The power of two samples for Generative Adversarial Networks

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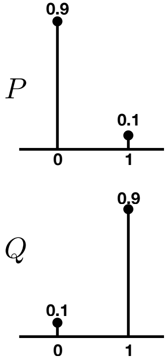
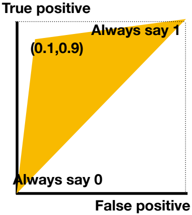
Simplest form of decision making: Binary hypothesis testing

- two hypotheses: 0 or 1
- observe one samples from P or Q
- decide 0 or 1

Example: Binary Symmetric Channel





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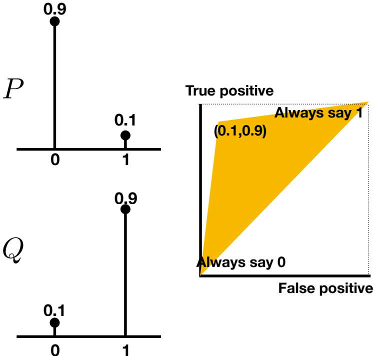
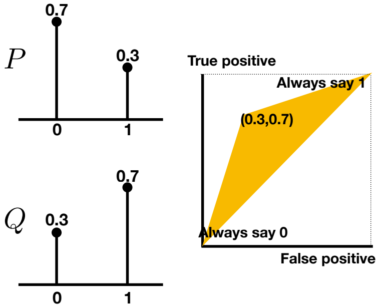
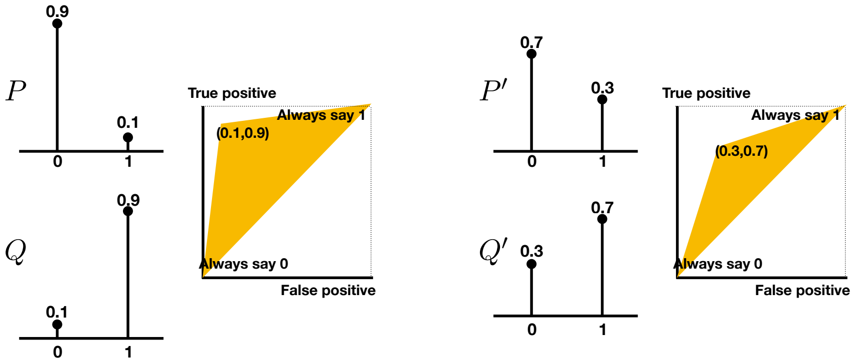




Blackwell's comparison theorem [1953]

- which experiment is better?



dominance in the region $\iff$ stochastic dominance

Binary hypothesis testing is at the center of Differential Privacy

Alice	22
Bob	45
⋮	⋮
Me	23

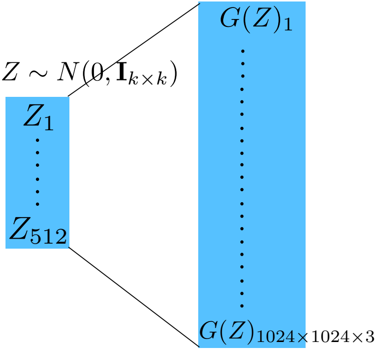
Alice	22
Bob	45
⋮	⋮

strongest **adversary**
who knows all the other entries

measure privacy with accuracy of
binary hypothesis testing

"The composition theorem for differential privacy", Peter Kairouz, Sewoong Oh, Pramod Viswanath

Generative models

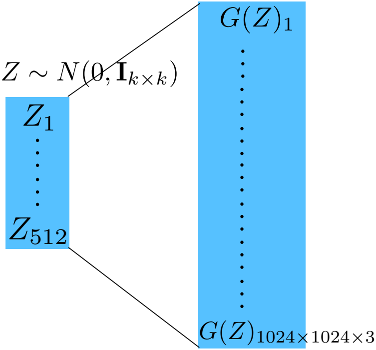


$G(Z) \in \mathbb{R}^{1024\times 1024\times 3}$

- A generative model is a black box that takes a random vector $Z \in \mathbb{R}^k$ and produces a sample vector $G(Z) \in \mathbb{R}^n$

[“Progressive Growing of GANs for Improved Quality, Stability, and Variation”, T. Karras, T. Aila, S. Laine, J. Lehtinen 2017]

Generative models



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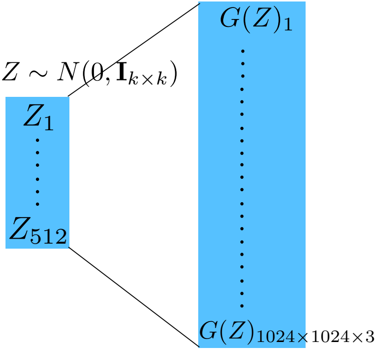

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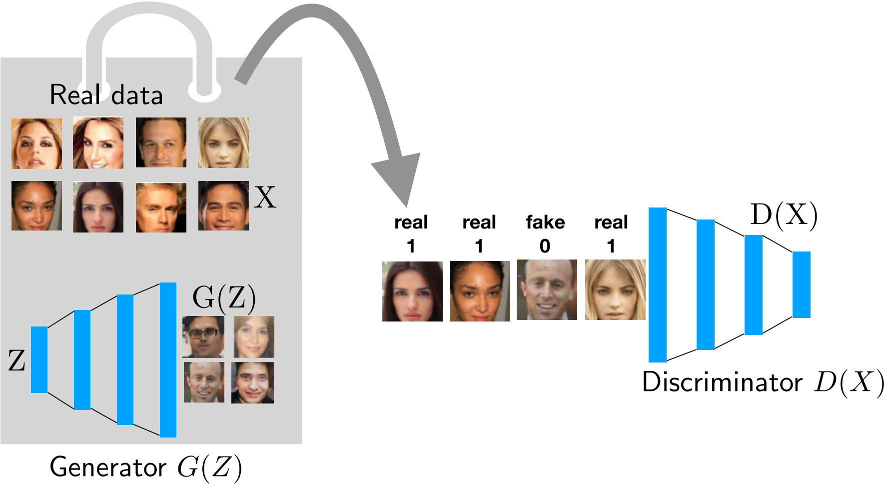
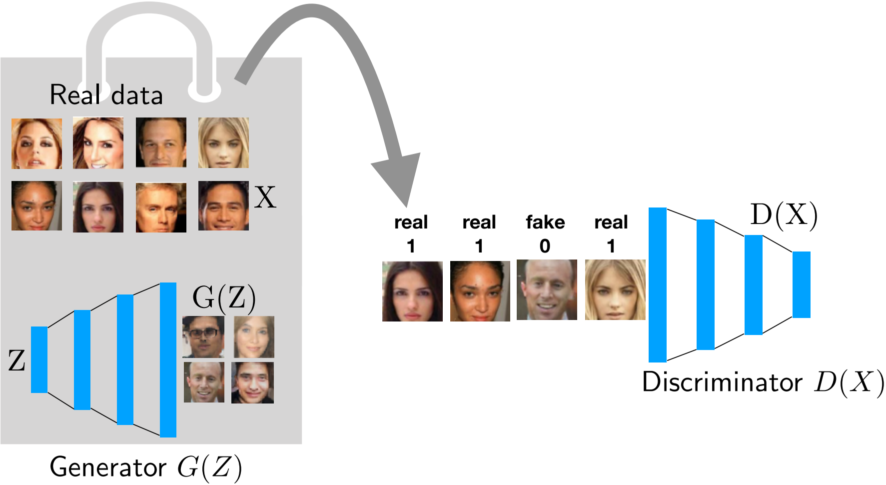
Generative Adversarial Networks (GAN)



$$\min_G \max_D V(G, D)$$

Generative Adversarial Networks (GAN)



$$\min_{G} \max_{D} V(G, D) \simeq \min_{Q} \underbrace{\frac{1}{2} D_{\mathrm{KL}}\left(Q \middle\| \frac{P+Q}{2}\right) + \frac{1}{2} D_{\mathrm{KL}}\left(P \middle\| \frac{P+Q}{2}\right)}_{\text{JS divergence}}$$

