

# Using Computational Methods to Improve Feedback for Learners

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# Selecting Formative Guidance



Provide  
experience X

**Algorithm**

$$3x - 2x + \frac{5}{2} = 6$$

$$3x - 2x = 6 - \frac{5}{2}$$

$$x = 6 - \frac{5}{2}$$

$$x = \frac{1}{2}$$



# Outline

- Personalizing feedback for learners using an inverse planning model to assess equation solving skills
- Improving feedback for all learners
- Future directions

# Interpreting Equation Solving: Bayesian Inverse Planning

Algebra  
skills ( $\theta_1$ )



$$3x + \frac{5}{2} = 2(x + 3)$$

$$3x + \frac{5}{2} = 2x + 6$$

$$3x - 2x + \frac{5}{2} = 6$$

$$3x - 2x = 6 - \frac{5}{2}$$

$$x = 6 - \frac{5}{2}$$

$$x = \frac{1}{2}$$

$$3x + \frac{5}{2} = 2(x + 3)$$

$$3x + \frac{5}{2} = 2x + 3$$

$$3x - 2x + \frac{5}{2} = 3$$

$$3x - 2x = 3 - \frac{5}{2}$$

$$x = 3 - \frac{5}{2}$$

$$x = \frac{1}{2}$$

Algebra  
skills ( $\theta_2$ )



$\Theta$  = space of possible understandings

$$p(\theta \mid \text{equations})$$

# Representing Understanding: $\Theta$

$\theta \in \Theta$ : 6-dimensional vector of parameters related to skill

Conceptual Mal-rules	$1+3x \Rightarrow 4x$ $3(2+5x) \Rightarrow 6+5x$
Arithmetic	$1+5.9x+3.2x \Rightarrow 1+8.1x$ $-3+5+x \Rightarrow -2+x$
Planning	$3x+5x+4 = 2 \Rightarrow 3x+4 = -5x+2$

e.g., Sleeman, 1984; Payne & Squibb, 1990; Koedinger & MacLaren, 1997

# Bayesian Inverse Planning

Algebra  
skills ( $\theta_1$ )



$$3x + \frac{5}{2} = 2(x + 3)$$

$$3x + \frac{5}{2} = 2x + 6$$

$$3x - 2x + \frac{5}{2} = 6$$

$$3x - 2x = 6 - \frac{5}{2}$$

$$x = 6 - \frac{5}{2}$$

$$x = \frac{1}{2}$$

$$p(\theta \mid \text{equations}) \propto p(\theta) \underbrace{p(\text{equations} \mid \theta)}$$

