Exercise 1

1 Loading the Dataset

In [263]:
```
import numpy as np
import numpy.linalg as LA
import matplotlib.pyplot as plt
from sklearn.datasets import load_digits
from sklearn.linear_model import LogisticRegression
from sklearn.model_selection import train_test_split,KFold
import warnings
warnings.filterwarnings('ignore')
```

In [264]:
```
#load dataset
digits = load_digits ()
data = digits["data"]
images = digits["images"]
target = digits["target"]
target_names = digits["target_names"]

#data filtering
num_1, num_2 = 3, 8
mask = np.logical_or(target == num_1, target == num_2)
data = data[mask]
target = target[mask]

#add column of 1's
data = np.hstack((data,np.ones((len(data),1))))

#relabel targets
target[target == num_1] = 1
target[target == num_2] = -1
```

1.1 Classification with sklearn

In [265]:
```
def hyperpar(lambdaSpace,k_splits=20):
    #grid search
    means=[]
    for i in range(len(lambdaSpace)):
        kf = KFold(n_splits=k_splits)
        kf.get_n_splits(data)
        scores=[]
        for train_index, test_index in kf.split(data):
            logReg=LogisticRegression(C=lambdaSpace[i],solver='lbfgs')
            logReg.fit(data[train_index],target[train_index])
            scores.append(logReg.score(data[test_index],target[test_index]))
        means.append(np.mean(scores))
    return means
```

In [266]:
```
lam=np.logspace(-2,5,8)
hyperpar(lam)
```

Out[266]:
```
[0.9859477124183007,
 0.9915032679738562,
 0.9942810457516339,
 0.9942810457516339,
 0.9942810457516339,
 0.9942810457516339,
 0.9942810457516339,
 0.9972222222222221]
```

Since the means are all similar or even equal we choose $\lambda = 100$

1.2 Optimization Methods

In [267]:
```
def sigmoid(z):
    return 1/(1+np.exp(-z))

def gradient(beta,X,y,lam=100):
    if len(X.shape)>1:
        return beta-lam/len(X)*np.sum(np.multiply(sigmoid(-np.multiply(y,X@beta)),np.multi
ply(y,X.T)),axis=1)
    else:
        return beta-lam*sigmoid(-y*X@beta)*y*X.T

def predict(beta,X):
    return np.sign(X@beta)

def zero_one_loss(y_pred,y_truth):
    return np.count_nonzero(y_pred!=y_truth)
```