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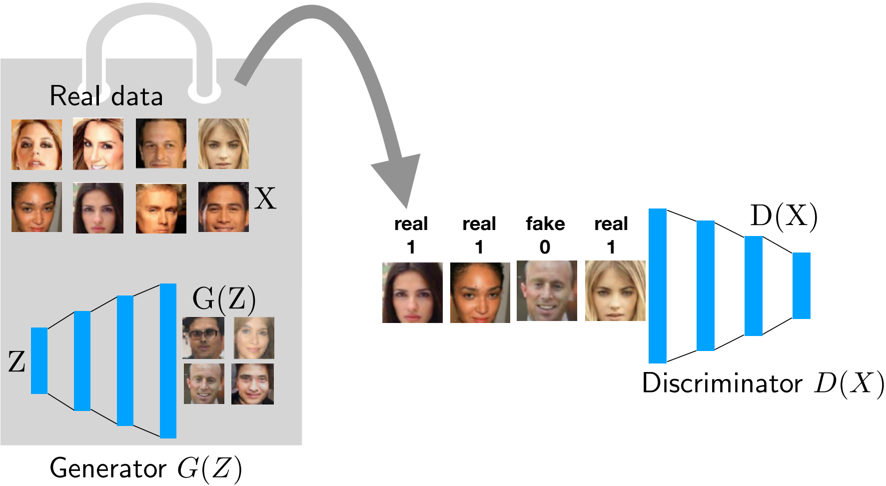
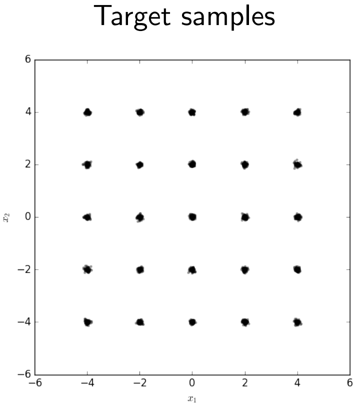
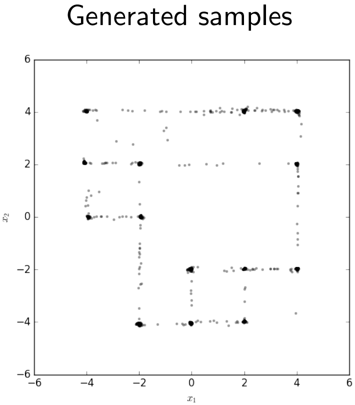


$$\min_G \max_D V(G,D) \simeq \min_Q \underbrace{D_{\mathrm{TV}}(P,Q)}_{\text{Total variation}}$$

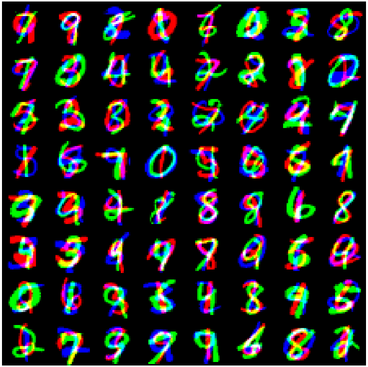
"Mode collapse" is a main challenge



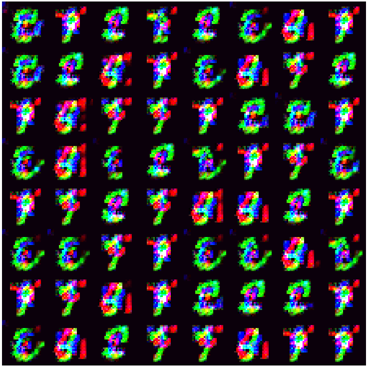


"Mode collapse" is a main challenge

Target samples



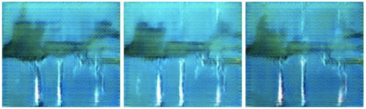
Generated samples



"Mode collapse" is a main challenge

- "A man in a orange jacket with sunglasses and a hat ski down a hill."

- "This guy is in black trunks and swimming underwater."



- "A tennis player in a blue polo shirt is looking down at the green court."

["Generating interpretable images with controllable structure", by Reed et al., 2016]

Formal mathematical characterization of mode collapse

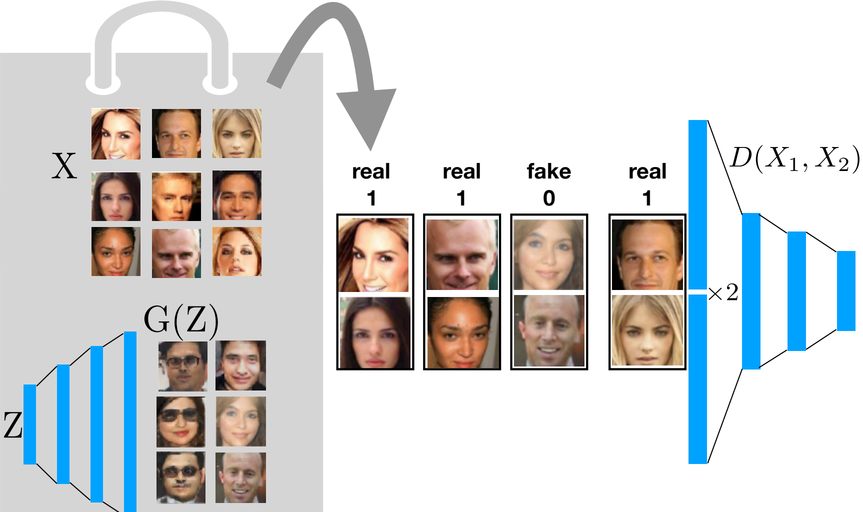
- we introduce a formal mathematical notion of mode collapse based on binary hypothesis testing and ROC curves
- we use this definition to make the following folklore precise

Lack of diversity is easier to detect
if the discriminator sees multiple sample jointly

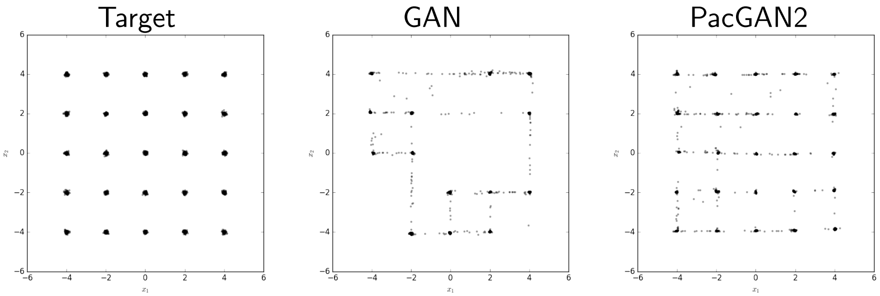
- this leads to a new architecture for tackling mode collapse

New framework: PacGAN

- lightweight overhead
- experimental results
- principled



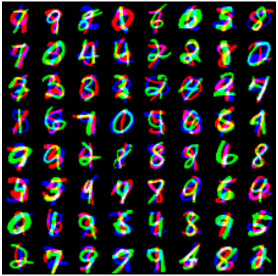
Benchmark tests



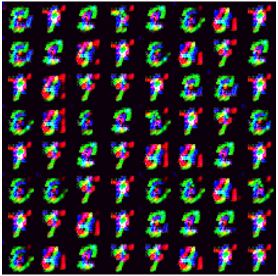
	Modes (Max 25)
GAN	17.3
PacGAN2	23.8
PacGAN3	24.6
PacGAN4	24.8

