CSC 7 Discrete Math **Name:** ____________________
Fall 2023
Study Guide
Week of Finals
Time Limit: Week before Exam

This Guide contains 5 pages and 25 example questions. Total points possible are 200.

1. (5 points) What is the order of operations for the following operators $|, \downarrow, \rightarrow, \leftrightarrow, \wedge, \vee, \sim$. Rank from top to bottom and if they have the same order then put on the same line to show equivalence. Why do they have this order?

2. (10 points) Determine the validity of the following argument. Show exactly why it is invalid or valid. Explicitly show why and explain.

$$(p \rightarrow q) \vee \sim r$$

$$(q \rightarrow p) \wedge r$$

$$\therefore p \rightarrow r$$

3. (10 points) Determine the validity of the following argument. Show exactly why it is invalid or valid. Explicitly show why and explain.

$$q \rightarrow p \wedge r$$

$$p \rightarrow q \vee \sim r$$

$$\therefore p \rightarrow r$$

4. (10 points) Show the truth table and determine if it is logically equivalent.

$$(p \rightarrow q) \wedge (q \rightarrow p) \equiv p \leftrightarrow q$$

5. (5 points) Show the logic circuit for $((p \wedge \sim q) \vee (p \wedge q)) \wedge q$

6. (10 points) Simplify $((p \wedge q) \vee (p \wedge \sim q)) \wedge q$ using Theorem 2.1.1

7. (10 points) Represent the previous simplified result with pierce arrow operators only. (the simplification of $((p \wedge q) \vee (p \wedge \sim q)) \wedge q$ using Theorem 2.1.1).

8. (5 points) Negate the following:

$$(\exists x \in D(\exists y \in E(P(X,Y))))$$

$$(\forall x \in D(\forall y \in E(P(X,Y))))$$

9. (5 points) Convert ACF_{16} to Binary, Octal and Decimal.

10. (5 points) Derive the 2's complement, 1 byte integer of 57_{10}. Display the result in Binary, Octal, Decimal and Hex.

11. (5 points) If today is Wednesday, what day of the week will be 999 days from today?

12. (10 points) Derive the summation series deductively and prove inductively.

$$\sum_{i=1}^{n} i = \frac{n * (n+1)}{2}$$

13. (5 points) Sum the integers from 53 to 124. Use the summation formula to derive the result.

14. (10 points) Sum the sequence $\{195, 200, 205, \ldots, 880, 885\}$. Again use the summation formula.

15. (10 points) Given the sequence $t_1 = 3, t_2 = 4$ and the recursive definition of the sequence $t_k = 2t_{k-1} - t_{k-2}$, find $t_n = f(n)$.

16. (10 points) Given the sequence generating function $C_n = f(n) = 2^n - 1$ develop a recursive function for $C_i, i \in \{0, 1, 2, \ldots, n\}$

17. (5 points) Write the following as rational numbers. $1.\underline{7}_{10} = 1.7777\ldots$ and $1.4\underline{65}_{10}$.

18. (5 points) Given a rational number represented by $A/B = R \in Q$ what is the maximum length of the repeated decimal? Justify your answer.

19. (5 points) Given P=.4 and Q=.6 calculate the sum when n=4:

$$\sum_{i=0}^{n} \binom{n}{i} P^i Q^{n-i}$$

20. (10 points) Turn the summation series into a simple function of n.

$$\sum_{i=0}^{n} \binom{n}{i} = f(n)?$$

and

$$\sum_{i=0}^{n} \binom{n}{i} 2^i = f(n)?$$

21. (10 points) Find $\Omega(g(n))$ and $O(g(n))$, the upper and lower asymptotic bound of the following function:

$$f(n) = \sum_{i=1}^{n} \sqrt{i}$$

22. (10 points) Given 2 Dice are randomly thrown and equally weighted, $D_1 = \{1, 2, 3, 4, 5, 6\}$ and $D_2 = \{1, 2, 3, 4, 5, 6\}$, determine the following probabilities:

$$P((D_1 = 4) \wedge (D_2 = 4))$$

$$P((D_1 = 4) \vee (D_2 = 4))$$

23. (5 points) What is the degree of the following graph?