$$3x + \frac{5}{2} = 2(x + 3)$$

$$3x + \frac{5}{2} = 2x + 3$$

$$3x - 2x + \frac{5}{2} = 3$$

$$3x - 2x = 3 - \frac{5}{2}$$

$$x = 3 - \frac{5}{2}$$

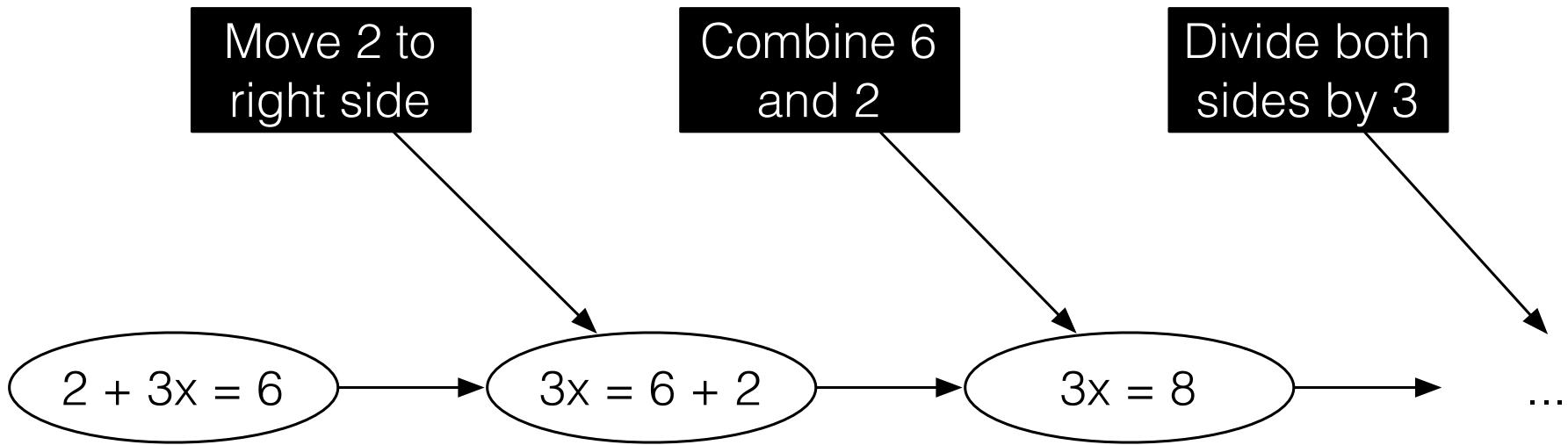
$$x = \frac{1}{2}$$

Algebra  
skills ( $\theta_2$ )



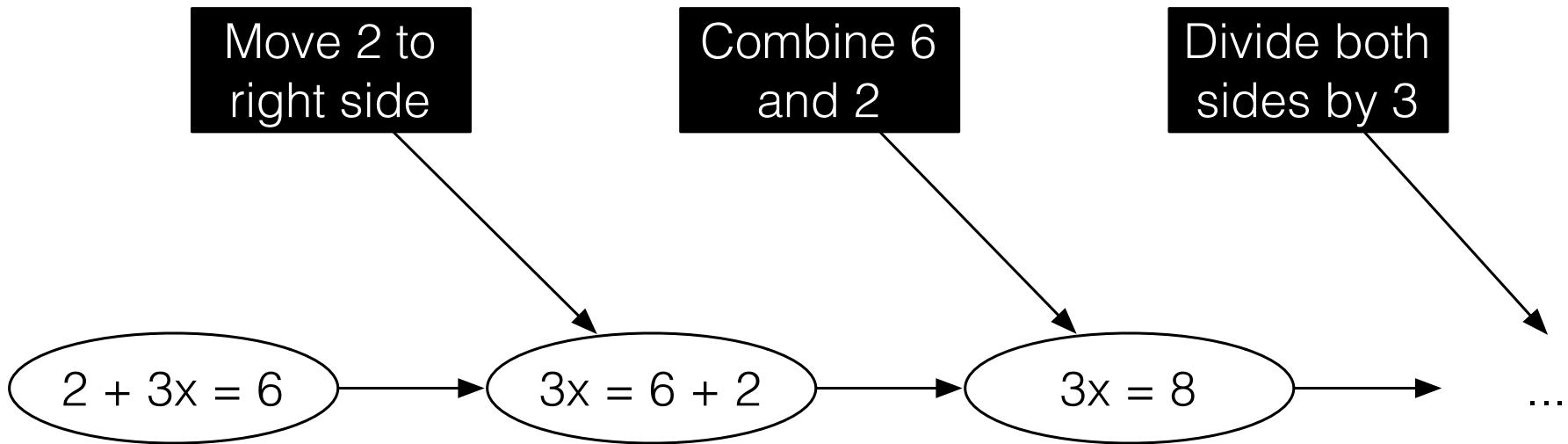
Model of equation solving  
given an understanding

