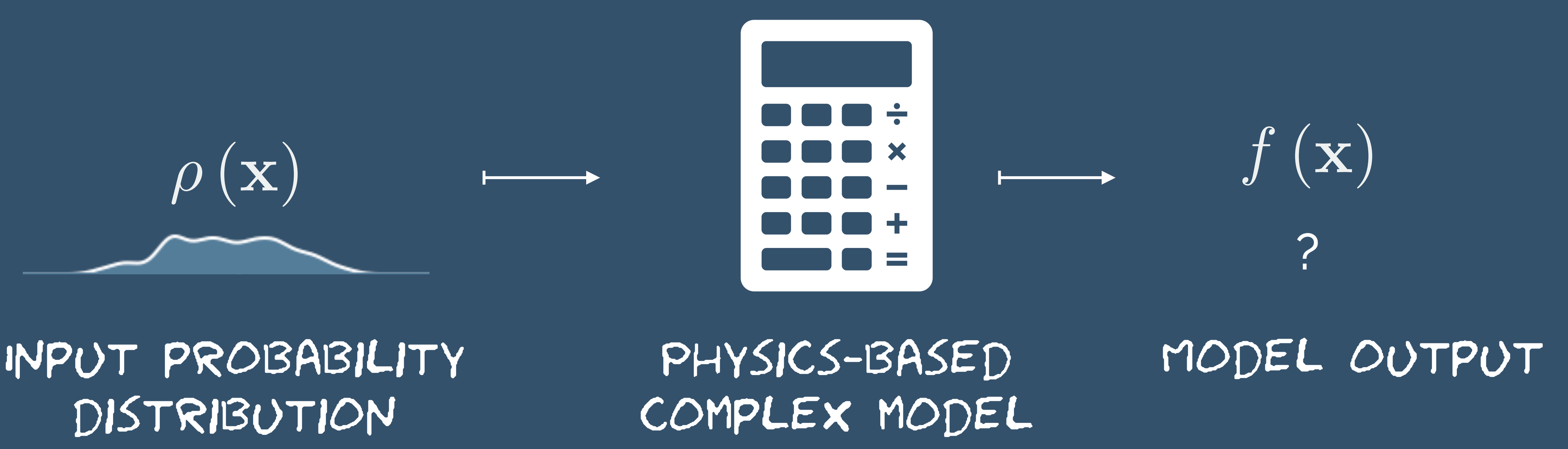
Bayesian polynomial chaos

A bayesian take on polynomial chaos — offering new and rigorous ways for engineers to interpret and work with simulation and measurement data.

1 Background & motivation

Polynomial chaos* [1] seeks to quantify the impact of an **aleatory** uncertainty ρ in a model's inputs $\mathbf{x}$, on its outputs f. This is done by creating a polynomial surrogate of the model, where the basis terms are from a family of orthogonal polynomials (orthogonal to $\rho(\mathbf{x})$).



The model is evaluated at quadrature points (tensor, sparse and effectively subsampled in higher dimensions [2,3]), facilitating rapid estimates of the mean, variance and skewness in f based on the uncertainty.

While extremely successful — with significant industrial uptake — two limitations of the paradigm are:

i. the inability to account for **epistemic** uncertainty in the model;
ii. the inability to exploit prior information or even experimental data (beyond those used as model boundary conditions).

2 Bayesian polynomial chaos

Bayesian polynomial chaos is a Gaussian process [4] with a mean, and two-point covariance function, both formed with orthogonal polynomials

$$g \sim \mathcal{N}\left(\underbrace{\boldsymbol{\mu}_g(\mathbf{x})}_{\text{mean function}}, \underbrace{\boldsymbol{\Sigma}_g(\mathbf{x}, \mathbf{x}')}_{\text{covariance function}}\right)$$

