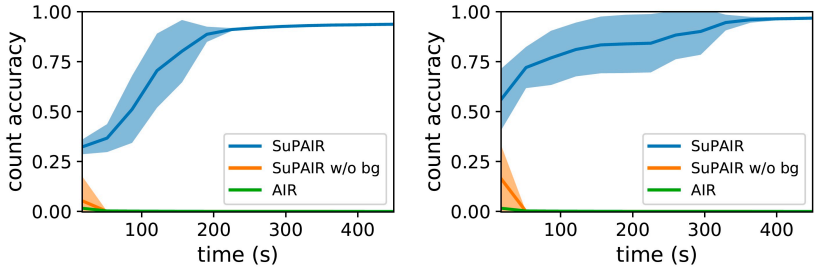
Consider e.g. unsupervised scene understanding using a generative model implemented in a neural fashion



[Attend-Infer-Repeat (AIR) model, Hinton et al. NIPS 2016]



(a) MNIST (b) Sprites

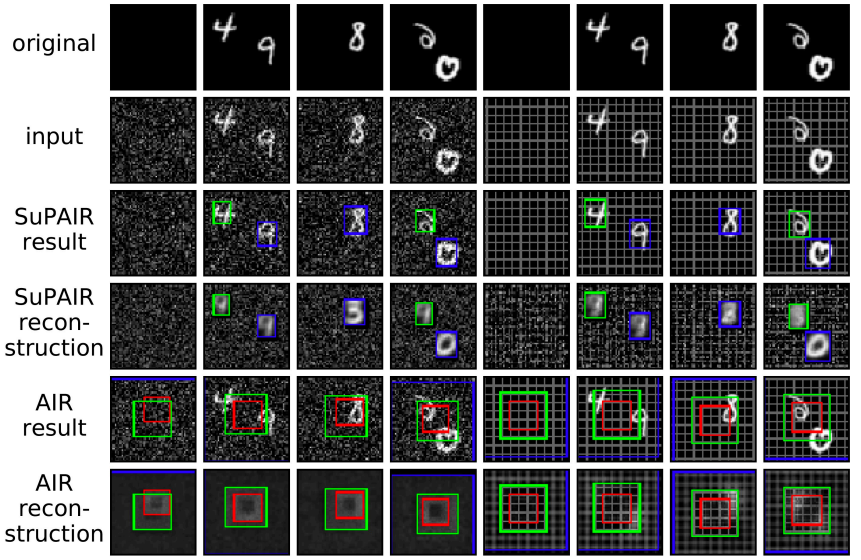


(c) Noisy MNIST (d) Grid MNIST

Replace VAE by SPN as object model

- infinite mixture model
- intractable density
- intractable posterior

- "large" but finite mixture model
- tractable density
- tractable marginals [Peharz et al., 2015]
- tractable posterior [Vergari et al., 2017]



Unsupervised physics learning

[Kossen, Stelzner, Hussing, Voelcker, Kersting ICLR 2020]

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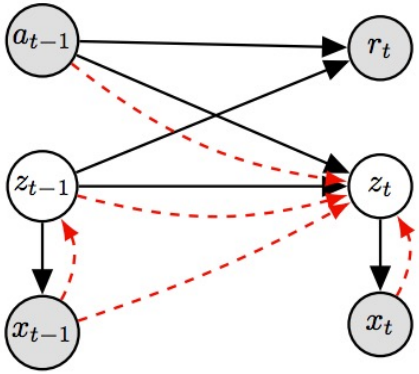
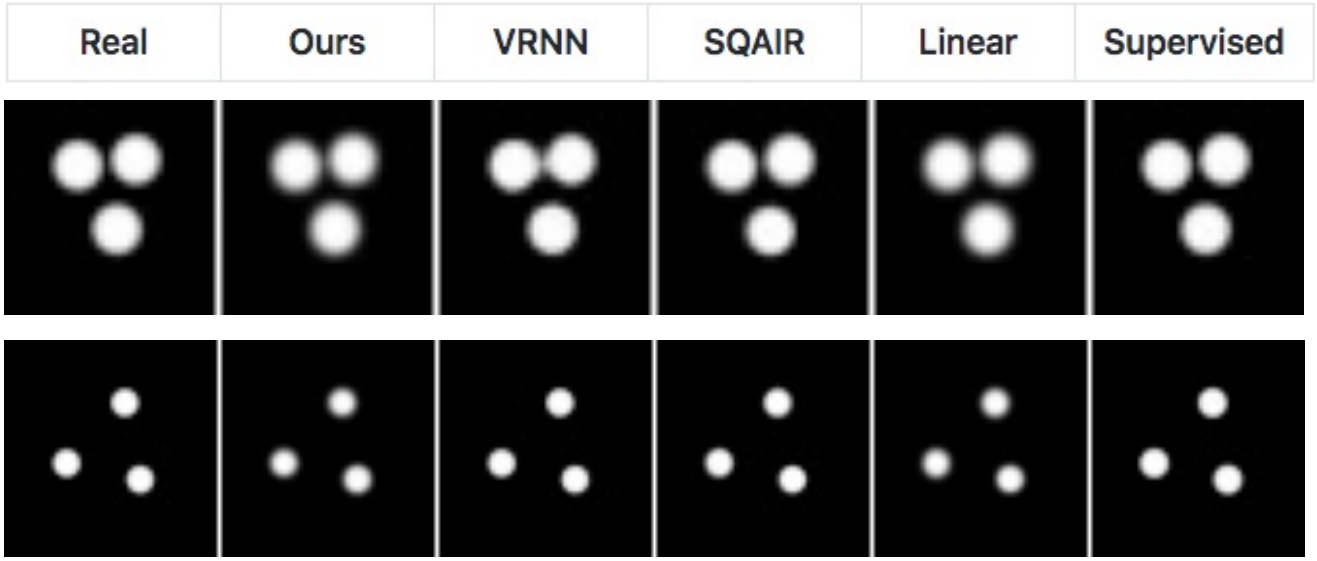
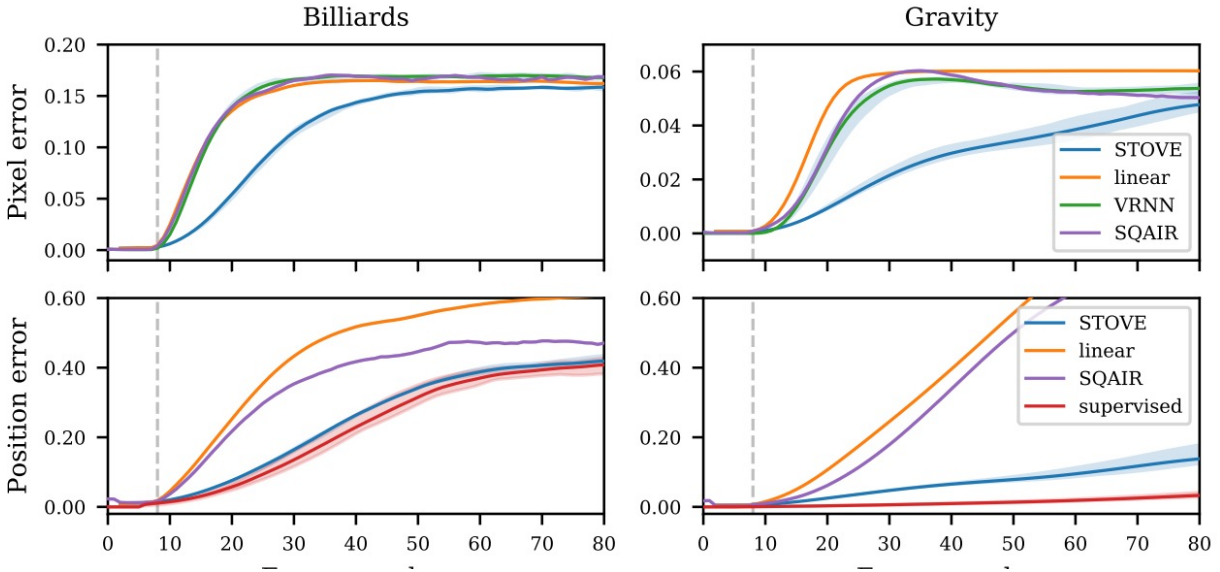




putting structure and tractable inference into deep models



Semi-ring structure allows one to go beyond the standard settings

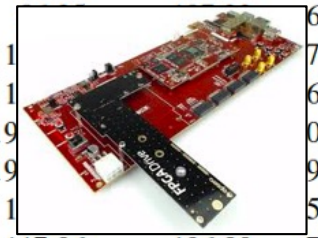
Edge AI, Crypto, Rankings, AutoML

[Sommer, Oppermann, Molina, Binnig, Kersting, Koch ICDD 2018, Weber, Sommer, Oppermann, Molina, Kersting, Koch FPT 2019]

