Shoot - Out Problem
Compare Calculations to Simulation

$\tilde{A}BC$ $2/3$ $AB\bar{C}$ $1/2$ $AB\bar{C}$ 1 $\bar{A}C$ $2/3$ $A\bar{C}$ 1 C

Column ①:

$1/3$ $A\bar{B}$ $1/2$ —— B

$1/2$ $\bar{A}B$ $1/3$ —— A

$2/3$ $A\bar{B}$ $1/2$ —— B

$1/2$ $\bar{A}B$ $1/3$ —— A

$2/3$ $A\bar{B}$ $1/2$ —— B

$1/2$ $\bar{A}B$ $1/3$ —— A

$2/3$ $A\bar{B}$ $1/2$ —— B

$1/2$ ①

Column ②:

$1/2$ $\bar{A}B$ $1/3$ —— A

$2/3$ $\tilde{A}\bar{B}$ $1/2$ —— B

$1/2$ $\bar{A}B$ $1/3$ —— A

$2/3$ $A\bar{B}$ $1/2$ —— B

$1/2$ $\bar{A}B$ $1/3$ —— A

$2/3$ $A\bar{B}$ $1/2$ —— B

$1/2$ $\bar{A}B$ $1/3$ —— A

$2/3$ ②

$1/3$ A (with $\bar{A}C$ branch)
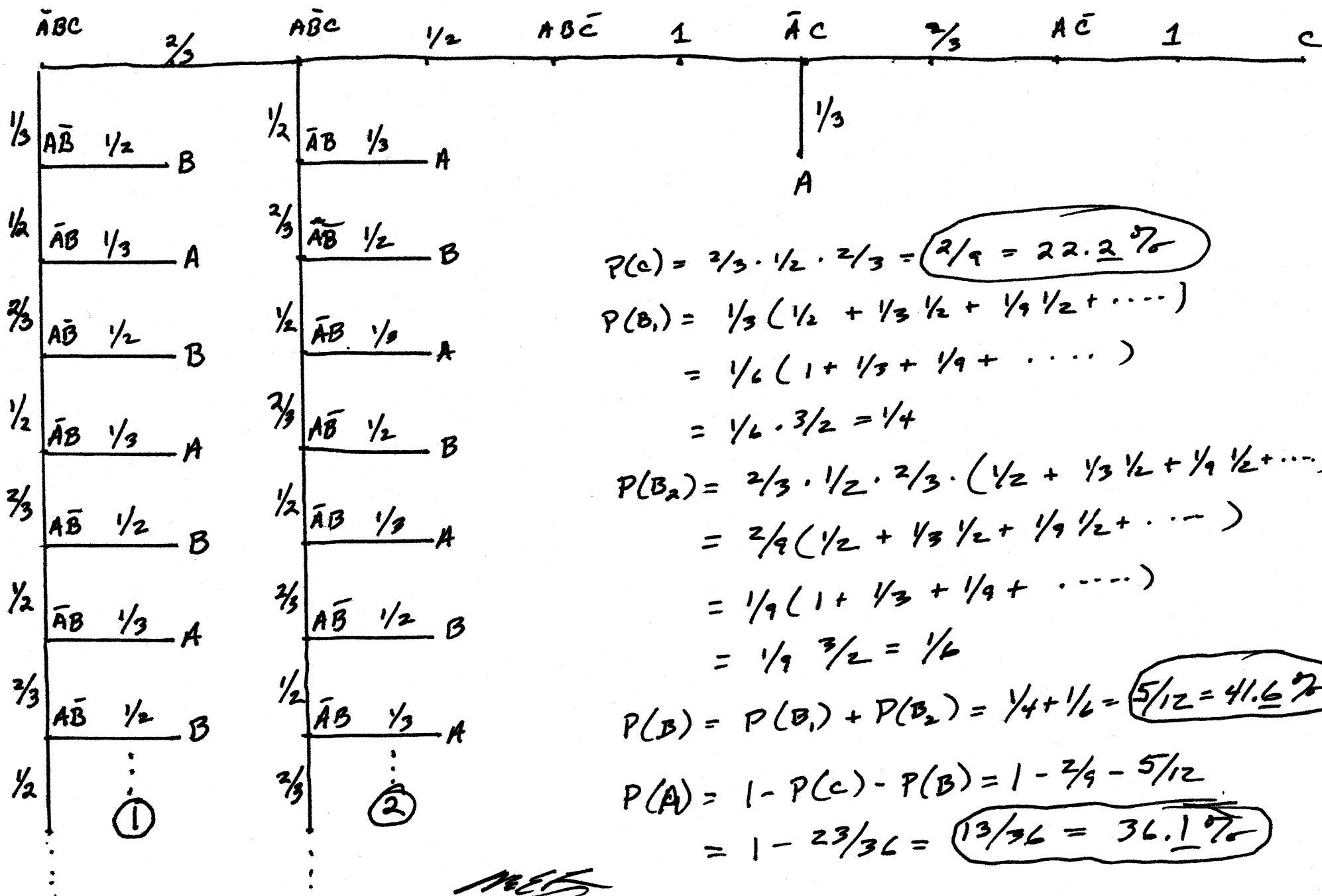
$$P(c) = 2/3 \cdot 1/2 \cdot 2/3 = \boxed{2/9 = 22.\underline{2}\%}$$