Benchmark datasets from VeeGAN paper

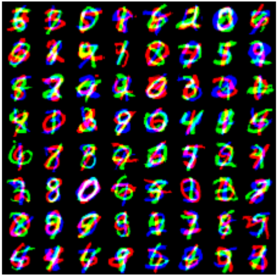
Real data



DCGAN



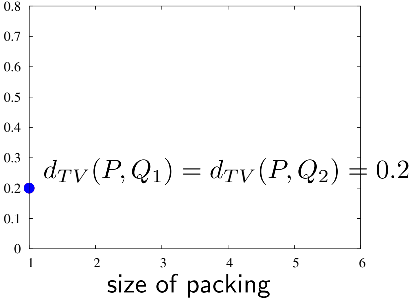
PacDCGAN2



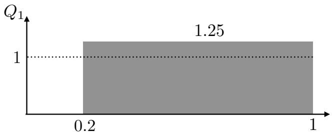
	Modes (Max 1000)
DCGAN	99.0
ALI	16.0
Unrolled GAN	48.7
VeeGAN	150.0
PacDCGAN2	1000.0
PacDCGAN3	1000.0
PacDCGAN4	1000.0

Intuition behind packing via toy example

Target distribution P

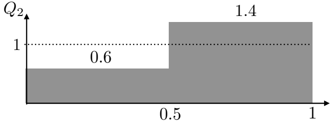


Generator Q_1
with mode collapse



$d_{\mathrm{TV}}(P, Q_1) = 0.2$

Generator Q_2
without mode collapse



$d_{\mathrm{TV}}(P, Q_2) = 0.2$

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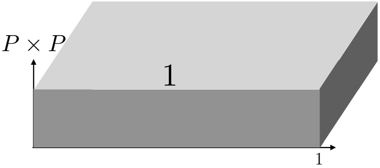

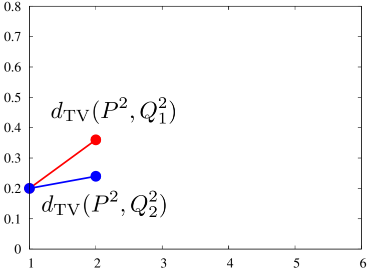

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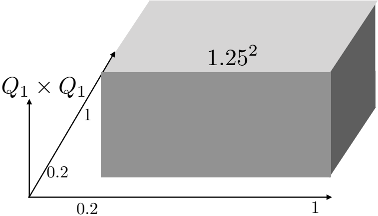
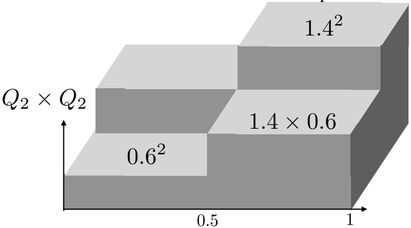


$d_{\mathrm{TV}}(P \times P, Q_1 \times Q_1) = 0.36$

Generator Q_2
without mode collapse



$d_{\mathrm{TV}}(P \times P, Q_2 \times Q_2) = 0.24$

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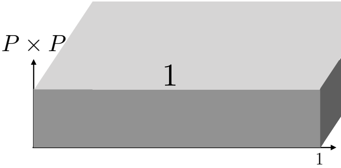
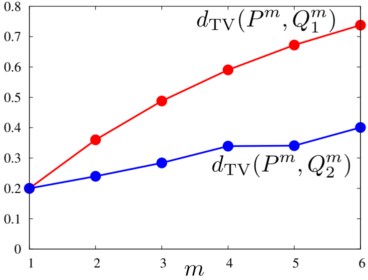
Target distribution P





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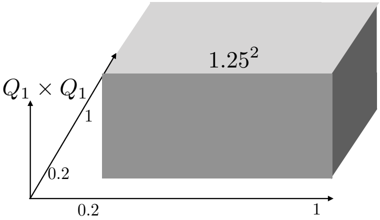

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Generator Q_2
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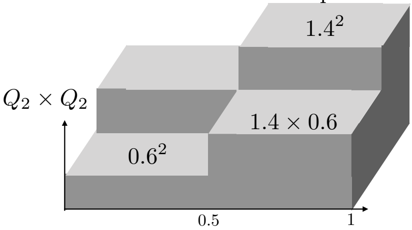

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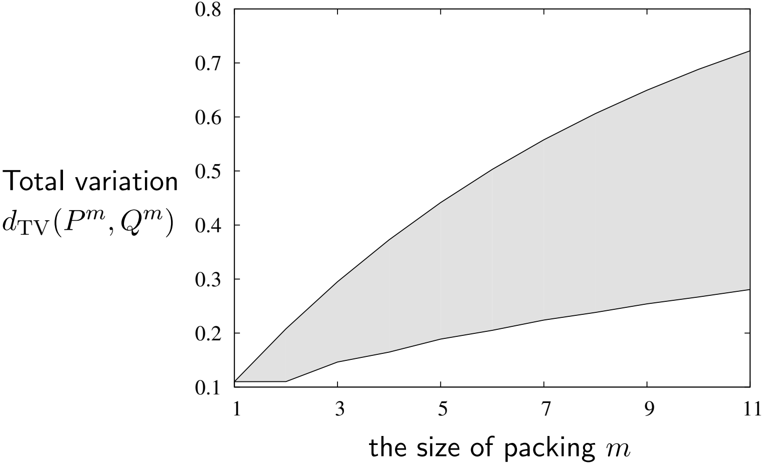
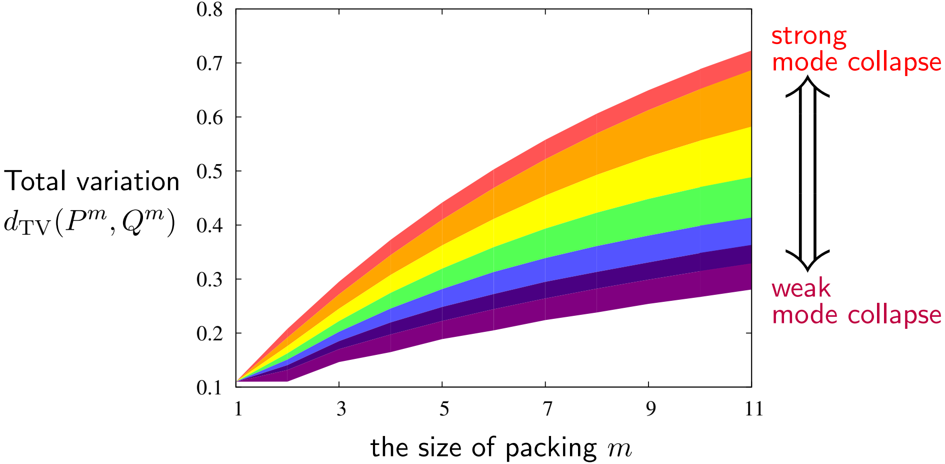
Evolution of TV distances

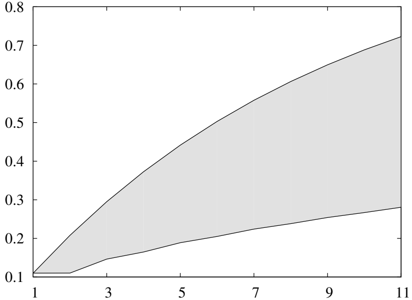


Evolution of TV distances through the prism of packing



Through packing, the target-generator pairs are expanded over the strengths of the mode collapse

$d_{\mathrm{TV}}(P^m, Q^m)$



degree of packing m

$$\begin{array}{ll} \max\limits_{P,Q} / \min\limits_{P,Q} & d_{\mathrm{TV}}(P^2, Q^2) \\ \text{subject to} & d_{\mathrm{TV}}(P, Q) = \tau \end{array}$$

- we focus on $m = 2$ for this talk

Intuition from Blackwell

Definition [mode collapse region]

