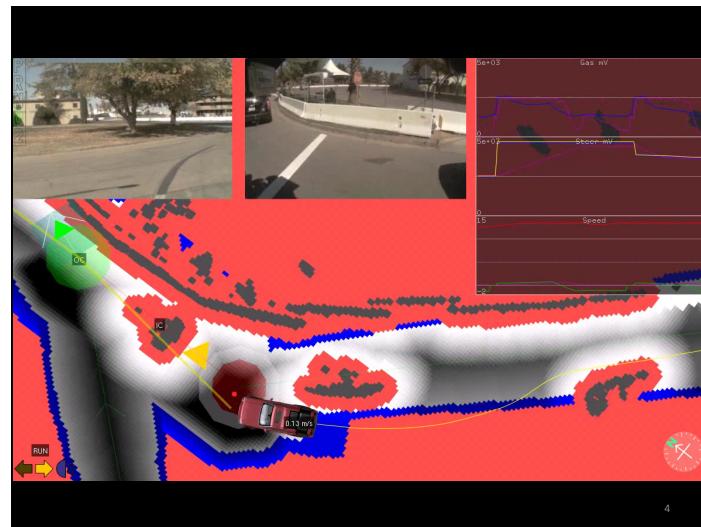


2



4



## A Driver's License for Autonomous Vehicles



- Earning a driver's license implies competency and safety in a *large variety of driving scenarios*.
- It is a skill assessment used to *bound the risk of operating a vehicle*.

6

## A Driver's License for Autonomous Vehicles



- Under what *criteria* can we determine that an *autonomous vehicle is safe?*
- How can we generalize its actions *beyond simple tests?*

7

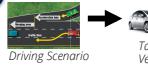
## The APEX Approach

How can we provide a driver's license for autonomous vehicles?



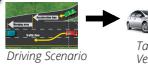
## The APEX Approach

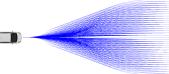
How can we provide a driver's license for autonomous vehicles?

- 1 Explore the modeling of subsets of autonomous driving missions as scenarios
 
- 2 Map driving scenarios to collections of agents represented as hybrid systems
 

## The APEX Approach

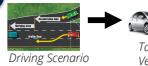
How can we provide a driver's license for autonomous vehicles?

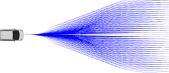
- 1 Explore the modeling of subsets of autonomous driving missions as scenarios
 
- 2 Map driving scenarios to collections of agents represented as hybrid systems
 

- 3 Implement planning stack in C++
 

## The APEX Approach

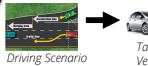
How can we provide a driver's license for autonomous vehicles?

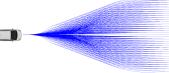
- 1 Explore the modeling of subsets of autonomous driving missions as scenarios
 
- 2 Map driving scenarios to collections of agents represented as hybrid systems
 

- 3 Implement planning stack in C++
 
- 4 Integrate ROS based vehicle OS
 

## The APEX Approach

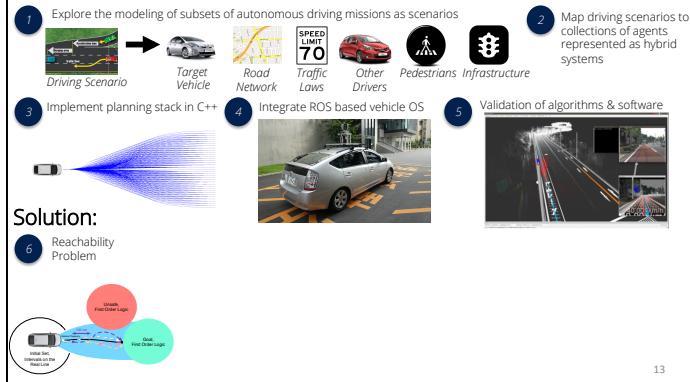
How can we provide a driver's license for autonomous vehicles?

- 1 Explore the modeling of subsets of autonomous driving missions as scenarios
 
- 2 Map driving scenarios to collections of agents represented as hybrid systems
 

- 3 Implement planning stack in C++
 
- 4 Integrate ROS based vehicle OS
 
- 5 Validation of algorithms & software
 

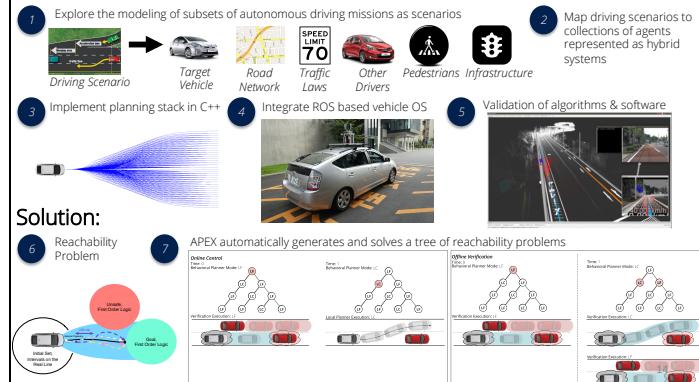
## The APEX Approach

How can we provide a driver's license for autonomous vehicles?