# Generative Model of Equation Solving: Markov Decision Processes



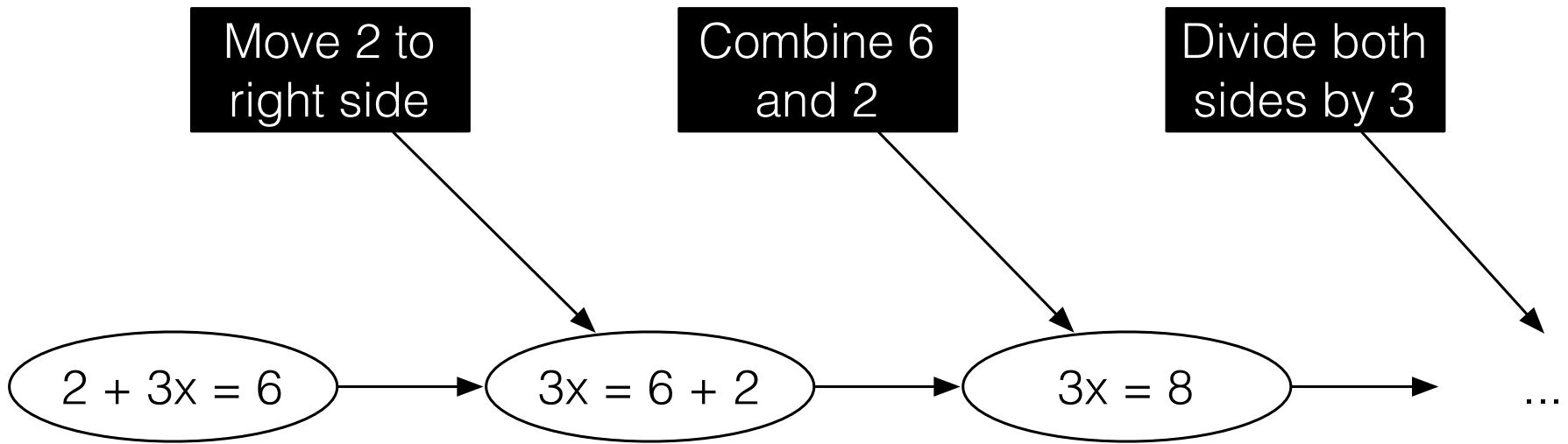
4 dimensions of  $\theta$  affect probability of transitions

# Generative Model of Equation Solving: Markov Decision Processes



1 dimension affects action set

# Modeling Learners' Policies



Assume a noisily optimal policy:

$$p(a | s) \propto \exp(\theta_\beta \cdot Q(s, a))$$

Long term expected value:

$$Q(s, a) = \sum_{s' \in S} p(s'|s, a) \left( R(s, a) + \gamma \sum_{a' \in A} p(a'|s') Q(s', a') \right)$$

# Bayesian Inverse Planning

Algebra  
skills ( $\theta_1$ )



$$3x + \frac{5}{2} = 2(x + 3)$$

$$3x + \frac{5}{2} = 2x + 6$$

$$3x - 2x + \frac{5}{2} = 6$$

$$3x - 2x = 6 - \frac{5}{2}$$

$$x = 6 - \frac{5}{2}$$

$$x = \frac{1}{2}$$

$$p(\theta \mid \text{equations}) \propto p(\theta)p(\text{equations} \mid \theta)$$

Algebra  
skills ( $\theta_2$ )