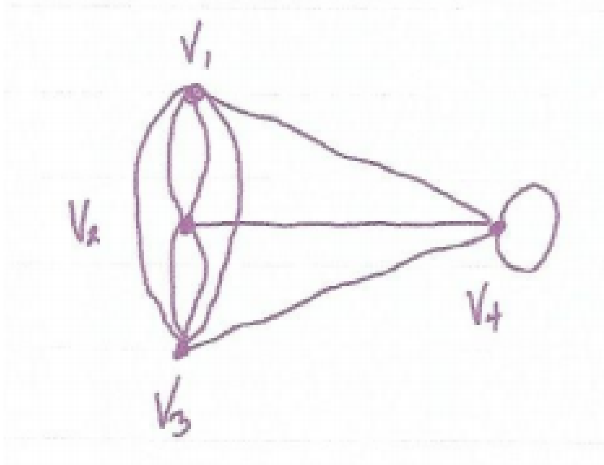

Figure 1: Graph Degree?

24. (10 points) Determine by examining the arrow diagrams, which relationships are functions, what type of function the diagram represents and any other outstanding and notable features. Discuss in detail and define your terms. Associate your explanation with the label for each diagram.

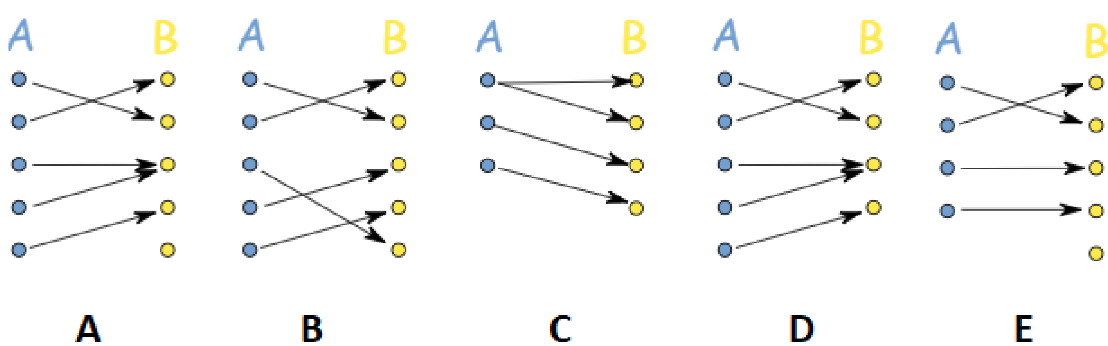

Figure 2: Function Types?

25. (15 points) From the 5 premises, derive the conclusion. Use deductive logic and the rules of inference. Define variables, reorder, negate, use transitivity. Label each step and show the inference used in each step.

   a. When I work a logic example without grumbling, you maybe sure it is one I understand.
   b. The argument in these examples are not arranged in regular order like the ones I am used to.
   c. No easy example makes my headache.
   d. I cannot understand examples if the arguments are not arranged in regular order like the ones I am used to.
   e. I never grumble at an example unless it gives me a headache.

$\therefore$ These examples are not easy!

Theorem 2.1.1 Logical Equivalences

Given any statement variables p, q, and r, a tautology $\mathbf{t}$ and a contradiction $\mathbf{c}$, the following logical equivalences hold.

1. *Commutative laws:*	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. *Associative laws:*	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. *Distributive laws:*	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. *Identity laws:*	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
5. *Negation laws:*	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
6. *Double negative law:*	$\sim(\sim p) \equiv p$	
7. *Idempotent laws:*	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. *Universal bound laws:*	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
9. *De Morgan's laws:*	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
10. *Absorption laws:*	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. *Negations of* $\mathbf{t}$ *and* $\mathbf{c}$*:*	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$

Modus Ponens	$p \to q$ p $\therefore q$	**Elimination**	**a.** $p \vee q$ $\sim q$ $\therefore p$ **b.** $p \vee q$ $\sim p$ $\therefore q$
Modus Tollens	$p \to q$ $\sim q$ $\therefore \sim p$	**Transitivity**	$p \to q$ $q \to r$ $\therefore p \to r$
Generalization	**a.** p $\therefore p \vee q$ **b.** q $\therefore p \vee q$	**Proof by Division into Cases**	$p \vee q$ $p \to r$ $q \to r$ $\therefore r$
Specialization	**a.** $p \wedge q$ $\therefore p$ **b.** $p \wedge q$ $\therefore q$		
Conjunction	p q $\therefore p \wedge q$	**Contradiction Rule**	$\sim p \to \mathbf{c}$ $\therefore p$

Valid Argument Forms

Figure 3: Theorem 2.1.1 and Theorem 2.3.1