In [269]:
```
def gradient_decent(m, X, y, tau=.1, gamma=.01,beta=0, lam = 100):
    N,d=X.shape
    if type(beta)!=np.ndarray:
        beta=np.zeros(d) if beta==0 else np.array(beta)
    for iteration in range(m):
        beta = beta - tau/(1 + gamma*iteration) * gradient(beta,X,y)
    return beta

def SG(m, X, y, tau=.1, gamma=.01,beta=0, lam = 100):
    N,d=X.shape
    if type(beta)!=np.ndarray:
        beta=np.zeros(d) if beta==0 else np.array(beta)
    for iteration in range(m):
        instance = np.random.randint(N)
        beta = beta - tau/(1 + gamma*iteration) * gradient(beta,X[instance],y[instance])
    return beta

def SG_minibatch(m, X, y, tau=.1, gamma=.01,batchsize=16,beta=0, lam = 100):
    N,d=X.shape
    if type(beta)!=np.ndarray:
        beta=np.zeros(d) if beta==0 else np.array(beta)
    indices = np.arange(len(X))
    for iteration in range(m):
        instances = np.random.permutation(len(X))[:batchsize]
        beta = beta - tau/(1 + gamma*iteration) * gradient(beta,X[instances],y[instances])
    return beta

def SG_momentum(m,X,y,tau=.1,gamma=.01,mu=.5,beta=0,lam=100):
    N,d=X.shape
    if type(beta)!=np.ndarray:
        beta=np.zeros(d) if beta==0 else np.array(beta)
    g=np.zeros(d)
    for t in range(m):
        #choose random instance
        i = np.random.randint(N)
        g=mu*g+(1-mu)*gradient(beta,X[i],y[i])
        beta=beta-tau/(1+gamma*t)*g
    return beta

def ADAM(m,X,y,tau=1e-4,epsilon=1e-8,mu1=.9,mu2=.999,beta=0,lam=100):
    #initialize with 0 see original paper
    #(https://arxiv.org/pdf/1412.6980.pdf)
    N,d=X.shape
    if type(beta)!=np.ndarray:
        beta=np.zeros(d) if beta==0 else np.array(beta)
    g=q=np.zeros(d)
    for t in range(m):
        #without replacement
        index=np.random.randint(len(X))
        l=gradient(beta,X[index],y[index])
        g=(1-mu1)*l+mu1*g
        q=(1-mu2)*np.square(l)+mu2*q
        g_til=g/(1-mu1**(t+1))
        q_til=q/(1-mu2**(t+1))
        beta=beta-tau*np.divide(g_til,np.sqrt(q)+epsilon)
    return beta

def stochastic_average_gradient(m,X,y,tau_0=.1,gamma=.01,beta=0,lam=100):
    #initialization
    N,d=X.shape
    if type(beta)!=np.ndarray:
        beta=np.zeros(d) if beta==0 else np.array(beta)
    g_stored=-np.multiply(np.multiply(sigmoid(-np.multiply(y,X@beta)),y),X.T)
    g=np.sum(g_stored,axis=1)/N
    for t in range(m):
        i=np.random.randint(N)
        g_i=-y[i]*np.multiply(X[i].T,sigmoid(-y[i]*X[i]@beta))
        g=g+(g_i-g_stored.T[i])/N
```

```
        g_stored.T[i]=g_i
        tau_t=tau_0/(1+gamma*t)
        beta=beta*(1-tau_t/lam)-tau_t*g
    return beta

def dual_coordinate_ascent(m,X,y,beta=0,lam=100,epsilon=1e-8):
    N,d=X.shape
    if type(beta)!=np.ndarray:
        beta=np.zeros(d) if beta==0 else np.array(beta)
    alpha=np.random.uniform(size=N)
    beta = lam/N * np.sum(np.multiply(np.multiply(alpha,y),X.T),axis = 1)
    for t in range(m):
        i=np.random.randint(N)
        f_p=y[i]*X[i]@beta+np.log(alpha[i]/(1-alpha[i]))
        f_pp=lam/N*X[i]@X[i].T+1/(alpha[i]*(1-alpha[i]))
        next_alpha_i=np.clip(alpha[i]-f_p/f_pp,a_max=1-epsilon,a_min=epsilon)
        beta=beta+lam/N*y[i]*X[i].T*(next_alpha_i-alpha[i])
        alpha[i]=next_alpha_i
    return beta

def newton(m,X,y,beta=0,lam=100):
    N,d=X.shape
    if type(beta)!=np.ndarray:
        beta=np.zeros(d) if beta==0 else np.array(beta)
    z,y_weighted,W=None,None,None
    for t in range(m):
        z=X@beta
        y_weighted=np.divide(y,sigmoid(y*z))
        W=np.diag(lam/N*np.multiply(sigmoid(z),sigmoid(-z)))
        beta=LA.inv(np.identity(d)+X.T@W@X)@X.T@W@(z+y_weighted)
    return beta
```

1.3 Comparison

In [270]:
```
X,X_test,y,y_test = train_test_split(data,target,test_size=0.3,random_state=0)
```

Learning Rate

Not all algorithms need all three hyper parameters

- gradient descent needs τ and γ
- stochastic gradient needs τ and γ
- SG minibatch needs τ and γ
- SG momentum needs τ, γ and μ
- ADAM needs τ (and μ_1 and μ_2 but they stay fixed)
- stochastic average gradient needs τ and γ
- dual coordinate as needs nothing
- Newton nedds nothing

