Algorithm Analysis

Mark Sort

```
for(int i=0; i<N-1; i++){
    for(int j=i+1; j<N; j++){
        If(a[i] > a[j]){
            temp = a[i]
            a[i] = a[j]
            a[j] = temp;
        }
    }
}
```

$O_b \quad O_i \quad O_j \quad PO_s$

$$O_b + \sum_{i=0}^{N-2}\left(O_i + \sum_{j=i+1}^{N-1}\left(O_j + PO_s\right)\right)$$

O represents operations which equates $T(O) \rightarrow$ clock cycles

$$\sum_{j=x}^{y} 1 = (y-x)+1$$

Let $O_j + PO_s = O_{js}$

So $\rightarrow$ $((N-1)-(i+1)+1)O_{js}$

$(N-i-1)O_{js}$

$$O_b + \sum_{i=0}^{N-2}\left(O_i + (N-1-i)\,O_{ts}\right)$$

$$\sum_{i=0}^{x} i = x\cdot(x+1)/2$$

$$\sum_{i=0}^{x}\left((i+1)^2 - i^2\right) = \begin{aligned}&(\cancel{1^2} - 0^2)\\ +&(\cancel{2^2} - \cancel{1^2})\\ +&(3^2 - \cancel{2^2})\\ &\vdots\\ &(\cancel{x^2} - (\cancel{x-1})^2)\\ +&((x+1)^2 - \cancel{x^2})\end{aligned}$$

$$\sum_{i=0}^{x}\left(\cancel{i^2} + 2i + 1 - \cancel{i^2}\right) = (x+1)^2$$

$$\sum_{i=0}^{x}(2i+1)$$

$$2\sum_{i=0}^{x} i + \sum_{i=0}^{x} 1$$

$$\sum_{i=0}^{x} i = \frac{(x+1)^2 - (x+1)}{2}$$

$$= \frac{x^2 + 2x + 1 - x}{2}$$

$$= \frac{x^2 + x + 1}{2}$$

$$= \frac{(x+1)(x+1-1)}{2}$$

$$= (x+1)\,x/2$$

$$O_b + \sum_{i=0}^{N-2} \left(O_i + (N-1-i) O_{ds} \right)$$

$$O_b + \sum_{i=0}^{N-2} \left(O_i + (N-1) O_{ds} - i\, O_{ds} \right)$$

$$O_b + (N-1)\left(O_i + (N-1) O_{ds} \right) - O_{ds} \sum_{i=0}^{N-2} i$$

$$(N-1)^2 O_{ds} + (N-1) O_i + O_b - \frac{(N-2)(N+1)}{2} O_{ds}$$

$f(N) =$ second order polynomial in N

$$\left(O_{ds} - \frac{O_{ds}}{2} \right) N^2 + \left(-2 O_{ds} + O_i + \tfrac{1}{2} O_{ds} \right) N + \left(2 O_{ds} - O_i + O_b \right)$$

$$c'' N^2 + c' N + c$$

$$c'' = (O_{ds}/2) \qquad c' = (O_i - \tfrac{3}{2} O_{ds})$$

$$c = (2 O_{ds} - O_i + O_b)$$