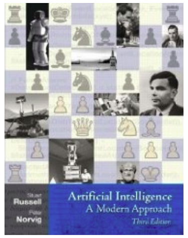
Outline

- Best-first search
  - Greedy best-first search
  - A* search
  - Heuristics
- Local search algorithms
  - Hill-climbing search
  - Beam search
  - Simulated annealing search
  - Genetic algorithms
- Constraint Satisfaction Problems
  - Constraints
  - Constraint Propagation
  - Backtracking Search
  - Local Search



Many slides based on Russell & Norvig's slides [Artificial Intelligence: A Modern Approach]

Local Search Algorithms

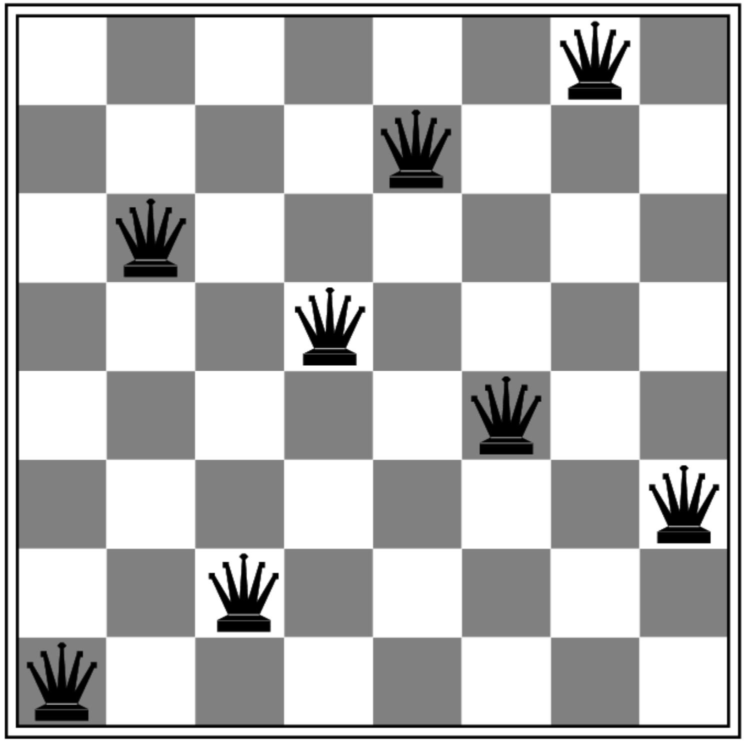
- In many optimization problems, the path to the goal is irrelevant
  - the goal state itself is the solution

- State space:
  - set of "complete" configurations
- Goal:
  - Find a configuration that satisfies all constraints

- Examples:
  - n-queens problem, travelling salesman, …

- In such cases, we can use local search algorithms

N-queens Problem



We do not want the path to the goal. The solution is all what matters.

Local Search

- **Approach**
  - keep a single "current" state (or a fixed number of them)
  - try to improve it by maximizing a heuristic evaluation
  - using only **„local“ improvements**
    - i.e., only modifies the current state(s)
  - paths are typically not remembered
  - similar to solving a puzzle by hand
    - e.g., 8-puzzle, Rubik's cube

- Advantages
  - uses very little memory
  - often quickly finds solutions in large or infinite state spaces

- Disadvantages
  - no guarantees for completeness or optimality

Optimization Problems

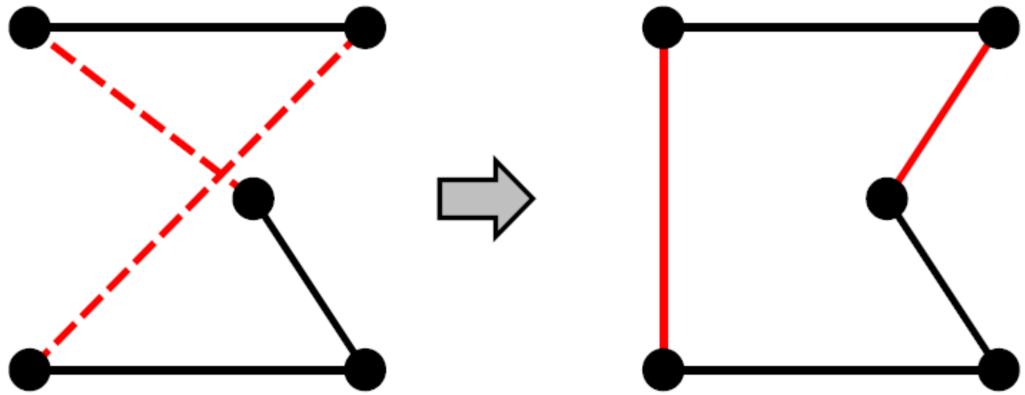
- Goal:
  - optimize some evaluation function (objective function)
- there is no goal state, and no path costs
  - hence A* and other algorithms we have discussed so far are not applicable

Named like this because it defines the „objective" that has to be met

- Example:
  - Darwinian evolution and survival of the fittest may be regarded as an optimization process

Example: Travelling Salesman Problem

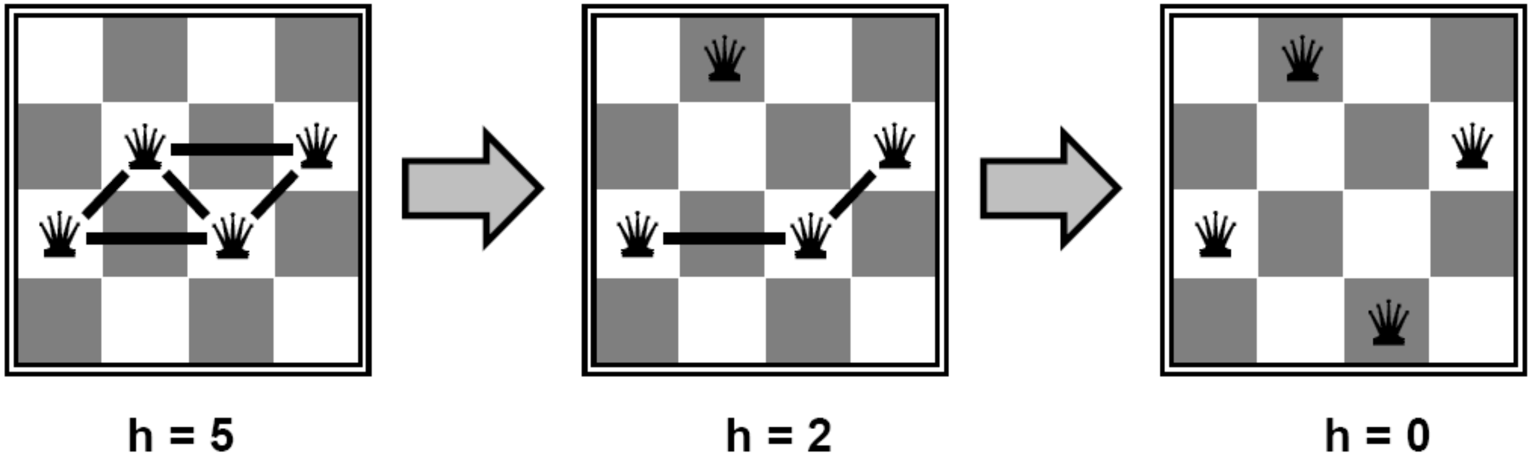
- Basic Idea:
  - Start with a complete tour
  - perform pairwise exchanges



  - variants of this approach get within 1% of an optimal solution very quickly with thousands of cities

Example: n-Queens Problem

- Basic Idea:
  - move a queen so that it reduces the number of conflicts



  - almost always solves n-queens problems almost instantaneously for very large n (e.g., n = 1,000,000)

Hill-climbing search

- Algorithm:
  - expand the current state (generate all neighbors)
  - move to the one with the highest evaluation
  - until the evaluation goes down

```
function HILL-CLIMBING( problem) returns a state that is a local maximum
   inputs: problem, a problem
   local variables: current, a node
                    neighbor, a node
   current ← MAKE-NODE(INITIAL-STATE[problem])
   loop do
       neighbor ← a highest-valued successor of current
       if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
       current ← neighbor
   end
```

Hill-climbing search
(aka Greedy Local Search)