## The APEX Approach

How can we provide a driver's license for autonomous vehicles?



## Outline

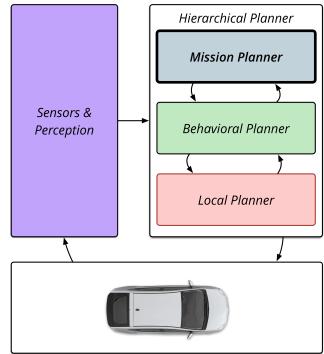
- Part 1: APEX Framework (SAE 2016) 10:00-11:30 AM**
  - Development of an AV model
  - Development of hybrid systems framework to accommodate AV model
  - Scenario: lane change
  - Technical details, proofs, and discussion
- Part 2: Demo: Agile AV Platform (RTSS WIP 2015) 11:30-12:00**
  - F1/10 demo
  - Hardware and algorithms overview
- Lunch**
- Part 3: APEX Benchmarks and Complexity (ARCH 2016) 1:30-2:30**
  - Development of AV model part 2
  - Pure Pursuit
  - Differential Inclusions
  - Composition of Agent Automata
  - Complex Scenario from Agents
- Part 4: Co-design of Control and Perception (RTSS 2015) 2:30-3:00**
  - Algorithms for Low Cost AV
- Part 5: Next Steps and Discussion 3:00-4:00**
  - Final Report Format
  - Development of framework for scenario creation from Open Street Maps.
  - Scenario authoring tools and agent building blocks.
  - Support for AV model in Unity and RViz.

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## Development of an AV Model

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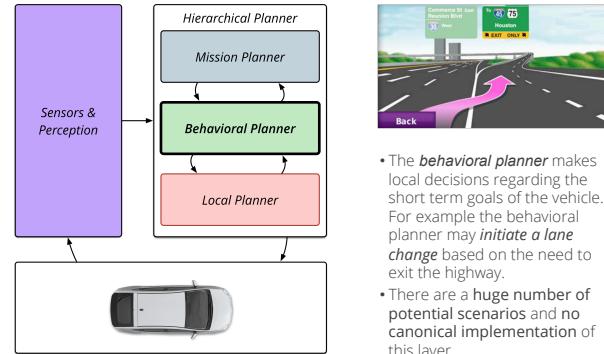
### Basic Architecture: Mission Planner



- The **mission planner** determines the series of road network links that the AV will follow in order to accomplish the **mobility goals** of the vehicle's occupants.
- Such information defines the **set of waypoints** that the behavioral planner will reason about. This is not a current focus of our research.

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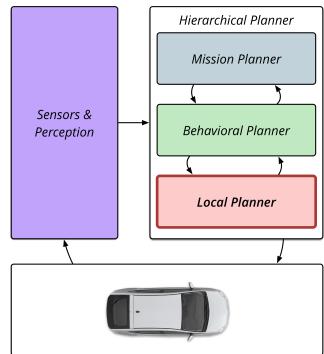
### Basic Architecture: Behavioral Planner



- The **behavioral planner** makes local decisions regarding the short term goals of the vehicle. For example the behavioral planner may **initiate a lane change** based on the need to exit the highway.
- There are a huge number of potential scenarios and no canonical implementation of this layer.

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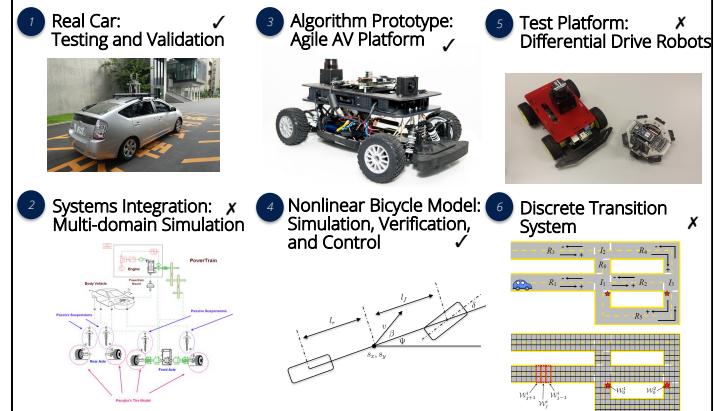
### Basic Architecture: Local Planner



- The **local planner** is responsible for trajectory generation and evaluation.
- Based on the goal selection provided by the behavioral planner, the local planner may use a variety of **optimal control techniques** to compute **dynamically and kinematically feasible trajectories**.
- It is essential to our framework because bad events are **defined by the vehicle state**.

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### Which Vehicle Model?