We say a pair (P, Q) of a target distribution P and a generator distribution Q has (ε, δ)-**mode collapse** if there exists a set S such that

$$P(S) \geq \delta \ , \qquad \text{and} \qquad Q(S) \leq \varepsilon \ .$$

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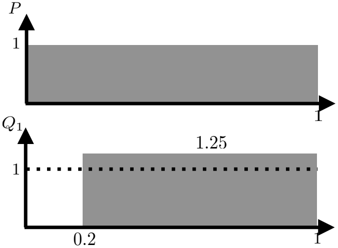

Generator Q_1
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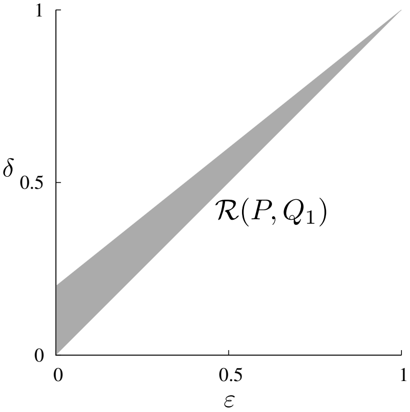

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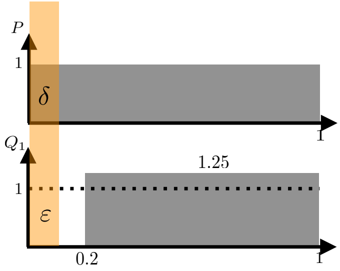

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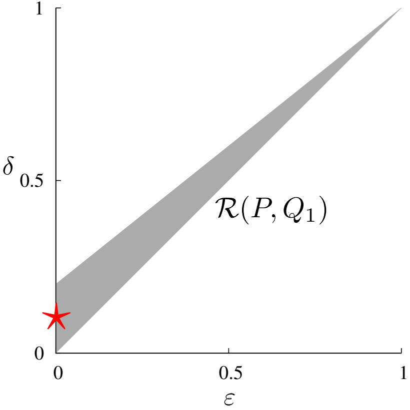

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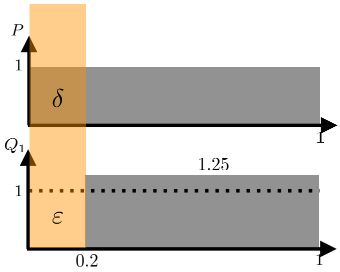

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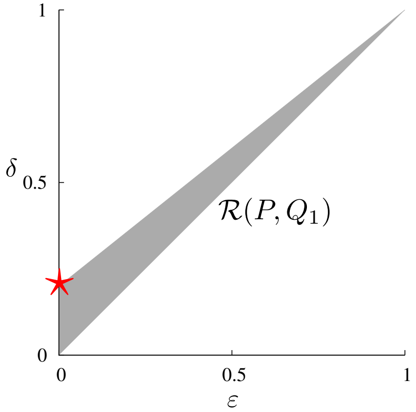

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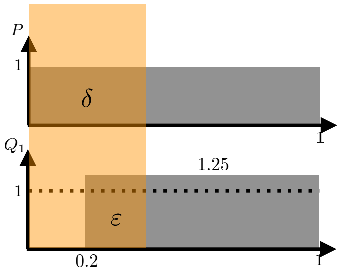

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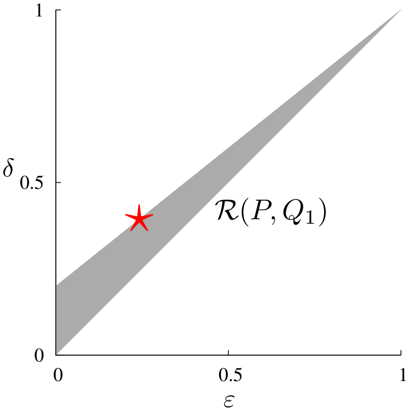

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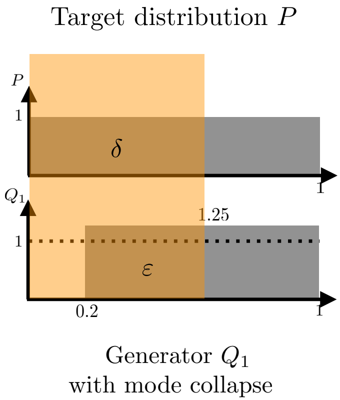

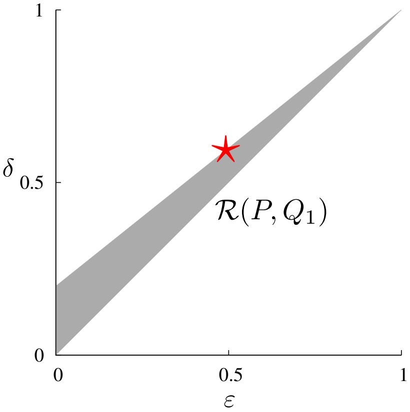

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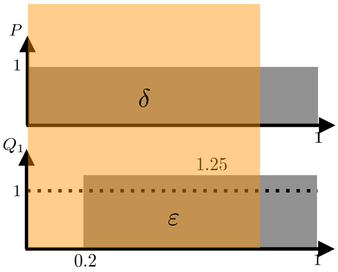

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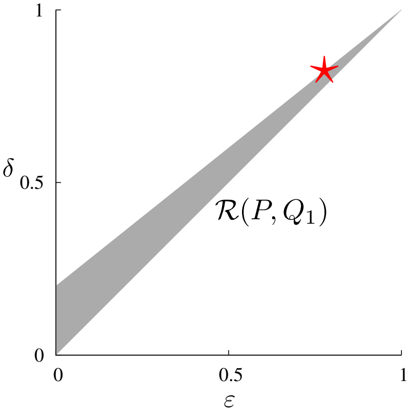

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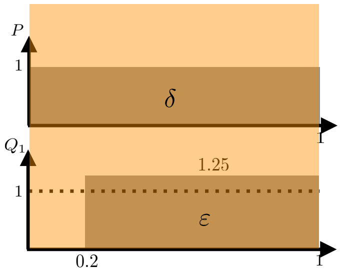

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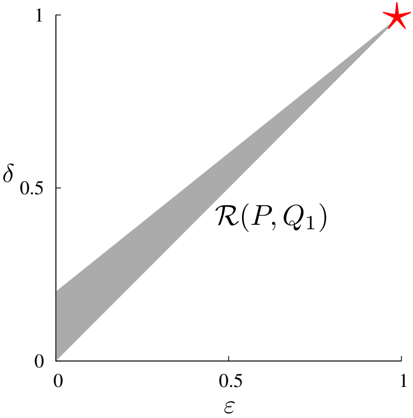

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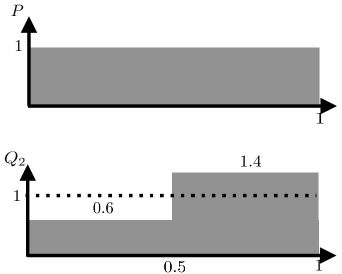
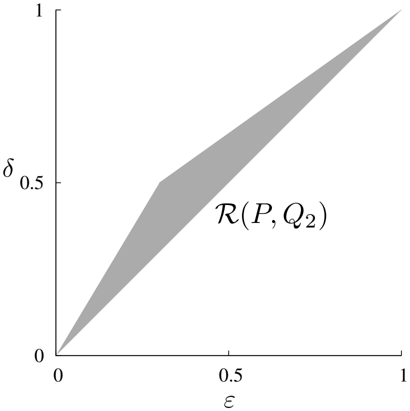


Generator Q_2
without mode collapse



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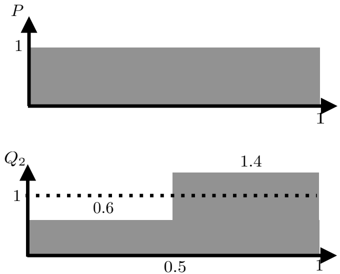