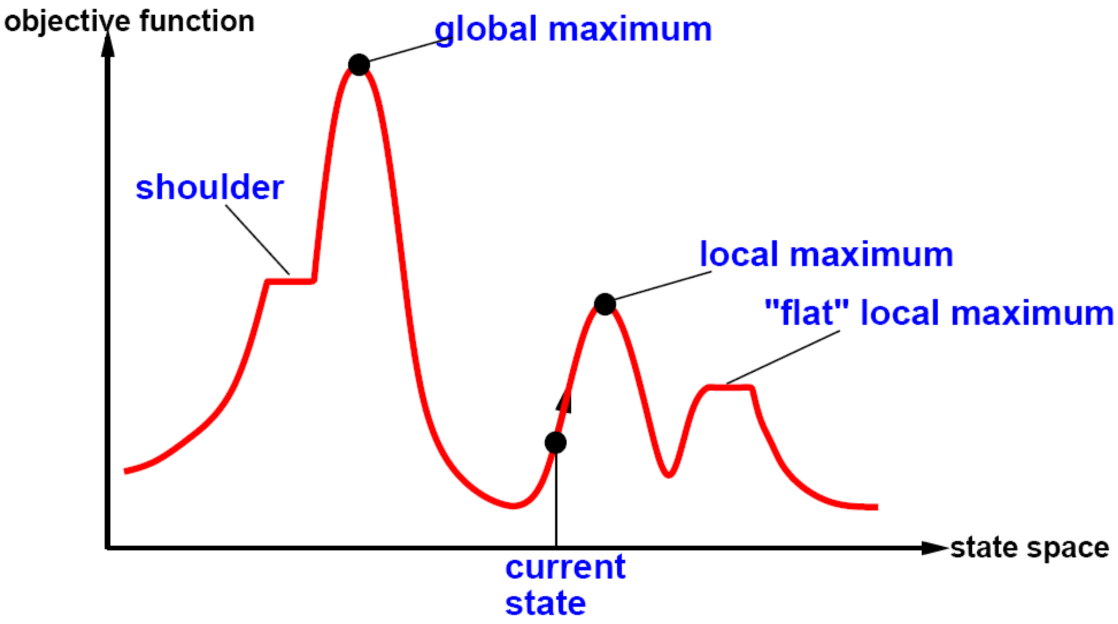
- Algorithm:
  - expand the current state (generate all neighbors)
  - move to the one with the highest evaluation
  - until the evaluation goes down
- Main Problem: Local Optima
  - the algorithm will stop as soon as is at the top of a hill
  - but it is actually looking for a mountain peak

"Like climbing Mount Everest in thick fog with amnesia"

- Other problems:
  - ridges
  - plateaux
  - shoulders

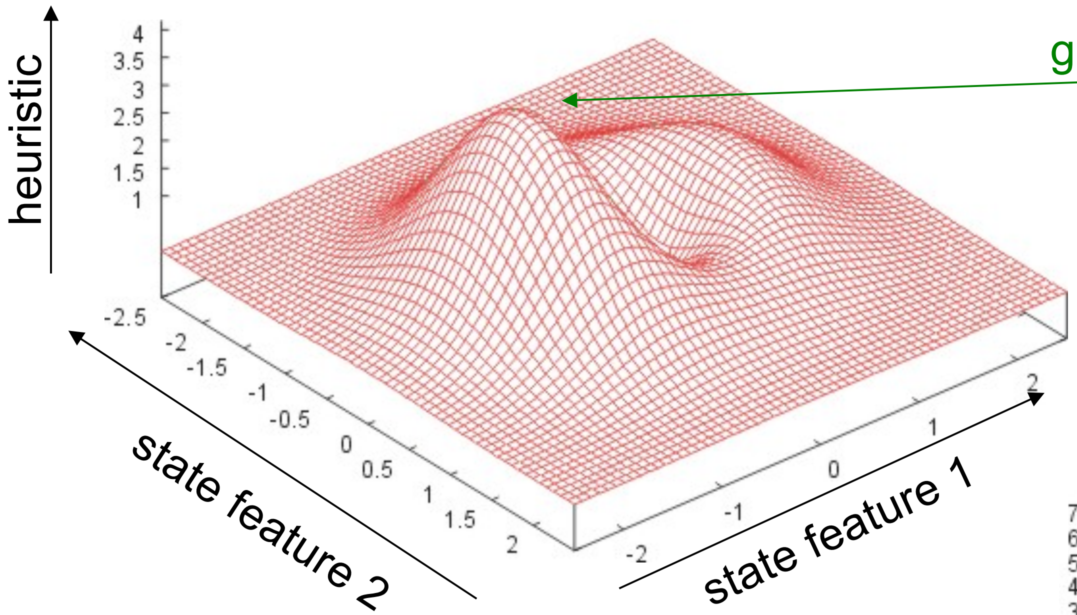
State Space Landscape

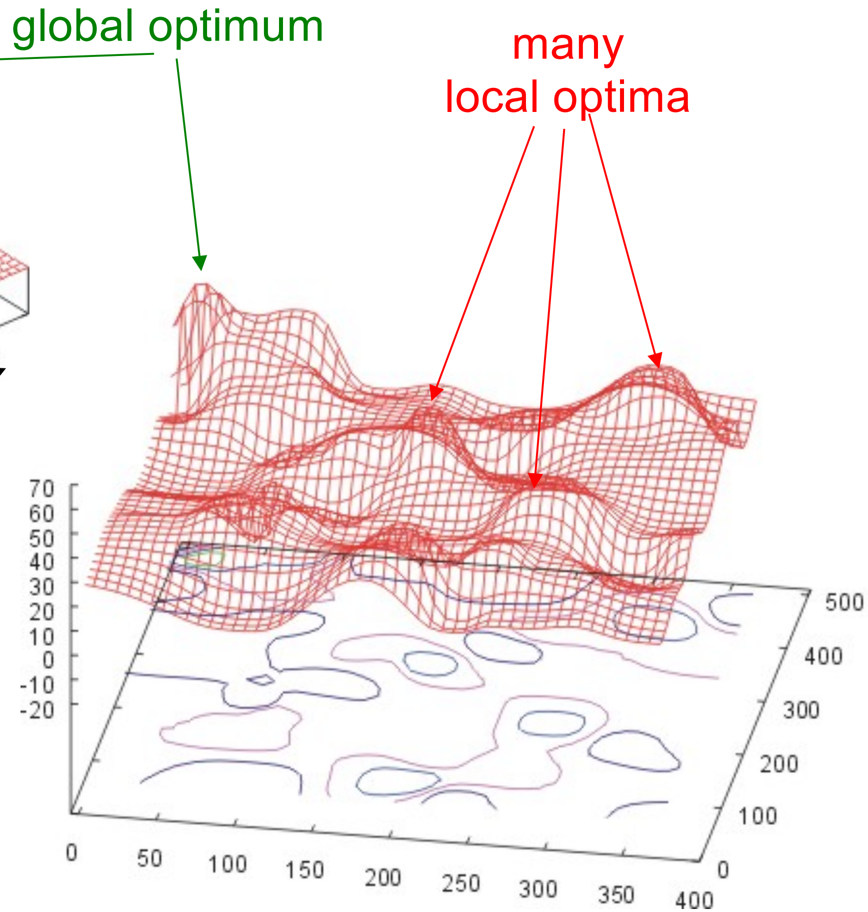
- state-space landscape
  - location: states
  - elevation: heuristic value (objective function)
- Assumption:
  - states have some sort of (linear) order
  - continuity regarding small state changes



