AARHUS
UNIVERSITET

Computer Vision & Machine Learning

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This week

Introduction to Image Processing:

- images as signal and functions

- Digital images

- Image enhancement

- Color image enhancement

Slides based on:

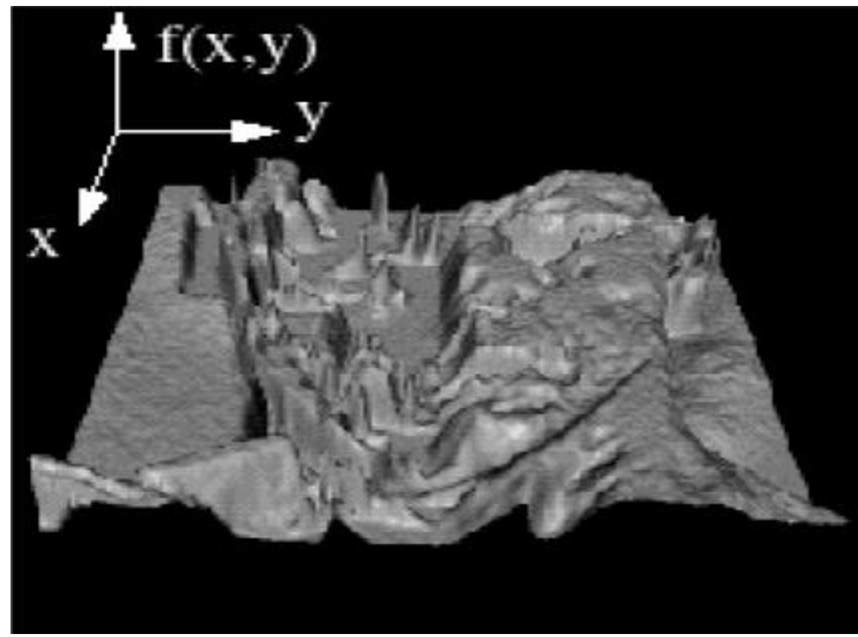
- S. Rafael, C. Gonzalez, E.R. Woods, Digital Image Processing, 3rd ed. 2008, Addison Wesley

Images as functions

Image

Images as functions

Image is a function of intensities (colors)



Images as functions

Consider a monochrome image

- We represent the spectral intensity distribution of the image by a continuous function f(x,y)
- f(x,y) is proportional to the grey level of the image at the point location (x,y)

$$\textbf{(black)}\ 0 \le f(x,y) \le f_{max}\ \textbf{(white)}$$

- The limit cases:
  - Lower bound is because light intensity is a real positive quantity
  - Upper bound is due to the fact that in all practical imaging systems, the physical system imposes some restrictions on the maximum intensity level of an image, e.g., film saturation and cathode ray tube phosphor heating
- Intermediate values between 0 and f_{max} are called shades of gray varying from black to white

Digital images

How do we get a digital image?

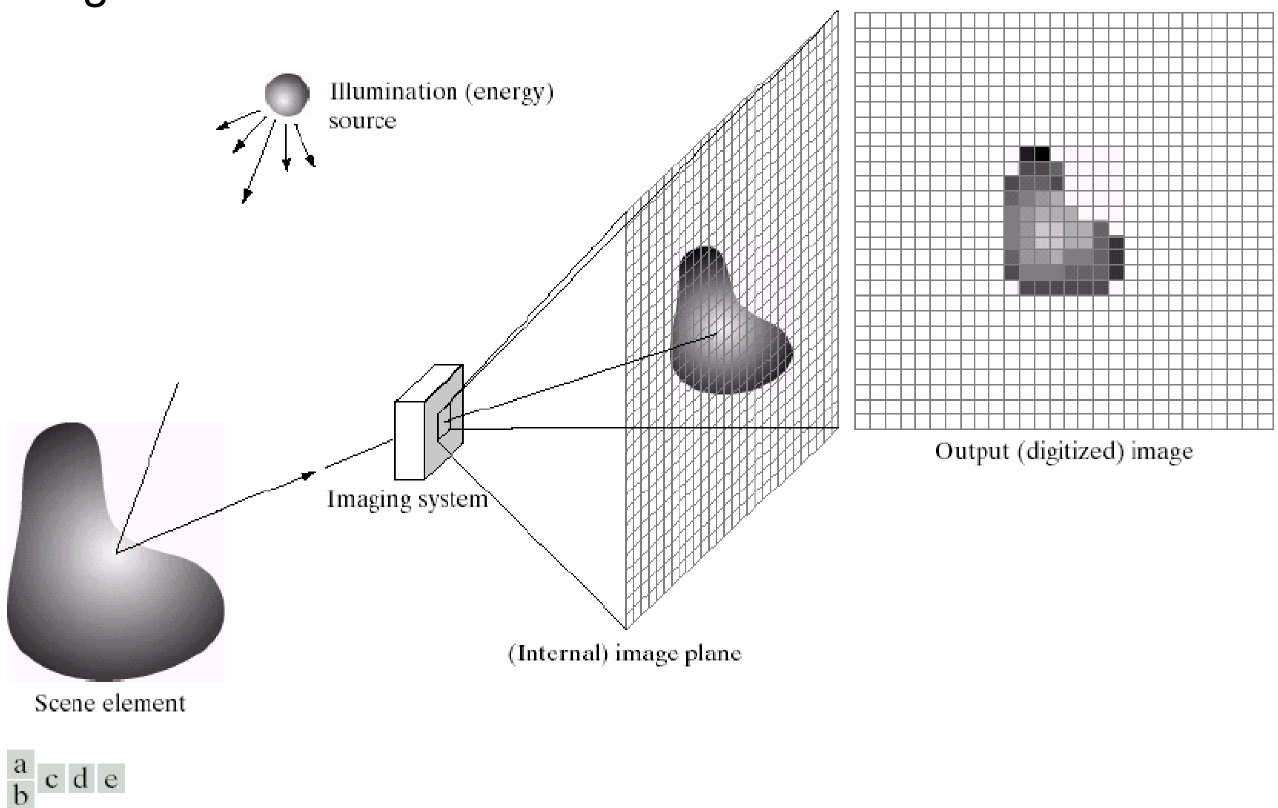


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

Digital images

How do we get a digital image:
- Spatial sampling

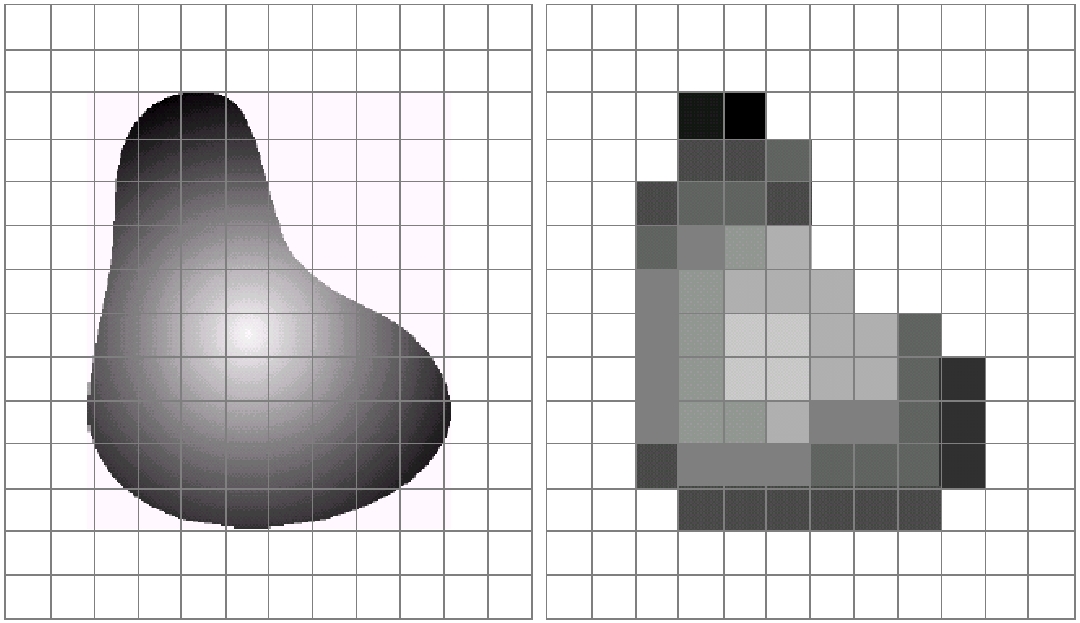


FIGURE 2.17 (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.