$$3x + \frac{5}{2} = 2(x + 3)$$

$$3x + \frac{5}{2} = 2x + 3$$

$$3x - 2x + \frac{5}{2} = 3$$

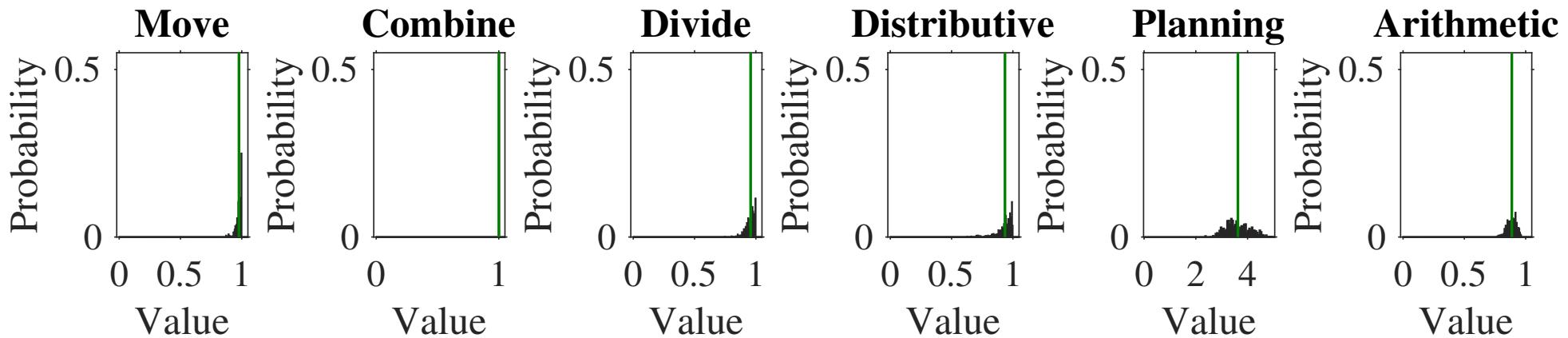
$$3x - 2x = 3 - \frac{5}{2}$$

$$x = 3 - \frac{5}{2}$$

$$x = \frac{1}{2}$$

Approximate via MCMC

# Output for One Learner



- In simulation, estimated parameters close to true parameters
- Correlated with estimates from teachers about individual skills for four of the dimensions

# Testing Personalized Feedback

**experimentAccount726**

Tutorial About the Berkeley Algebra Tutor Contact Info

There are some things that you can do in one step, however. For instance, if you want to divide both sides by a constant, you don't need to write everything out like the picture on the left; you can divide and simplify at the same time, just like the picture on the right.

Step	Equation	Add/Remove	TypeSet Equation
1	$-5x = -5$		$-5x = -5$
2	$\frac{-5x}{-5} = \frac{-5}{-5}$	+	$x = 1$
3	$x = 1$	x	$x = 1$

**X** It isn't necessary to show the middle line. **✓** You can divide by a constant and simplify all in one line.

Continue

You completed enough problems to get feedback from the tutor! The more problems you do, the more you'll improve your algebra skills. The computer analyzed the way that you have been solving problems to identify skills that you're close to mastering and skills where you still need some improvement.

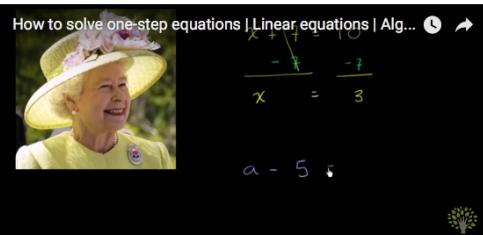
## Distributive property

Great! You're able to turn terms like  $3(x + 4)$  into  $3x + 12$ , distributing the 3 over the terms in parentheses. You know that you have to multiply **all** of the terms in parentheses by the coefficient.

## Moving terms

You sometimes have difficulty moving a term from one side of the equation to the other. When you move a term, you're subtracting that term from both sides. This means that the term will change signs when it moves from one side to the other.

You will now be reviewing a skill that you haven't quite mastered. Click continue to begin!



Show interface help.

Step Equation Add/Remove Typeset Equation

1  $-2y = -5 - 7(7y - 9)$   $-2y = -5 - 7(7y - 9)$

2  $-2y = -5 - 49y + 63$   $+ -2y = -5 - 49y + 63$

3  $-2y = -57 - 49y$   $+ -2y = -57 - 49y$

4  $-2y - 49y = -57$   $+ -2y - 49y = -57$

5  $-51y = -57$   $+ -51y = -57$

6  $y = -\frac{57}{51}$   $\times y = -\frac{57}{51}$

Add step

Done solving!

most points for getting the problem right, but you still earn some points for trying! Remember, you can click the "Interface reference" tab to get a reminder.

u're ready to continue!

Show interface help

It looks like you made an error. The first row where you have an error is highlighted. Check your work, making sure you always keep the two sides equal to one another. You might want to delete all of the rows below the one with the error, so that you won't be confused by your previous attempt.

Remember, the more you practice your skills, the better you'll perform!

Try again!

Step Equation

1  $-4$

2  $2$

3

4  $x = -\frac{24}{5}$   $\times x = -\frac{24}{5}$

Add step

Show interface help.