Multi-Dimensional State-Landscape

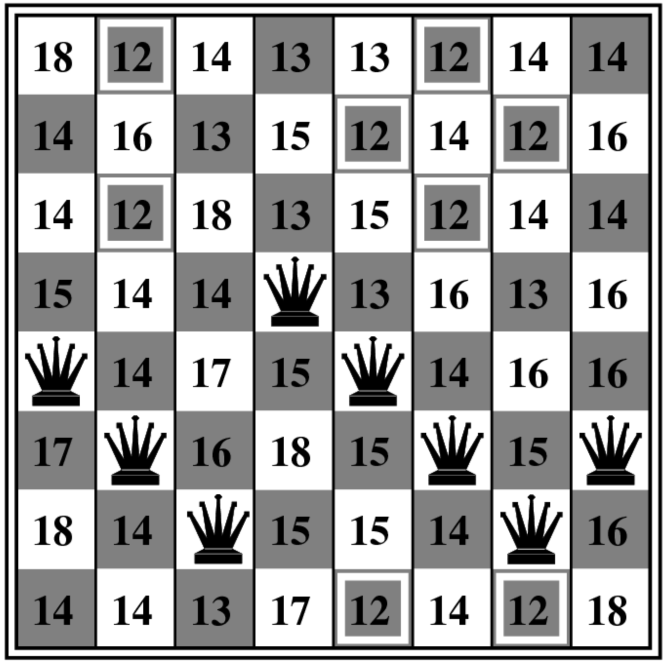
- States may be refined in multiple ways
  → similarity along various dimensions





Example: 8-Queens Problem

- Heuristic h:
  - number of pairs of queens that attach each other
- Example state: $h = 17$



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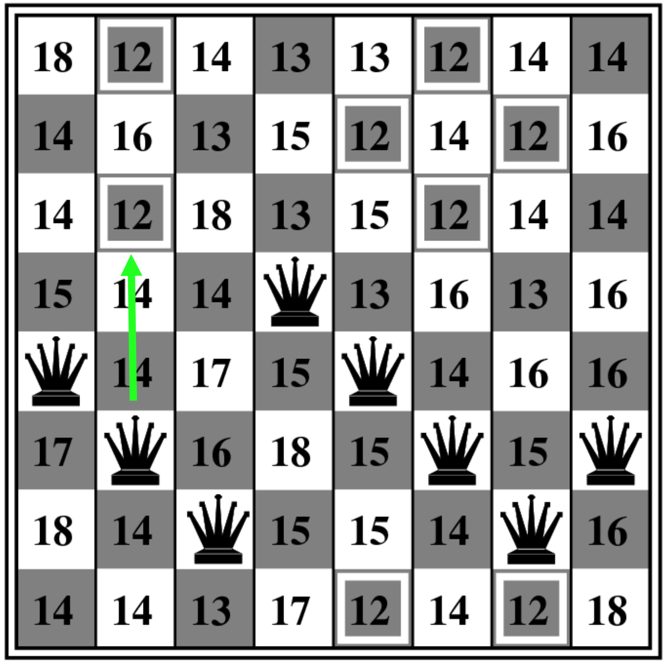
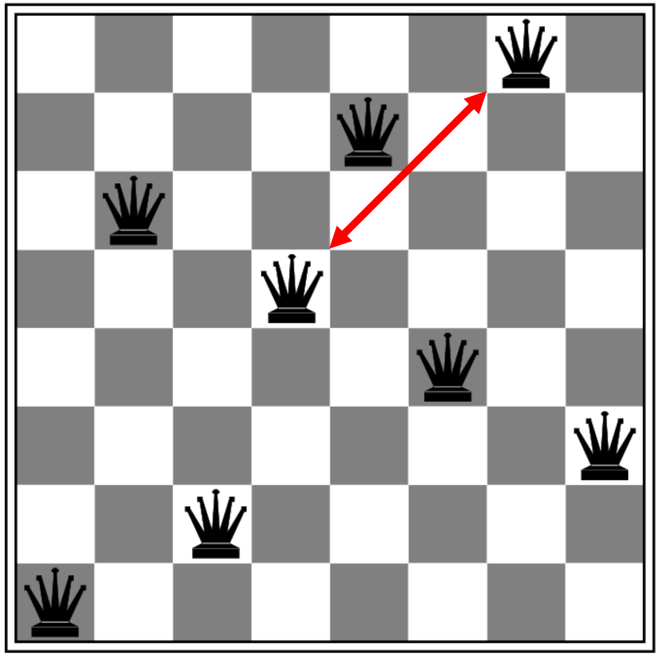


- Best Neighbor(s): $h = 12$

- Local optimum with $h = 1$



- no queen can move without increasing the number of attacked pairs

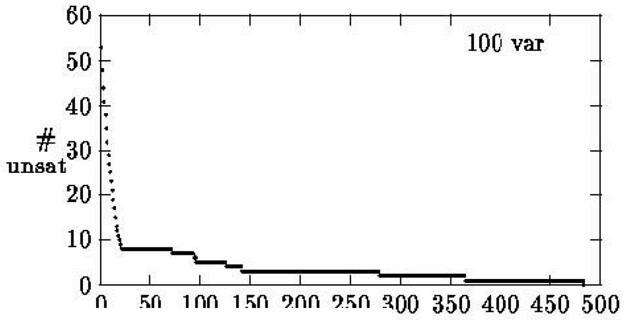
Randomized Hill-Climbing Variants

- Random Restart Hill-Climbing
  - Different initial positions result in different local optima

  $\rightarrow$ make several iterations with different starting positions
- Example:
  - for 8-queens problem the probability that hill-climbing succeeds from a randomly selected starting position is ≈ 0.14

  $\rightarrow$ a solution should be found after about $1/0.14 \approx 7$ iterations of hill-climbing
- Stochastic Hill-Climbing
  - select the successor node ramdomly
  - better nodes have a higher probability of being selected

Another example: Greedy SAT



Task: Find a satifying configuration I of a propositional formula **Δ**

auxiliary functions:

- violated(Δ, I): number of clauses in Δ not satisfied by I
- flip(I, v): assignment that results from I when changing the valuation of proposition v

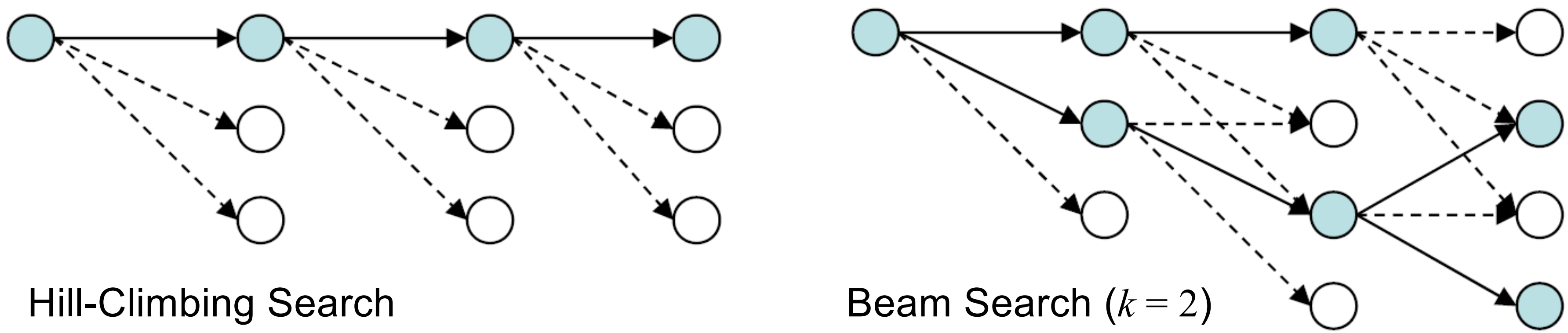
function GSAT(Δ):

repeat *max-tries* **times**:
  $I :=$ a random assignment
  repeat *max-flips* **times**:
    if $I \models \Delta$:
      return I
    $V_{\text{greedy}} :=$ the set of variables v occurring in Δ
      for which violated$(\Delta, \text{flip}(I, v))$ is minimal
    randomly select $v \in V_{\text{greedy}}$
    $I := \text{flip}(I, v)$
return no solution found

Beam Search

- Keep track of k states rather than just one
  - k is called the beam size

- Algorithm
  - Start with k randomly generated states
  - At each iteration, all the successors of all k states are generated
  - select the k best successors from the complete list and repeat



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  - select the k best successors from the complete list and repeat.
- Implementation

  Can be implemented similar to the Tree-Search algorithm:
  - sort the queue by the heuristic function h (as in greedy search)
  - but limit the size of the queue to k
  - and expand all nodes in queue simultaneously

Beam Search

- Keep track of k states rather than just one
  - k is called the beam size
- Note
  - Beam search is different from k parallel hill-climbing searches!
  - Information from different beams is combined
- Effectiveness
  - suffers from lack of diversity of the k states
    - e.g., if one state has better successors than all other states
    - thus it is often no more effective than hill-climbing