Digital images

How do we get a digital image:
- Spatial sampling
- Quantization

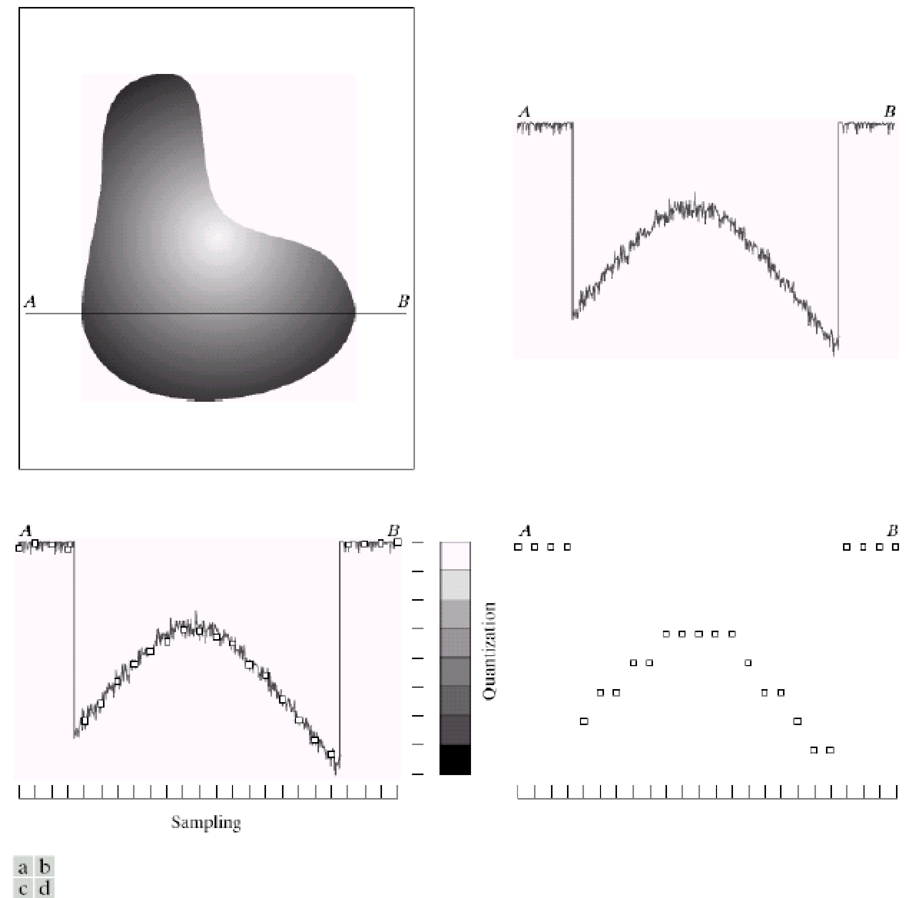


FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Digital images

How do we get a digital image:

- Spatial sampling
  - partitioning xy plane into a grid
  - the coordinate of the center of each grid is a pair of elements from the Cartesian product ZxZ (Z^2)
  - Z^2 is the set of all ordered pairs of elements (a,b) with a and b being integers from Z

Digital images

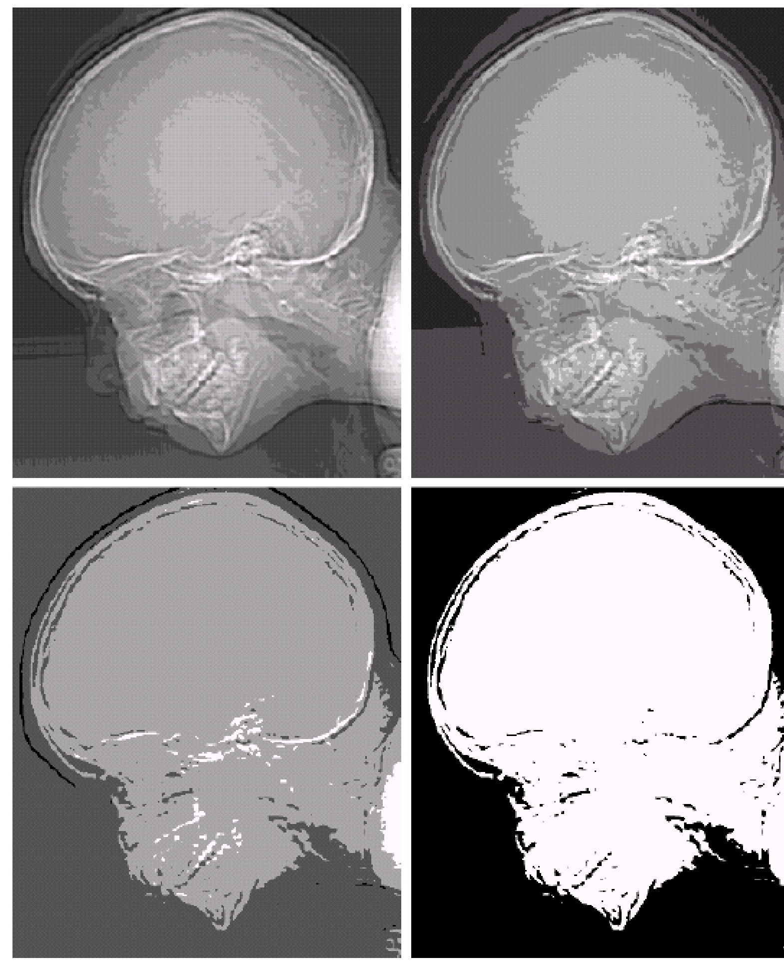
How do we get a digital image:

- Spatial sampling
  - partitioning xy plane into a grid
  - the coordinate of the center of each grid is a pair of elements from the Cartesian product ZxZ (Z^2)
  - Z^2 is the set of all ordered pairs of elements (a,b) with a and b being integers from Z

- Quantization
  - gray levels are usually integers (L in number)
  - the (discrete) levels are equally spaced between 0 and L-1 in the gray scale
  - the number of gray levels is (usually) a power of 2 ($256 = 2^8$)
- Keep in mind that many times we might transform the gray levels to real values $0 \leq f(x,y) \leq 1$ for applying our computations

Digital images

Digital images are not only those captured by (digital) cameras!



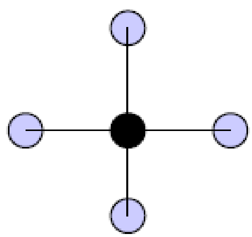
e f
g h

FIGURE 2.21 *(Continued)* (e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)

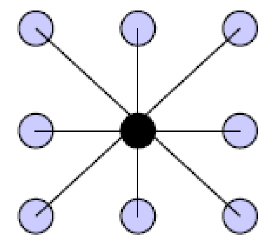
Digital images

Commonly used terminology

Neighbors of a pixel $p=(i,j)$



$N_4(p)=\{(i-1,j),(i+1,j),(i,j-1),(i,j+1)\}$



$N_8(p)=\{(i-1,j),(i+1,j),(i,j-1),(i,j+1),$
$(i-1,j-1),(i-1,j+1),(i+1,j-1),(i+1,j+1)\}$

Adjacency

4-adjacency: p,q are 4-adjacent if p is in the set $N_4(q)$

8-adjacency: p,q are 8-adjacent if p is in the set $N_8(q)$

Note that if p is in $N_{4/8}(q)$, then q must be also in $N_{4/8}(p)$

Digital images

Commonly used distance definitions (for the image grid)

Euclidean distance
(2-norm)

$2\sqrt{2}$	$\sqrt{5}$	2	$\sqrt{5}$	$2\sqrt{2}$
$\sqrt{5}$	$\sqrt{2}$	1	$\sqrt{2}$	$\sqrt{5}$
2	1	0	1	2
$\sqrt{5}$	$\sqrt{2}$	1	$\sqrt{2}$	$\sqrt{5}$
$2\sqrt{2}$	$\sqrt{5}$	2	$\sqrt{5}$	$2\sqrt{2}$

D_4 distance
(city-block distance)

4	3	2	3	4
3	2	1	2	3
2	1	0	1	2
3	2	1	2	3
4	3	2	3	4

D_8 distance
(checkboard distance)

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Image enhancement

Enhancement in the spatial domain:

- Transformation of the form $g(x,y) = T[f(x,y)]$

  f(x,y) is the original image

  T[.] is an operator applied (locally) at (a neighborhood of) position (x,y)

- Special case when the size of the neighborhood is equal to 1. Then, T[.] is called intensity transformation function