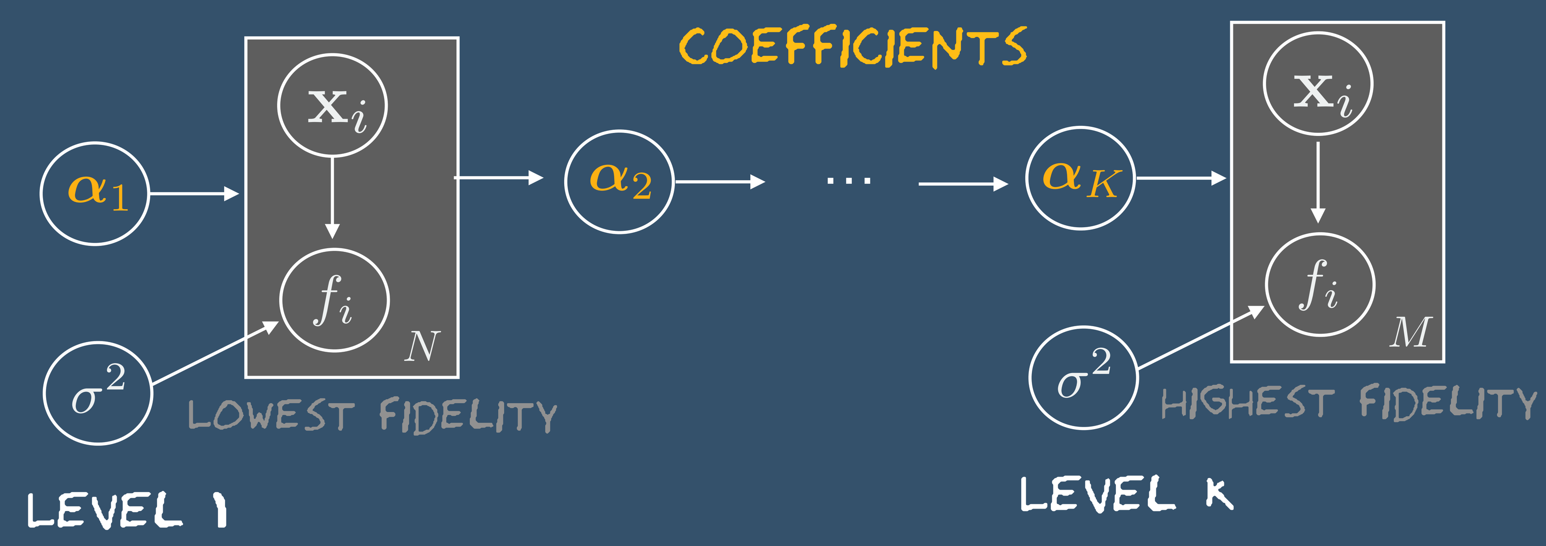
where these quantities are given by

$$\boldsymbol{\mu}_g(\mathbf{x}) = \underbrace{\mathbf{V}^T(\mathbf{x})}_{\text{Vandermonde type matrix}} \boldsymbol{\mu}_{\alpha} \qquad \boldsymbol{\Sigma}_g(\mathbf{x}, \mathbf{x}') = \mathbf{V}^T(\mathbf{x}) \boldsymbol{\Sigma}_{\alpha} \mathbf{V}^T(\mathbf{x}')$$

MEAN & COVARIANCE OF COEFFICIENTS

Whilst this paradigm naturally addresses the epistemic uncertainty issue, it offers three key ideas.

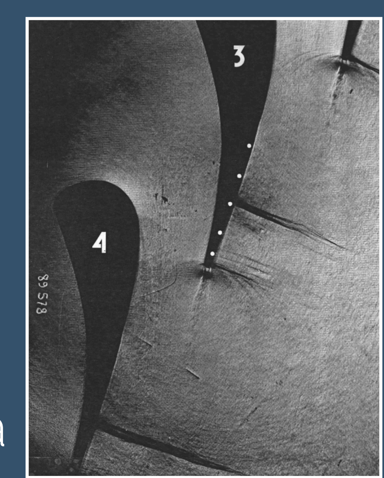
i. A **multi-fidelity framework**, where the posterior coefficients of a lower-fidelity polynomial, become the prior coefficients for a higher-fidelity polynomial.