- Stochastic Beam Search
  - chooses k successors at random
  - better nodes have a higher probability of being selected

Simulated Annealing Search

- combination of hill-climbing and random walk
- Idea:
  - escape local maxima by allowing some "bad" moves
  - but gradually decrease their frequency (the *temperature*)
- Effectiveness:
  - it can be proven that if the temperature is lowered slowly enough, the probability of converging to a global optimum approaches 1
  - Widely used in VLSI layout, airline scheduling, etc

- Note:
  - Annealing *in metallurgy and materials science, is a heat treatment wherein the microstructure of a material is altered, causing changes in its properties such as strength and hardness. It is a process that produces equilibrium conditions by heating and maintaining at a suitable temperature, and then cooling very slowly.*

Simulated Annealing Search

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Simulated Annealing Search

- combination of hill-climbing and random walk

function SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state
    inputs: *problem*, a problem
            schedule, a mapping from time to "temperature"
    local variables: *current*, a node
            next, a node
            T, a "temperature" controlling prob. of downward steps

    current $\leftarrow$ MAKE-NODE(INITIAL-STATE[*problem*])
    for $t \leftarrow 1$ **to** ∞ **do**
        $T \leftarrow$ *schedule*[t]
        if $T = 0$ **then return** *current*
        next $\leftarrow$ a randomly selected successor of *current*
        $\Delta E \leftarrow$ VALUE[*next*] – VALUE[*current*]
        if $\Delta E > 0$ **then** *current* $\leftarrow$ *next*
        else *current* $\leftarrow$ *next* only with probability $e^{\Delta E/T}$

Genetic Algorithms

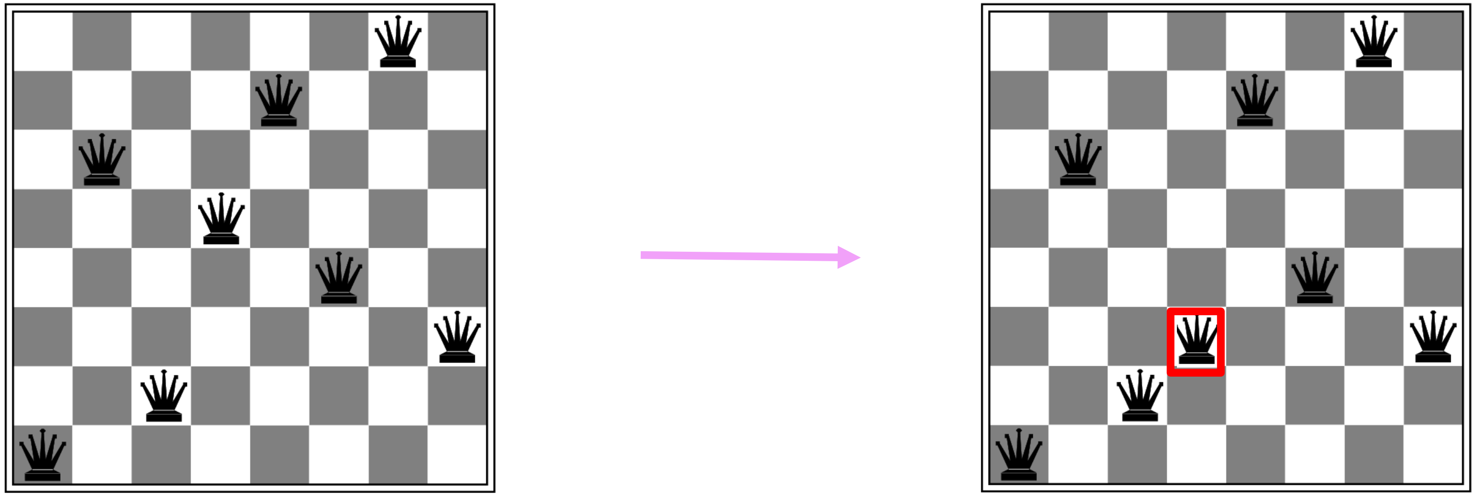
- Same idea as in Stochastic Beam Search
  - but uses „sexual“ reproduction (new nodes have two parents)
- Basic Algorithm:
  - Start with k randomly generated states (population)
  - A state is represented as a string over a finite alphabet
    - often a string of 0s and 1s
  - Evaluation function (fitness function)
  - Produce the next generation by selection, cross-over, and mutation

Mutation

- Modelled after mutation of DNA
  - take one parent strings
  - modify a random value

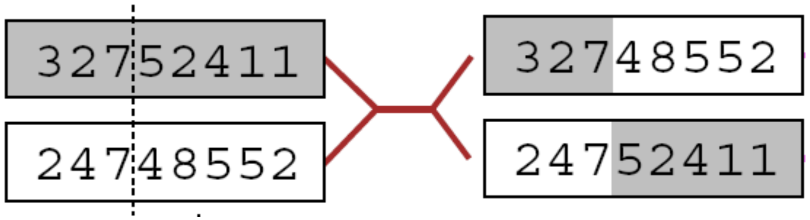
16257583 → 16237583

- comparable to a stochastic hill-climbing step

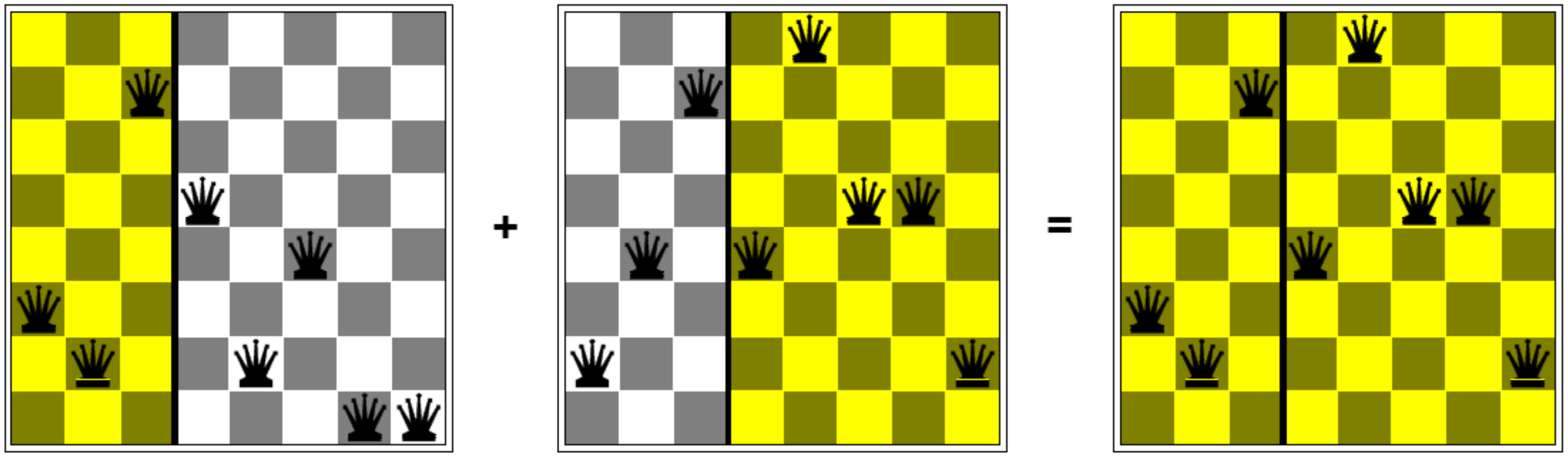


Cross-Over

- Modelled after cross-over of DNA
  - take two parent strings
  - cut them at cross-over point
  - recombine the pieces