Step	Equation	Add/Remove	Typeset Equation
1	$-2y = -5 - 7(7y - 9)$		$-2y = -5 - 7(7y - 9)$
2	$-2y = -5 - 49y + 63$	+	$-2y = -5 - 49y + 63$
3	$-2y = -57 - 49y$	+	$-2y = -57 - 49y$
4	$-2y - 49y = -57$	+	$-2y - 49y = -57$
5	$-51y = -57$	+	$-51y = -57$
6	$y = -\frac{57}{51}$	x	$y = -\frac{57}{51}$

$$-2y = -5 - 7(7y - 9)$$

$$-2y = -5 - 49y + 63$$

$$-2y = -57 - 49y$$

$$-2y - 49y = -57$$

$$-51y = -57$$

$$y = -\frac{57}{51}$$

Add step

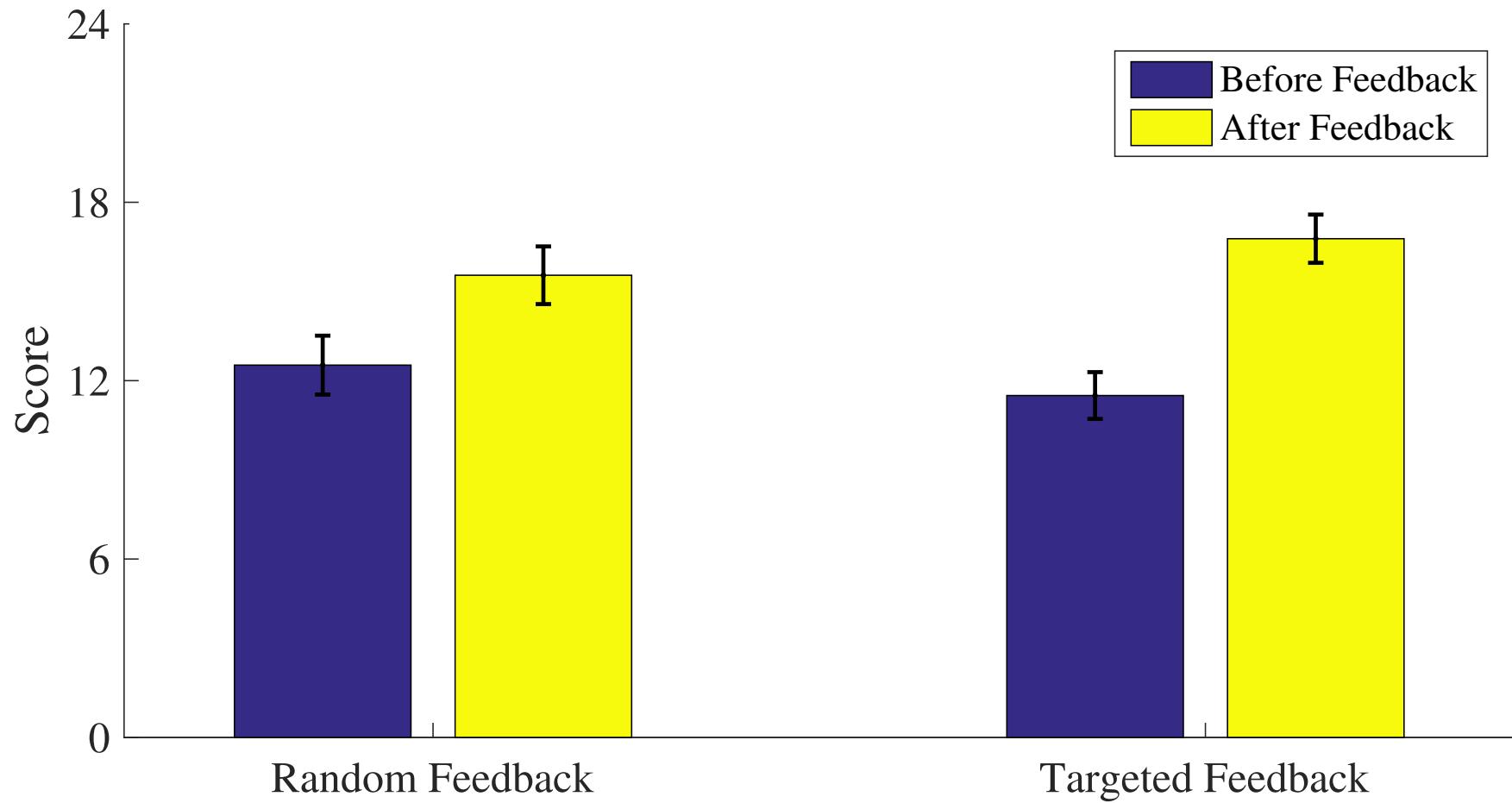
Done solving!

