

# Probabilities and Counting

## Terminology

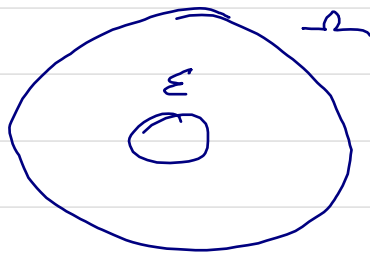
$P \rightarrow$  probability

$\mathcal{E} \rightarrow$  event

$m \rightarrow$  the count

$\Omega \rightarrow$  sample space

## Diagram - Venn



$$P(\mathcal{E}) = \frac{m(\mathcal{E})}{m(\Omega)} \rightarrow 0 \leq P(\mathcal{E}) \leq 1$$

## Ways to count

$$N! = \prod_{i=1}^N i = 1 \cdot 2 \cdot 3 \cdots N$$

$$\log_e N! = \log_e \prod_{i=1}^N i = \sum_{i=1}^N \log_e i$$

$$N! = e^{\log_e N!} = e^{\sum_{i=1}^N \log_e i}$$

Let  $P$  = permutation  $\rightarrow$  ordered  
 $C$  = combination  $\rightarrow$  unordered  
 $r$  = with replacement allowed

Define

$${}_n P_m^r = N^m$$

$${}_n C_m^r = \frac{(n+m-1)!}{(n-1)! m!}$$

$${}_n P_m = \frac{n!}{(n-m)!}$$

$${}_n C_m = \frac{n!}{(n-m)! m!}$$



Permutations  
with and without  
replacement



Combinations  
with and without  
replacement

Example problems

- 1) Calculate how many pin numbers are possible using 3 digits base 8 digits  $\{0, 1, 2, \dots, 7\}$

$${}_n P_m^r = N^m = 8^3 \rightarrow \{000, 001, \dots, 776, 777\}$$

512

2) Using 1) how many pin numbers have no duplicate digits

$${}_nP_m = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 8 \cdot 7 \cdot 6 = \underline{\underline{336}}$$

3) What is probability of receiving a pin number from 2) & 1) where no numbers are repeated

$$P(E) = \frac{n(E)}{n(\Omega)} = \frac{{}_nP_m / (n-m)!}{n^m} = \frac{336}{512} \approx \underline{\underline{66\%}}$$

4) If you have 3 gallons of ice cream, a gallon each of chocolate, vanilla, and strawberry. How many different triple cones are possible where order doesn't matter. List the combinations.

$${}_nC_m = \frac{5!}{2! 3!} = 10$$

CVS, CCV, CCS, VVC, VVS, SSC, SSV,  
CCC, VVV, SSS.

5) Power ball lottery

5 numbers of 69 in any order  
and 1 number of 26.

a) How many different ways to pull  
a number

$$n(\Omega) = {}_{69}C_5 {}_{26}C_1$$

b) What is probability of winning

$$P(E) = \frac{n(E)}{n(\Omega)} = \frac{{}_5C_5 {}_1C_1}{{}_{69}C_5 {}_{26}C_1}$$

$$\text{or odds} = 1 : \frac{1}{P(E)} = 1 : {}_{69}C_5 {}_{26}C_1$$

c) What is probability or odds of 4 numbers

$$P(E) = \frac{{}_5C_4 {}_{64}C_1 {}_{25}C_1}{{}_{69}C_5 {}_{26}C_1} = 1 : \frac{{}_{69}C_5 {}_{26}C_1}{{}_5C_4 {}_{64}C_1 {}_{25}C_1}$$