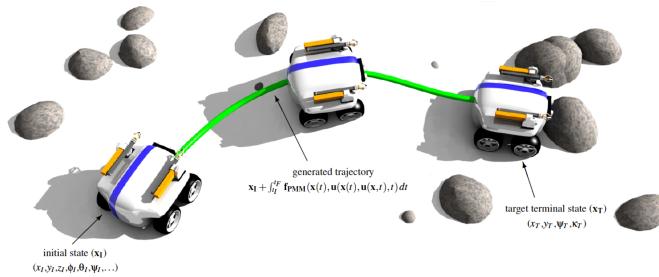
## Vehicle Model

- The **bicycle model** is used as the intermediate representation (IR) because it supports verification of *low level trajectory trackers* and analysis with respect to first order logic necessary for *formal verification*.
- Model captures the effect of steering input and longitudinal acceleration on the vehicle's trajectory.

$$\begin{aligned}\dot{\beta} &= \left( \frac{C_r l_r - C_f l_f}{mv^2} \right) \dot{\psi} + \left( \frac{C_f}{mv} \right) \delta - \left( \frac{C_f + C_r}{mv} \right) \beta \\ \ddot{\psi} &= \left( \frac{C_r l_r - C_f l_f}{I_z} \right) \beta - \left( \frac{C_f l_f^2 - C_r l_r^2}{I_z} \right) \left( \frac{\dot{\psi}}{v} \right) + \left( \frac{C_f l_f}{I_z} \right) \delta \\ \dot{v}_x &= a_x \\ \dot{s}_x &= v \cos(\beta + \psi) \\ \dot{s}_y &= v \sin(\beta + \psi) \\ \dot{\delta} &= v_w\end{aligned}$$

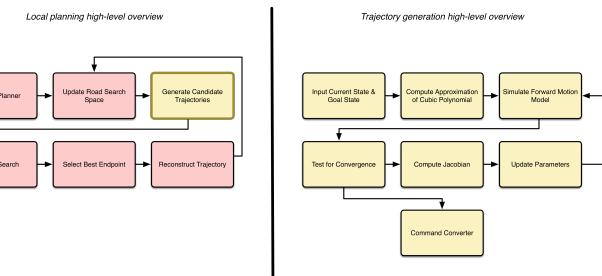
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## The Local Planning Problem



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## Local Planning Details



## Problem Definition

- The state  $x$  of a vehicle is described by the following vector:
$$x = [s_x, s_y, s_z, \phi, \theta, \rho, v_x, v_y, v_z, \omega_x, \omega_y, \omega_z, \kappa]^t$$
- Define the initial state as  $x_I$  and the target state as  $x_T$
- Note that  $\kappa$ , the curvature is simply the rate of change of  $\theta$  with respect to distance traveled
- For this exercise we will consider a vehicle simplified to a 2D model:
$$x = [s_x, s_y, \theta, v, \kappa]$$
- Which evolves according to the following set of ordinary nonlinear differential equations:
$$\dot{x} = f(x, u(p, x))$$
- Here,  $u(p, x)$  is the parameterized control input

## Methodology

- We will be concerned with computing trajectories which lead from an initial state  $x_i$  to a final state  $x_f$  within a finite time horizon.
- The final state state boundary constraints which are given by the mission planner are defined by the following vector:

$$x_c = [s_x, s_y, \theta]$$

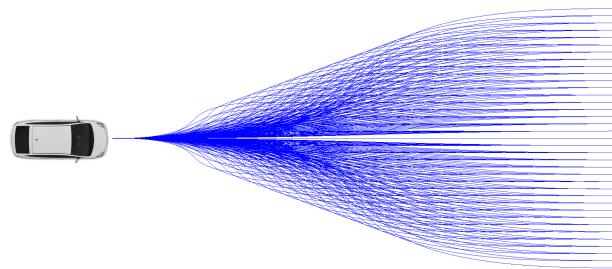
- A prediction of the vehicles final state (using Euler's method) due to the control input is:

$$x_f(p, x) = x_i + \int_{t_0}^{t_f} \dot{x}(x, p) dt$$

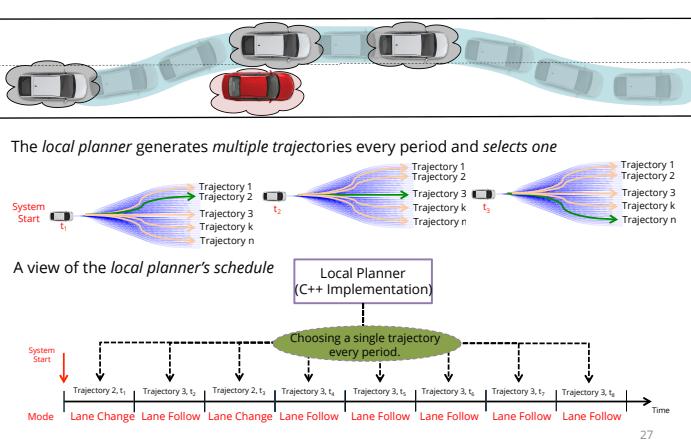
- The goal of the algorithm is to drive the following equation to zero:

$$C(x, p) = x_c - x_f(p, x)$$

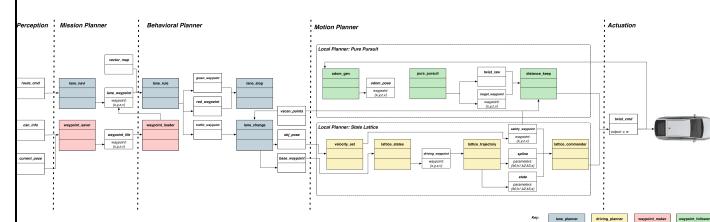
## Trajectory Generation



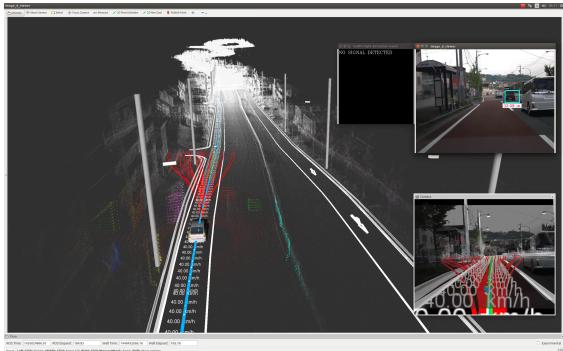
## Trajectory Generation



## Planning Architecture

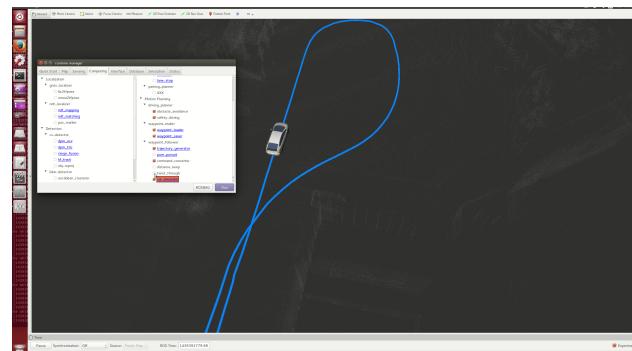


Simulation Environment



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Vehicle Test



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Vehicle Test



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Development of Hybrid Systems Framework

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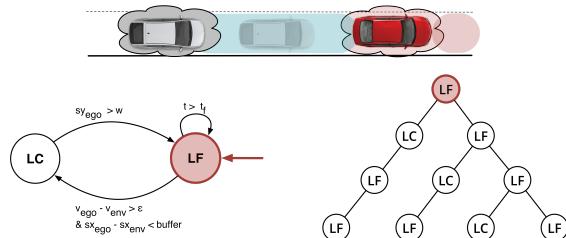
## Why go this route?

- Autonomous vehicles are *mixed discrete-continuous systems*.
- Their controllers make discrete decisions which cause the *vehicle* to evolve in a continuous state space according to ordinary differential equations.
- Hybrid systems are the natural framework to express this behavior.
- Utilizing the formal semantics of hybrid systems will allow the definition of simulations and verification tasks in the same language and framework.
- Hybrid systems provide a rich enough *intermediate representation* to allow for further abstraction if necessary.
- This modeling framework most closely corresponds with the controllers which will run on real vehicles.

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## Behavior Controller Execution

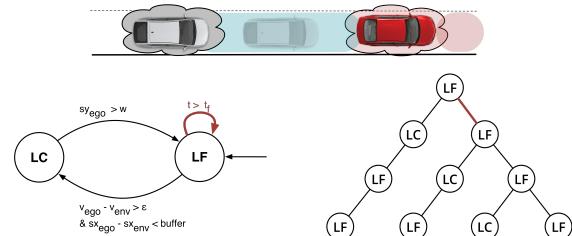
Verification Execution: LF

Start in the *Lane Following* mode.

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## Behavior Controller Execution

Verification Execution: LF

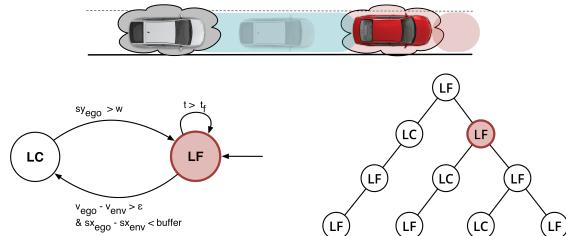


If the current time is greater than the *schedule period*  
update the *reference trajectory*...