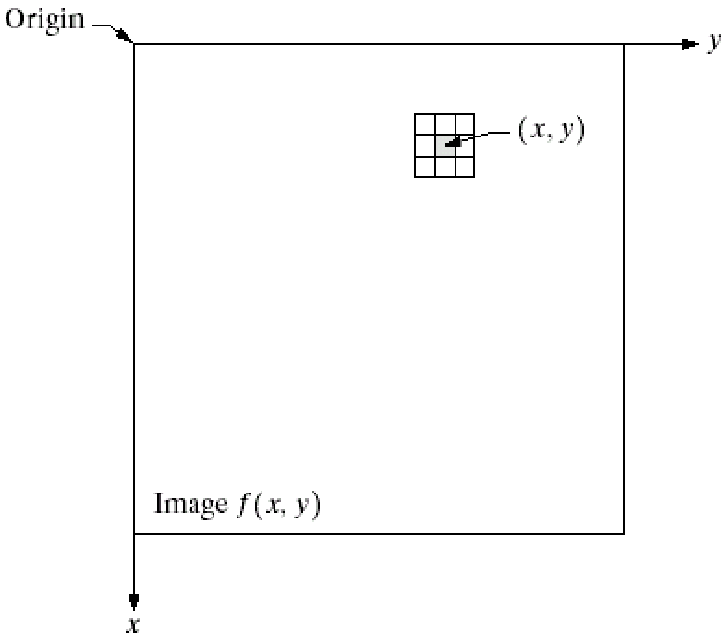


Image enhancement

Linear filtering

- Use a linear combination of the intensity values of pixels residing in the current pixel's neighborhood

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

$$g(x,y) = \frac{\sum_{m=-M}^{M} \sum_{n=-N}^{N} w(m,n) f(x+m, y+n)}{\sum_{m=-M}^{M} \sum_{n=-N}^{N} w(m,n)}$$

Image enhancement

Linear filtering

- Use a linear combination of the intensity values of pixels residing in the current pixel's neighborhood
- Example filters

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

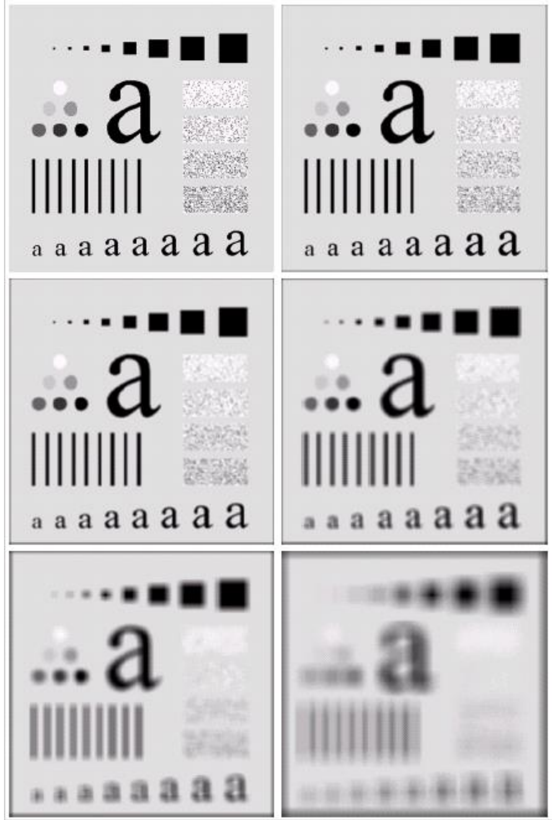
a b

FIGURE 3.34 Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to the sum of the values of its coefficients, as is required to compute an average.

Image enhancement

Linear filtering

- Example results



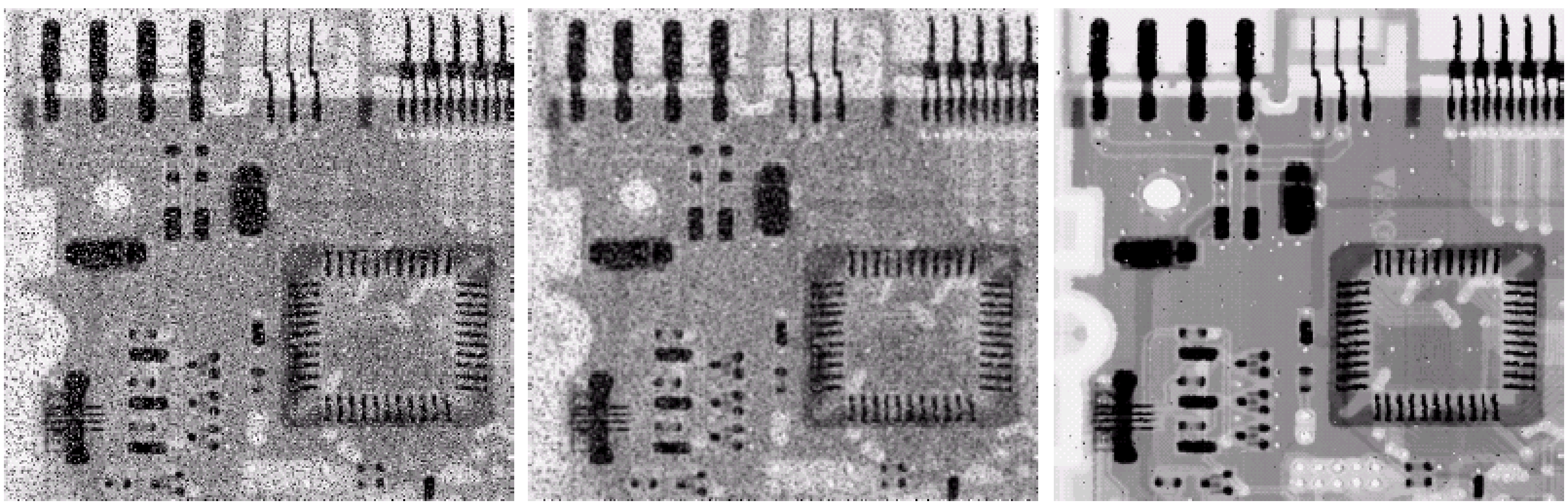
a b
c d
e f

FIGURE 3.35 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $n = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

Image enhancement

Non-linear filtering

- Use a nonlinear function to calculate the result of the intensity in the new image
- An example is Median filtering



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Image enhancement

Intensity transformation

- Apply the transformation g(x,y) = T[f(x,y)] using only the value of f(x,y)
- The transformation T[.] can be hand-crafted or not

FIGURE 3.3 Some basic gray-level transformation functions used for image enhancement.

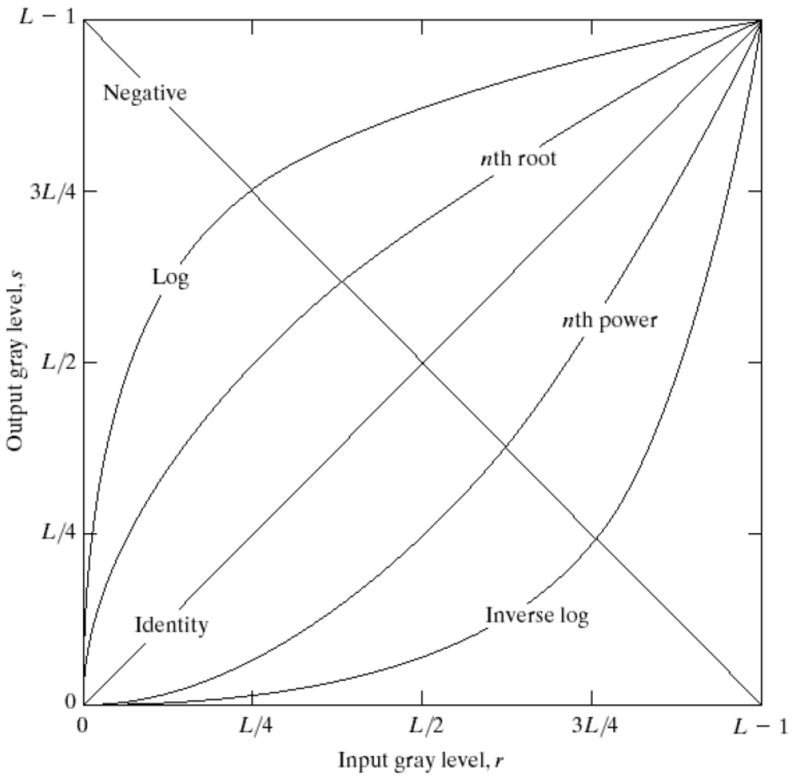


Image enhancement

Intensity histogram

- The histogram of an image with gray levels in the range [0,L-1] is a discrete function $h(r_k) = n_k$

  r_k: the k-th gray level
  n_k: number of pixels in the image having gray level r_k.

