multiple integration

- The definite integral at its core is in infinite sum (limit as the rectangle's width tend to 0). One interpretation as we've seen is areas.
- But if we have $z = f(x, y)$, we can use the same concepts of over estimating and underestimating the volume (in the 3-D case, just like area in the 2-D case) and squeeze out the error and get a result that helps us compute the volume:
  $lim_{\Delta x_i, \Delta y_j \to 0} \sum_{j=1}^{m} \sum_{i=1}^{n} f(c, d)\Delta x_i \Delta y_j$(also called a double sum) which turns out to be an integral when we do the procedure of "squeezing out the error". We denote that integral as $\int\int_R f(x, y)dA$. This exists if f is piecewise continuous. The way to evaluate this integral is as follows:
  $\int\int_R f(x, y)dA = \int_{x_1}^{x_2} [\int_{g(x_1)}^{g(x_2)} f(x, y)dy]\, dx.$
  To evaluate the above, we just we simply integrate the inner one first, which will give us a function of just x. Then we do the outer integral. That will give us a number.

  just like how we proved that the infinite sums lead to the integral (first fundamental theorem of integral calculus), we can prove similarly that the infinite sum $lim_{\Delta x_i, \Delta y_j \to 0} \sum_{j=1}^{m} \sum_{i=1}^{n} f(c, d)\Delta x_i \Delta y_j$ is the double integral $\int\int_R f(x, y)dA = \int_{x_1}^{x_2} [\int_{g(x_1)}^{g(x_2)} f(x, y)dy]\, dx$. ("squeeze out the error").
- Random fact: area of a parallelogram = $|AB \times AC| = side\ a \times side\ b \times sin(\theta)$
- A line integral is an integral, taken along a line, of any function that has a continuously varying value along that line. it is denoted by$\oint_c \vec{f}(\vec{R}) \cdot d\vec{R}$. Once you parametrize the multiple variables with a single parameter t and take the dot product, it is basically a regular integral but notice how it graphically is taken over a line. Also, notice how this is different from the multiple integral $\int\int_R f(x, y)dA = \int_{x_1}^{x_2} [\int_{g(x_1)}^{g(x_2)} f(x, y)dy]\, dx.$
- A region is called a connected region if you can get from any place in the region to any other place in the region without having to leave the region. Both the below regions are connected. The first is a simply connected region. The second is a multiply connected region. The difference is that for a simply connected region, its complement is a connected region. If the complement of a connected region is not connected, that region is called a multiply connected region.
- Green's Theorem: *If R is simply connected with boundary C, then* $\oint_c Mdx + Ndy = \int\int_R (\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y})dA$. *Simple proof is here.*
- The above can be applied to multiply connected regions by expressing the multiply connected regions as a sum of simply connected regions.