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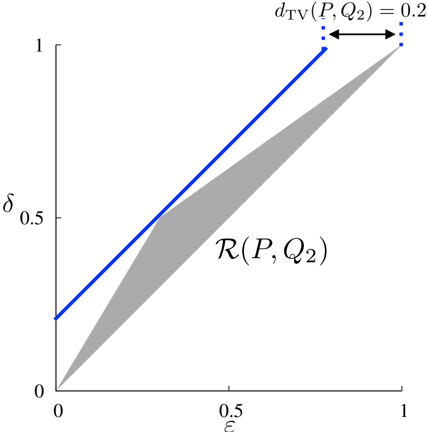
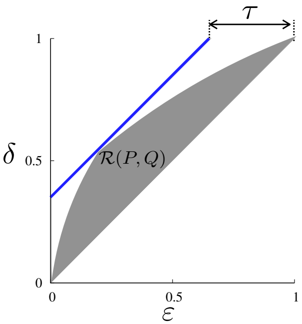


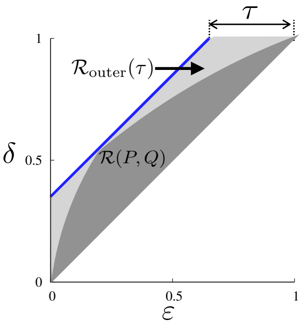
Upper bound



$$\max_{P,Q} \quad d_{\mathrm{TV}}(P^2, Q^2)$$

$$\text{subject to} \quad d_{\mathrm{TV}}(P,Q) = \tau$$

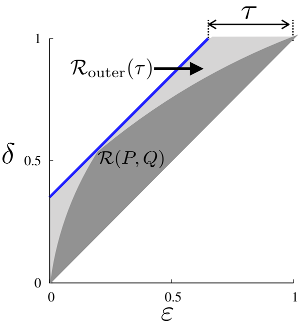
Upper bound

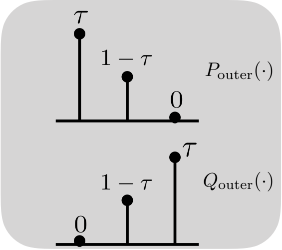


$$\max_{P,Q} \quad d_{\text{TV}}(P^2, Q^2)$$
$$\text{subject to} \quad d_{\text{TV}}(P, Q) = \tau$$

$$\mathcal{R}(P,Q) \subseteq \mathcal{R}_{\text{outer}}(\tau)$$

Upper bound

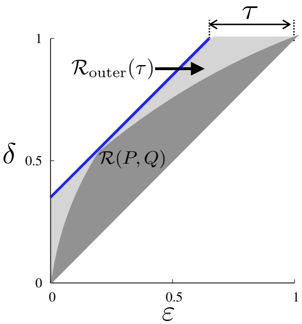


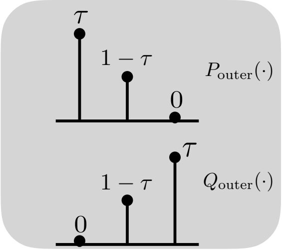


$$\begin{aligned} \max_{P,Q} \quad & d_{\mathrm{TV}}(P^2, Q^2) \\ \text{subject to} \quad & d_{\mathrm{TV}}(P,Q) = \tau \end{aligned}$$

$$\mathcal{R}(P,Q) \subseteq \mathcal{R}_{\text{outer}}(\tau)$$

Upper bound





$$\max_{P,Q} \quad d_{\text{TV}}(P^2, Q^2)$$

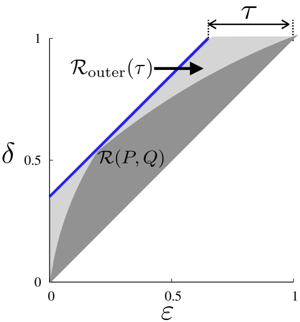
$$\text{subject to} \quad d_{\text{TV}}(P, Q) = \tau$$

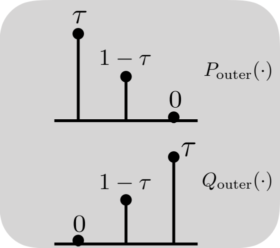
$$\begin{aligned} \mathcal{R}(P,Q) &\subseteq \mathcal{R}_{\text{outer}}(\tau) \\ \mathcal{R}(P^2,Q^2) &\subseteq \mathcal{R}(P_{\text{outer}}^2, Q_{\text{outer}}^2) \end{aligned}$$

Blackwell's theorem

$$\begin{aligned} & \mathcal{R}(P,Q) \subseteq \mathcal{R}(P',Q') \\ \Rightarrow \quad & \mathcal{R}(P^2,Q^2) \subseteq \mathcal{R}(P'^2,Q'^2) \end{aligned}$$

Upper bound





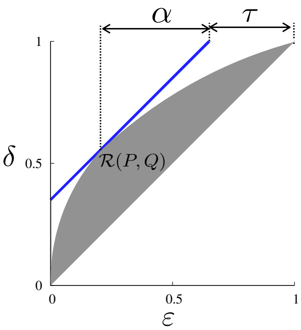
$$\begin{aligned} \max_{P,Q} \quad & d_{\text{TV}}(P^2, Q^2) \\ \text{subject to} \quad & d_{\text{TV}}(P, Q) = \tau \end{aligned}$$

$$\begin{aligned} \mathcal{R}(P,Q) &\subseteq \mathcal{R}_{\text{outer}}(\tau) \\ \mathcal{R}(P^2,Q^2) &\subseteq \mathcal{R}(P^2_{\text{outer}}, Q^2_{\text{outer}}) \\ d_{\text{TV}}(P^2,Q^2) &\leq \underbrace{d_{\text{TV}}(P^2_{\text{outer}}, Q^2_{\text{outer}})}_{1-(1-\tau)^2} \end{aligned}$$

Blackwell's theorem

$$\begin{aligned} & \mathcal{R}(P,Q) \subseteq \mathcal{R}(P',Q') \\ \Rightarrow \quad & \mathcal{R}(P^2,Q^2) \subseteq \mathcal{R}(P'^2,Q'^2) \end{aligned}$$

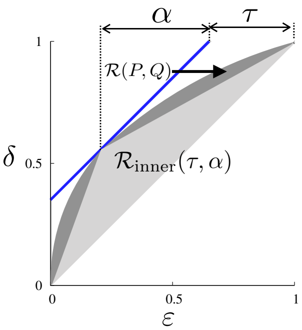
Lower bound



$$\min_{P,Q} \quad d_{\mathrm{TV}}(P^2, Q^2)$$

$$\text{subject to} \quad d_{\mathrm{TV}}(P, Q) = \tau$$

Lower bound

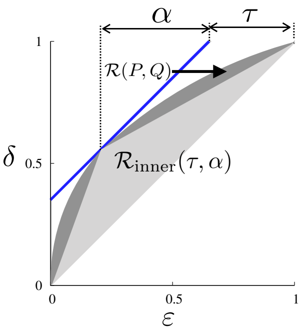


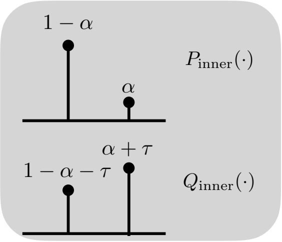
$$\min_{P,Q} \quad d_{\mathrm{TV}}(P^2,Q^2)$$

subject to $\quad d_{\mathrm{TV}}(P,Q)=\tau$

$$\mathcal{R}_{\mathrm{inner}}(\tau,\alpha) \quad \subseteq \quad \mathcal{R}(P,Q)$$