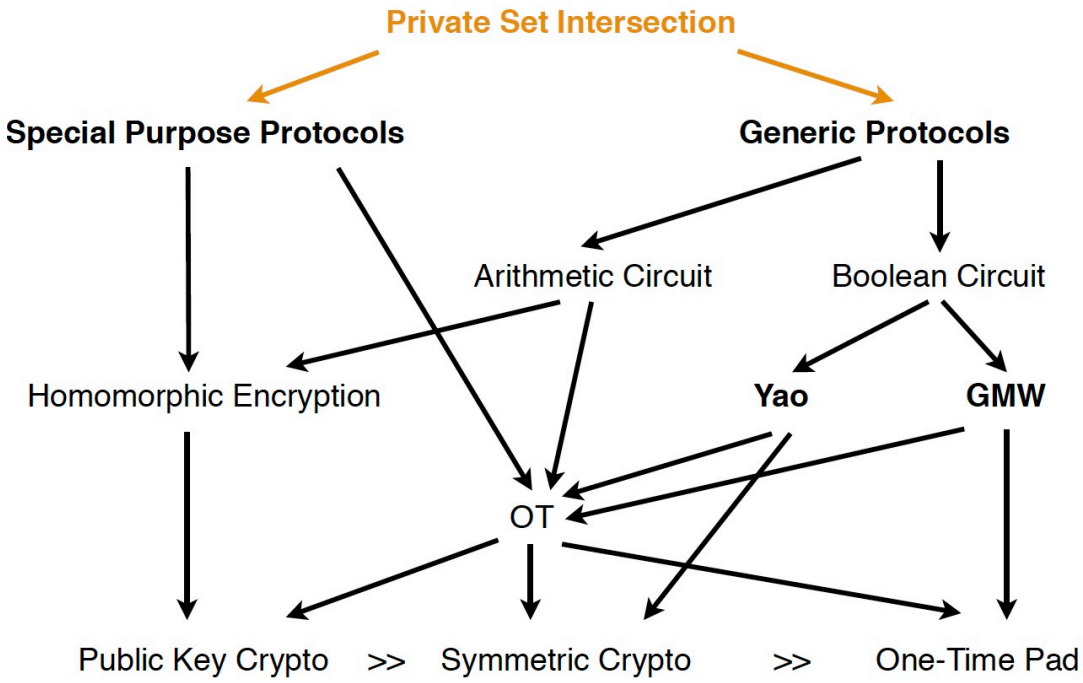
IEEE International Conference on Computer Design

IEEE International Conference on Field-Programmable Technology (FPT)

Dataset	Rows	CPU (μs)	T-CPU (rows/μs)	CPUF (μs)	T-CPUF (rows/μs)	GPU (μs)	T-GPU (rows/μs)	FPGA Cycle Counter	FPGAC (μs)	T-FPGAC (rows/μs)	FPGA (μs)	T-FPGA (rows/μs)
Accidents	17009	2798.27			7.87	63090.94	0.27	17249			696.00	**24.44**
Audio	20000	4271.78			5.4			20317	1		761.00	**26.28**
Netflix	20000	4892.22			4.8			20322	1		654.00	**30.58**
MSNBC200	388434	15476.05			30.5			388900	19		008.00	**77.56**
MSNBC300	388434	10060.78			41.2			388810	19		933.00	**78.74**
NLTCS	21574	791.80			31.3			21904	1		566.00	**38.12**
Plants	23215	3621.71	6.41	3521.04	6.59	67004.41	0.35	23592	117.96	196.80	778.00	**29.84**
NIPS5	10000	25.11	**398.31**	26.37	379.23	8210.32	1.22	10236	51.18	195.39	337.30	29.65
NIPS10	10000	83.60	**119.61**	84.39	118.49	11550.82	0.87	10279	51.40	194.57	464.30	21.54
NIPS20	10000	191.30	52.27	182.73	**54.72**	18689.04	0.54	10285	51.43	194.46	543.60	18.40
NIPS30	10000	387.61	25.80	349.84	**28.58**	25355.93	0.39	10308	51.80	193.06	592.30	16.88
NIPS40	10000	551.64	18.13	471.26	**21.22**	30820.49	0.32	10306	51.53	194.06	632.20	15.82
NIPS50	10000	812.44	12.31	792.13	12.62	36355.60	0.28	10559	52.80	189.41	720.60	**13.88**
NIPS60	10000	1046.38	9.56	662.53	**15.09**	40778.36	0.25	12271	61.36	162.99	799.20	12.51
NIPS70	10000	1148.17	8.71	1134.80	8.81	46759.26	0.21	14022	70.11	142.63	858.60	**11.65**
NIPS80	10000	1556.99	6.42	1277.81	7.83	63217.99	0.16	14275	78.51	127.37	961.80	**10.40**



How do we do deep learning offshore?



There are generic protocols to validate computations on authenticated data without knowledge of the secret key

DNA MSPN
Gates: 298208 Yao Bytes: 9542656 Depth: 615

DNA PSPN
Gates: 228272 Yao Bytes: 7304704 Depth: 589

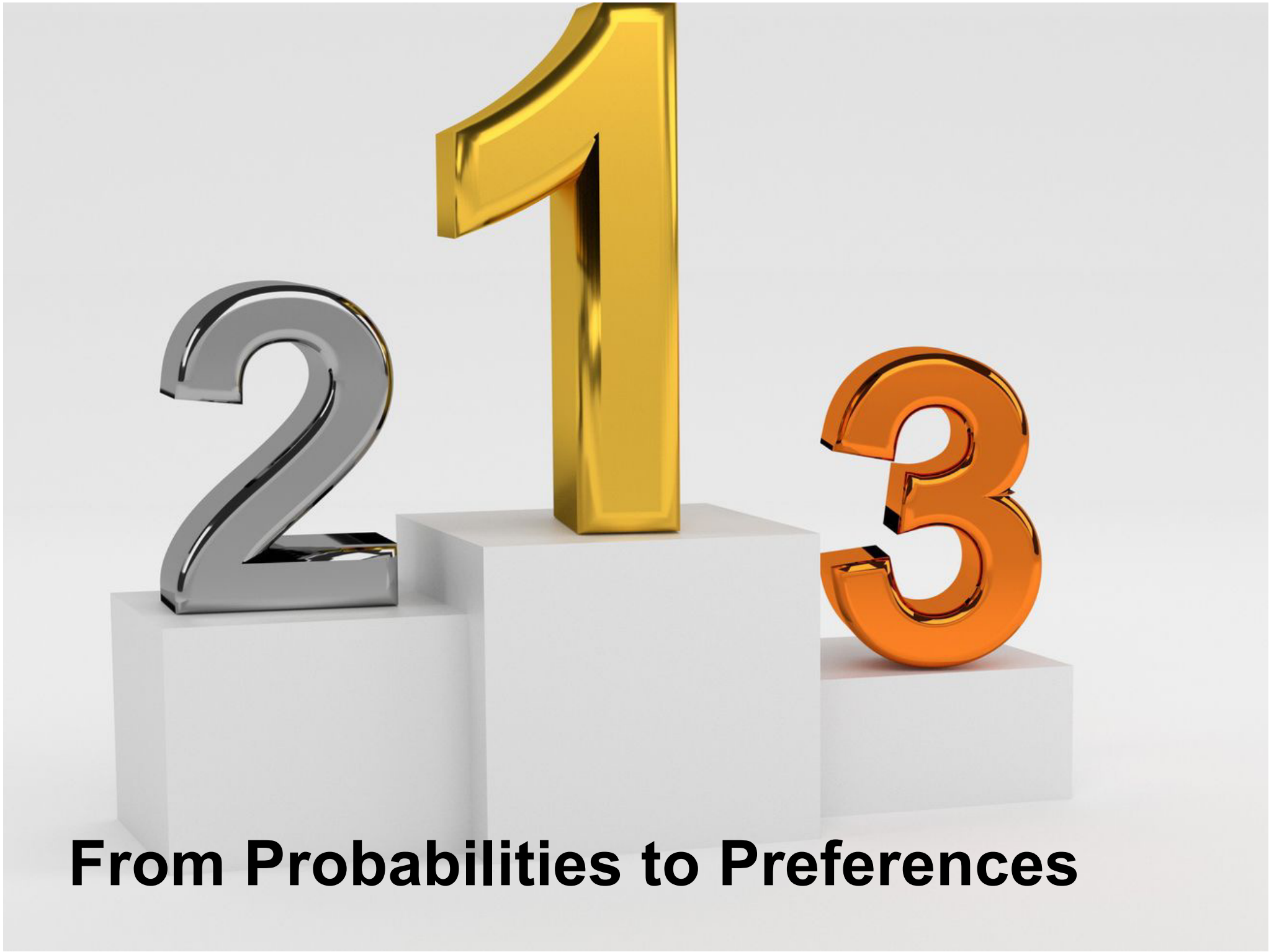
NIPS MSPN
Gates: 1001477 Yao Bytes: 32047264 Depth: 970

Secure multi-party computation for SPNs?