- Probability Density Function (PDF) of image intensity values:
- Normalized histogram → $p(r_k) = n_k / n$

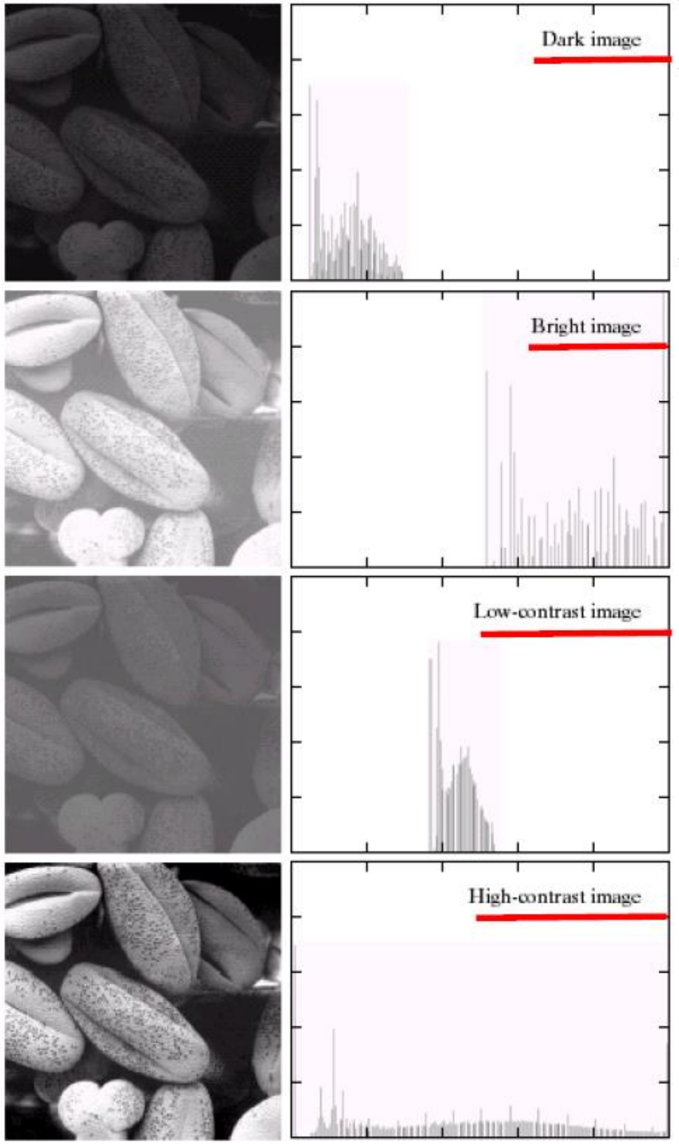


Image enhancement

Contrast stretching

- Poor contrast is the most common defect in images and is caused by reduced and/or nonlinear amplitude range or poor lighting conditions.
- A typical contrast stretching transformation is shown below:

  u1 → v1

  u1: is the intensity value of a pixel in the original (input) image

  v1: is the intensity value of the same pixel in the transformed (output) image

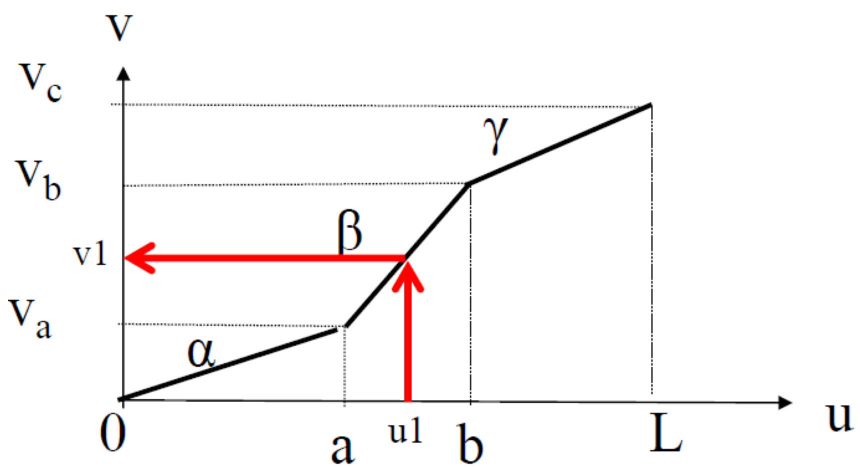
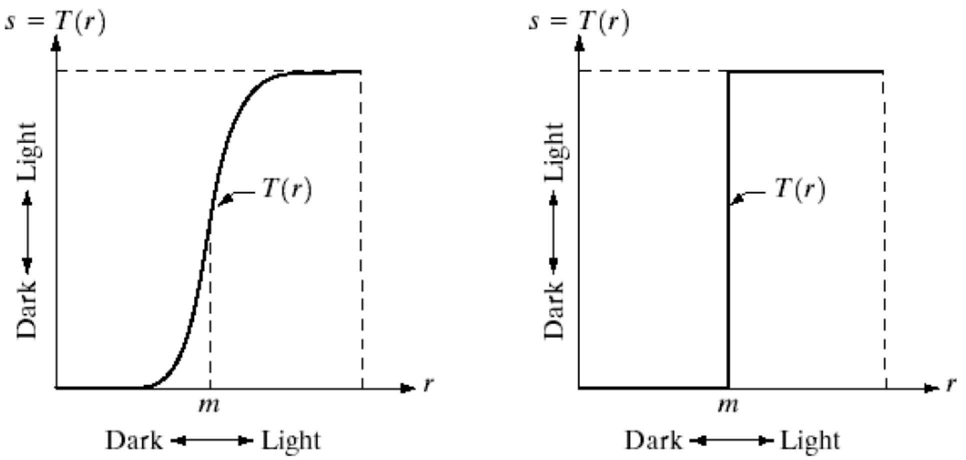


Image enhancement

Contrast stretching

- Poor contrast is the most common defect in images and is caused by reduced and/or nonlinear amplitude range or poor lighting conditions.
- Special case is the thresholding operator



a b

FIGURE 3.2 Gray-level transformation functions for contrast enhancement.

Image enhancement

Histogram equalization:

- Re-scales the intensity values of an image so that the enhanced image histogram follows some desired form

Image enhancement

Histogram equalization:

- Re-scales the intensity values of an image so that the enhanced image histogram follows some desired form

- Goal is to transfer the gray levels so that the histogram of the resulting image is equalized to be constant
  h[i] = const., for all i

- Used for equally using all available gray levels

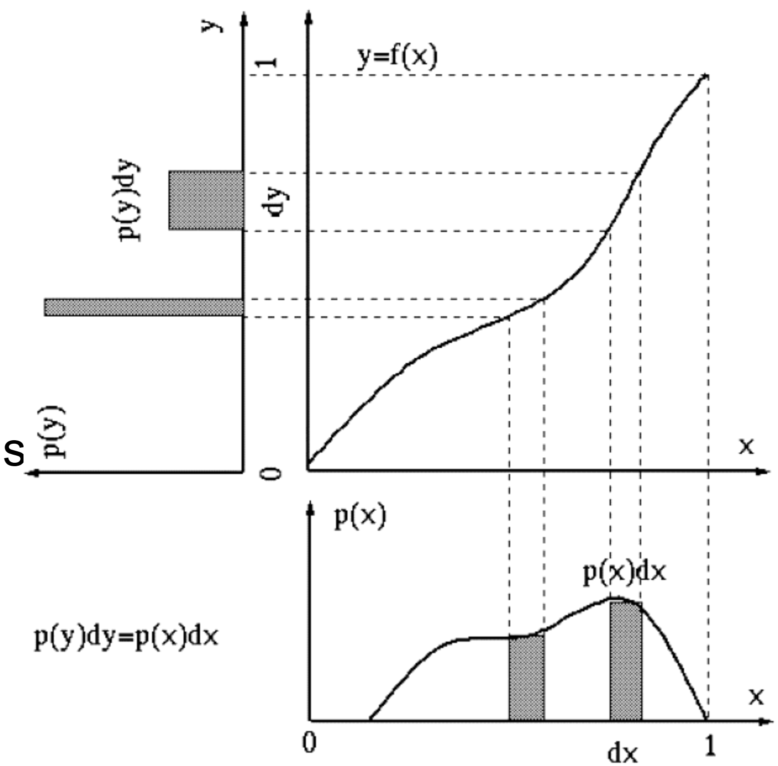


Image enhancement

Histogram equalization:

- We let p(y) be a constant c. In particular, if the gray levels are assumed to be in the ranges between 0 and 1 (0<x<1, 0<y<1), then we have

$$c\,dy = p(x)dx, \quad \text{or } c\frac{dy}{dx} = p(x)$$

- This means that the mapping function for histogram equalization is

$$y = \int_0^x p(u)du = F(x) - F(0) = F(x)$$

where $F(x) = \int_0^x p(u)du, \quad F(0) = 0$ is the cumulative probability distribution of the input image, which monotonically increases

Image enhancement

Histogram equalization:

- For discrete gray levels, the gray level of the input takes one of the discrete values and the continuous mapping function $y = f(x) \triangleq \int_0^x p(x)dx$ becomes discrete $y' = f(x) \triangleq \sum_{i=0}^{x} P_i$

  where P_i is the probability for the gray level of any given pixel to be (0 ≤ i ≤ L-1)

$$P_i = \frac{n_i}{\sum_{i=0}^{L-1} n_i} = \frac{n_i}{N} \quad \text{and} \quad \sum_{i=0}^{L-1} P_i = 1$$

