Accelerating Simulation of Stiff Systems with Continuous Time Echo State Networks

Ranjan Anantharaman[1], Yingbo Ma[2], Shashi Gowda[1], Chris Laughman[3], Viral Shah[2], Alan Edelman[1] and Chris Rackauckas[1]

MIT[1], Julia Computing[2], Mitsubishi Electric Research Labs[3]

- **General, data-driven method** to generate “surrogates” (an approximate model)
- Replicates the physics with **high accuracy**
- **Fast: 100x acceleration** over baseline

Consider

$$x' = f(x, p, t)$$

- “p” - parameters of the system, belonging to parameter space “P”
- “t” - time
- “f” - nonlinear function

Let x(p*,t) denote a solution of this system for some p* in P.

Construct a high dimensional ODE (called a reservoir):

$$r' = \tanh(Ar + W_{in}x(p^*, t))$$

Next, compute a matrix W_out, which is the **least squares projection** from this reservoir to *any other* solution in this parameter space x(p,t) :

$$x(p, t) = W_{out}r(t)$$

If we sample “n” sets of parameters {p_1,..., p_n} from P, we can construct a set of matrices:

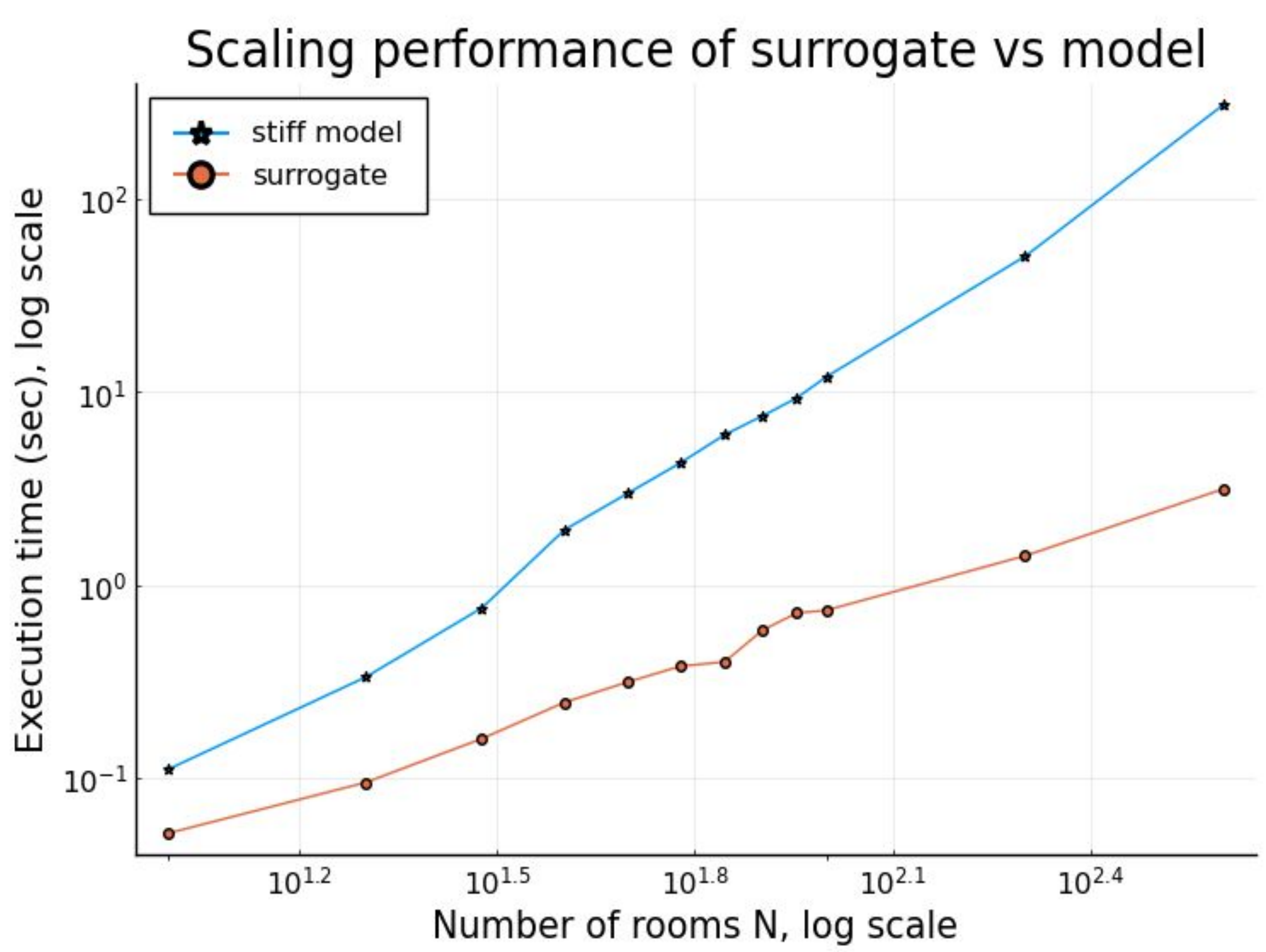
$$\{W_{out}(p_1), \ldots, W_{out}(p_n)\}$$

With these matrices, construct an interpolating object (radial basis function):

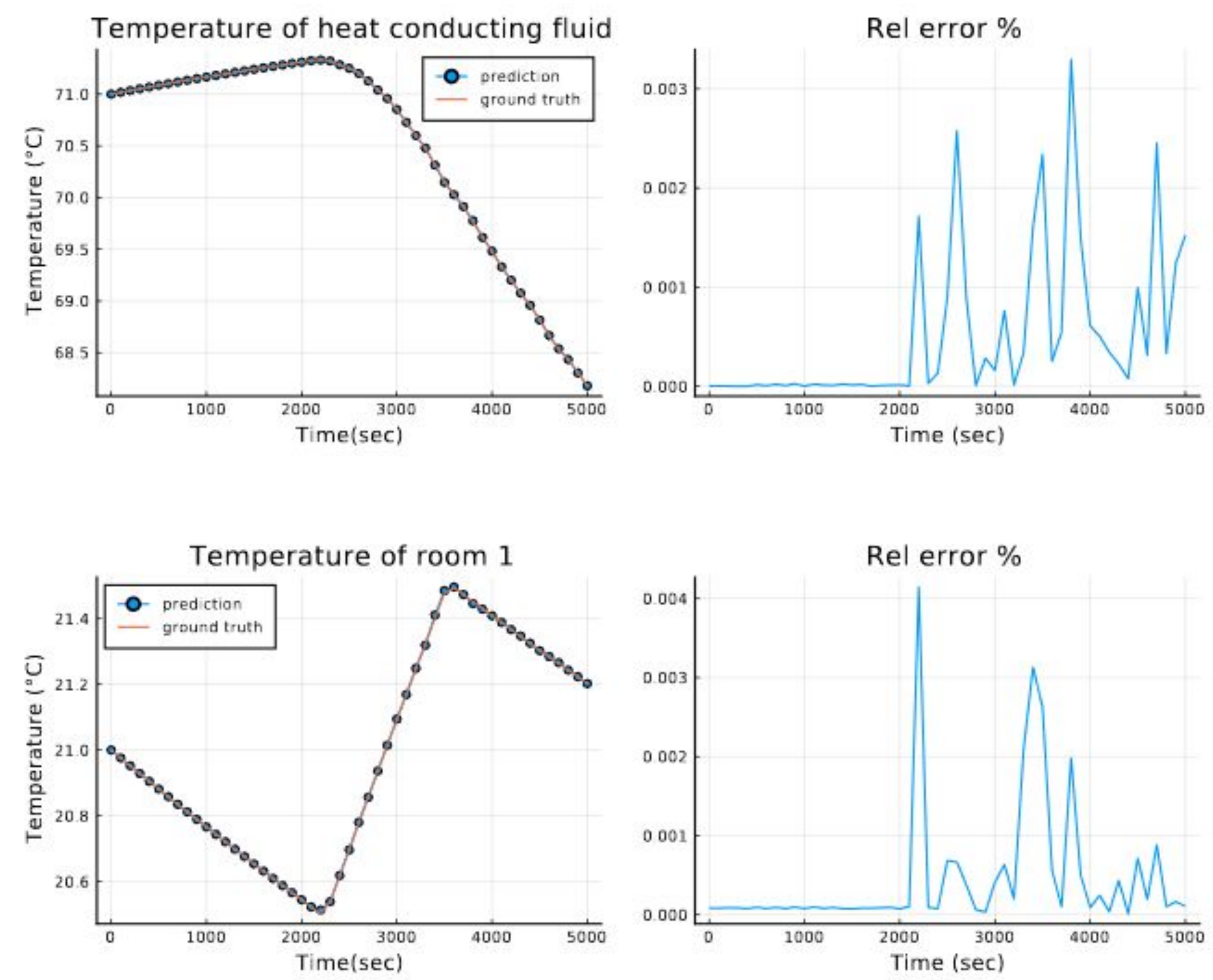
$$p \mapsto W_{out}$$

And define the following predictor:

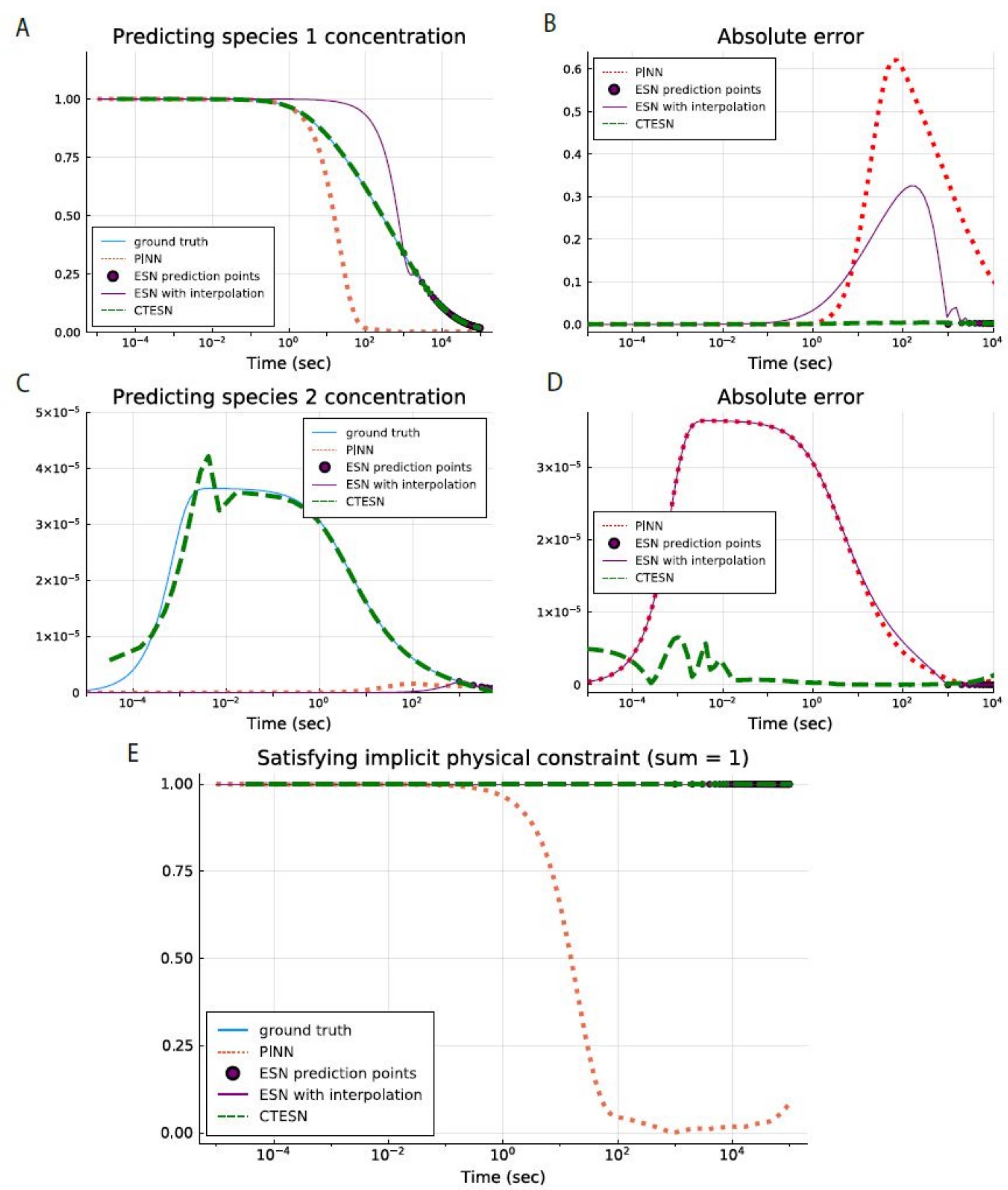
$$\hat{x}(\hat{p}, t) = W_{out}(\hat{p})r(t)$$



Scalable Heating System:
Scaling performance of our surrogate
100x acceleration over baseline of 801 ODEs



Scalable Heating system:
Predictive performance on parameters the surrogate has not seen in training. (10 rooms in this system)



Comparison of Physics Informed Neural Networks vs (Discrete) Echo State Networks vs CTESN on Robertson’s Equations -
CTESNs are able to capture both slow and fast transients, improving on the current state of the art. CTESN uses infromation from the ODE solver during training, which enables it to capture multi-scale dynamics