Crypto SPNs

[Molina, Weinert, Treiber, Schneider, Kersting ECAI 2020]

ECAI 2020 Santiago de Compostela



From Probabilities to Preferences

Rain	R	T	T	T	T	T	T	T	T	F	F	F	F	F	F	F	F
Neighbour's Grad	N	T	T	T	T	F	F	F	F	T	T	T	T	F	F	F	F
Grass	H	T	T	F	F	T	T	F	F	T	T	F	F	T	T	F	F
Sprinkler	S	T	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F
	Rank	4	1	∞	∞	∞	∞	∞	∞	5	∞	8	2	3	∞	6	0

Let us abstract probabilities into rankings

- **Not surprising events = rank 0**
- **Surprising = rank 1**
- **Very surprising = rank 2**

…

From Probabilities to Preferences

[Shao, Skryagin, Yu, Rienstra, Thimm SUM 2020]

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The 14th International Conference on Scalable Uncertainty Management

September 23rd-25th 2020 - Bozen-Bolzano, Italy

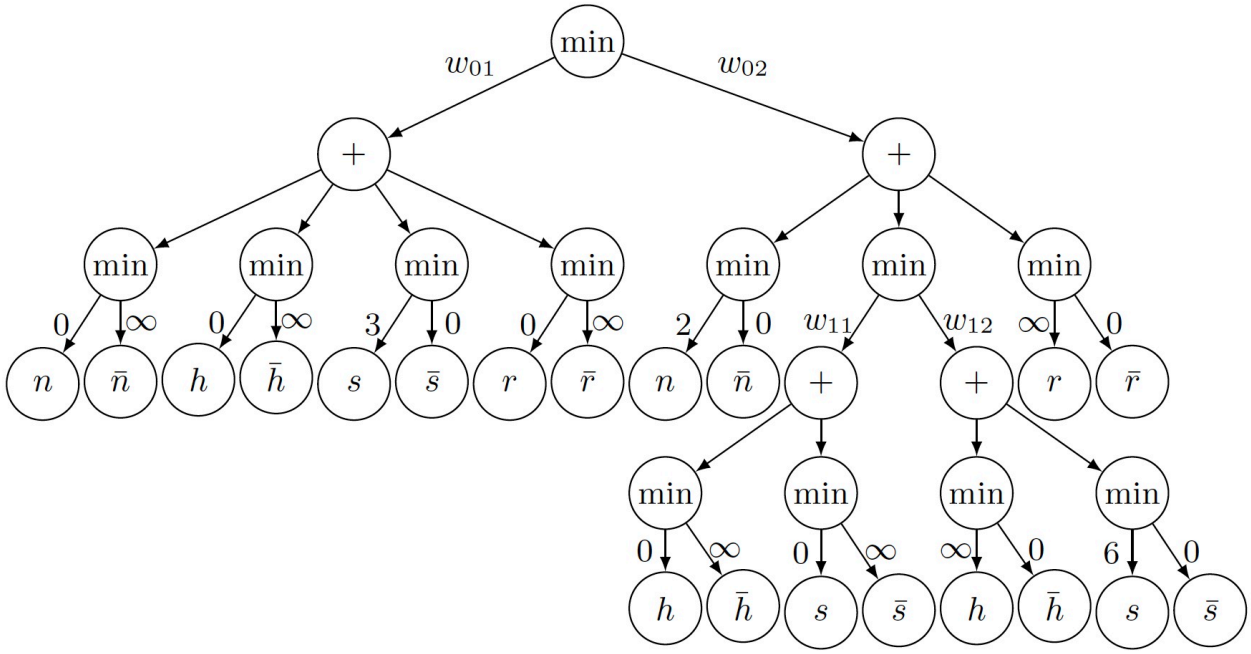
Min-Sum Networks

Rain	R	T	T	T	T	T	T	T	T	F	F	F	F	F	F	F	F
Neighbour's Grad	N	T	T	T	T	F	F	F	F	T	T	T	T	F	F	F	F
Grass	H	T	T	F	F	T	T	F	F	T	T	F	F	T	T	F	F
Sprinkler	S	T	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F
	Rank	4	1	∞	∞	∞	∞	∞	∞	5	∞	8	2	3	∞	6	0

Let us abstract probabilities

- **Not surprising events = r**
- **Surprising = rank 1**
- **Very surprising = rank 2**

…



From Probabilities to Preferences

[Shao, Skryagin, Yu, Rienstra, Thimm SUM 2020]

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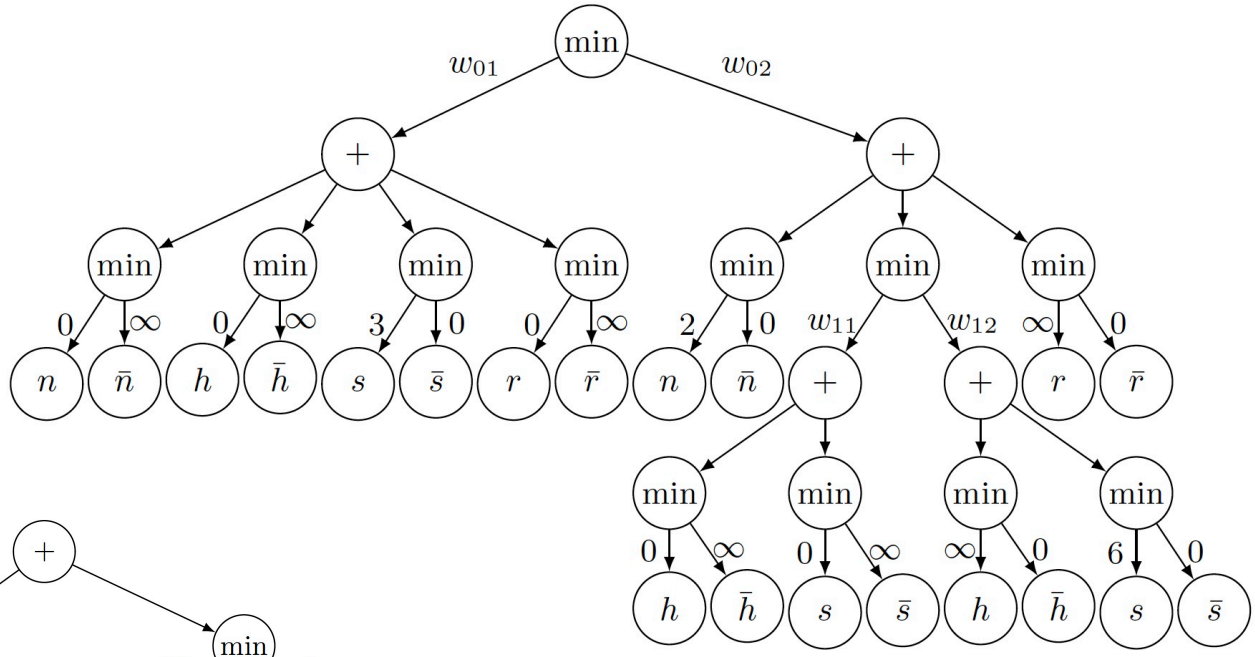
Min-Sum Networks

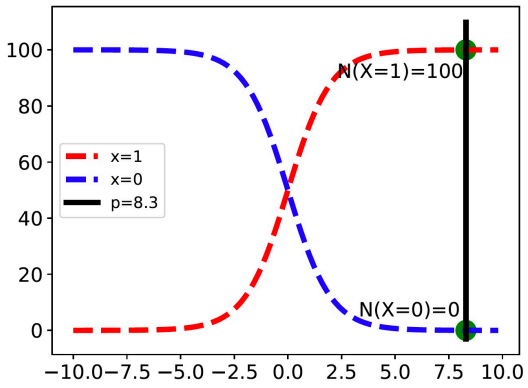
Rain	R	T	T	T	T	T	T	T	T	F	F	F	F	F	F	F	F
Neighbour's Grad	N	T	T	T	T	F	F	F	F	T	T	T	T	F	F	F	F
Grass	H	T	T	F	F	T	T	F	F	T	T	F	F	T	T	F	F
Sprinkler	S	T	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F
	Rank	4	1	∞	∞	∞	∞	∞	∞	5	∞	8	2	3	∞	6	0

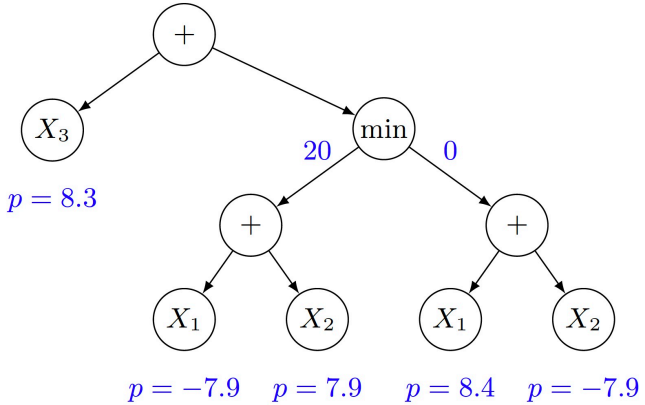
Let us abstract probabilities

- **Not surprising events = r**
- **Surprising = rank 1**
- **Very surprising = rank 2**

…



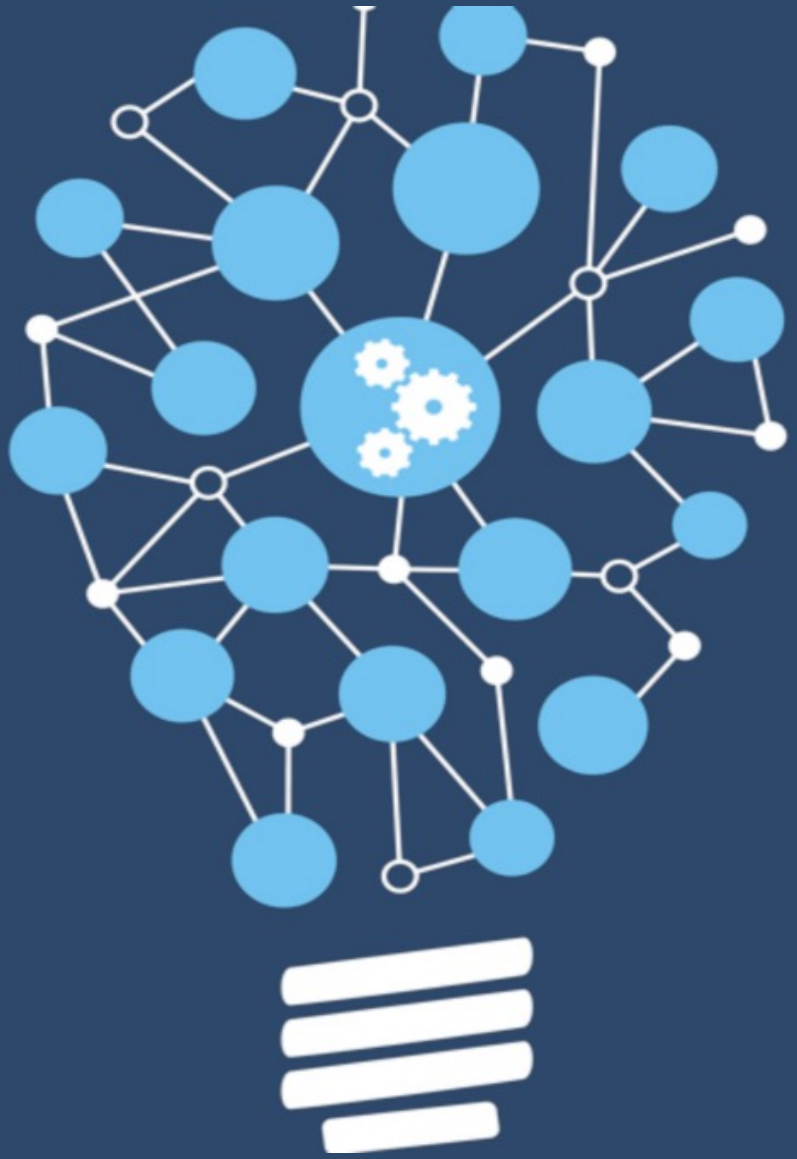




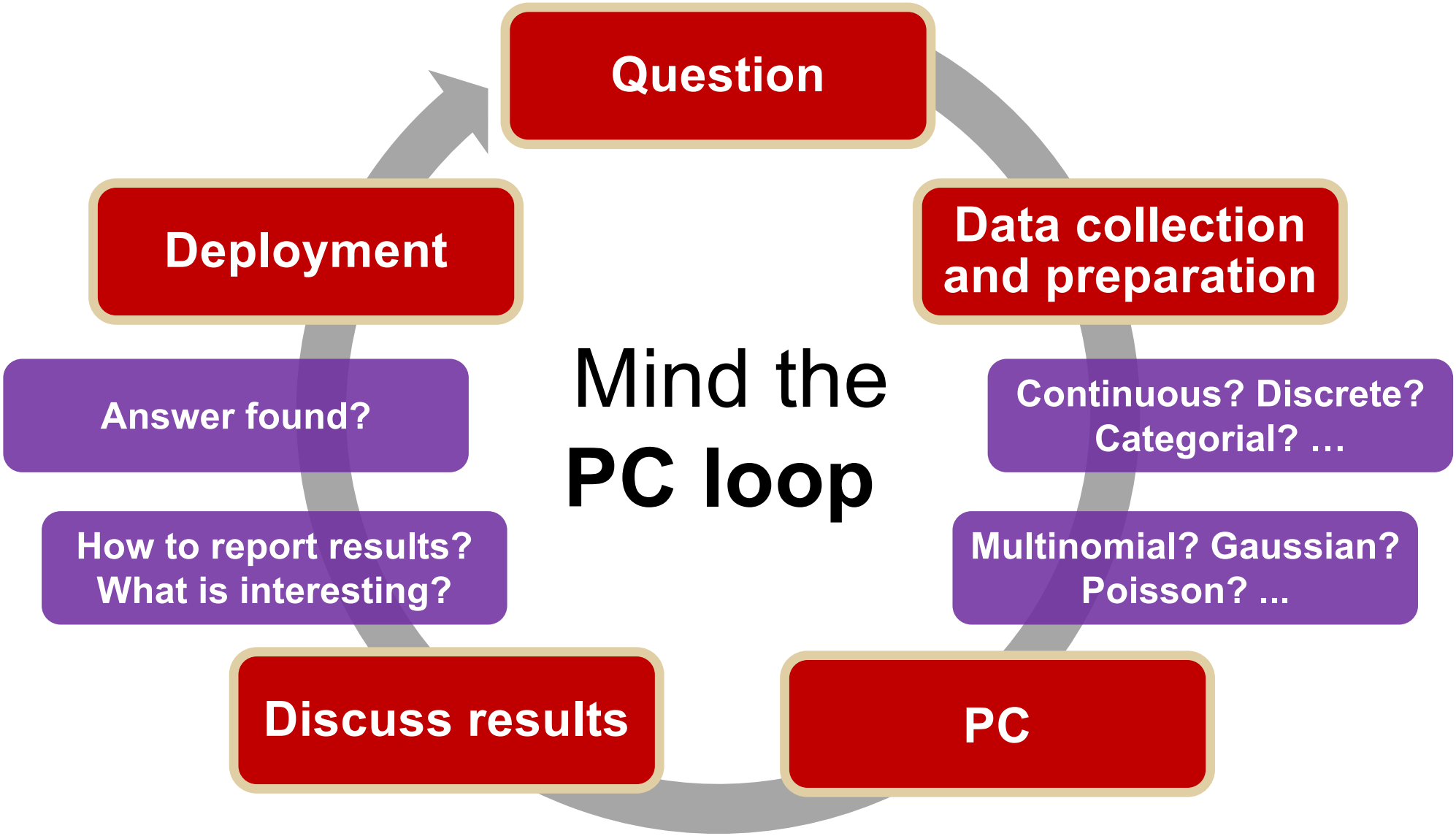
$$\hat{\theta} \triangleq \arg\min_{\theta} N(D|\theta)$$

$$N(D|\theta) = \sum_{i \in m} N(\mathbf{x}_i|\theta) \text{ with}$$

$$(2 * (\text{sigmoid}(p) - 0.5) * (X - 0.5) + 0.5)$$



Automated Density Estimation


Question
Data collection and preparation
Continuous? Discrete? Categorial? …
Multinomial? Gaussian? Poisson? ...
PC
Discuss results
How to report results? What is interesting?
Answer found?
Deployment
Mind the
PC loop

[Molina, Natarajan, Vergari, Di Mauro, Esposito, Kersting AAAI 2018]

Distribution-agnostic Deep Probabilistic Learning

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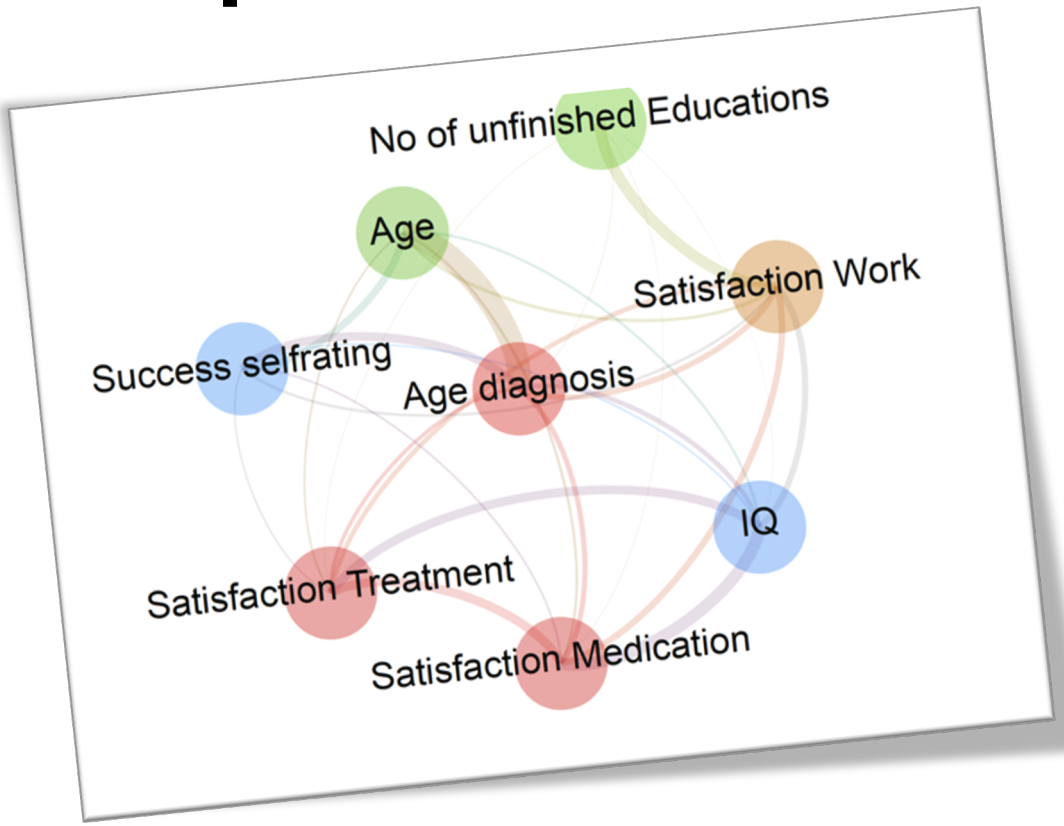
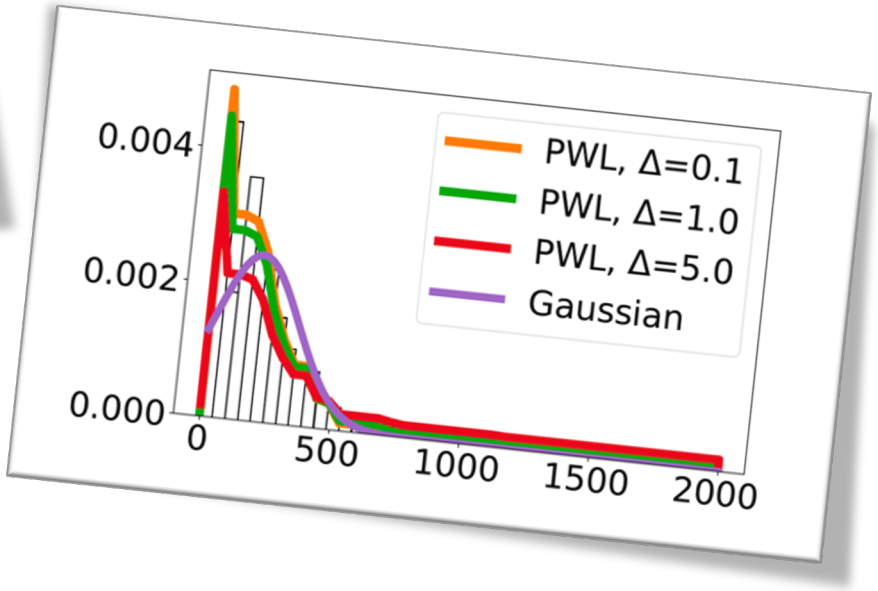
INDIANA UNIVERSITY



Use nonparametric independency tests and piece-wise linear approximations



[Molina, Natarajan, Vergari, Di Mauro, Esposito, Kersting AAAI 2018]

Distribution-agnostic Deep Probabilistic Learning

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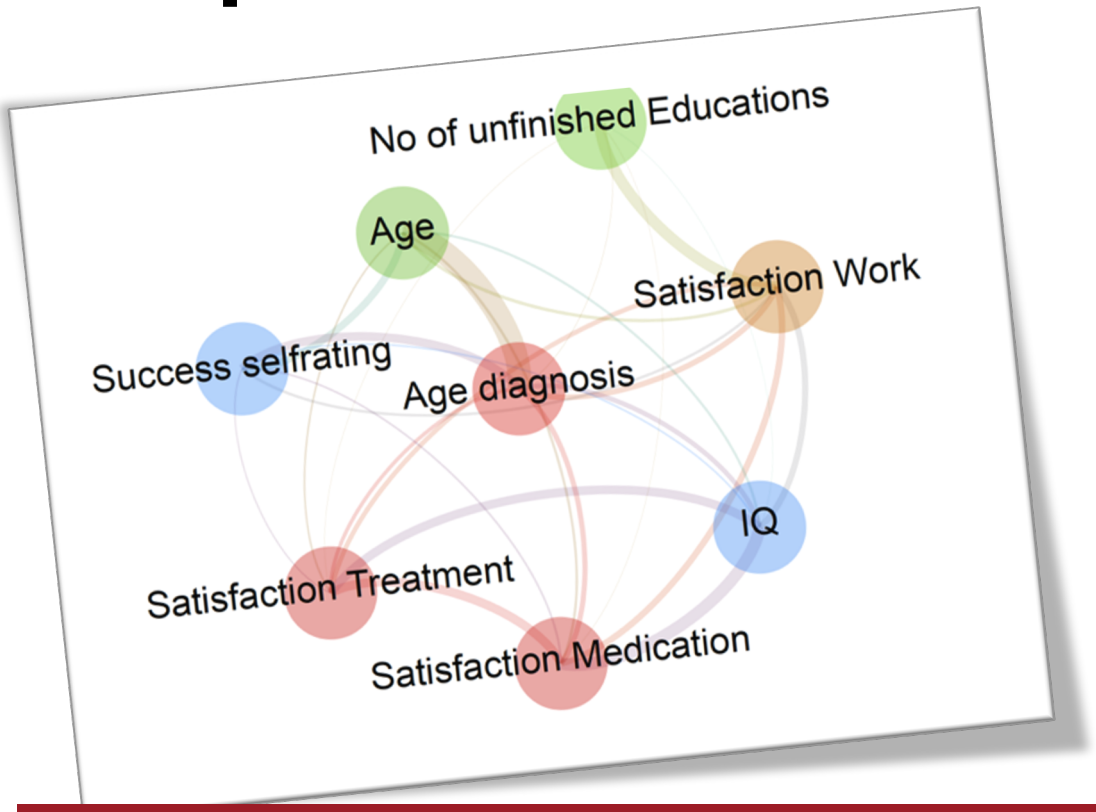

Use nonparametric independency tests and piece-wise linear approximations

However, we have to provide the statistical types and do not gain insights into the parametric forms of the variables. **Are they Gaussians? Gammas? …**

[Vergari, Molina, Peharz, Ghahramani, Kersting, Valera AAAI 2019]

The Explorative Automatic Statistician

UBER AI Labs

UNIVERSITY OF CAMBRIDGE

Max Planck Institute for Intelligent Systems

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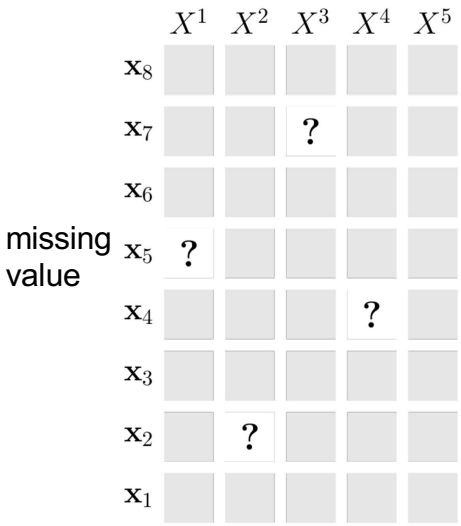
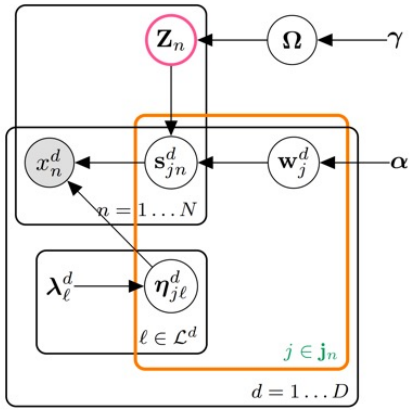
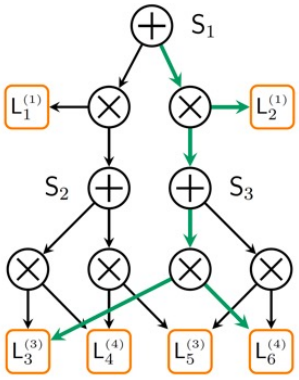


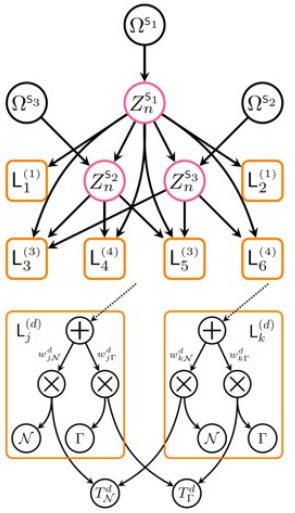
We can even automatically discovers the statistical types and parametric forms of the variables



Bayesian Type Discovery



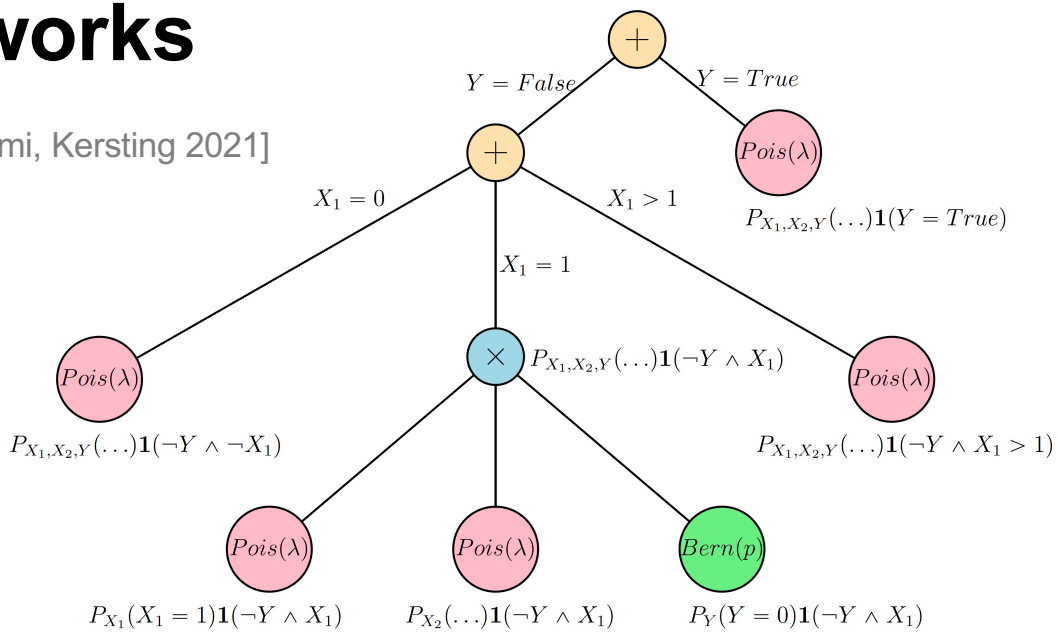
Mixed Sum-Product Network



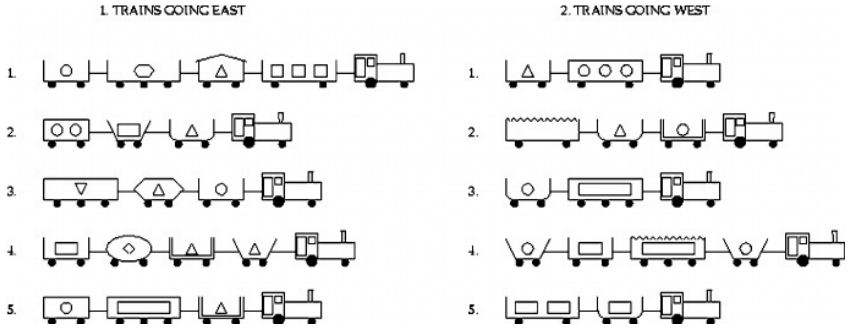
Automatic Statistician

Generative Clausal Networks

[Correia, Peharz, de Campos NeurIPS 2020, Ventola, Dhami, Kersting 2021]



Propositionalization



First-order feature construction

f1(T):-hasCar(T,C),clength(C,short).

f2(T):-hasCar(T,C), hasLoad(C,L),
loadShape(L,circle)

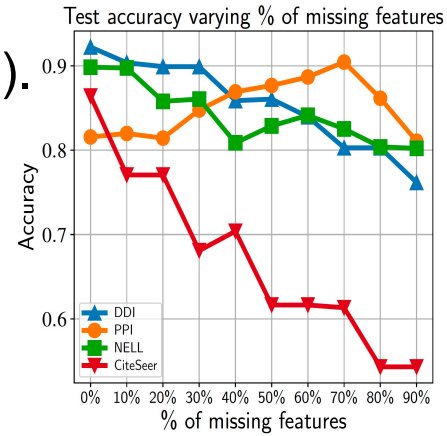
f3(T) :- …

Propositional Learning:

t(T) ← f1(T), f4(T)

PC learning:

#f1(T), #f2(T), #f2(T)



Data	Methods	Accuracy	AUC-ROC	AUC-PR	Data	Methods	Accuracy	AUC-ROC	AUC-PR
DDI	LR	**93.79**	**87.62**	83.64	*NELL*	LR	83.96	58.90	28.27
	GB	86.77	86.17	67.95		GB	87.69	75.28	48.07
	NN	87.54	85.03	69.21		NN	88.26	74.42	48.35
	DT	85.52	83.22	65.12		DT	87.27	71.77	45.71
	TILDE	72.52	73.43	70.79		TILDE	81.48	86.27	78.23
	RDN-B	75.54	82.87	83.13		RDN-B	81.26	88.47	83.41
	MLN-B	63.80	79.83	78.40		MLN-B	60.54	**89.44**	**85.30**
	GCLN-B	85.05	70.57	70.84		GCLN-B	80.74	38.72	23.15
	GCLN-P	92.22 ↑	87.53 ↑	**83.81** ↑		GCLN-P	**89.83** ↑	**89.44** ↑	56.39 ↑
PPI	LR	78.13	81.54	52.44	*CiteSeer*	LR	76.33	83.78	53.30
	GB	77.21	78.25	49.54		GB	96.05	**97.21**	87.24
	NN	76.94	75.75	47.49		NN	**96.50**	96.88	88.10
	DT	76.47	77.52	48.71		DT	95.33	96.79	85.38
	TILDE	62.20	62.87	58.27		TILDE	91.47	83.33	73.45
	RDN-B	67.15	72.84	74.02		RDN-B	94.72	97.11	**89.23**
	MLN-B	54.87	74.39	73.34		MLN-B	81.98	94.67	80.54
	GCLN-B	79.72	82.75	59.43		GCLN-B	77.18	71.34	41.20
	GCLN-P	**81.56** ↑	**91.16** ↑	**80.08** ↑		GCLN-P	86.42 ↑	71.57 ↑	42.57 ↑

Partial Counts As observations:

$$P(X > k) = 1 - \sum_{i=0}^{k} P(X = i)$$

Deep Databases

[Hilprecht, Schmidt, Kulessa, Molina, Kersting, Binnig VLDB 2020]

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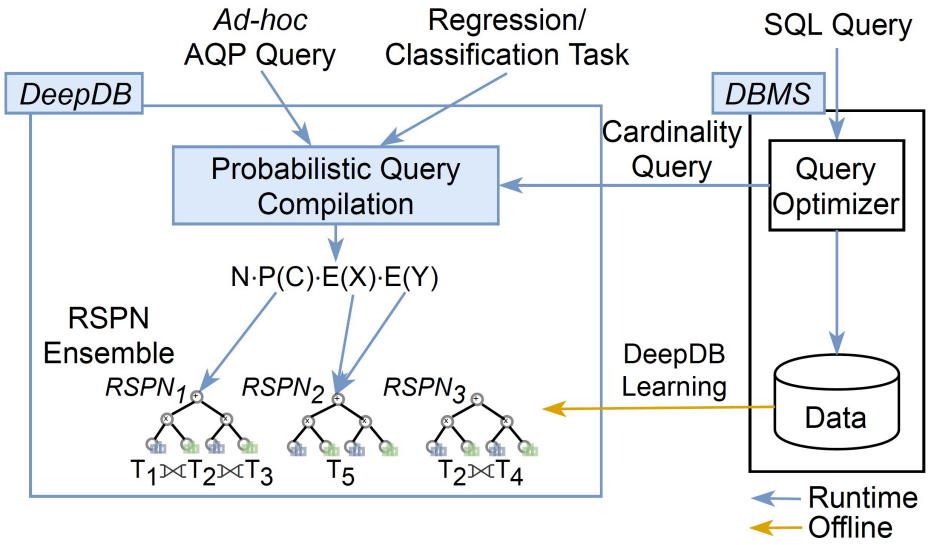
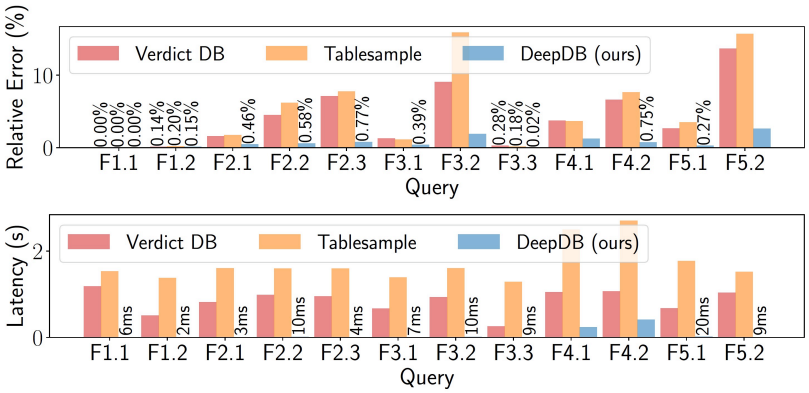
KIT Karlsruher Institut für Technologie



VLDB 2020 TOKYO AUG.31 SEP.04

46TH INTERNATIONAL CONFERENCE ON VERY LARGE DATA BASES



Hybrid AI: Sum Product Logic

= Probabilistic Circuits + (Deep NNs + (Probabilities + Logic Programming))

Sum Product Logic

DeepProbLog

ProbLog

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[Skryagin, Stelzner, Molina, Ventola, Yu, Kersting BBPR-GNN+ 2020]

Multi-Output GP-SPNs

[Yu, Zhu, Trapp, Skryagin, Kersting UAI 2021]

37th Conference on Uncertainty in Artificial Intelligence
July 27-30, 2021
Online
uai2021

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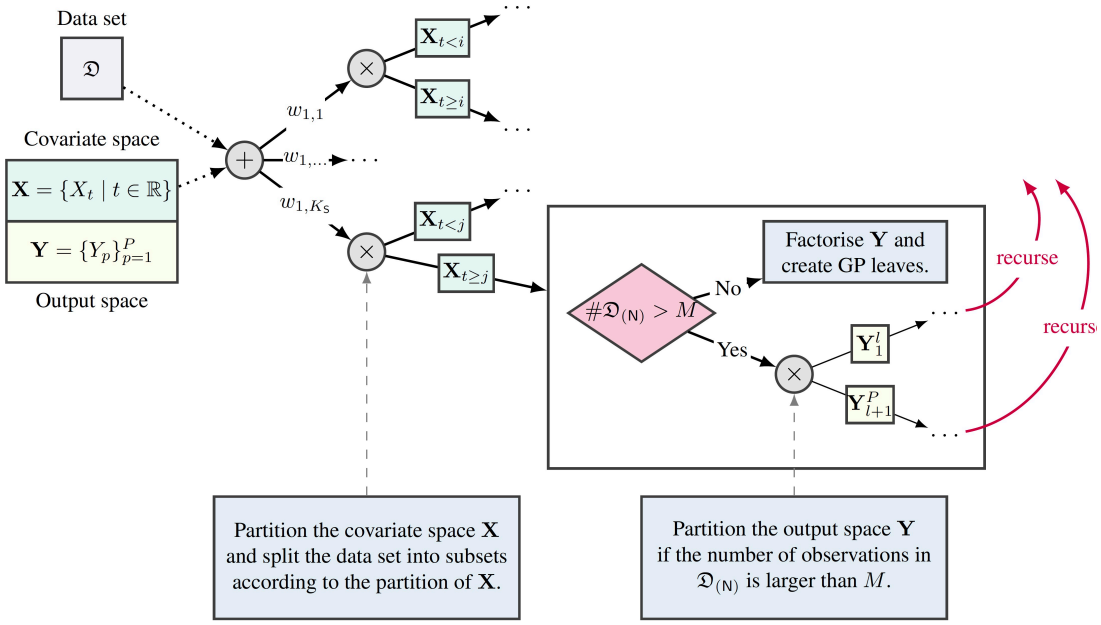
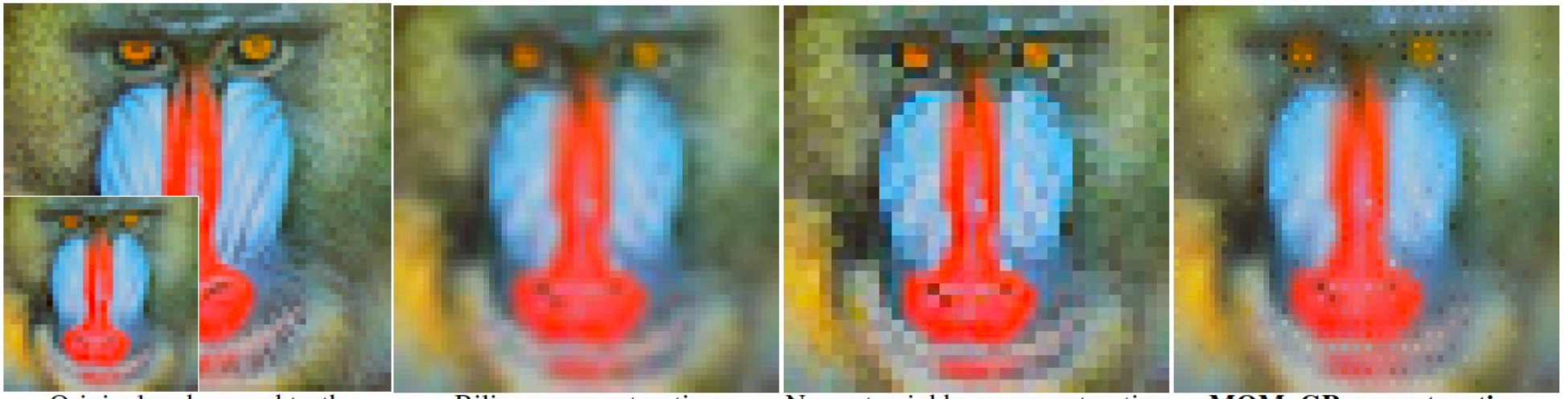
Aalto University

Nanjing University of Aeronautics & Astronautics



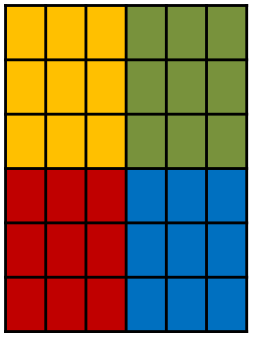
Data Set		LR	GP	MOGP	MOSVGP	DSMGP	sumGP	**MOMoGP**
Parkinsons	RMSE	0.974	0.783	0.793	0.864	**0.774**	0.784	0.775↓
	MAE	0.816	0.610	**0.603**	0.708	0.604	0.610	0.605↓
	NLPD	2.787	2.389	**1.766**	2.515	2.319	2.388	2.208↑
scm20d	RMSE	0.854	0.832	**0.816**	0.824	0.839	0.829	0.820↑
	MAE	0.652	0.643	**0.630**	0.636	0.646	0.641	**0.630**↑
	NLPD	21.876	21.013	21.310	17.319	19.059	14.859	**11.416**↑
WindTurbine	RMSE	0.391	**0.133**	0.139	0.302	0.143	**0.133**	0.143
	MAE	0.311	0.074	0.080	0.236	**0.073**	0.074	**0.073**
	NLPD	1.435	-2.649	2.594	0.104	**-8.749**	-2.627	-7.467↓
Energy	RMSE	0.752	NA	NA	0.659	**0.547**	NA	0.556↓
	MAE	0.605	NA	NA	0.516	0.400	NA	**0.398**↑
	NLPD	14.775	NA	NA	14.169	11.745	NA	**8.610**↑
usFlight	RMSE	0.983	NA	NA	0.955	**0.927**	NA	0.934↓
	MAE	0.529	NA	NA	0.494	**0.492**	NA	0.505↓
	NLPD	2.331	NA	NA	2.251	2.178	NA	**2.091**↑



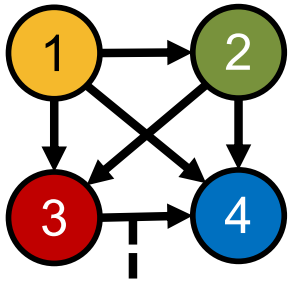
Original and ground truth | Bilinear reconstruction | Nearest neighbour reconstruction | **MOMoGP reconstruction**

Putting a little bit of structure into SPN models allows one to realize autoregressive deep models akin to PixelCNNs [van den Oord et al. NIPS 2016]

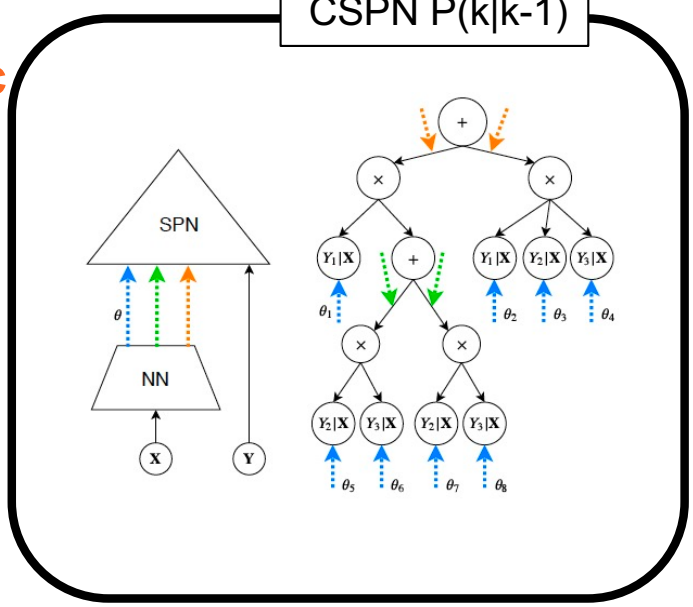
CSPNs
PixelCNNs



chain rule of probabilities



CSPN P(k|k-1)



