We are concerned with the variance of standard normal random variables transformed by the sine. Therefore we consider a mean of a particular transformed Wiener process

$$f(t) = \mathbb{E}\left[\sin^2(W_t)\right].$$

Knowing the derivatives

$$\frac{d}{dx}\sin^2(x) = 2\sin(x)\cos(x) = \sin(2x)$$

.

$$\frac{d^2}{dx^2}\sin(x)^2 = 2\cos(2x),$$

we may apply Itô's lemma

$$\begin{aligned}\sin^2(W_t) &= \int_0^t \sin(2W_s)dW_s + \frac{1}{2}\int_0^t 2\cos(2W_s)ds = \\ &= \int_0^t \sin(2W_s)dW_s + \int_0^t \cos(2W_s)ds.\end{aligned}$$

Consequently

$$f(t) = \int_0^t \mathbb{E}\left[\cos(2W_s)\right]ds.$$

Now, consider

$$g(t,u) = \mathbb{E}\left[\cos(uW_t)\right].$$

Applying Itô's lemma we obtain

$$\begin{aligned}g(t,u) = \mathbb{E}\left[\cos(u(W_s))\right] &= \mathbb{E}\left[\frac{1}{2}\int_0^t -u^2\cos(uW_s)ds\right] = \\ &= -\frac{1}{2}u^2\int_0^t \mathbb{E}\left[\cos(uW_s)\right]ds = -\frac{1}{2}u^2\int_0^t g(s,u)ds.\end{aligned}$$

Thus

$$\frac{\partial g}{\partial t}(t,u) = -\frac{1}{2}u^2 g(t,u)$$

and

$$g(t,u) = C(u)e^{-\frac{1}{2}u^2 t}$$

.

Since

$$g(0,u) = 1$$

We have

$$C(u) = 1$$

and thus that

$$g(t,u) = e^{-\frac{1}{2}u^2 t}.$$

Thus

$$f(t) = \int_0^t \mathbb{E}\left[\cos(2W_s)\right] ds = \int_0^t g(t,2) = \int_0^t e^{-\frac{1}{2}4t} = \int_0^t e^{-2t} = \frac{e^{-2t}-1}{-2} =$$
$$= \frac{1-e^{-2t}}{2}.$$

We can conclude that

$$\mathbb{E}\left[\sin(Z)^2\right] = \frac{1-e^{-2}}{2} \approx 0.43233235.$$