In [271]:
```
tau_space=np.logspace(-3,-1,3)
mu_space=[.1,.2,.5]
gamma_space=np.logspace(-4,-2,3)
```

In [272]:
```python
def hyperSeach(func,spaces,m,X,y):
    '''
    func: Optimization method as function
    spaces: a list of all hyper parameter spaces to be checked
            needs to be in order: tau, gamma, mu (leave out what is not needed)
    m: number of iterations
    X: data
    y: targets

    returns tuple of the best found hyper parameter in spaces
    '''
    N=len(spaces)
    kf = KFold(n_splits=10)
    hyper_par=[None]*N
    best_error=np.inf
    #perform exhaustive grid search
    for hyper in zip(*spaces):
        error=0
        for train_index ,validation_index in kf.split(X):
            X_train ,X_validation = X[train_index],X[validation_index]
            y_train ,y_validation = y[train_index],y[validation_index]
            #optimize
            beta=func(m,X_train,y_train,*list(hyper))
            error+=zero_one_loss(y,np.sign(X@beta))
        if error<best_error:
            hyper_par=list(hyper)
    return tuple(hyper_par)
```

In [273]:
```python
#gradient descent
t,g=hyperSeach(gradient_decent,[tau_space,gamma_space],10,data,target)
print('parameters with lowest error rate: tau=%.2f, gamma=%.2f'%(t,g))
```

```
parameters with lowest error rate: tau=0.10, gamma=0.01
```

In [274]:
```python
#SG
t,g=hyperSeach(SG,[tau_space,gamma_space],150,data,target)
print('parameters with lowest error rate: tau=%.2f, gamma=%.2f'%(t,g))
```

```
parameters with lowest error rate: tau=0.10, gamma=0.01
```

In [275]:
```python
#SG minibatch
t,g=hyperSeach(SG_minibatch,[tau_space,gamma_space],150,data,target)
print('parameters with lowest error rate: tau=%.2f, gamma=%.2f'%(t,g))
```

```
parameters with lowest error rate: tau=0.10, gamma=0.01
```

In [276]:
```python
#SG momentum
t,g,m=hyperSeach(SG_momentum,[tau_space,gamma_space,mu_space],150,data,target)
print('parameters with lowest error rate: tau=%.2f, gamma=%.2f, mu=%.2f'%(t,g,m))
```

```
parameters with lowest error rate: tau=0.10, gamma=0.01, mu=0.50
```

In [277]:
```python
#ADAM
t=hyperSeach(ADAM,[tau_space],150,data,target)
print('parameters with lowest error rate: tau=%.2f'%t)
```

```
parameters with lowest error rate: tau=0.10
```