Learn Conditional SPN (CSPNs) by non-parametric conditional independence testing and conditional clustering [Zhang et al. UAI 2011; Lee, Honovar UAI 2017; He et al. ICDM 2017; Zhang et al. AAAI 2018; Runge AISTATS 2018]
encoded using gating functions

Conditional SPNs

[Shao, Molina, Vergari, Peharz, Liebig, Kersting PGM 2020]

PGM 2020 AALBORG

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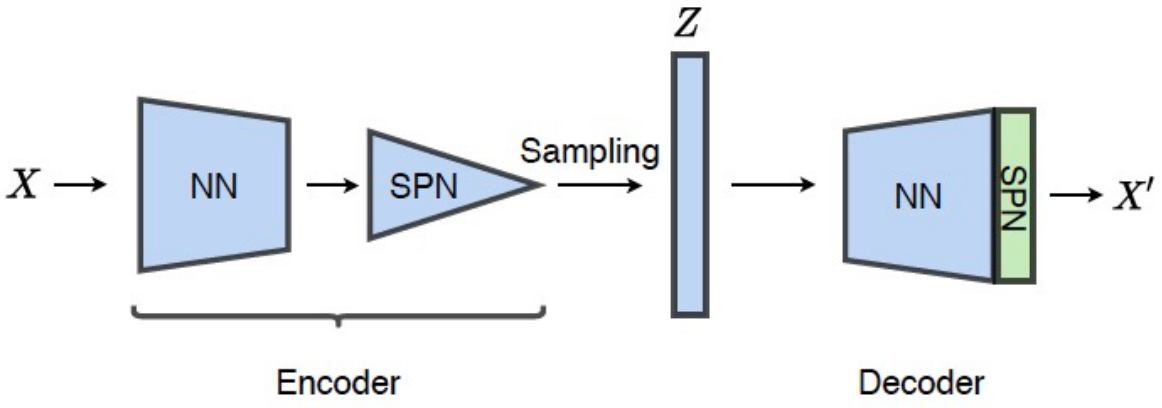
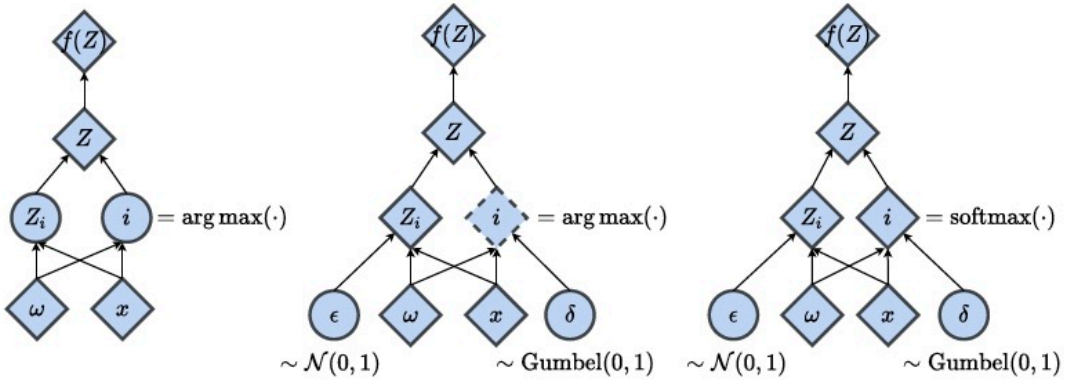
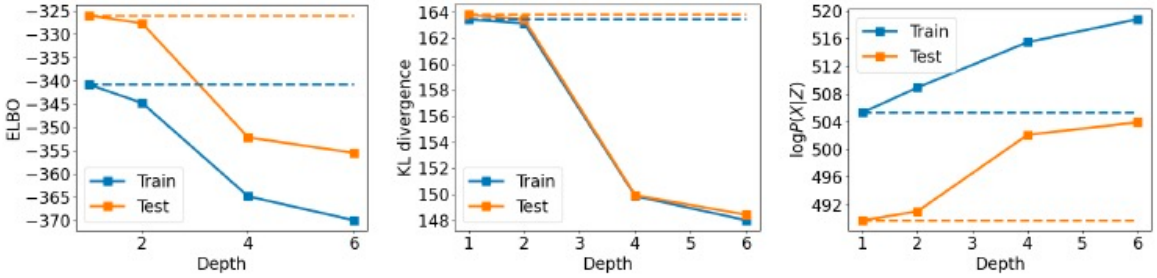
Use a (neural) CPSN as endcoder and decoder together with differentiable sampling







(e) On CelebA. Prior: isotropic multivariate Gaussian.

Sum-Product VAE

[Shao, Molina, Vergari, Peharz, Liebig, Kersting 2021]

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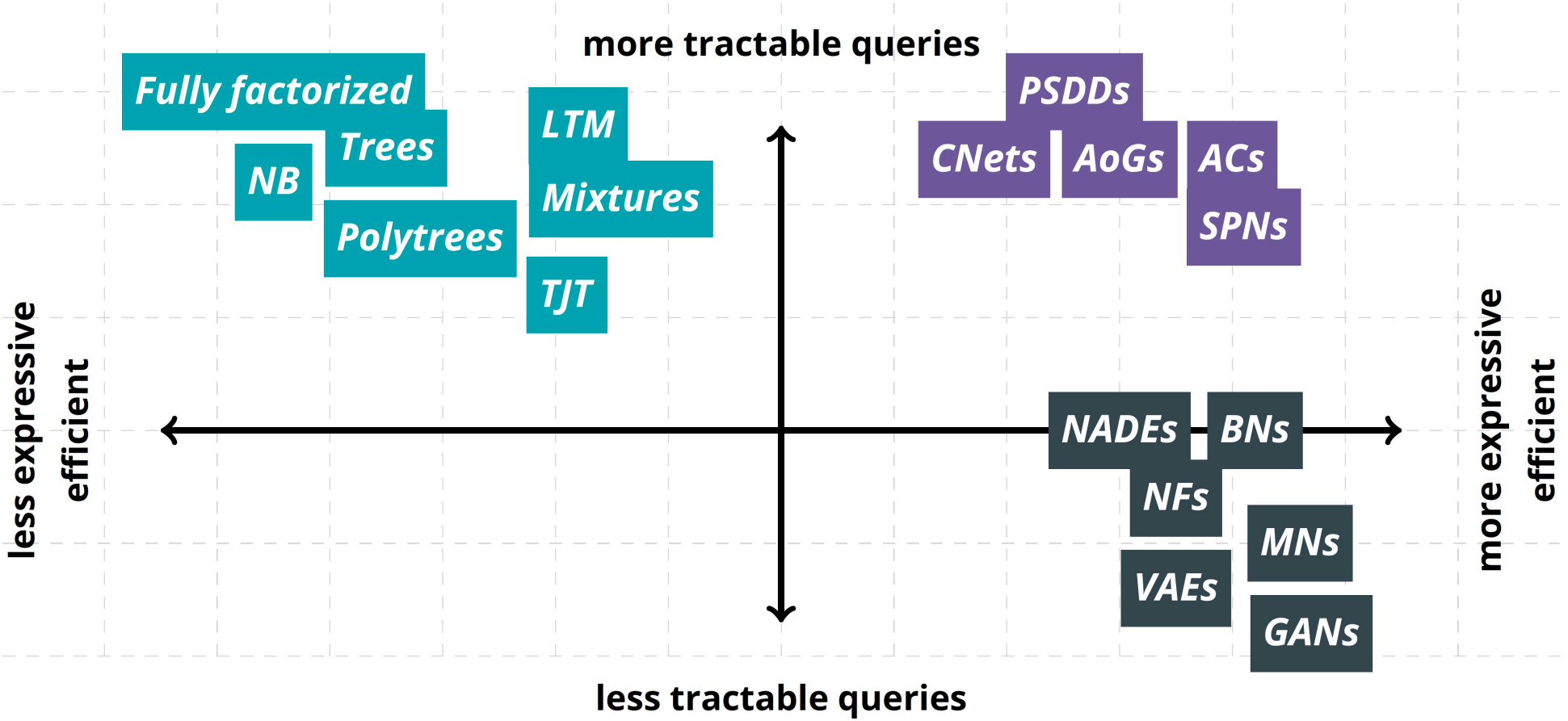
Sum-Product Networks are an example of tractable probabilistic models. There are many others, and on typically speaks of probabilistic cirxuits



What have we learnt about SPNs?

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Sum-product networks (SPNs)

- DAG of sums and products
- They are instances of Arithmetic Circuits (ACs)
- Compactly represent partition function
- Learn many layers of hidden variables

Efficient marginal inference

Easy learning

Can outperform well-known alternatives