- The resulting function y' is in the range of 0 ≤ y' ≤ 1 and it needs to be converted to the gray levels 0 ≤ y ≤ L-1 by one of the two following ways

$$y = \lfloor y'(L-1) + 0.5 \rfloor \qquad y = \lfloor \frac{y' - y'_{min}}{1 - y'_{min}}(L-1) + 0.5 \rfloor$$

floor operator

Image enhancement

Histogram equalization example:

- Assume the images have pixels in 8 gray levels

x_i	n_i	$P_i = n_i/N$	$y' = F_i$	y_j^1	P_j^1 PDF	F_j^1 CDF	y_j^2	P_j^2 PDF	F_j^2 CDF
0/7	790	0.19	0.19	1/7	0.19	0.19	0/7	0.19	0.19
1/7	1023	0.25	0.44	3/7	0.25	0.44	2/7	0.25	0.44
2/7	850	0.21	0.65	5/7	0.21	0.65	4/7	0.21	0.65
3/7	656	0.16	0.81	6/7			5/7	0.16	0.81
4/7	329	0.08	0.89	6/7	0.24	0.89	6/7	0.08	0.89
5/7	245	0.06	0.95	7/7			7/7		
6/7	122	0.03	0.98	7/7			7/7		
7/7	81	0.02	1.00	7/7	0.11	1.00	7/7	0.11	1.00
	4096								

Image enhancement

Histogram equalization example:
- Assume the images have pixels in 8 gray levels

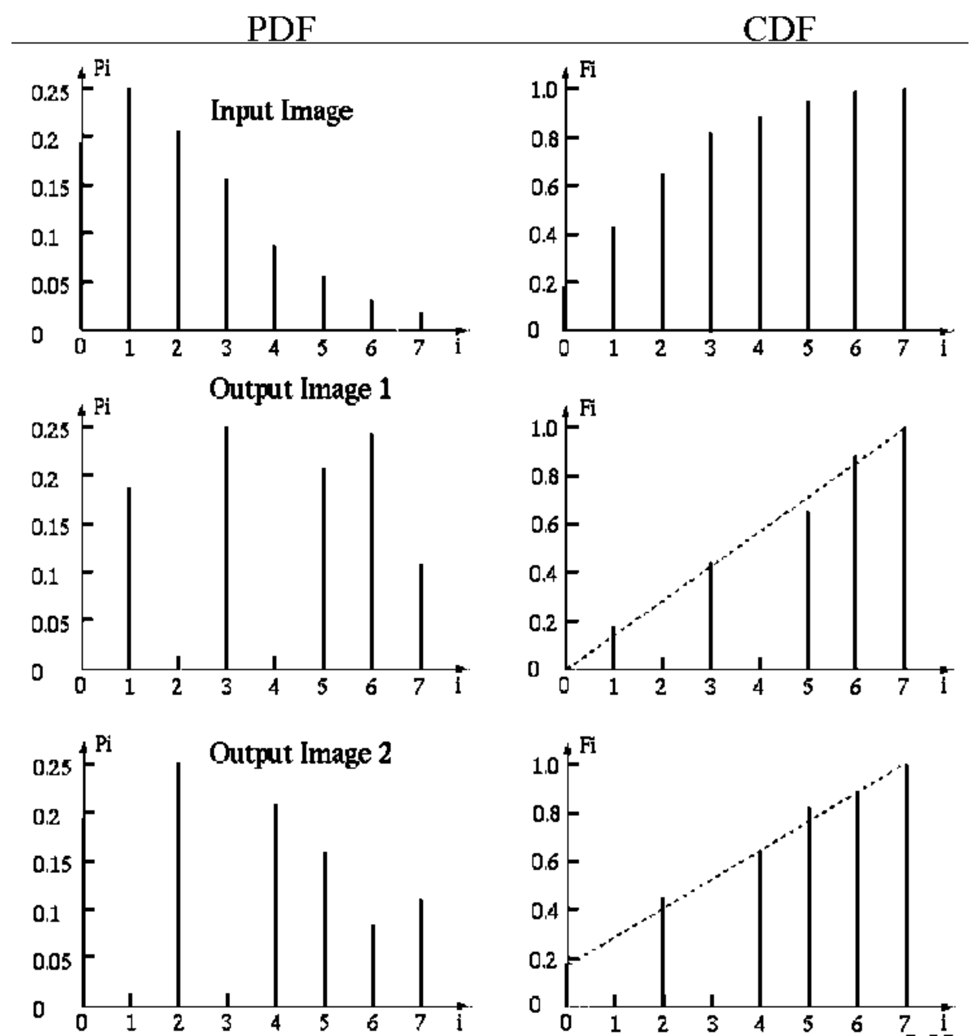


Image enhancement

Histogram equalization example:

- the histogram of a given image is equalized. Although the resulting histogram may not look constant, but the cumulative histogram is a exact linear ramp indicating that the density histogram is indeed equalized
- The density histogram is not guaranteed to be a constant because the pixels of the same gray level cannot be separated to satisfy a constant distribution

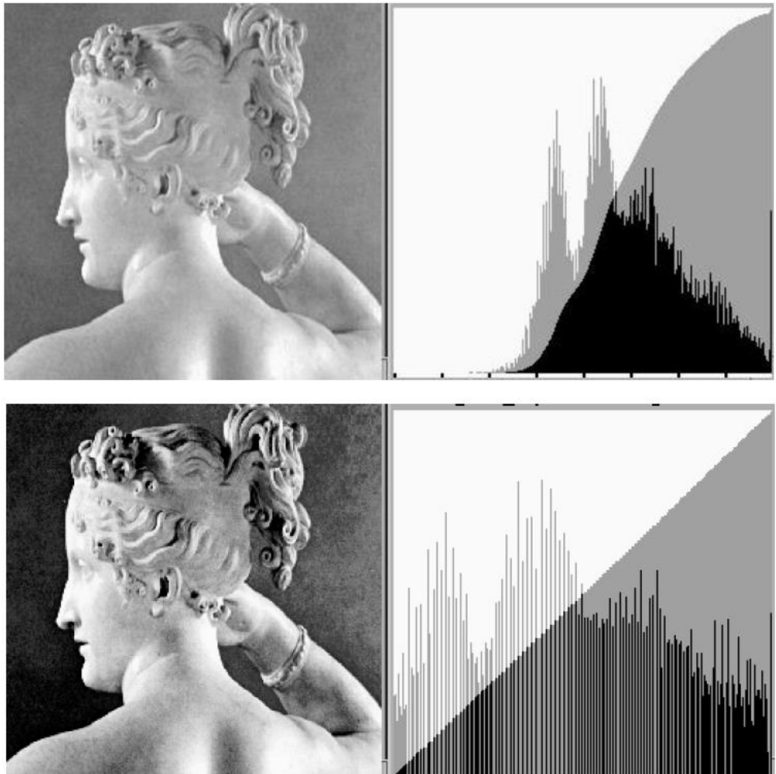
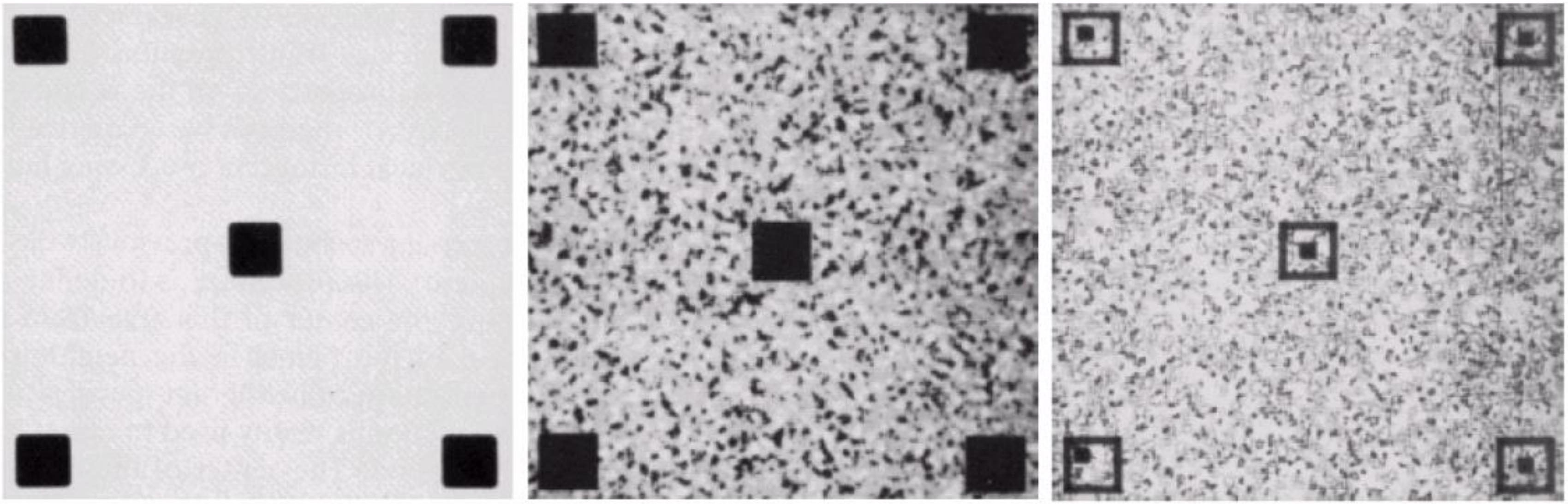


Image enhancement

Localized histogram equalization example:



a b c

FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7 × 7 neighborhood about each pixel.

Color images

What is a color image:

- Due to these absorption characteristics, colors are seen as variable combinations of so called “primary colors” red, green and blue.
- In 1931, CIE designated the following:

  Blue = 435.8nm

  Green = 546.1nm

  Red = 700nm

CIE: International Commisssion on Illumination

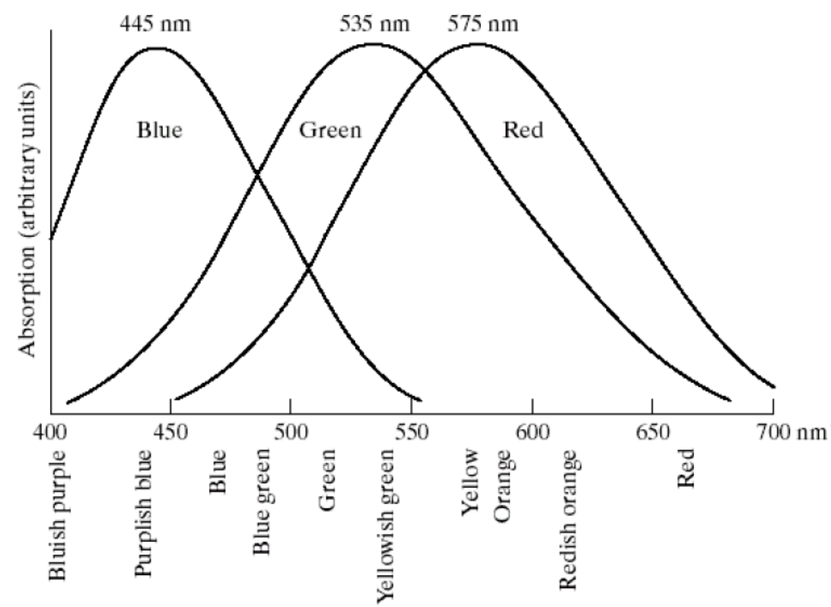


FIGURE 6.3 Absorption of light by the red, green, and blue cones in the human eye as a function of wavelength.

Color images

What is a color image:

- Primary colors can be added in pairs to produce secondary colors of light: {magenta, cyan, yellow}
- Mixing the three primary colors produces white color
- A primary color of pigments or colorants is defined as one that subtracts or absorbs a primary color of light and reflects the other two
- Primary colors of pigments are {magenta, cyan, yellow} and their secondary colors are {red, green, blue}

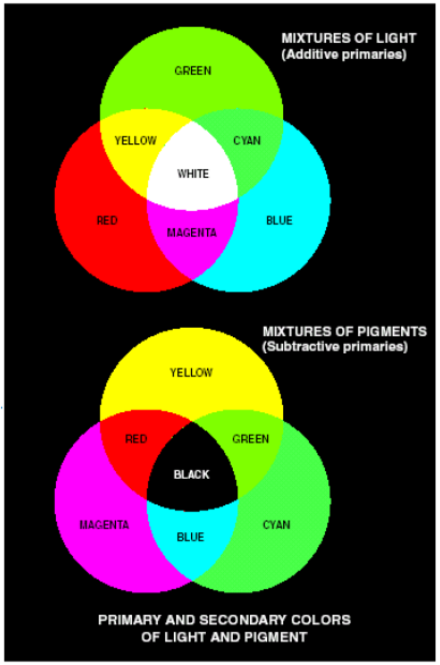


FIGURE 6.4 Primary and secondary colors of light and pigments. (Courtesy of the General Electric Co., Lamp Business Division.)

Color images

Color models

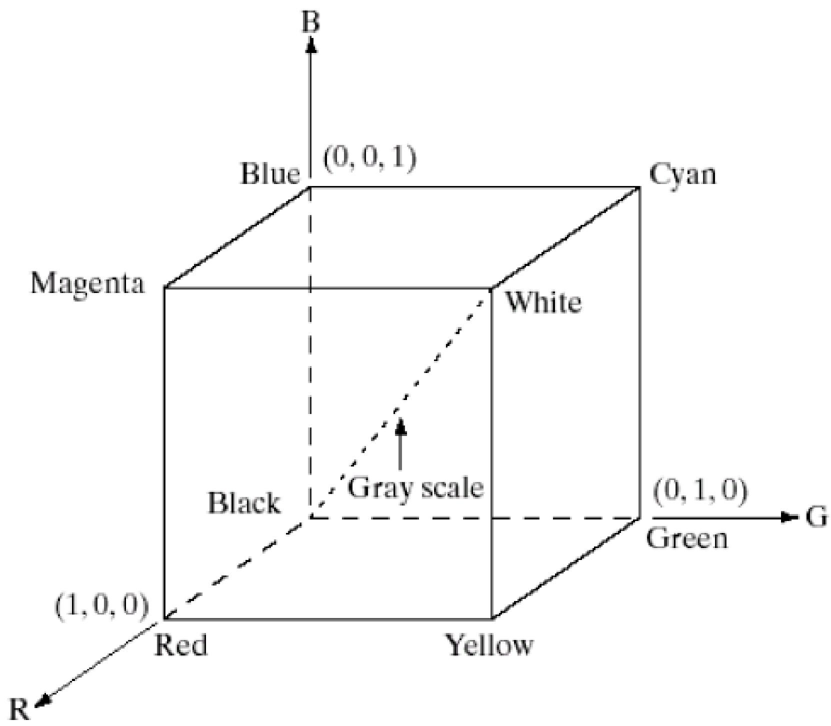
- Color models or color spaces refer to a color coordinate system in which each point represents one color
- Different models are defined (standardized) for different purposes, e.g.

  Hardware oriented models:
  - RGB for color monitors (CRT and LCD) and video cameras,
  - CMYK (cyan, magenta, yellow and black) for color printers

  Color manipulation models:
  - HSI (hue, saturation and brightness) is closest to the human visual system
  - Lab is most uniform color space
  - YCbCr (or YUV) is often used in video where chroma is down-sampled (the human visual system is much more sensitive to luminance than to color)
  - XYZ is known as the raw format
  - others

Color images

The RBG color model



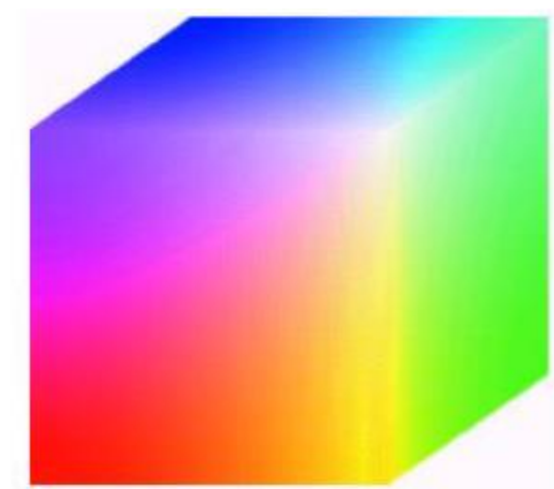
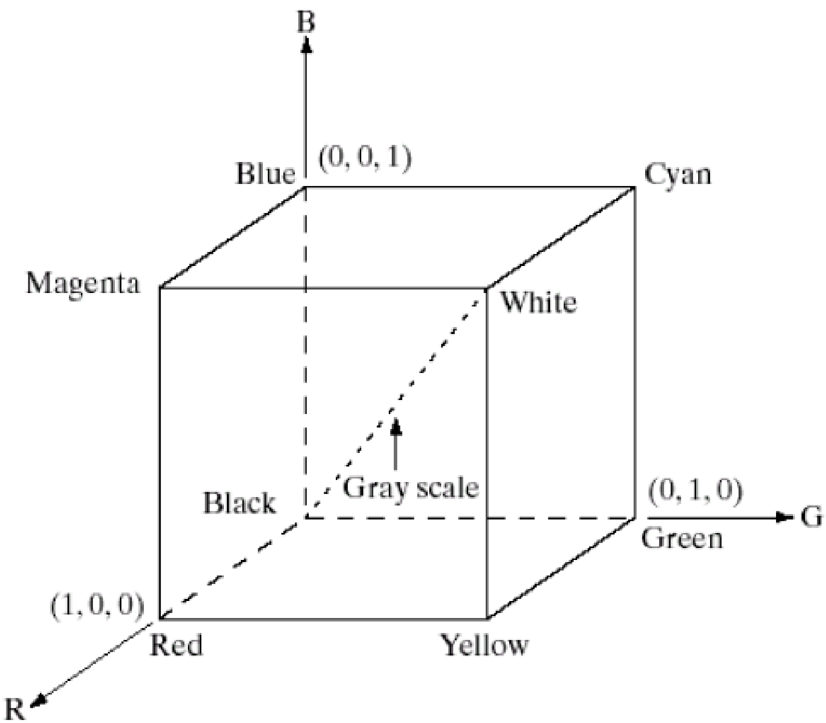


FIGURE 6.8 RGB 24-bit color cube.

Color images

The RBG color model



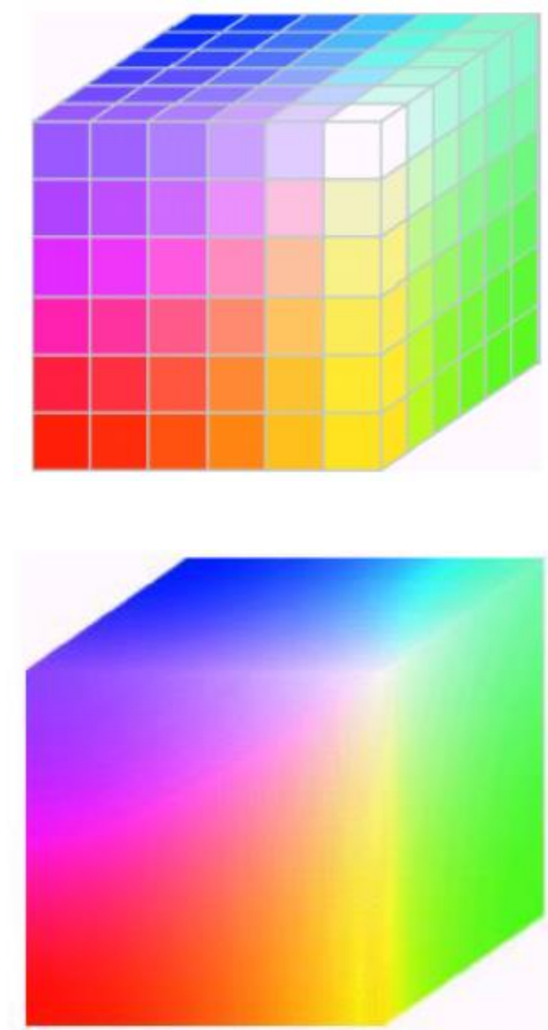
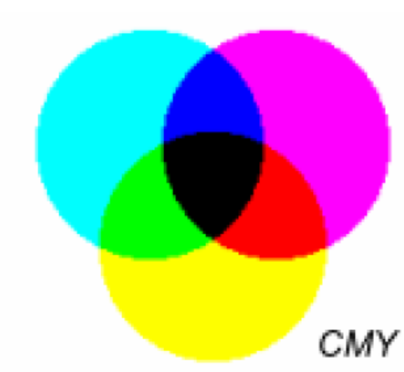


FIGURE 6.8 RGB 24-bit color cube.

Color images

Additive vs Subtractive colors:

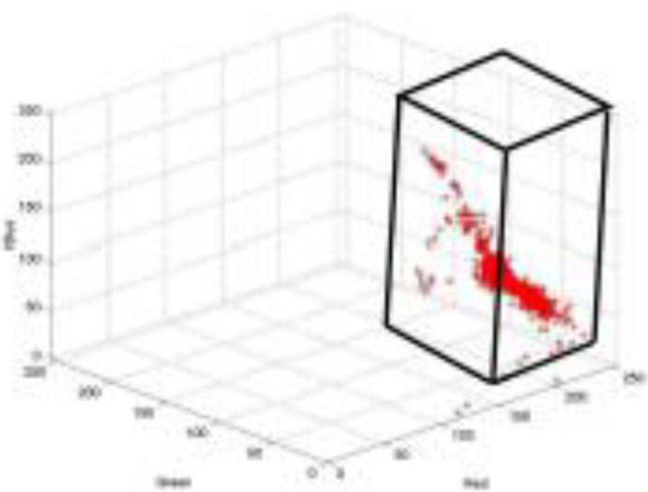
- When light from different sources reaches our eyes directly, the colors are additive
- In printing, we use subtractive colors {cyan, magenta, yellow}
- Cameras use additive colors {red, green, blue}



$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

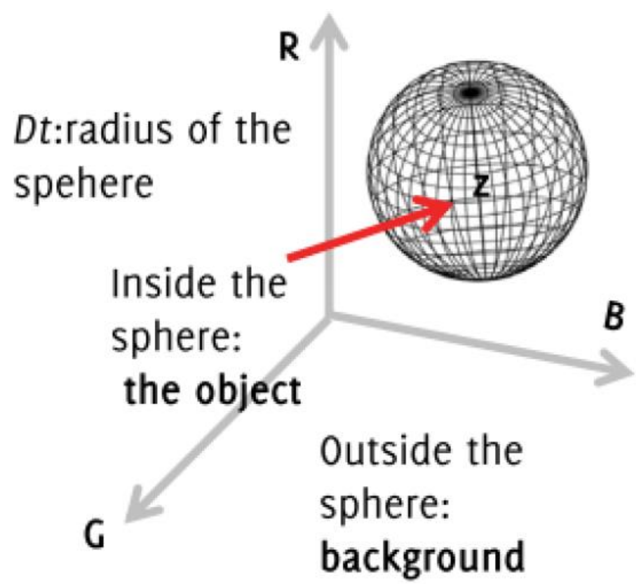
Color images

Segmentation of a region with homogeneous color
- Using color thresholds (one per color)



Color images

Segmentation of a region with homogeneous color
- Using distance from the prototype color



Color images

HSV color space:

- Intensity or Value or Brightness: varies along the vertical axis and measures the extent to which an area appears to exhibit light. It is proportional to the electromagnetic energy radiated by the source
- Hue: denoted by H and varies along the circumference. It measures the extent to which an area matches colors red, orange, yellow, blue or purple (or a mixture of any two). In other words, hue is a parameter which distinguishes the color of the source, i.e., is the color red, yellow, blue, etc
- Saturation: the quantity which distinguishes a pure spectral light from a pastel shade of the same hue. It is simply a measure of white light added to the pure spectral color. In other words, saturation is the colorfulness of an area judged in proportion to the brightness of the object itself. Saturation varies along the radial axis.

Color images

HSV color space:

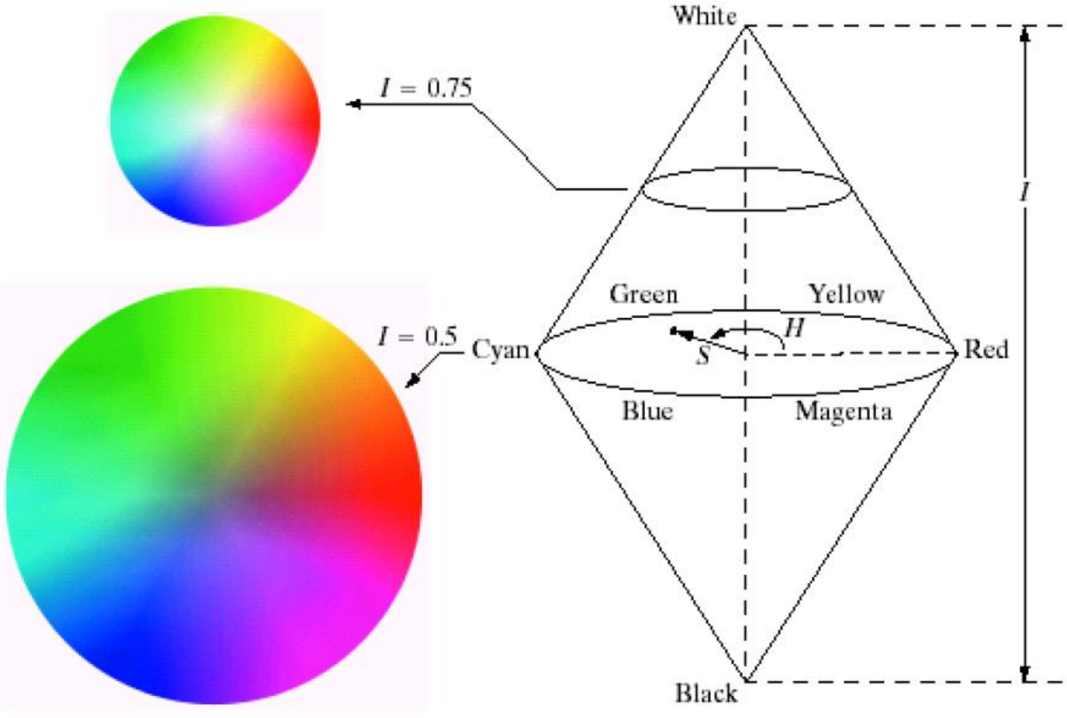
- RGB to HSV conversion

$$H = \begin{cases} \theta & \text{if} \quad B \le G \\ 360 - \theta & \text{if} \quad B > G \end{cases}$$

$$\theta = \cos^{-1}\left\{ \frac{\frac{1}{2}[(R-G)+(R-B)]}{[(R-G)^2 + (R-B)(G-B)]^{1/2}} \right\}$$

$$S = 1 - \frac{3}{(R+G+B)}[\min(R,G,B)]$$

$$I = \tfrac{1}{3}(R+G+B)$$



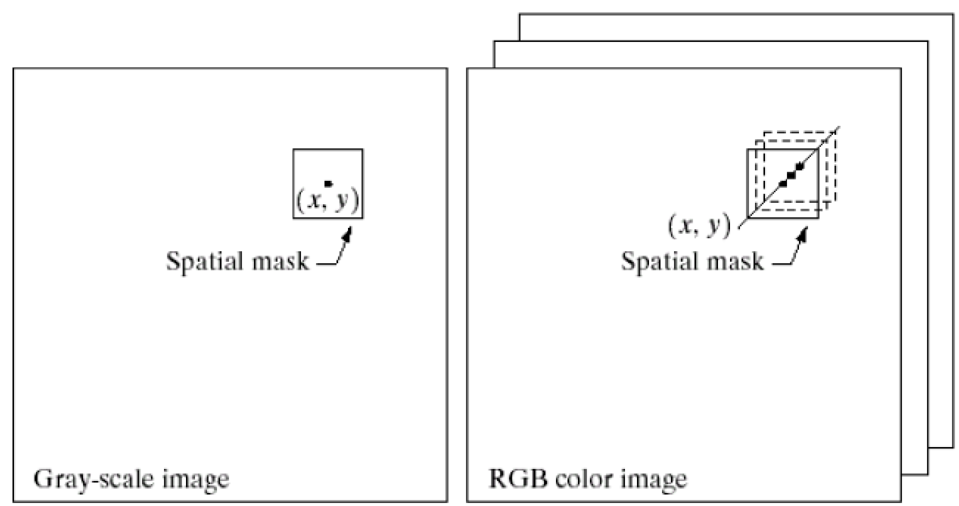
Color image enhancement

Color image is formed by three gray-scale images (one per each RGB color)

- color transformation g(x,y) = T[f(x,y)]

a b

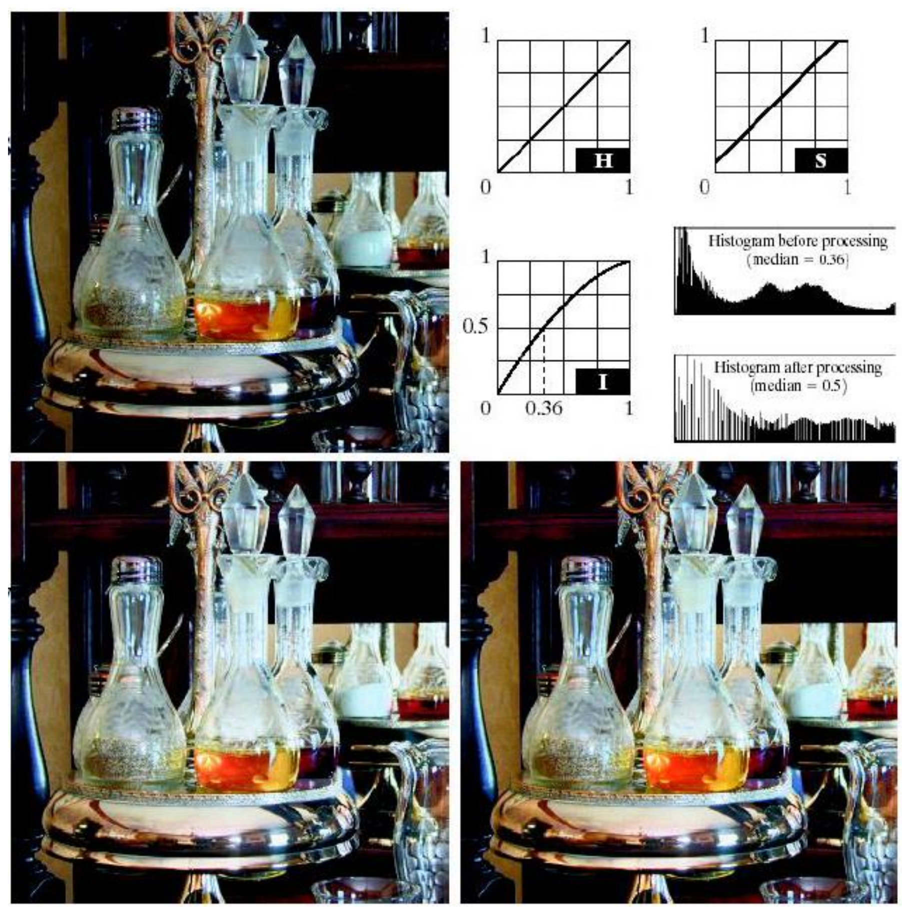
FIGURE 6.29 Spatial masks for gray-scale and RGB color images.



Color image enhancement

Histogram equalization

- Would it be wise to equalize color components indepen-dently?



a b
c d

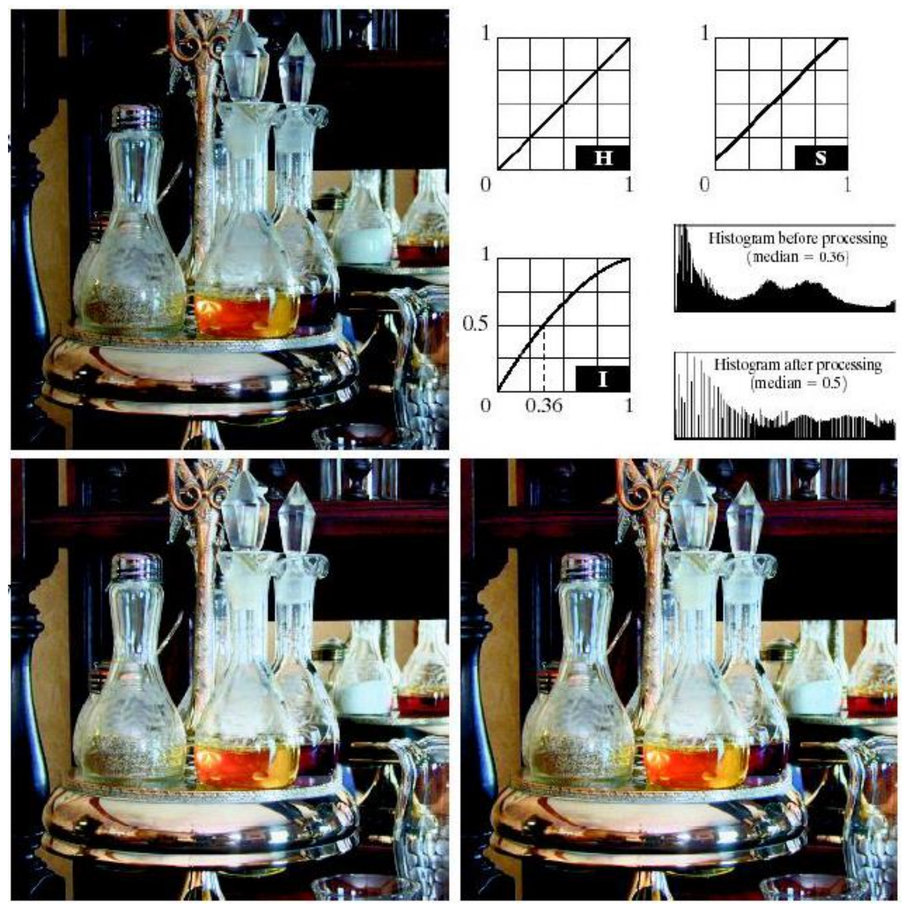
FIGURE 6.37 Histogram equalization (followed by saturation adjustment) in the HSI color space.

Color image enhancement

Histogram equalization

- Would it be wise to equalize color components independently? <u>No, this way colors change</u>

- Equalize intensity component in HSV color space



a b
c d

FIGURE 6.37 Histogram equalization (followed by saturation adjustment) in the HSI color space.