Lower bound



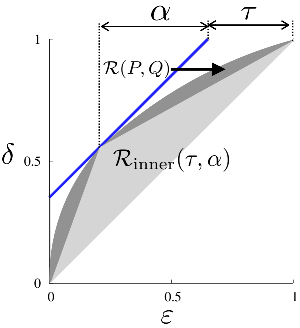


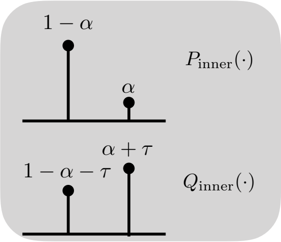
$$\min_{P,Q} \quad d_{\mathrm{TV}}(P^2, Q^2)$$

$$\text{subject to} \quad d_{\mathrm{TV}}(P,Q) = \tau$$

$$\mathcal{R}_{\text{inner}}(\tau,\alpha) \subseteq \mathcal{R}(P,Q)$$

Lower bound





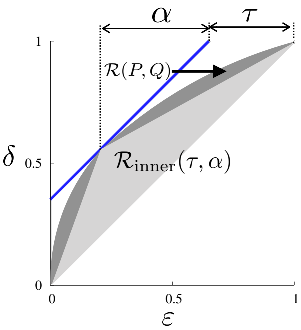
$$\begin{aligned} \min_{P,Q} \quad & d_{\text{TV}}(P^2, Q^2) \\ \text{subject to} \quad & d_{\text{TV}}(P,Q) = \tau \end{aligned}$$

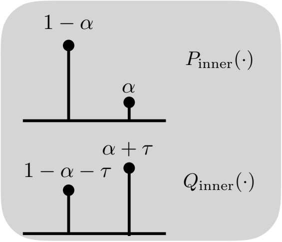
$$\begin{aligned} \mathcal{R}_{\text{inner}}(\tau,\alpha) \quad &\subseteq \quad \mathcal{R}(P,Q) \\ \mathcal{R}(P_{\text{inner}}^2, Q_{\text{inner}}^2) \quad &\subseteq \quad \mathcal{R}(P^2,Q^2) \end{aligned}$$

Blackwell's theorem

$$\begin{aligned} & \mathcal{R}(P,Q) \subseteq \mathcal{R}(P',Q') \\ \Rightarrow \quad & \mathcal{R}(P^2,Q^2) \subseteq \mathcal{R}(P'^2,Q'^2) \end{aligned}$$

Lower bound



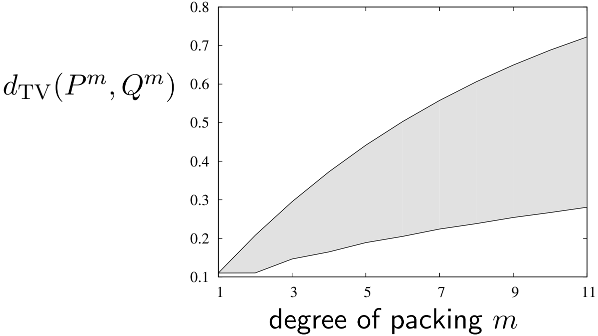


$$\begin{aligned} \min_{P,Q} \quad & d_{\text{TV}}(P^2, Q^2) \\ \text{subject to} \quad & d_{\text{TV}}(P,Q) = \tau \end{aligned}$$

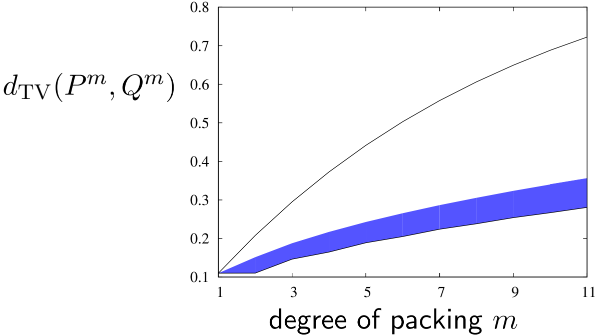
$$\begin{aligned} \mathcal{R}_{\text{inner}}(\tau,\alpha) & \subseteq \mathcal{R}(P,Q) \\ \mathcal{R}(P_{\text{inner}}^2, Q_{\text{inner}}^2) & \subseteq \mathcal{R}(P^2,Q^2) \\ \underbrace{\min_{\alpha} \; d_{\text{TV}}(P_{\text{inner}}^2, Q_{\text{inner}}^2)}_{\tau} & \leq d_{\text{TV}}(P^2,Q^2) \end{aligned}$$

Blackwell's theorem

$$\begin{aligned} & \mathcal{R}(P,Q) \subseteq \mathcal{R}(P',Q') \\ \Rightarrow \quad & \mathcal{R}(P^2,Q^2) \subseteq \mathcal{R}(P'^2,Q'^2) \end{aligned}$$

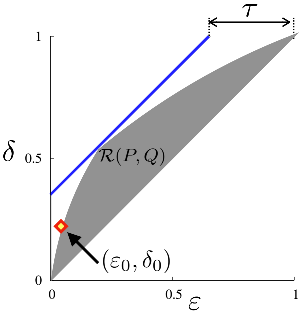


$$\begin{array}{ll} \max\limits_{P,Q} \,/\, \min\limits_{P,Q} & d_{\mathrm{TV}}(P^2, Q^2) \\ \text{subject to} & d_{\mathrm{TV}}(P, Q) = \tau \end{array}$$



$$
\begin{aligned}
\max_{P,Q} \quad & d_{\mathrm{TV}}(P^2, Q^2) \\
\text{subject to} \quad & d_{\mathrm{TV}}(P, Q) = \tau \\
& \text{no } (\varepsilon_0, \delta_0)\text{-mode collapse}
\end{aligned}
$$

Upper bound without $(\varepsilon_0, \delta_0)$-mode collapse

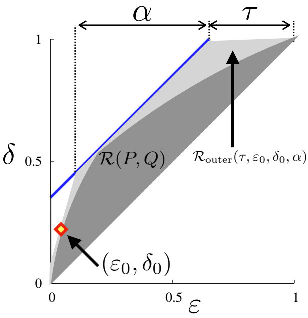


$$\max_{P,Q} \quad d_{\mathrm{TV}}(P^2, Q^2)$$

subject to $\quad d_{\mathrm{TV}}(P, Q) = \tau$

no $(\varepsilon_0, \delta_0)$-mode collapse

Upper bound without $(\varepsilon_0, \delta_0)$-mode collapse



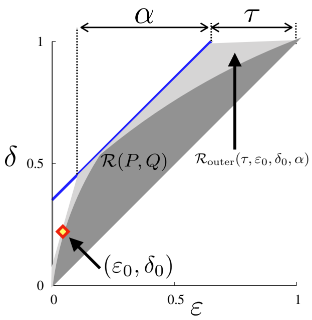
$$\max_{P,Q} \quad d_{\text{TV}}(P^2, Q^2)$$

subject to $\quad d_{\text{TV}}(P, Q) = \tau$

no $(\varepsilon_0, \delta_0)$-mode collapse

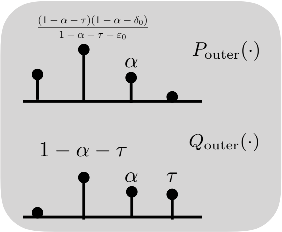
$$\mathcal{R}(P,Q) \subseteq \mathcal{R}_{\text{outer}}(\tau, \varepsilon_0, \delta_0, \alpha)$$

Upper bound without $(\varepsilon_0, \delta_0)$-mode collapse

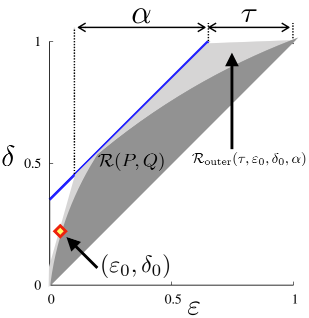


$$
\begin{array}{rl}
\max\limits_{P,Q} & d_{\mathrm{TV}}(P^2, Q^2) \\
\text{subject to} & d_{\mathrm{TV}}(P, Q) = \tau \\
 & \text{no } (\varepsilon_0, \delta_0)\text{-mode collapse}
\end{array}
$$

$$\mathcal{R}(P, Q) \quad \subseteq \quad \mathcal{R}_{\text{outer}}(\tau, \varepsilon_0, \delta_0, \alpha)$$