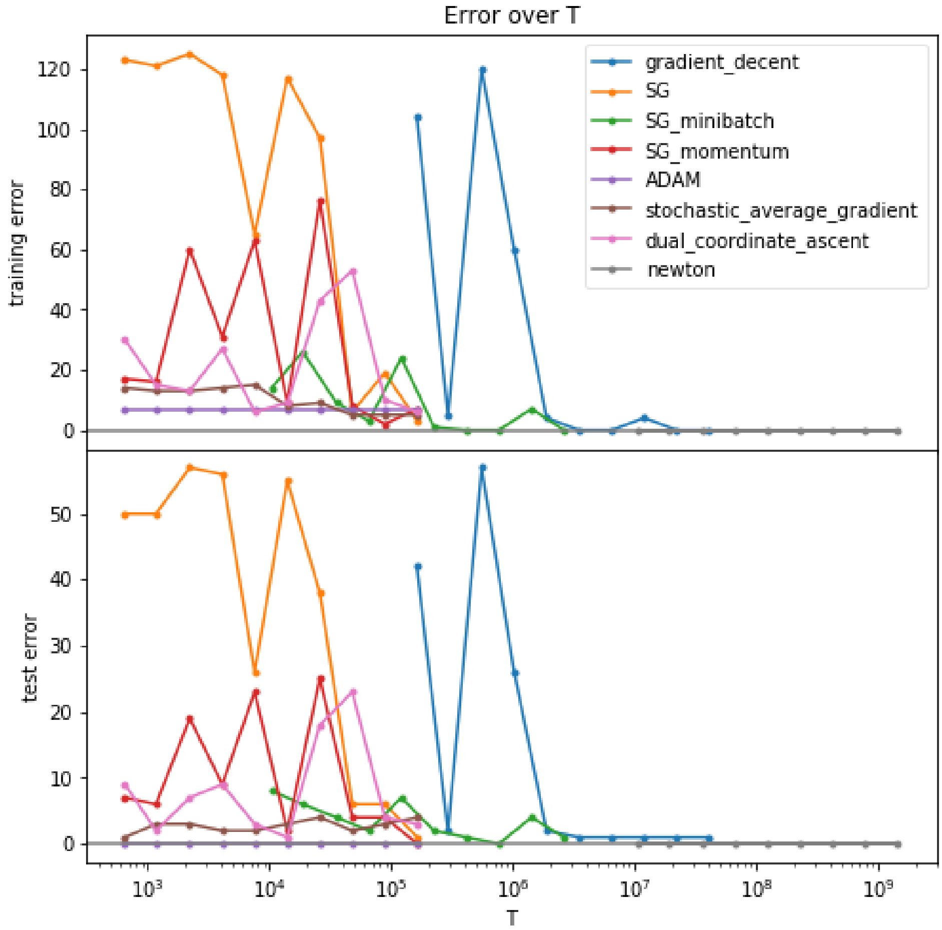
In [278]:
```python
#stochastic average gradient
t,g=hyperSeach(ADAM,[tau_space,gamma_space],150,data,target)
print('parameters with lowest error rate: tau=%.2f, gamma=%.2f'%(t,g))
```

```
parameters with lowest error rate: tau=0.10, gamma=0.01
```

Speed

In [283]:
```
functions = [gradient_decent,SG,SG_minibatch,SG_momentum,ADAM,stochastic_average_gradient,
dual_coordinate_ascent,newton]
train_err = []
test_err = []
iterations = np.logspace(1,3.4,10,dtype=int)
for i in range(len(functions)):
    f_i,f_it=[],[]
    for m in iterations:
        function = functions[i]
        if i != 4 :
            beta = function(m,X,y)
        f_i.append(zero_one_loss(predict(beta,X),y))
        f_it.append(zero_one_loss(predict(beta,X_test),y_test))
    train_err.append(f_i)
    test_err.append(f_it)
```

In [284]:
```python
N,d = X.shape
B = 16
grad_time = N*d*iterations
stoch_time = d*iterations
mini_time = B*d*iterations
newton_time = N*d**2*iterations
T = np.array([grad_time,stoch_time,mini_time,stoch_time,stoch_time,stoch_time,stoch_time,n
ewton_time])
plt.figure(figsize=(8,8))
F=len(functions)
for i in range(F):
    plt.subplot(2, 1, 1)
    plt.semilogx(T[i], train_err[i], '.-',label=functions[i].__name__)
    plt.title('Error over T')
    plt.ylabel('training error')
    plt.legend(loc='best',framealpha=.5)
    plt.subplot(2, 1, 2)
    plt.subplots_adjust(hspace=0)
    plt.semilogx(T[i], test_err[i], '.-',label=functions[i].__name__)
    plt.xlabel('T')
    plt.ylabel('test error')
```



The best (lowest error after complete traing) concergence is reached by newton, stochastic average decent, gradient decent and minibatch.The other algorithms (exept ADAM) tend to fluctuate a lot. ADAM reaches its lowest error after the first iteration and stays constant (at a rather low value). The fastest to konverge seems to be stochastic average gradient. Remarkable is that the newton algorithm always reaches the lowest test error but also is very slow.

In [ ]: