$$CC_0 \longrightarrow CC_1 \longrightarrow \cdots \longrightarrow CC_T$$

Newton's uniform movement: $= S_t = S_{t-1} + V_t$

$$CC_t = \begin{bmatrix} P_t \\ V_t \end{bmatrix} = \begin{bmatrix} P_{t-1} + V_{t-1}\,\Delta t \\ V_{t-1} \end{bmatrix} \overset{\Delta t = 1}{=} \begin{bmatrix} P_{t-1} + V_{t-1} \\ V_{t-1} \end{bmatrix}$$

$$= M\, CC_{t-1} = M \begin{bmatrix} P_{t-1} \\ V_{t-1} \end{bmatrix}$$

$$\Rightarrow M = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow M\, CC_{t-1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_{t-1} \\ V_{t-1} \end{bmatrix} = \begin{bmatrix} P_{t-1} + V_{t-1} \\ V_{t-1} \end{bmatrix}$$

$$CC_t = M\, CC_{t-1}$$

$$CC_t \sim N\left(M\, CC_{t-1},\ \sigma_{cc}^2\right)$$

Initial cc

mean-cc0, sd-cc

$$CC_0 \sim N\left(\mu_{cc_0},\ \sigma_{cc}^2\right)$$

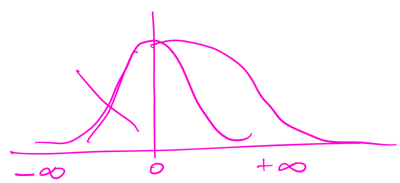
mean-mean-cc0, sd-mean-cc0

Trainable

$$\mu_{cc} \sim N\left(\mu_{\mu cc_0},\ \sigma^2_{\mu cc_0}\right)$$ → Hyper-prior distribution

Fixed, Fixed

$$\sigma_{cc} \sim HN\left(\sigma_{\sigma cc}\right) \Rightarrow$$

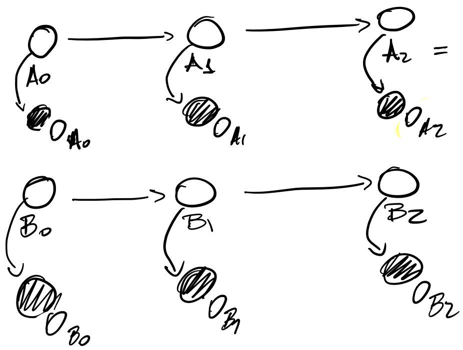


Trainable, Fixed

sd_sd_cc

Latent System component

No coordination



$$A_2 = \begin{bmatrix} P_{A_2} \\ V_{A_2} \end{bmatrix}$$

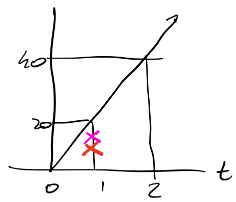
$$A_t = \begin{bmatrix} P_{A_{t-1}} + V_{A_{t-1}} \\ V_{A_{t-1}} \end{bmatrix}$$

$$F_{A_t} = P_{A_t}$$

$$A_0 = \begin{bmatrix} 0 \\ 20 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 20 \\ 20 \end{bmatrix}$$

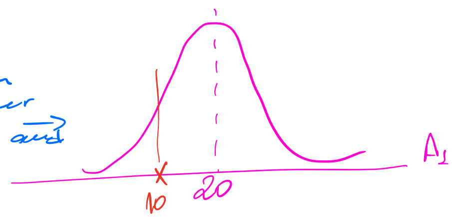
$$A_2 = \begin{bmatrix} 40 \\ 20 \end{bmatrix}$$



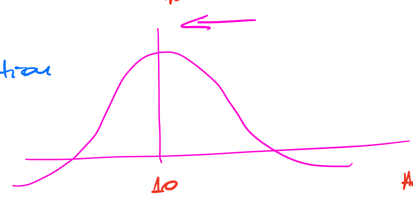
$$O_1 = [10]$$

$$A_1 = \mathcal{N}\left(\begin{bmatrix} 20 \\ 20 \end{bmatrix}, \underset{\Sigma}{\sigma_{cc}} \right)$$

Expectation based on our assumptions and dynamics →

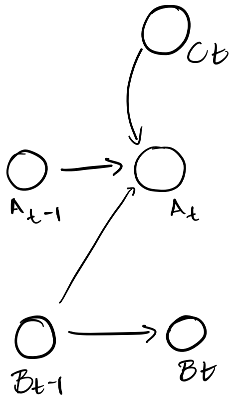


Correction



$$O_t \sim \mathcal{N}\left(A_t, \underset{\Sigma}{\sigma_O^2}\right)$$

With coordination



No coordination

$$A_t = \begin{bmatrix} p_{A_{t-1}} + v_{A_{t-1}} \\ v_{A_{t-1}} \end{bmatrix}$$

w/ coordination

$$A_t = \begin{bmatrix} p_{A_{t-1}} + v_{A_{t-1}} \\ (1-C_t)\, v_{A_{t-1}} + C_t\, v_{B_{t-1}} \end{bmatrix}$$