- it is helpful if the substrings are meaningful subconcepts



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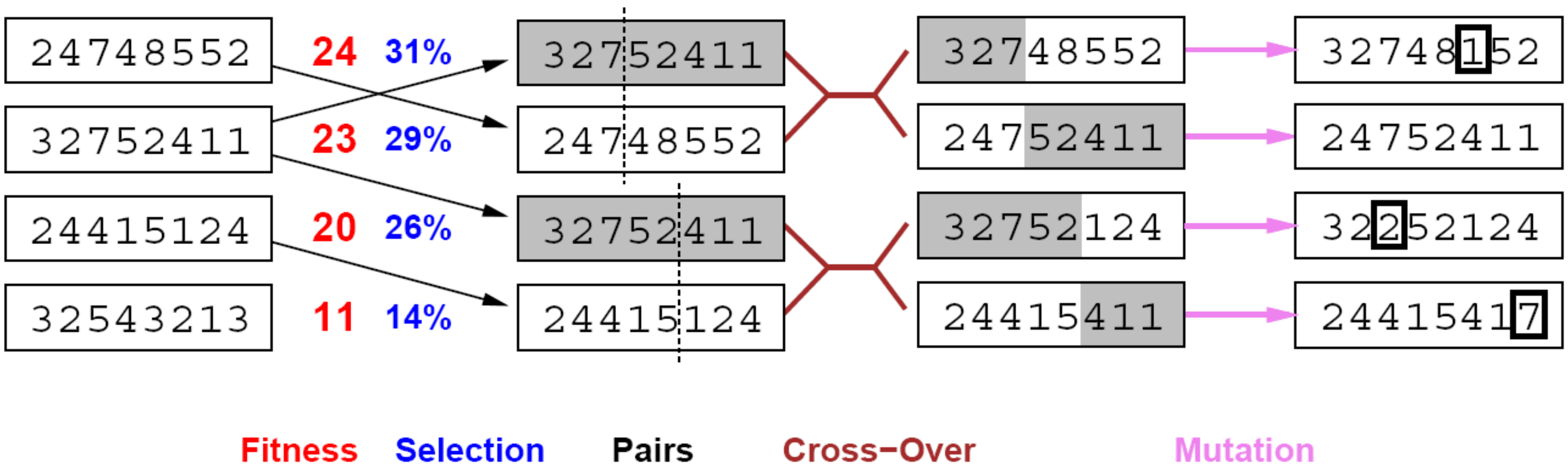
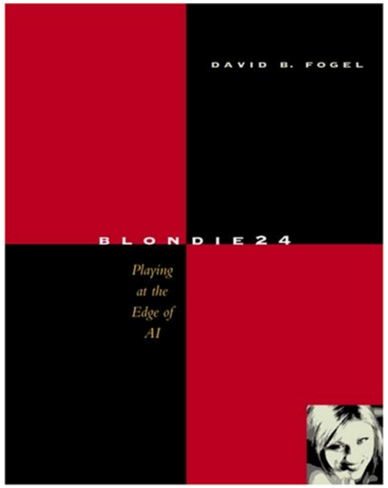


Genetic Algorithm

function GENETIC_ALGORITHM(*population,* FITNESS-FN) **return** an individual
  input: *population*, a set of individuals
    FITNESS-FN, a function which determines the quality of the individual
  repeat
    new_population ← empty set
    loop for i **from** 1 **to** SIZE(*population*) **do**
      x ← RANDOM_SELECTION(*population*, FITNESS_FN)
      y ← RANDOM_SELECTION(*population*, FITNESS_FN)
      child ← REPRODUCE(*x,y*)
      if (small random probability) **then** *child* ← MUTATE(*child*)
      add *child* to *new_population*
    population ← *new_population*
  until some individual is fit enough or enough time has elapsed
  return the best individual in *population*, according to FITNESS_FN

Genetic Algorithms

- Evaluation
  - attractive and popular
    - easy to implement general optimization algorithm
    - easy to explain to laymen (boss)
  - perform well
    - unclear under which conditions they work well
    - other randomized algorithms perform equally well (or better)
- Numerous applications
  - optimization problems
    - circuit layout
    - job-shop scheduling
  - game playing
    - checkers program Blondie24 (David Fogel)
      - nice and easy read, but shooting a bit over target in its claims...



Genetic Programming

- popularized by John R. Koza

> *Genetic programming is an automated method for creating a working computer program from a high-level problem statement of a problem. It starts from a high-level statement of "what needs to be done" and automatically creates a computer program to solve the problem.*

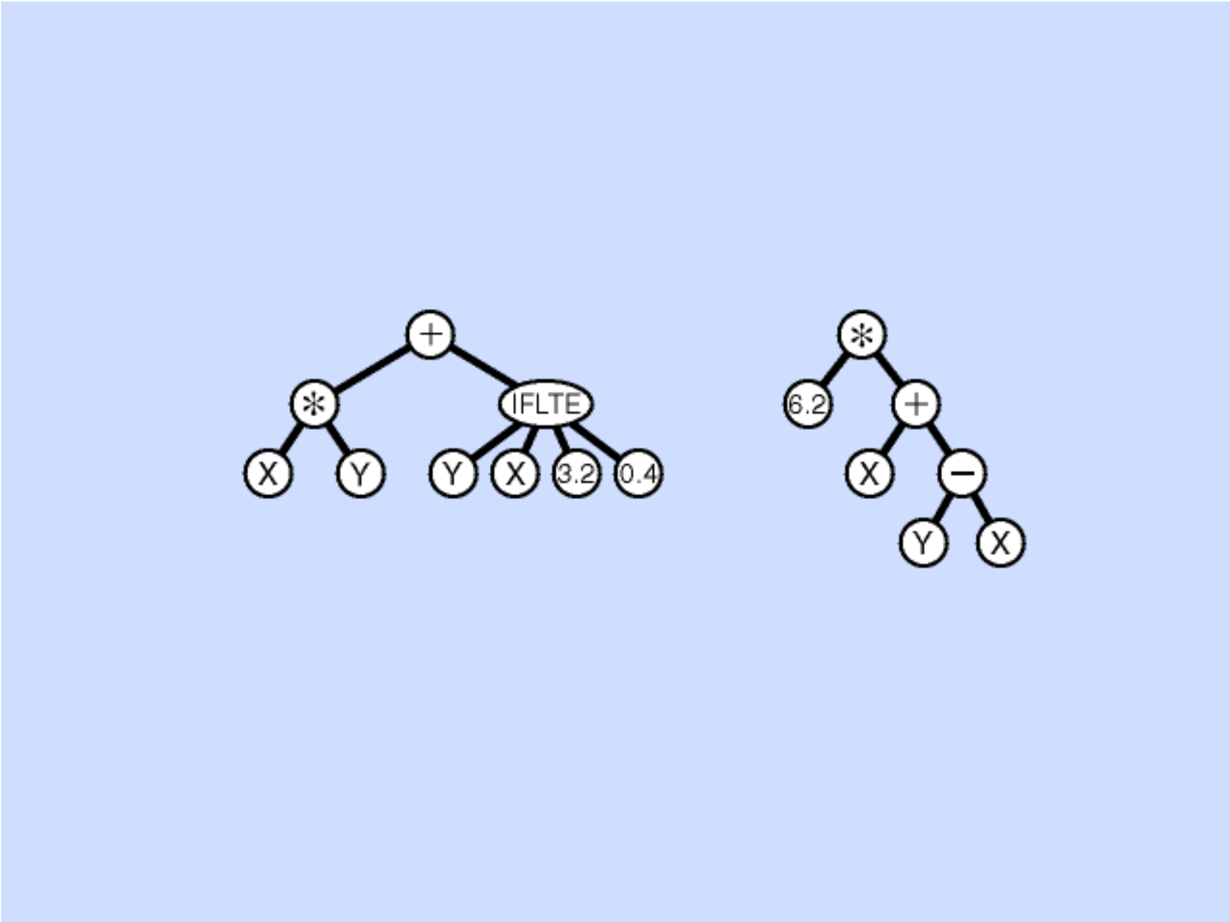
- applies Genetic Algorithms to program trees
  - Mutation and Cross-over adapated to tree structures
  - special operations like
    - inventing/deleting a subroutine
    - deleting/adding an argument,
    - etc.