Session 1:  
Website Problem  
Solving and Multiple  
Choice Test

Session 2:  
Feedback Activity

Session 3:  
Website Problem  
Solving and Multiple  
Choice Test

# Performance Change for Participants with Varying Skill Levels



Reliable difference in amount of improvement by condition.

# Improving Feedback Quality

## Question 2

Correct

Mark 1.00 out of  
1.00

 Flag question  
 Edit question

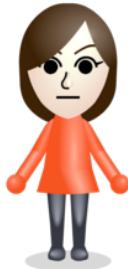
Chris has a cookie jar that contains 5 chocolate cookies and 3 oatmeal cookies. He will draw two cookies from the jar, one at a time without replacing the first cookie. What is the probability that Chris gets a chocolate cookie on his first draw and an oatmeal cookie on his second draw?

Enter your answer below.

15/56

When you have 8 cookies in the jar and 5 are chocolate you have a  $5/8$  chance of the cookie you draw being chocolate. When there are 7 cookies in the jar and 3 are oatmeal you have a  $3/7$  chance of drawing the oatmeal cookie. To get the overall probability you need to multiply  $5/8$  by  $3/7$  which results in overall probability of  $15/56$

# Measuring and Optimizing Feedback Quality



Chris has a cookie jar that contains 5 chocolate cookies and 3 oatmeal cookies. He will draw two cookies from the jar, one at a time without replacing the first cookie. What is the probability that Chris gets a chocolate cookie on his first draw and an oatmeal cookie on his second draw?

Enter your answer below.

View  
feedback

When you have 8 cookies in the jar and 5 are chocolate you have a  $5/8$  chance of the cookie you draw being chocolate. When there are 7 cookies in the jar and 3 are oatmeal you have a  $3/7$  chance of drawing the oatmeal cookie. To get the overall probability you need to multiply  $5/8$  by  $3/7$  which results in overall probability of  $15/56$

How helpful do you think this explanation is for learning?

Absolutely  
Unhelpful



Perfect

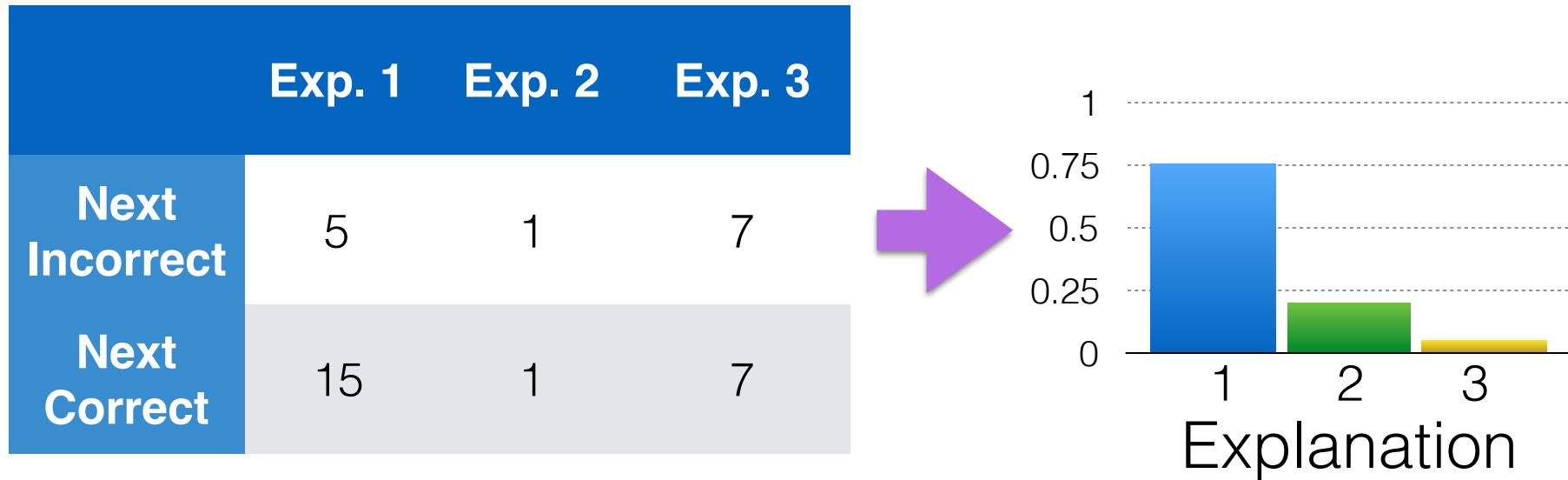
Question 2  
Correct  
Mark 1.00 out of  
1.00  
 Flag question  
 Edit question

Rate feedback

Answer next question

Select explanations using bandit algorithm

# Thompson Sampling



Maintain estimate of effectiveness for each explanation (here, Beta distribution), and update for each learner.