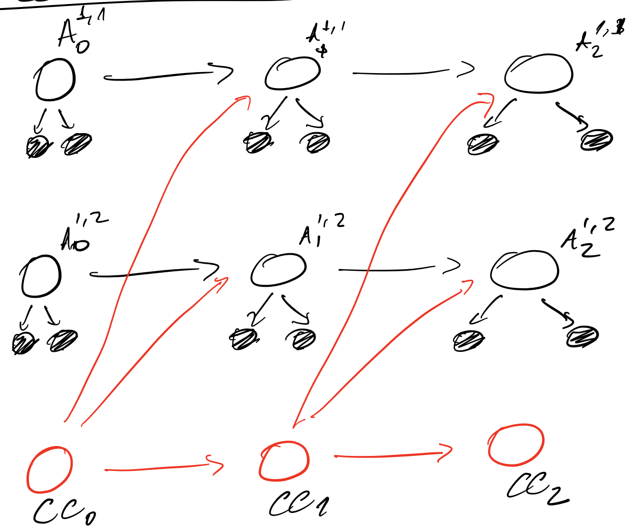
$$A_t = U A_{t-1} + F B_{t-1}$$

$$A_t = \begin{bmatrix} 1 & 1 \\ 0 & (1-C_t) \end{bmatrix} A_{t-1} + \begin{bmatrix} 0 & 0 \\ 0 & C_t \end{bmatrix} B_{t-1}$$

$$O_{A_t} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} p_{A_t} \\ v_{A_t} \end{bmatrix} = p_{A_t}$$

$$O_{A_t} = \begin{bmatrix} 1 & 0 \\ \vdots & \\ 1 & 0 \end{bmatrix} \begin{bmatrix} p_{A_t} \\ v_{A_t} \end{bmatrix} = \left.\begin{bmatrix} p_{A_t} \\ \vdots \\ p_{A_t} \end{bmatrix}\right\} \#\text{channels}$$

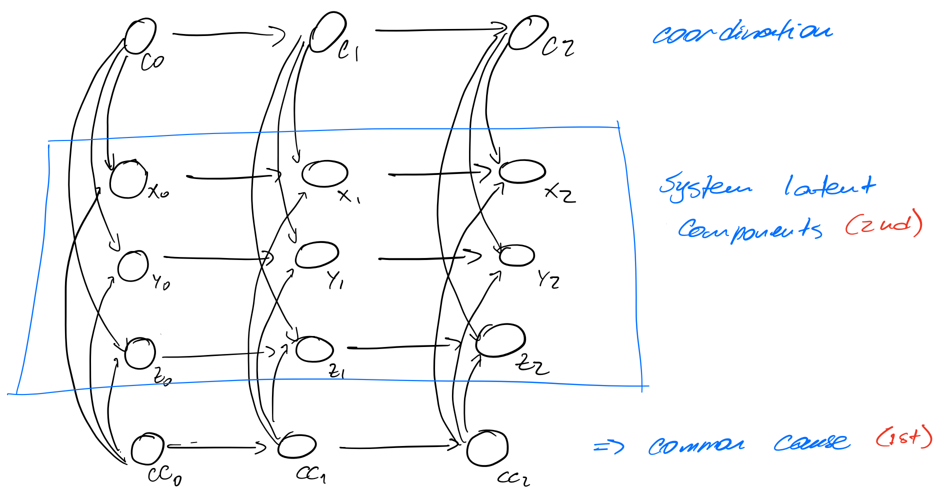
Adding the common cause



w/ coordination and common cause

$$A_t = \begin{bmatrix} p_{A_{t-1}} + v_{A_{t-1}} \\ (1-c_t) v_{A_{t-1}} + c_t \cancel{v_{B_{t-1}}} \end{bmatrix}$$

v_{CC_t}



1st test case

$$p(cc_0, cc_1, cc_2) = p(cc_0)\, p(cc_1 | cc_0)\, p(cc_2 | cc_1)$$

$$\log p = \log p(cc_0) + \log p(cc_1 | cc_0) + \log(cc_2 | cc_1)$$

2nd test case

$$\underline{p(x_{0:2}, y_{0:2}, z_{0:2})} = p(x_0 | c_0, cc_0)\, p(y_0 | c_0, cc_0)\, p(z_0 | c_0, cc_0)$$
$$p(x_1 | x_0, cc_1, c_1)\, p(y_1 | y_0, cc_1, c_1)\, p(z_1 | z_0, cc_1, c_1)$$
$$p(x_2 | x_1, cc_2, c_2)\, p(y_2 | y_1, cc_2, c_2)\, p(z_2 | z_1, cc_2, c_2)$$

$$p(x_0 | c_0, cc_0) = \mathcal{N}\left(c_0 cc_0 + (1 - c_0)\mu_x,\ \sigma_A^2\right)$$

$$p(x_1 | c_1, cc_1) = \mathcal{N}\left(\underline{c_1 cc_1 + (1 - c_1) x_0},\ \sigma_A^2\right)$$

$$p(x_t | c_t, cc_t) = \mathcal{N}\left(c_t cc_t + (1 - c_t) x_{t-1},\ \sigma_A^2\right)$$

$$p\left(x_1 \mid c_1, cc_1, x_0\right) = p\left(\begin{bmatrix} x_1^p \\ x_1^v \end{bmatrix} \middle| \, c_1, \begin{bmatrix} cc_1^p \\ cc_1^v \end{bmatrix}, \begin{bmatrix} x_0^p \\ x_0^v \end{bmatrix}\right)$$

$$= p\left(\begin{bmatrix} x_1^p \\ x_1^v \end{bmatrix} \middle| \begin{bmatrix} x_0^p + x_0^v \\ (1-c_1)x_0^v + c_1 cc_1^v \end{bmatrix}\right)$$

$$= p\left(\begin{bmatrix} x_1^p \\ x_1^v \end{bmatrix} \middle| \underbrace{\begin{bmatrix} 1 & 1 \\ 0 & (1-c_1) \end{bmatrix}}_{F = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}} \begin{bmatrix} x_0^p \\ x_0^v \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & c_1 \end{bmatrix}}_{U} \begin{bmatrix} cc_1^p \\ cc_1^v \end{bmatrix}\right)$$