Homework 1 – Lecture 1. Vlad Ifjiu.

4. n people to form a ring

$$\left.\begin{matrix}1234,\ 2134,\ 2314, \\ 2341,\ 2431,\ 3421\end{matrix}\right\}\ 6 \quad (n-1)!$$

$4321 = 1234$

if we have $4321 = 1234$ because of being a circular surface. $\Rightarrow \frac{n!}{n}$ ways of rotating.

therefore $\frac{n!}{n} = (n-1)!$

2. $C_{(n,r)} = C_{(n,n-r)}$

$$C_{(n,r)} = \frac{n!}{r!(n-r!)} \qquad C_{(n,n-r)} = \frac{n!}{(n-r)!(n-n+r)!}$$

$$\left.\begin{matrix} C_{(n,n-r)} = \dfrac{n!}{(n-r)!\,r!} \\ C_{(n,n-r)} = \dfrac{n!}{(n-r)!\,r!}\end{matrix}\right\} \Rightarrow C_{(n,r)} = C_{(n,n-r)}$$

5. permutations $\underbrace{ABCDEFGH}_{8\text{ chars}}$ continuing ABC

take ABC – fixed

<u>ABC</u> EFGH, D<u>ABC</u>EFGH, DE<u>ABC</u>FGH, DEF<u>ABC</u>GH ...

X can only take one spot at a time

$\Rightarrow 1*2*3*4*5*6 = 720$ ways

6. n bit string with length n that contain exactly r ones.?

$n = 3$

$r=1 \Rightarrow 001, 010, 100 \longrightarrow 3$

$r=2 \Rightarrow 110, 101, 011 \longrightarrow 3$

$r=3 \Rightarrow 111 \longrightarrow 1$

$$\frac{3!}{1!(2!)} = \frac{3!}{2!} = 3$$

$$\frac{3!}{2!\,1!} = 3$$

$$\frac{3!}{3!\,0!} = 1$$

therefore $(\forall)\, n, r$, $\frac{n!}{r!(n-r)!} = C_n^r$

3. 100 people

how many ways to chose 1st, 2nd, 3rd prize?

$100 \cdot 99 \cdot 98$

1. $C_n^r = C_{n-1}^{r-1} + C_{n-1}^{r}$

$$C_n^r = \frac{n!}{r!(n-r)!}$$

$$C_{n-1}^{r-1} = \frac{(n-1)!}{(r-1)!\,(n-1-(r-1))!} = \frac{(n-1)!}{(r-1)!\,(n-r)!}$$

$$C_{n-1}^{r} = \frac{(n-1)!}{r!\,(n-1-r)!}$$

$$C_{n-1}^{r-1} + C_{n-1}^{r} = \frac{(n-1)!}{(r-1)!\,(n-r)!} + \frac{(n-1)!}{r!\,(n-1-r)!} =$$

$$= \frac{(n-1)!}{(r-1)!\,(n-r-1)!\cdot(n-r)} + \frac{(n-1)!}{(r-1)!\cdot r(n-r-1)!} =$$

$$= \frac{r(n-1)! + (n-r)(n-1)!}{r\cdot(r-1)!\cdot(n-r-1)!\,(n-r)} = \frac{(n-1)!(\cancel{r}+n-\cancel{r})}{r(r-1)!\,(n-r-1)!(n-r)} =$$

$$= \frac{(n-1)!\,n}{r(r-1)!\,(n-r-1)!\,(n-r)} = \frac{n!}{r!\,(n-r)} = C_n^r$$