- Several successful applications
  - Annual awards for performance competitive to humans
    http://www.genetic-programming.com/humancompetitive.html
- More information at http://www.genetic-programming.org/

Random Initialization of Population

Animated Image taken from http://www.genetic-programming.com/gpanimatedtutorial.html

Mutation



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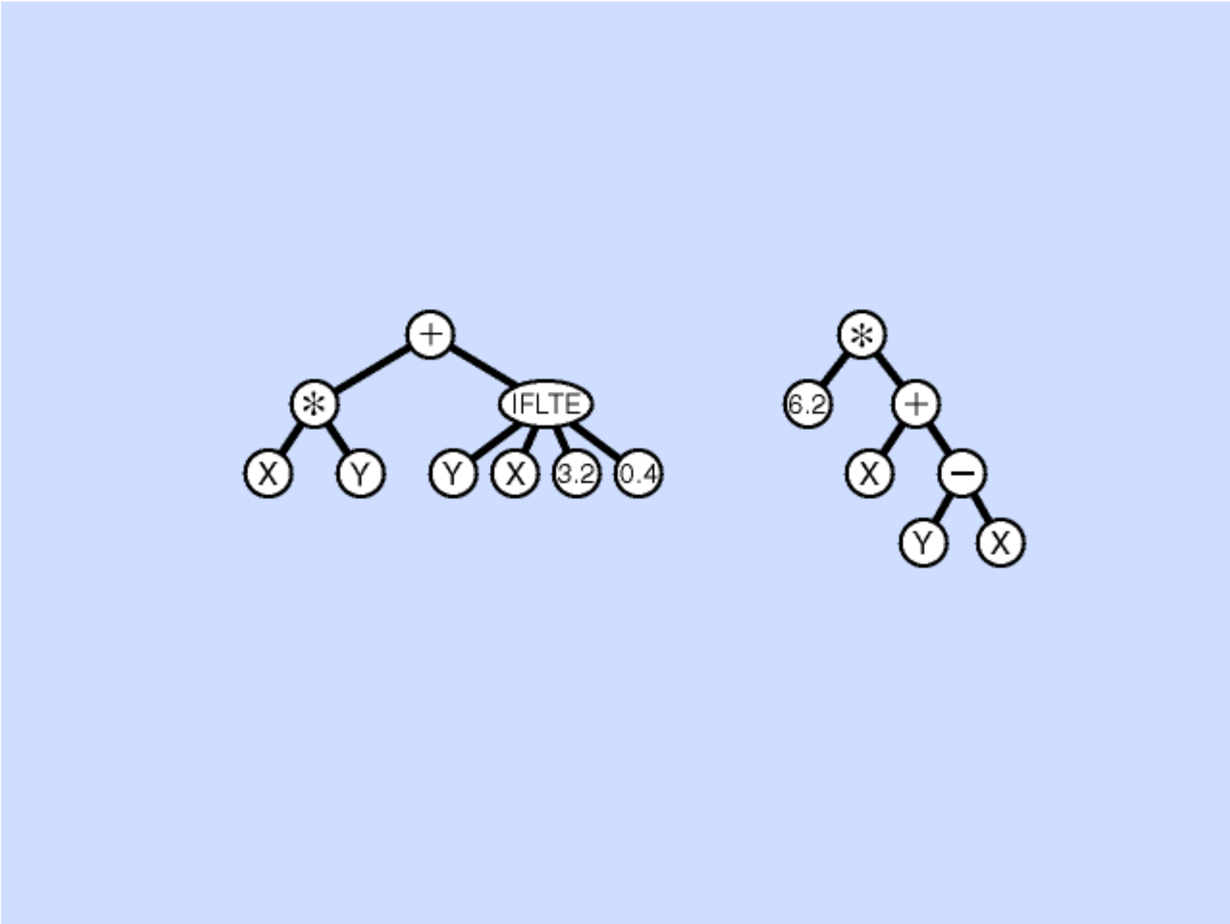
Cross-Over



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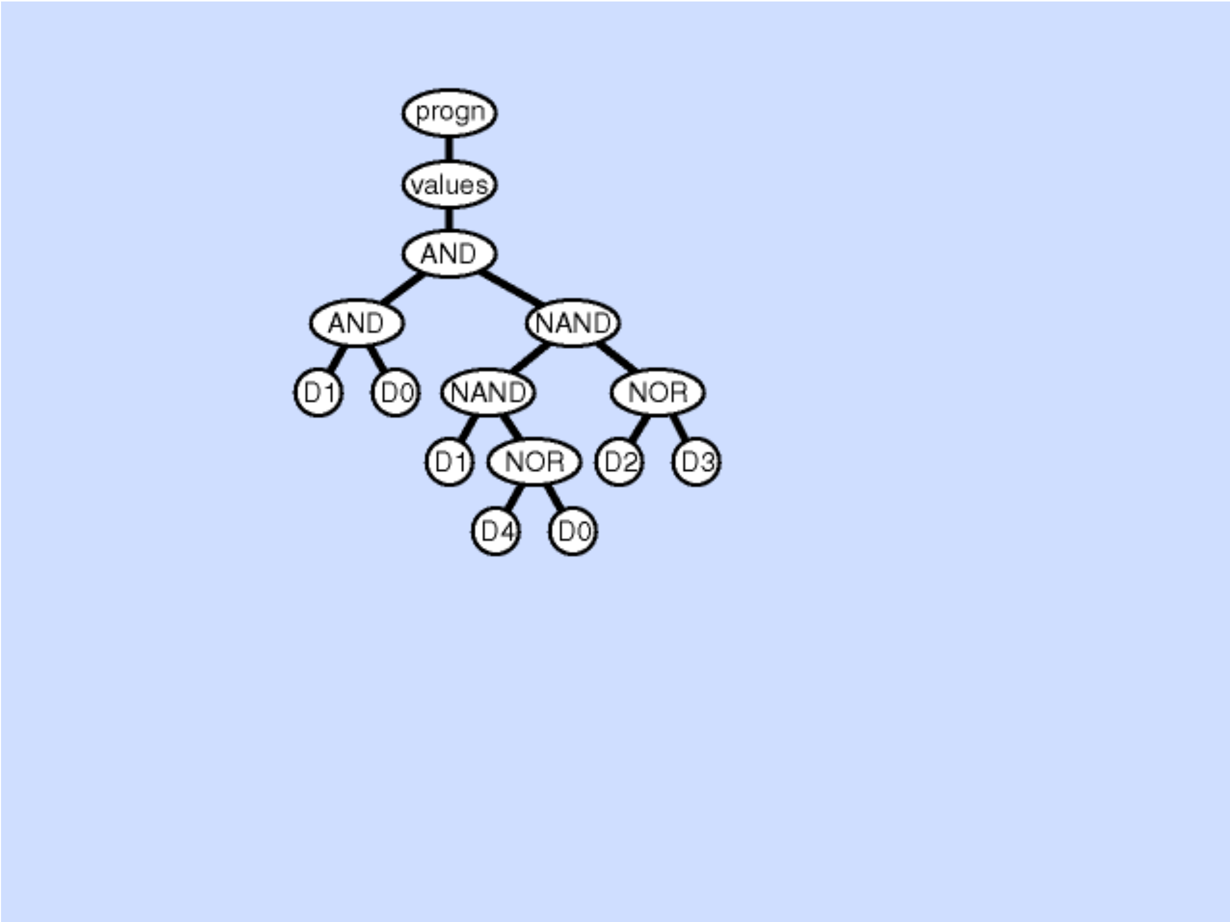
Create a Subroutine