# Generating Explanations

When you have 8 cookies in the jar and 5 are chocolate you have a 5/8

chance to get a chocolate cookie. This can be simulated by going to the jar and picking a cookie at random. If there are 8 cookies in the jar, there are 5 chocolate ones. The total number of cookies in the jar is 8.

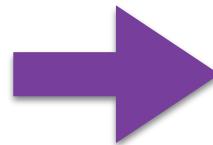
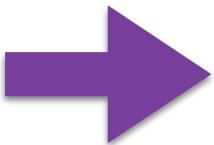
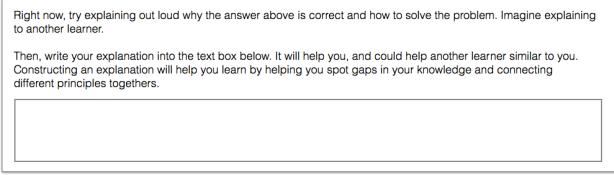
Since there are 5 chocolate cookies the probability that Chris gets an chocolate cookie is 5/8

Since Chris removed 1 cookie from the jar and did not replace it or put it back

Right now, try explaining out loud why the answer above is correct and how to solve the problem. Imagine explaining to another learner.

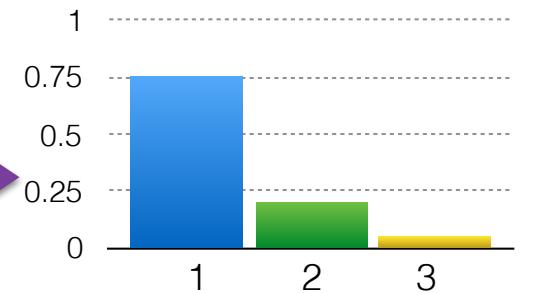
Then, write your explanation into the text box below. It will help you, and could help another learner similar to you. Constructing an explanation will help you learn by helping you spot gaps in your knowledge and connecting different principles together.