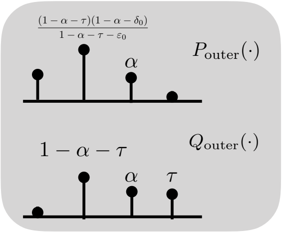


Upper bound without $(\varepsilon_0, \delta_0)$-mode collapse



$$
\begin{array}{ll}
\max\limits_{P,Q} & d_{\rm TV}(P^2, Q^2) \\
\text{subject to} & d_{\rm TV}(P,Q) = \tau \\
& \text{no } (\varepsilon_0, \delta_0)\text{-mode collapse}
\end{array}
$$

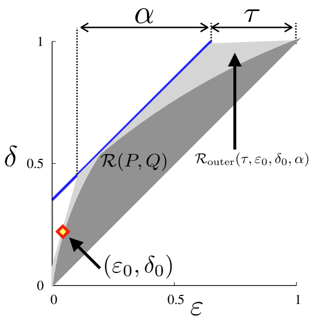
$$
\begin{aligned}
\mathcal{R}(P,Q) &\subseteq \mathcal{R}_{\rm outer}(\tau, \varepsilon_0, \delta_0, \alpha) \\
\mathcal{R}(P^2,Q^2) &\subseteq \mathcal{R}(P^2_{\rm outer}, Q^2_{\rm outer})
\end{aligned}
$$



Blackwell's theorem

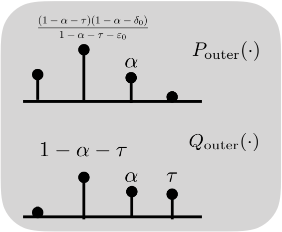
$$
\begin{aligned}
& \mathcal{R}(P,Q) \subseteq \mathcal{R}(P',Q') \\
\Rightarrow\ & \mathcal{R}(P^2,Q^2) \subseteq \mathcal{R}(P'^2,Q'^2)
\end{aligned}
$$

Upper bound without $(\varepsilon_0, \delta_0)$-mode collapse



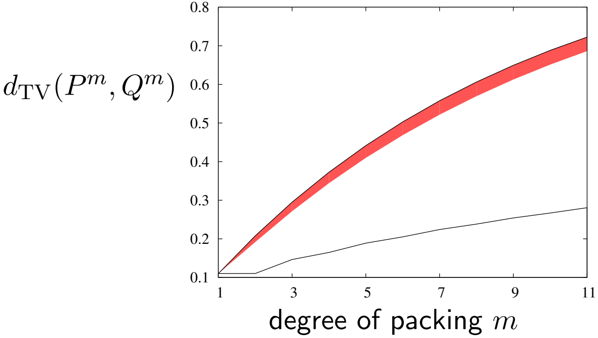
$$
\begin{array}{rl}
\max\limits_{P,Q} & d_{\mathrm{TV}}(P^2, Q^2) \\
\text{subject to} & d_{\mathrm{TV}}(P, Q) = \tau \\
 & \text{no } (\varepsilon_0, \delta_0)\text{-mode collapse}
\end{array}
$$

$$
\begin{array}{rcl}
\mathcal{R}(P,Q) & \subseteq & \mathcal{R}_{\mathrm{outer}}(\tau, \varepsilon_0, \delta_0, \alpha) \\
\mathcal{R}(P^2,Q^2) & \subseteq & \mathcal{R}(P^2_{\mathrm{outer}}, Q^2_{\mathrm{outer}}) \\
d_{\mathrm{TV}}(P^2,Q^2) & \leq & \underbrace{\max\limits_{\alpha}\ d_{\mathrm{TV}}(P^2_{\mathrm{outer}}, Q^2_{\mathrm{outer}})}_{\text{simple to evaluate}}
\end{array}
$$



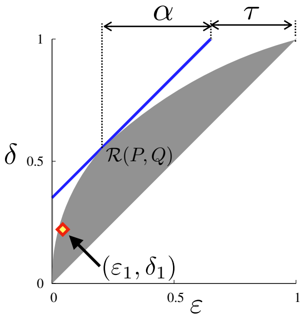
Blackwell's theorem

$$
\begin{array}{rl}
 & \mathcal{R}(P,Q) \subseteq \mathcal{R}(P', Q') \\
\Rightarrow & \mathcal{R}(P^2,Q^2) \subseteq \mathcal{R}(P'^2, Q'^2)
\end{array}
$$



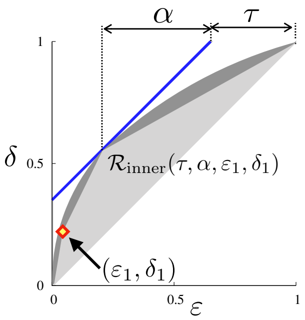
$$
\begin{array}{rl}
\min\limits_{P,Q} & d_{\mathrm{TV}}(P^2, Q^2) \\
\text{subject to} & d_{\mathrm{TV}}(P, Q) = \tau \\
& (\varepsilon_1, \delta_1)\text{-mode collapse}
\end{array}
$$

Lower bound with $(\varepsilon_1, \delta_1)$-mode collapse



$$\begin{array}{rl} \min_{P,Q} & d_{\mathrm{TV}}(P^2, Q^2) \\ \text{subject to} & d_{\mathrm{TV}}(P, Q) = \tau \\ & (\varepsilon_1, \delta_1)\text{-mode collapse} \end{array}$$

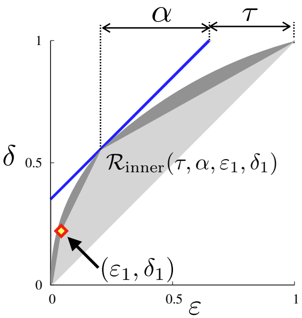
Lower bound with $(\varepsilon_1, \delta_1)$-mode collapse

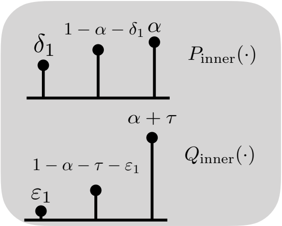


$$\begin{aligned} \min_{P,Q} \quad & d_{\text{TV}}(P^2, Q^2) \\ \text{subject to} \quad & d_{\text{TV}}(P, Q) = \tau \\ & (\varepsilon_1, \delta_1)\text{-mode collapse} \end{aligned}$$

$$\mathcal{R}_{\text{inner}}(\tau, \alpha, \varepsilon_1, \delta_1) \quad \subseteq \quad \mathcal{R}(P, Q)$$

Lower bound with $(\varepsilon_1, \delta_1)$-mode collapse

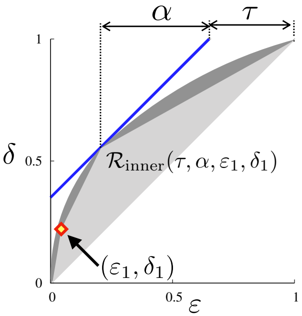




$$\begin{aligned} \min_{P,Q} \quad & d_{\text{TV}}(P^2, Q^2) \\ \text{subject to} \quad & d_{\text{TV}}(P, Q) = \tau \\ & (\varepsilon_1, \delta_1)\text{-mode collapse} \end{aligned}$$

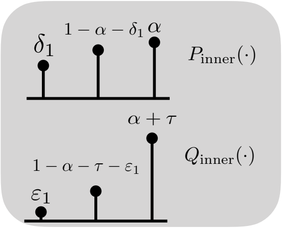
$$\mathcal{R}_{\text{inner}}(\tau, \alpha, \varepsilon_1, \delta_1) \quad \subseteq \quad \mathcal{R}(P, Q)$$