$$P(B_1) = 1/3 \left(1/2 + 1/3 \cdot 1/2 + 1/9 \cdot 1/2 + \dots \right)$$
$$= 1/6 \left(1 + 1/3 + 1/9 + \dots \right)$$
$$= 1/6 \cdot 3/2 = 1/4$$

$$P(B_2) = 2/3 \cdot 1/2 \cdot 2/3 \cdot \left(1/2 + 1/3 \cdot 1/2 + 1/9 \cdot 1/2 + \dots \right)$$
$$= 2/9 \left(1/2 + 1/3 \cdot 1/2 + 1/9 \cdot 1/2 + \dots \right)$$
$$= 1/9 \left(1 + 1/3 + 1/9 + \dots \right)$$
$$= 1/9 \cdot 3/2 = 1/6$$

$$P(B) = P(B_1) + P(B_2) = 1/4 + 1/6 = \boxed{5/12 = 41.6\%}$$

$$P(A) = 1 - P(c) - P(B) = 1 - 2/9 - 5/12$$
$$= 1 - 23/36 = \boxed{13/36 = 36.\underline{1}\%}$$

or calculate $P(A)$ directly

$$P(A_3) = \tfrac{2}{3} \cdot \tfrac{1}{2} \cdot 1 \cdot \tfrac{1}{3} = \underline{\underline{\tfrac{1}{9}}}$$

$$P(A_2) = \tfrac{2}{3} \cdot \tfrac{1}{2} \cdot (\tfrac{1}{3} + \tfrac{2}{3} \tfrac{1}{2} (\tfrac{1}{3} + \tfrac{2}{3} \cdot \tfrac{1}{2} \cdot (\tfrac{1}{3} \cdots \cdots) \cdots)$$

$$= \tfrac{1}{9} (1 + \tfrac{1}{3} + \tfrac{1}{9} \cdots \cdots)$$

$$= \tfrac{1}{9} \cdot \tfrac{3}{2} = \underline{\underline{\tfrac{1}{6}}}$$

$$P(A_1) = \tfrac{1}{3} \cdot \tfrac{1}{2} (\tfrac{1}{3} + \tfrac{2}{3} \cdot \tfrac{1}{2} \cdot \tfrac{1}{3} \cdots \cdots - - -$$

$$= \tfrac{1}{18} (1 + \tfrac{1}{3} + \tfrac{1}{9} \cdots \cdots - - -$$

$$= \tfrac{1}{18} \cdot \tfrac{3}{2} = \underline{\underline{\tfrac{1}{12}}}$$

$$P(A) = P(A_1) + P(A_2) + P(A_3)$$

$$= \tfrac{1}{9} + \tfrac{1}{6} + \tfrac{1}{12} = \frac{4 + 6 + 3}{36}$$

$$= \boxed{\underline{\underline{\tfrac{13}{36}}} = 36.1\% }$$