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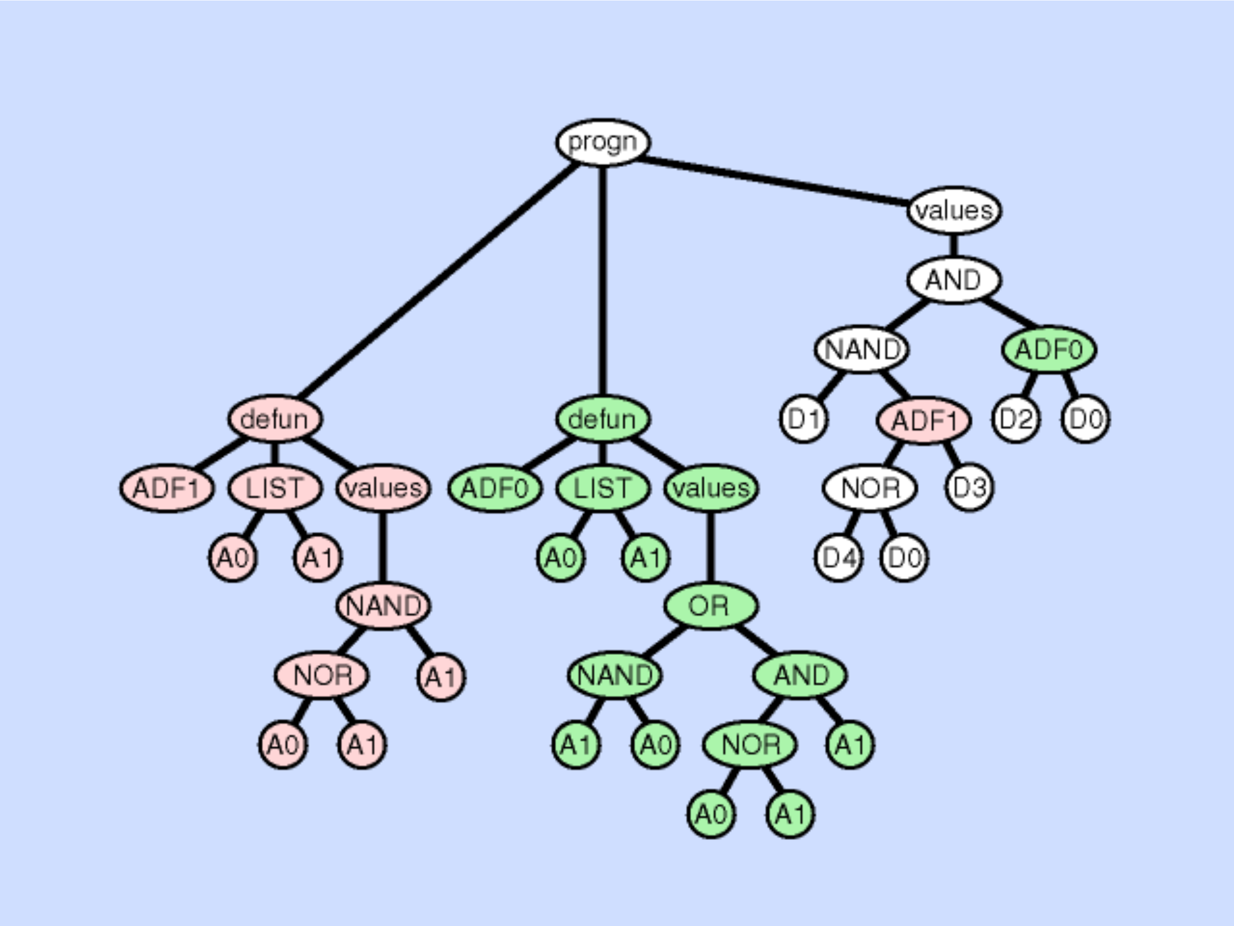
Delete a Subroutine



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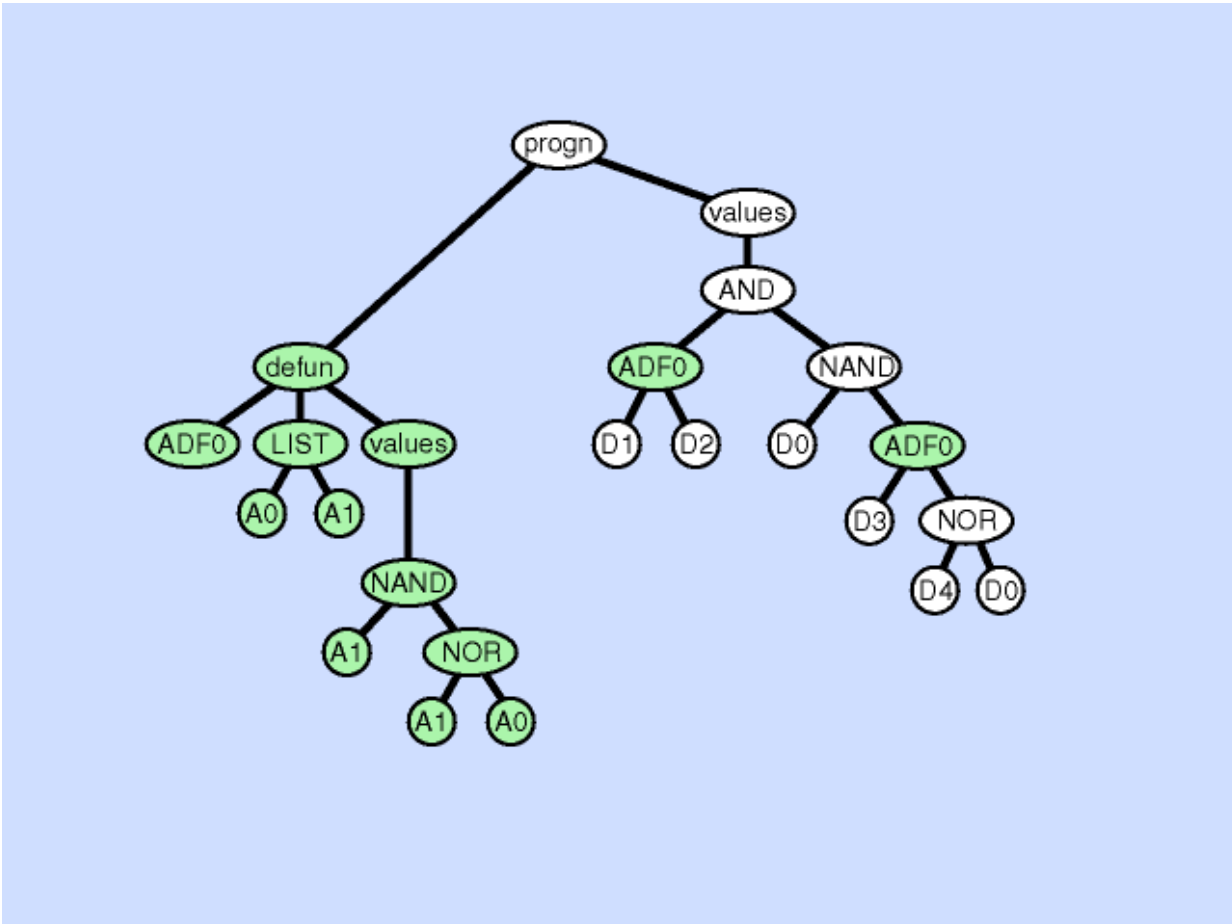
Duplicate an Argument



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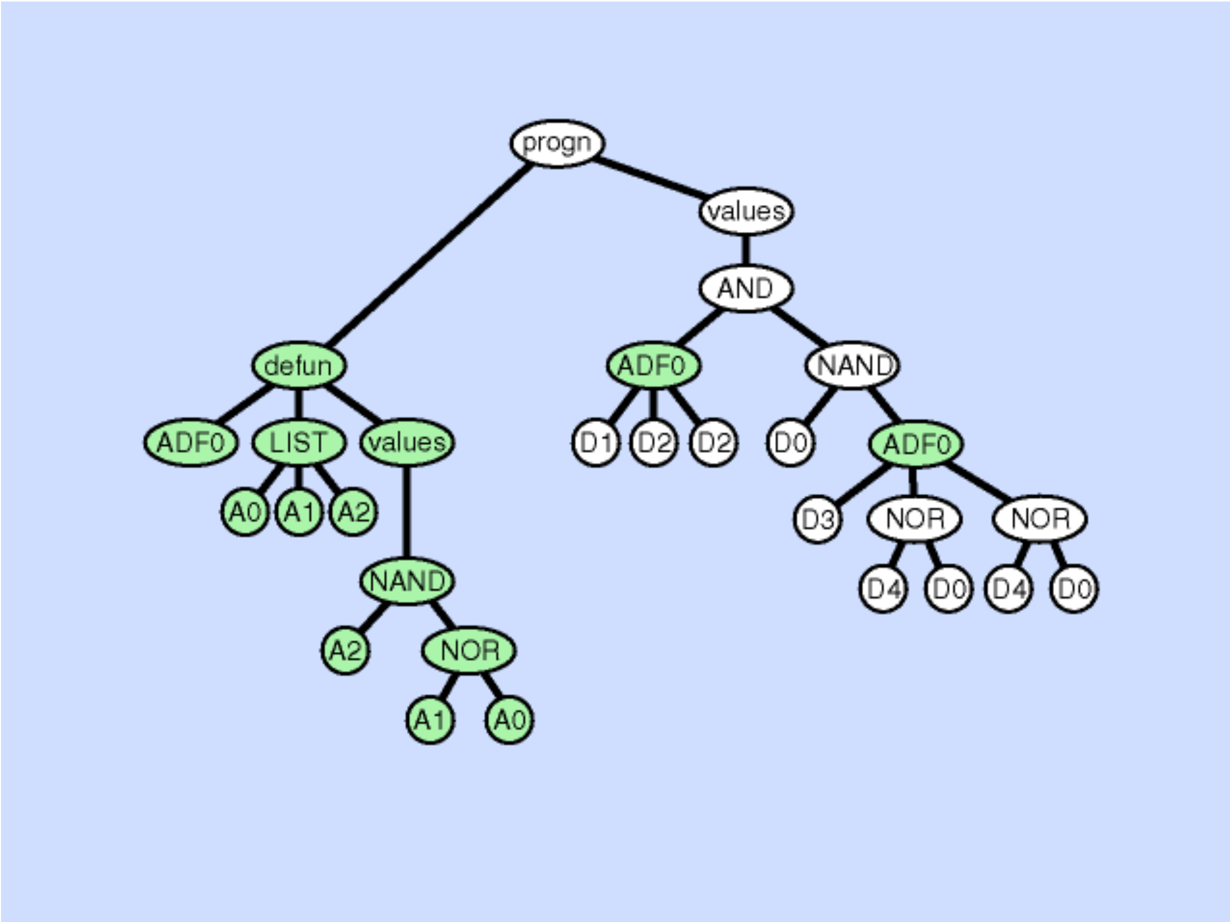
Delete an Argument