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## Behavior Controller Execution

Verification Execution: LF

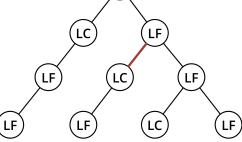
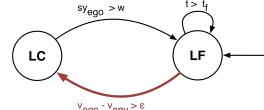
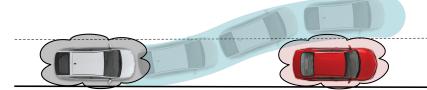


Continue in the *Lane Following* mode following updated  
reference trajectory.

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## Behavior Controller Execution

Verification Execution: LC

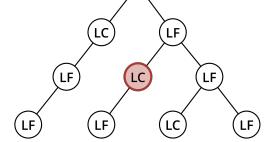
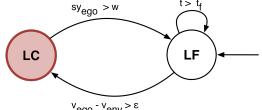
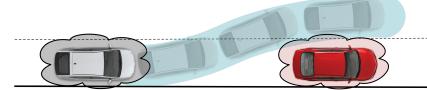


If the *ego-vehicle* is within the *buffer* and the difference in velocities is big enough *transition*.

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## Behavior Controller Execution

Verification Execution: LC

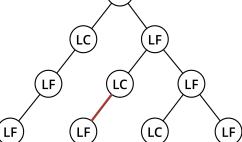
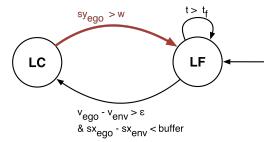


Enter *Lane Change* mode and follow new reference trajectory.

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## Behavior Controller Execution

Verification Execution: LF

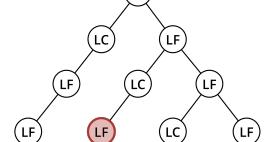
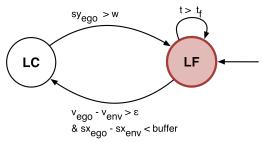


If the *ego-vehicle* has entered new lane update *reference trajectory* and execute *transition*.

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## Behavior Controller Execution

Verification Execution: LF



Continue in the *Lane Following* mode following updated reference trajectory.

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## Other Drivers



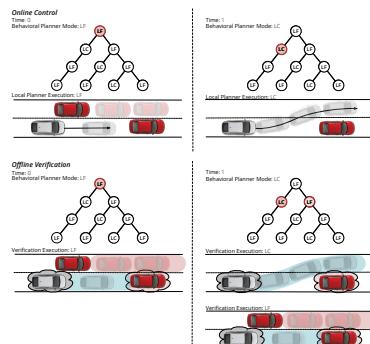
- For this study we begin with a simple model of the other cars in the environment.
- Initially expressivity of modeling language limited us to this representation.
- We do not know the control inputs for the other vehicles so here it is fruitless to include a more expressive dynamical model
- We only care about the path which the other vehicle will take
- In this case we allow non-determinism in the initial estimate of the vehicles velocity.
- Real multi-object tracking perception algorithms assume constant velocity between each position update, thus this is not a bad model of the ego-vehicles view of the world.
- We will show later how more complexity can be added to the non-deterministic behavior of the other agents in the environment by taking unknown control inputs as differential inclusions.

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## Verifying a Lane Change

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## Lane Change



- The ego vehicle is driving in the right lane of a uni-directional two lane road network.
- Another car is driving in front of the ego vehicle at a lower speed.
- We include the extreme case where the environmental vehicle stops.
- The ego vehicle's planner must eventually initiate a lane change maneuver, which involves moving to the left lane, in order to enter a left hand turn lane at a four way stop.
- We highlight that when there is significant uncertainty regarding the ego vehicles orientation and that it may deviate (initially) from the reference trajectory

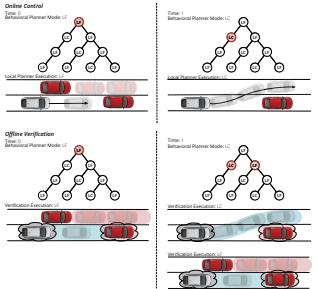
43

## Specification

- The ego vehicle travels at a velocity less than or equal to the speed limit  
 $\square (v_{ego} \leq v_{lim})$  (29)
- The ego vehicle does not drive backwards  
 $\square (v_{ego} \geq 0)$  (30)
- The ego vehicle does not collide with any of the  $n$  other objects in the environment  
 $\square (\sqrt{(s_{x_{ego}} - s_{x_{env}})^2 + (s_{y_{ego}} - s_{y_{env}})^2} \geq r) \quad \forall i = 1 \dots n$  (31)
- If a timed lane change request is invoked, the ego vehicle completes the lane change on time.  
 $\square (LC \rightarrow (s_{y_{ego}} > w) \wedge (t \leq t_{max}))$  (32)
- Acceleration ceases when some maximum velocity is reached.  
 $\square (v_{env} \geq v_{max} \rightarrow a = 0)$  (33)
- Other agents must drive in the proper direction according to their lane.  
 $\square (v_{env} \geq 0)$  (34)
- The accelerations of other agents are within those rates achievable by maximum engine power  
 $\square (a_{env} \leq a_{max})$  (35)
- Other agents maintain their lanes unless explicitly specified not to.  
 $\square (\neg LC \rightarrow (y_{min} \leq s_{y_{env}}) \wedge (y_{max} \geq s_{y_{env}}))$  (36)
- Lane changes by other agents are only permitted if the alternate lane is unoccupied or unless a degenerate scenario is being modeled.  
 $\square (LO \rightarrow \neg LC)$  (37)

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## Verification Algorithm



### Algorithm: External Trajectory Generation

```
safety_var := SAFE
search_depth := x

WHILE ( depth<search_depth)
    safety_var = Check kth trajectory
    IF safety_var=SAFE, then initialize k+1th
        trajectory with goal set of kth trajectory and
        update depth_counter if all nodes at a depth
        have been visited...
    ELSE IF safety_var:=UNSAFE, then
        return safety_var
    END WHILE, return safety_var
```

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## dReach Verification Instance

An example dReach file that is *automatically* converted from one trajectory generated from the local planner model 1.

```
// Dynamics
// Lane change motion primitive
model 1

int
/* X position must be greater than or equal to 0
(sx>0);
// Velocity must be greater than or equal to 0
(sv>0);

Row
/* TARGET VEHICLE */
const
/* (tau0=1); // Initial time
// Constant values
// Constants
#define m 1.723 // Mass of vehicle
#define rho 1.078 // Air density
#define C_f 108000.0 // Aerodynamic drag coefficient
#define C_d 1.25 // Drag coefficient
#define k_1 1.515 //簧常数
#define k_2 2.0 //簧刚度
#define k_3 1.0 //阻尼系数
#define k_4 2.0 //阻尼系数
#define k_5 1.0 //阻尼系数
#define k_6 1.0 //阻尼系数

// Scenario parameters
#define r 3.7 // Width of road
#define v 11.1 // Desired velocity
#define a_m 0.0 // Acceleration of environmental vehicle.
#define a_v 0.0 // Acceleration of ego vehicle.

// Helpers to clean up equations...
#define c1 (C_f - C_d)*v*v
#define c2 (Ps1_dot - Ps1_dot - k4*v*delta)

// Trigonometric inputs
// Rate of change of front wheel angle
#define w_k1 ((cos(Psi1_dot)*d1)*(ty1 - sy1) - sin(Psi1_dot)*(sx1 - sx))/k2*(Ps1_dot - Ps1_dot + k2*v*dot - k4*v*delta)
#define w_k2 ((Ps1_dot - Ps1_dot - k4*v*delta)/k2)

// Longitudinal acceleration
#define a_x ((k3*(cos(Psi1_dot)*d1)*(tx1 - sx) + sin(Psi1_dot)*(ty1 - sy)) + k6*v*dot - v*v)

// Cubic spline inputs
#define a_y ((k4.5*Psi1_dot*v*v*d1*v*v + 2*c1*v*dot*v*dot + 3*c2*v*dot*v*dot*v*dot + (j1*c1)*delta*v));
#define a_z ((k4.5*Psi1_dot*v*v*d1*v*v + 2*c1*v*dot*v*dot + 3*c2*v*dot*v*dot*v*dot + (j1*c1)*delta*v));
#define a_w ((k4.5*Psi1_dot*v*v*d1*v*v + 2*c1*v*dot*v*dot + 3*c2*v*dot*v*dot*v*dot + (j1*c1)*delta*v));

// Rate of change of yaw angle
#define d1 (Ps1_dot - v - d1*v*dot*v*dot*v*dot + 3*c1*v*dot*v*dot*v*dot*v*dot + (j1*c1)*delta*v);
#define d2 (Ps1_dot - v - d1*v*dot*v*dot*v*dot + 3*c1*v*dot*v*dot*v*dot*v*dot + (j1*c1)*delta*v);
#define d3 (Ps1_dot - v - d1*v*dot*v*dot*v*dot + 3*c1*v*dot*v*dot*v*dot*v*dot + (j1*c1)*delta*v);

// Other Vehicle
// d/dt(sx,env)=0;
jump:
j:
else:
@1 (and (dot > 0) (sx < 0.5) (sv >= 0) (sy >= 10.8) (v >= 11.1) (sx_d > 0) (sy_d > 0)
    (Ps1_dot > 0) (Ps1_dot > 0) (dot > 0) (Beta > 0) (delta > 0) (tau > 0)
    (Ps1_dot > 0) (dot > 0) (Beta > 0) (delta > 0) (tau > 0)
    (sx_gr > 15) (kappa_d > 0));
goal:
@0 (or (and (sy <= 0) (tau > 2) (tau < 3)) (and (sy >= 0) (sx >= sx_env)));
```

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## Lane Change

Symbol	Note	Old Assignment	New Assignment
w	Lane Width	-	3.7m
$\epsilon$	Threshold of the relative speed	-	1 m/s
Buffer	Distance from the ahead vehicle	-	15 m
tf	Max timestep to generate the trajectories	-	3.0 s
v	Velocity of Ego Vehicle	-	[10.8, 11.1]
Sx	Position of Ego Vehicle, X	-	[0.0, 0.5]
Sy	Position of Ego Vehicle, Y	-	[0.0, 0.1]
$\Phi$	Orientation of Ego Vehicle, rad	-	[0.0, 0.1]

- Scenario
  - The vehicle ahead suddenly stops during lane following
- Verification Result: **UNSAFE**
  - (i.e.) the vehicle may crash the ahead vehicle)
- Analysis (Based on the counter example)
  - Buffer (15m) is too small to change the lane
  - Initial position of the vehicle is heading slightly toward right before starting lane changing
  - The dotted line shows the intended trajectory, the solid line shows the actual trajectory
  - Error due to poor state estimation

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## Lane Change: Part 2

Symbol	Note	Old Assignment	New Assignment
w	Lane Width	3.7m	3.7m
$\epsilon$	Threshold of the relative speed	1 m/s	1 m/s
Buffer	Distance from the ahead vehicle	15 m	20 m
tf	Max timestep to generate the trajectories	3.0 s	3.0 s
v	Velocity of Ego Vehicle	[10.8, 11.1]	[10.9, 11]
Sx	Position of Ego Vehicle, X	[0.0, 0.5]	[0.0, 0.5]
Sy	Position of Ego Vehicle, Y	[0.0, 0.1]	[0.0, 0.05]
$\Phi$	Orientation of Ego Vehicle, rad	[0.0, 0.1]	[0.0, 0.1]

- Scenario
  - The vehicle ahead suddenly stops during lane following
- Verification Result: **SAFE**
  - (i.e.) the vehicle may crash the ahead vehicle)
- Analysis (Based on the counter example)
  - Buffer (20 m) is enough to change the lane
  - Initial position of the vehicle is heading slightly toward right before starting lane changing

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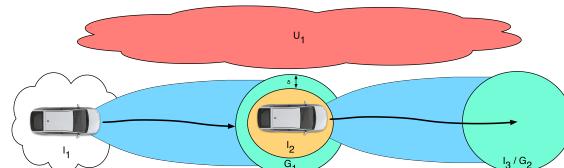
### Lane Change: Part 3

Symbol	Note	Old Assignment	New Assignment
w	Lane Width	3.7m	3.7m
$\epsilon$	Threshold of the relative speed	1 m/s	1 m/s
Buffer	Distance from the ahead vehicle	15 m	20 m
$t_f$	Max timestep to generate the trajectories	3.0 s	3.0 s
v	Velocity of Ego Vehicle	[10.8, 11.1]	[10.9, 11]
Sx	Position of Ego Vehicle, X	[0.0, 0.5]	[0.0, 0.5]
Sy	Position of Ego Vehicle, Y	[0.0, 0.1]	[0.0, 0.05]
$\Phi$	Orientation of Ego Vehicle, rad	[0.0, 0.1]	[0.0, 0.2]

- Scenario
  - The vehicle ahead suddenly stops during lane following
- Verification Result: **UNSAFE**
  - (i.e.) the vehicle may crash the ahead vehicle
- Analysis (Based on the counter example)
  - Uncertainty about ego vehicle orientation causes tracking controller to deviate from reference trajectory
  - Demonstrates ability to validate supplier specifications on test scenarios

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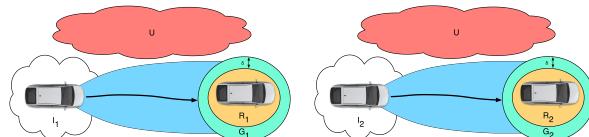
### Composition of Trajectories



- The ego-vehicle starts in  $I_1$ ,  $I_1$  does not intersect  $U_1$ .
- If verification instance 1 returns SAFE, then  $X$ , the ego vehicle's state, never intersects  $U_1$  and the ego-vehicle finishes the run within  $G_1$ .
- Verification instance 2 must begin within  $I_2$ ,  $I_2$  is a subset of  $G_1$ . Thus,  $I_2$  does not intersect  $U_1$ .
- If verification instance 2 returns SAFE, then  $X$ , the ego vehicle's state, never intersects  $U_1$  and the ego-vehicle finishes the run within  $G_2$ .
- Thus, the ego vehicle never intersects the unsafe region  $U_1$  and we show that the composition of two SAFE bounded time verification instances must imply that the ego vehicle can never intersect  $U_1$  over the combined horizon.
- Key takeaway:** If we verify a sequence of two trajectories, the composition of the trajectories is SAFE.
- Key takeaway:** The goal sets do not grow.
- Note: Delta is fixed through all verification instances.

