inverse differentiation

- We can think of all of differentiation as a single function. Given a function, we assign a unique function that we have called the derivative of the function. Can we think of the inverse of this function? In other words, given a function, what input(s) to the differentiation machine give the function as output? What functions are those for which the given function is the rate of change?
- How do we know that given an input to the inverse differentiation machine it has outputs that only differ by constants? This is because from the consequence of mean value theorem, if f′(b) - f′(a) = 0 (that is, if two functions have the same derivative), f(b) - f(a) is a constant. Thus proved.
- If we group all these functions in a group and make that one entity, we can make the inverse machine a function (single valued)
- $D^{-1}(f(x)) = \{G(x) : G'(x) = f(x)\}$. This is the implicit definition of the set.
- $D^{-1}(x^7) = \{G(x) : G(x) = \frac{x^8}{8} + c\}$
- Inverse differentiation also follows the addition rule: $D^{-1}[f(x) + g(x)] = D^{-1}[f(x)] + D^{-1}[g(x)]$.
- $D^{-1}(f(x)) = \int f(x)dx$. Think of the $\int blah\ dx$ as the inverse differentiation machine.
- Some rules:
  - $\int x^n dx = \frac{x^{n+1}}{n+1}$
  - $\int [f(x) + g(x)]dx = \int [f(x)]dx + \int [g(x)]dx$
  - $\int kx^n dx = k \int x^n dx$
- Notation: Why do we use the dx in the integral's symbol? Herbert Gross mentions something about treating $\frac{dy}{dx}$ as a fraction and cross multiplying, but I totally don't see why thats allowed, given what we've said so far. I'm hoping this will be clarified later.
- Inverse differentiation can be handled neatly just by knowing differentiation with a switch in emphasis. Then why do we study this? well, there are many cases, turns out, where we know the slope and want to find the function.
- This is called an indefinite integral and the word "indefinite" comes from the fact that the constant c is not known, unless a point (x_0, y_0) is given, which would fix the constant.
- if we know $\frac{dy}{dx}$ and we want to find out how much y changed by as we moved from $x = a$ to $x = b$, all we have to do is to compute $G(b) - G(a)$ where G is a function whose derivative is the given derivative. Here the constant c isn't required because it cancels out when we find change. Geometrically, the change ΔG is same for all curves specified by different c values.
- Once we invent $\int f(x)dx$ to denote all functions $\{G(x) : G' = f\}$, why not invent $\int_a^b f(x)dx$ to denote $G(b) - G(a)$? So we do! So $\int$ is indefinite integral and $\int_a^b$ is the definite integral.