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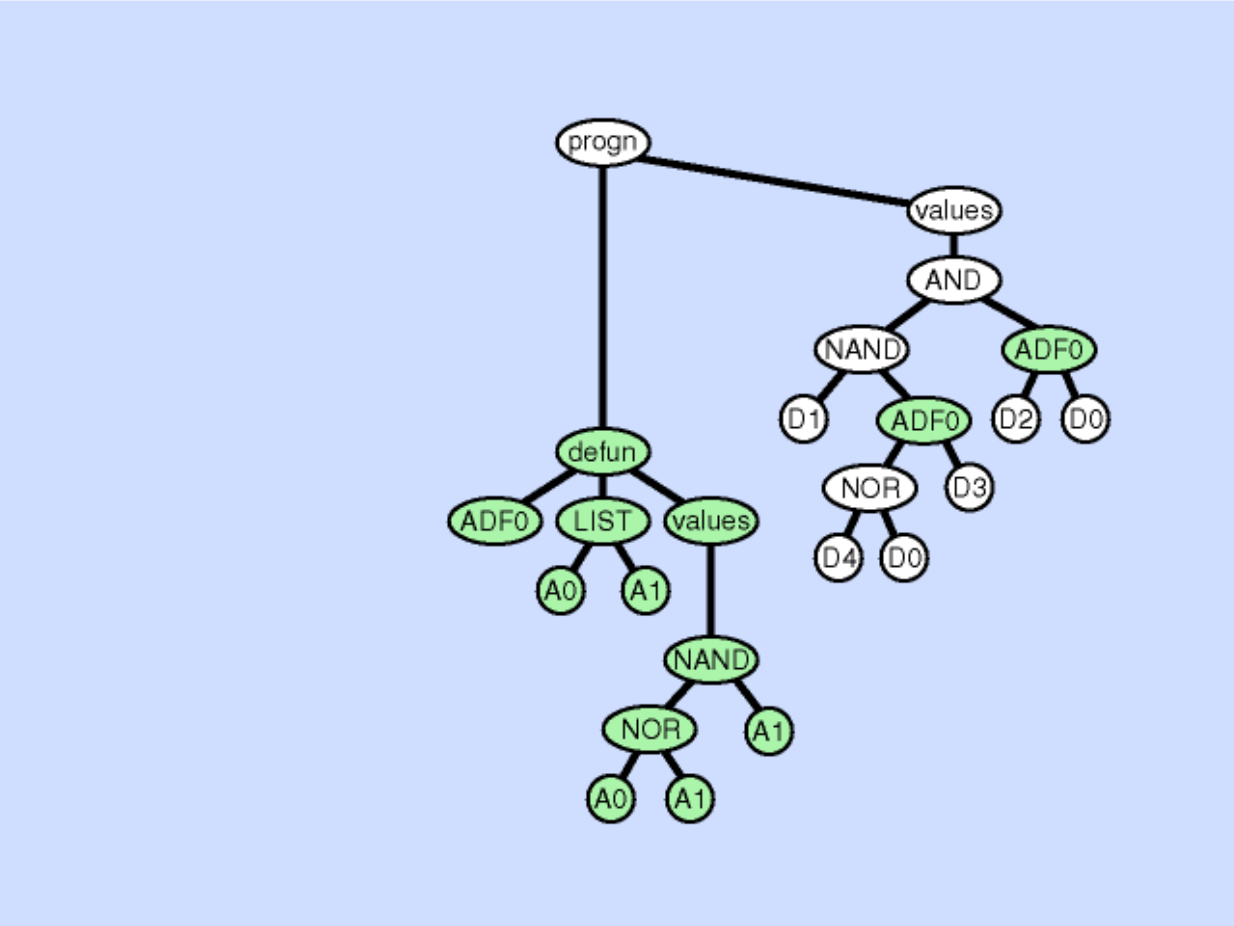
Create a Subroutine by Duplication



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Local Search in Continuous Spaces

- In many real-world problems the state space is continuous
  - Discretize the state space
    - e.g., assume only n different positions of a steering wheel or a gas pedal
  - Gradient Descent (Ascent)
    - hill-climbing using the gradient of the objective function f
    - f needs to be differentiable ($\rightarrow$ backpropagation in Neural Networks)
  - Empirical Gradient
    - empirically evaluate the response of f to small state changes
    - same as hill-climbing in a discretized space

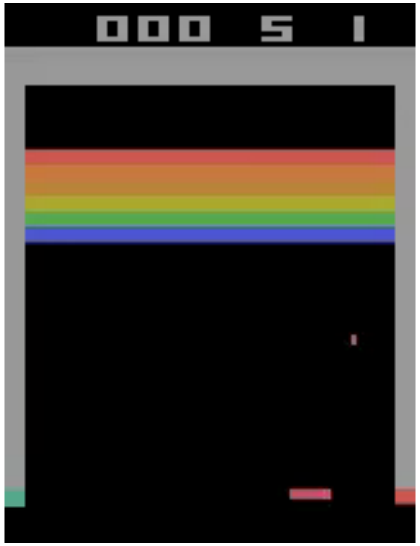
Might be relevant for Deep Reinforcement Learning

Improving Exploration in Evolution Strategies for Deep Reinforcement Learning via a Population of Novelty-Seeking Agents

Edoardo Conti* **Vashisht Madhavan*** **Felipe Petroski Such**
Joel Lehman **Kenneth O. Stanley** **Jeff Clune**
Uber AI Labs

Abstract

Evolution strategies (ES) are a family of black-box optimization algorithms able to train deep neural networks roughly as well as Q-learning and policy gradient methods on challenging deep reinforcement learning (RL) problems, but are much faster (e.g. hours vs. days) because they parallelize better. However, many RL problems require directed exploration because they have reward functions that are sparse or deceptive (i.e. contain local optima), and it is unknown how to encourage such exploration with ES. Here we show that algorithms that have been invented to promote directed exploration in small-scale evolved neural networks via populations of exploring agents, specifically novelty search (NS) and quality diversity (QD) algorithms, can be hybridized with ES to improve its performance on sparse or deceptive deep RL tasks, while retaining scalability. Our experiments confirm that the resultant new algorithms, NS-ES and two QD algorithms, NSR-ES and NSRA-ES, avoid local optima encountered by ES to achieve higher performance on Atari and simulated robots learning to walk around a deceptive trap. This paper



Edoardo Conti, Vashisht Madhavan, Felipe Petroski Such, Joel Lehman, Kenneth O. Stanley, Jeff Clune: Improving Exploration in Evolution Strategies for Deep Reinforcement Learning via a Population of Novelty-Seeking Agents. NeurIPS 2018: 5032-5043