Lower bound with $(\varepsilon_1, \delta_1)$-mode collapse



$$
\begin{array}{rl}
\min\limits_{P,Q} & d_{\mathrm{TV}}(P^2, Q^2) \\
\text{subject to} & d_{\mathrm{TV}}(P, Q) = \tau \\
 & (\varepsilon_1, \delta_1)\text{-mode collapse}
\end{array}
$$

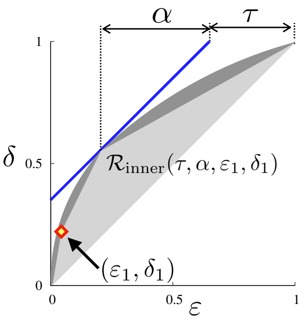
$$
\begin{array}{rcl}
\mathcal{R}_{\text{inner}}(\tau, \alpha, \varepsilon_1, \delta_1) & \subseteq & \mathcal{R}(P, Q) \\
\mathcal{R}(P^2_{\text{inner}}, Q^2_{\text{inner}}) & \subseteq & \mathcal{R}(P^2, Q^2)
\end{array}
$$



Blackwell's theorem

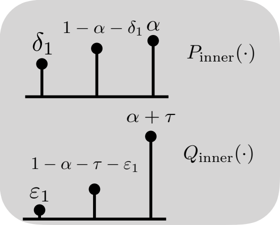
$$
\begin{array}{rl}
 & \mathcal{R}(P, Q) \subseteq \mathcal{R}(P', Q') \\
\Rightarrow & \mathcal{R}(P^2, Q^2) \subseteq \mathcal{R}(P'^2, Q'^2)
\end{array}
$$

Lower bound with $(\varepsilon_1, \delta_1)$-mode collapse



$$\begin{array}{rl} \min\limits_{P,Q} & d_{\text{TV}}(P^2, Q^2) \\ \text{subject to} & d_{\text{TV}}(P, Q) = \tau \\ & (\varepsilon_1, \delta_1)\text{-mode collapse} \end{array}$$

$$\begin{array}{rcl} \mathcal{R}_{\text{inner}}(\tau, \alpha, \varepsilon_1, \delta_1) & \subseteq & \mathcal{R}(P, Q) \\ \mathcal{R}(P^2_{\text{inner}}, Q^2_{\text{inner}}) & \subseteq & \mathcal{R}(P^2, Q^2) \\ \underbrace{\min\limits_{\alpha}\; d_{\text{TV}}(P^2_{\text{inner}}, Q^2_{\text{inner}})}_{\text{simple to evaluate}} & \leq & d_{\text{TV}}(P^2, Q^2) \end{array}$$

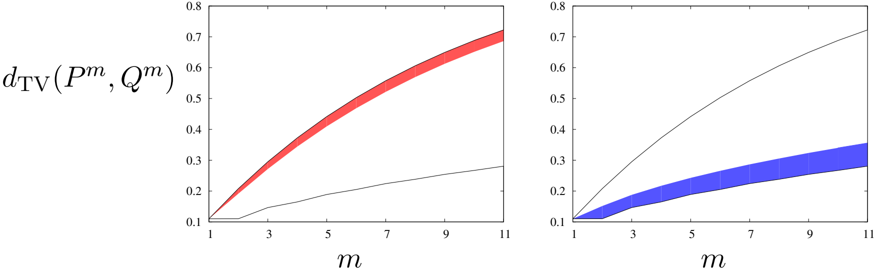


Blackwell's theorem

$$\begin{array}{rl} & \mathcal{R}(P, Q) \subseteq \mathcal{R}(P', Q') \\ \Rightarrow & \mathcal{R}(P^2, Q^2) \subseteq \mathcal{R}(P'^2, Q'^2) \end{array}$$

Achievable TV distances for distributions

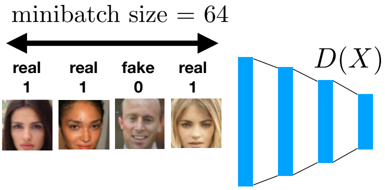
with $(\varepsilon_1, \delta_1)$-mode collapse   without $(\varepsilon_0, \delta_0)$-mode collapse



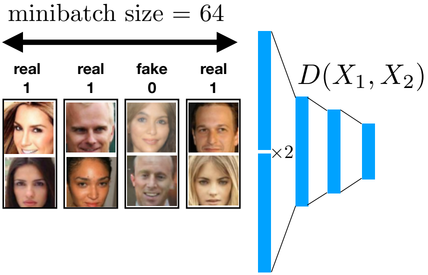
with packing, the discriminator naturally penalizes (P, Q) with severe mode collapses

Could we be cheating (hyper-parameter tuning)?

1. Discriminator size



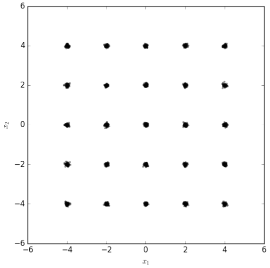
GAN



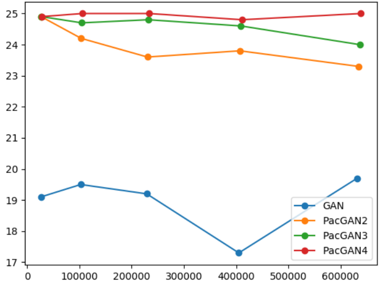
PacGAN2

Could we be cheating (hyper-parameter tuning)?

1. Discriminator size



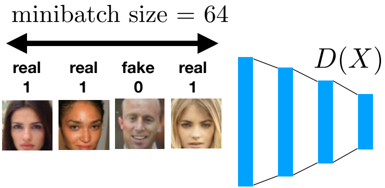
modes captured



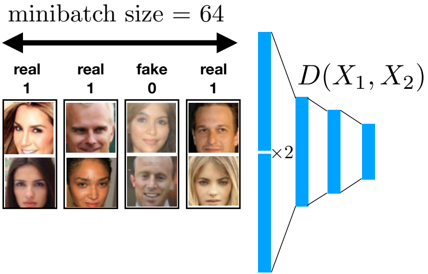
# of parameters in $D(\cdot)$

Could we be cheating (hyper-parameter tuning)?

2. Minibatch size



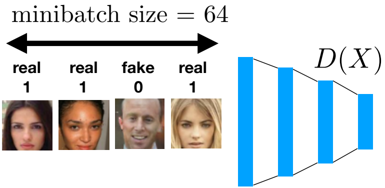
GAN



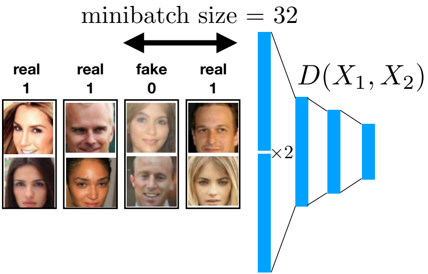
PacGAN2

Could we be cheating (hyper-parameter tuning)?

2. Minibatch size



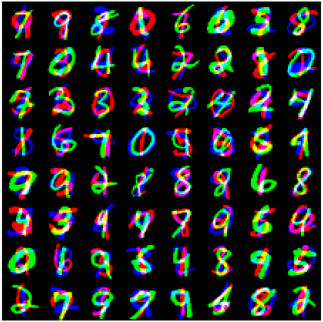
GAN



PacGAN2

Could we be cheating (hyper-parameter tuning)?

2. Minibatch size



	Modes
DCGAN	99.0
PacDCGAN2	1000.0

Our paper is: “PacGAN: the power of two samples in generative adversarial networks”

All codes for the experiments at:
https://github.com/fjxmlzn/PacGAN