# AXIS: Adaptive eXplanation Improvement System



Filter likely  
useless  
explanations

Learner-sourced  
explanations

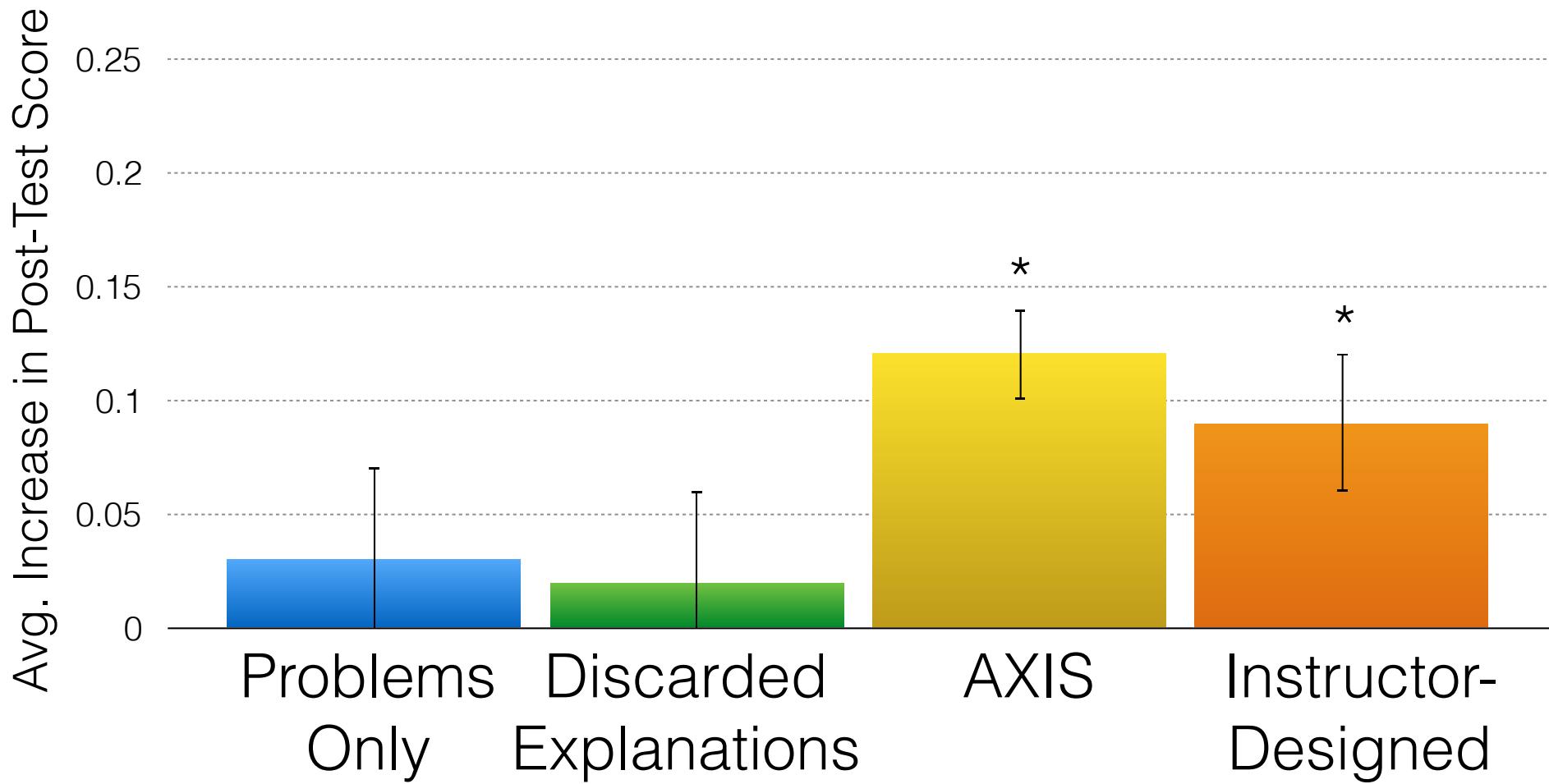


Choose next  
explanation via  
Thompson sampling

# Testing AXIS

- Initial 150 participants solved four math problems, each with an explanation selected by AXIS and with the opportunity to generate their own explanation
- Evaluate explanations: new participants solved the 4 problems, each followed by an explanation, and then were assessed using 12 new problems
- Participants in the evaluation received (1) no explanations, (2) explanations discarded in filtering, (3) explanations chosen by AXIS, or (4) explanations written by an instructional designer

# Improvement from Learning to Assessment Phase



# Conclusion

- Machine learning and computational modeling can provide domain-general frameworks for selecting feedback and improving it
- Next steps:
  - Combine dynamic improvement and personalization
  - Improving hypothesis space of cognitive model via data
  - Feedback on strategy

# Thank you!

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## Collaborators:

Tom Griffiths	Joseph Williams	Walter Lasecki
Rachel Jansen	Juho Kim	Neil Heffernan
Jonathan Brodie	Samuel Maldonado	
Sam Vinitsky	Krzysztof Gajos	

## Related papers:

Rafferty, A. N., LaMar, M. M., & Griffiths, T. L. (2015). Inferring learners' knowledge from their actions. *Cognitive Science*, 39(3): 584-618.

Rafferty, A. N., Jansen, R. A., & Griffiths, T. L. (2016) Using Inverse Planning for Personalized Feedback. *Proceedings of the 9th International Conference on Educational Data Mining*.

Williams, J. J., Kim, J., Rafferty, A. N., Maldonado, S., Gajos, K. Z., Lasecki, W. S., & Heffernan, N. (2016). AXIS: Generating Explanations at Scale with Learnersourcing and Machine Learning. *Proceedings of the Third (2016) ACM Conference on Learning @ Scale*.

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