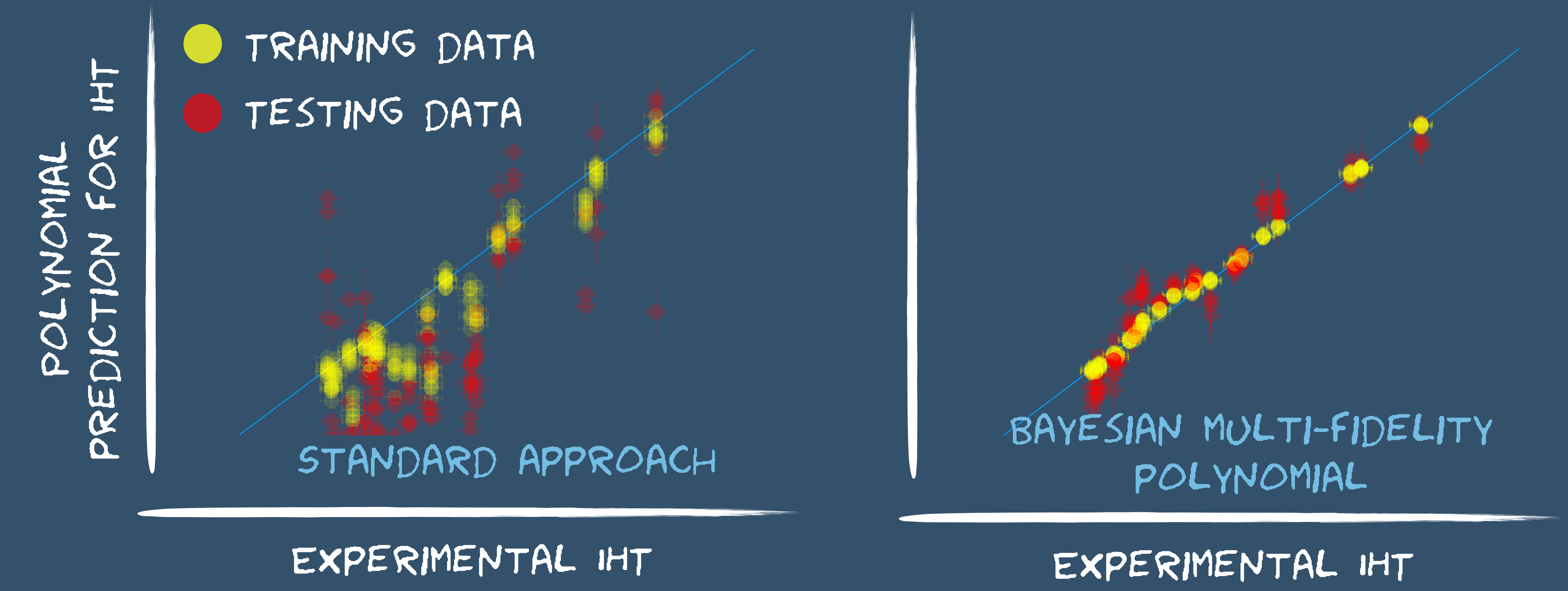


ii. Ability to **fuse additional linear operatic information**. (e.g., integrals and gradients) into the polynomial posterior, by creating a joint distribution, conditioning, and then marginalising over the additional information.
iii. Ability to construct **coregional polynomials** for related outputs from the physics-based model — reducing number of model evaluations.

3 Preliminary results

A demonstration of the multi-fidelity framework is given below. The objective is to predict experimental surface integrated heat transfer (IHT), for variations in Mach number, Reynolds number and turbulence intensity for a VKI turbine blade [5].





*A bit of a misnomer as there is nothing inherently chaotic about it.

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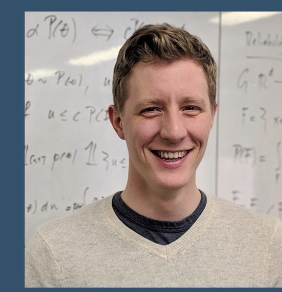
Pranay Seshadri
Imperial College London;
The Alan Turing Institute;
Effective Quadratures.
psesh.com

Andrew Duncan
Imperial College London;
The Alan Turing Institute;
Effective Quadratures.



Ashley Scillitoe
The Alan Turing Institute;
Effective Quadratures.

Imperial College London
imperial.ac.uk

The Alan Turing Institute
turing.ac.uk

Effective Quadratures
effective-quadratures.org

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[2] Constantine et al. (2012) *Computer Methods in App. Mechanics.* **Paper**

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