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### Verification Instances are Independent



**Theorem 1.** Given,  $I_1, I_2$  the initial sets,  $G_1, G_2$  the goal sets, and  $\delta$  the relaxation parameter of the verification instances  $V_1, V_2$ . The verification instances are independent if the initialization,  $I_2 \supseteq R_1$ . Where  $R_i$  is the reachable set of  $V_i$ .

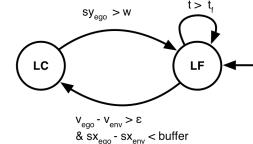
**Remark.** For each verification instance we call dReach to prove that  $R_i \subseteq G_i$ . If dReach returns that the program is SAFE, then we are assured that  $R_i \subseteq G_i$  and  $x_i$  never intersects  $U$ .

**Proof.** We ensure by construction that  $I_2 \supseteq R_1$  if  $I_2 = G_1 - \delta$ . Via the verification results we know that  $G_i$  does not intersect  $U$ , the unsafe region. Furthermore, we know that as the  $x$  evolves from  $t_0$  to  $t$  it never intersects  $U$  and ends in  $G_{i+1}$ .  $\square$

**Remark.** Independence in this sense implies that we can compose trajectories to develop safe trajectory sequences. Furthermore, it implies that verification instances may be run in parallel.

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### Results: Infinite Time Property



- Behavior planner can express runs of the form  $LF^*$  or  $(LF^* LC LF^*)^*$
- If the ego-vehicle were to operate on an infinitely long, straight road then the runs could be of infinite length.
- For a run  $(LF LC)$  we demonstrate that both LF and LC are safe.
- We have already shown that we can compose LF and LC if sequential trajectories are initialized in the goal set of the previous trajectory. Thus, a finite run can certainly be shown to be safe.
- Because the representation of the behavior planner and motion planner here only admits a finite set of trajectories which are exactly the same in every execution we can show LF, LF, LF, ... is safe.
  - on a straight road lane following and lane changing trajectories never change.
- Furthermore, it is easy to see that  $(LF^* LC LF^*)^*$  is also safe if LF, LC is safe.
- This property will not hold on arbitrarily curved roads.

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## Runtime and Complexity

Table 5: Verification Results

Symbol	Scenario 1	Scenario 2	Scenario 3
$w$	3.7	3.7	3.7
$B$	15	20	20
$\delta$	0.1	0.1	0.1
$v_{ego}$	[10.8, 11.1]	[10.9, 11]	[10.9, 11]
$s_{x,ego}$	[0.0, 0.5]	[0.0, 0.5]	[0.0, 0.5]
$s_{y,ego}$	[0.0, 0.1]	[0.0, 0.05]	[0.0, 0.05]
$\Psi$	[0.0, 0.1]	[0.0, 0.1]	[0.0, 0.2]
Search Depth	2	2	2
Verification Time (s)	30.821	373.924	36.166
Result	$\delta$ -UNSAFE	SAFE	$\delta$ -UNSAFE

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## Conclusions

- Why go to all the trouble to make an AV model?
  - Cannot perform verification without a model
  - Modeling using formal semantics such as “hybrid systems” will allow for bottom up approach to building more scenarios from agent blocks.
  - Modeling using formal semantics such as “hybrid systems” will allow for translation to simulation environments.
- First fully automatic verification of realistic lane changing algorithms and planning stack!
- The counterexamples are intuitive...
  - At the time of writing the modeling language of dReach and dReal did not support key features for true non-determinism.
    - This leaves us to hand engineer strange environment behaviors
  - Yes, this gives us confidence that we are finding real bugs
  - No, it is not good enough yet.
  - We need more complex scenarios, but building CTL like trees with external trajectory generation is not efficient.
  - In the sequel we will show how to get around these two deficits.

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## Early Autonomous Vehicles



In 2008 AV prototypes, were *ad-hoc, complex, and expensive...*

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## State of the Art AV

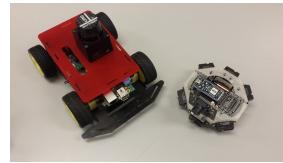


Today industrial grade prototypes have gotten better, but platforms are still *inaccessible* and *dangerous*...

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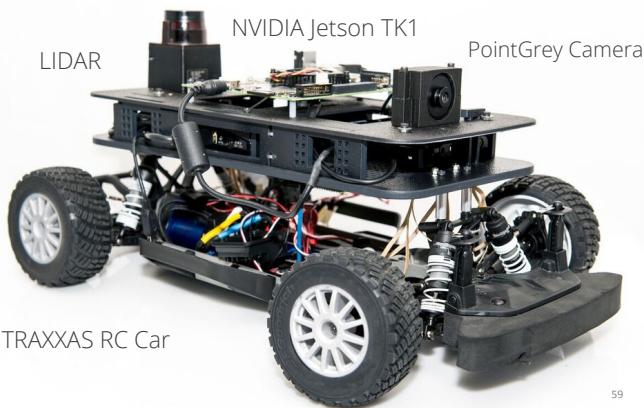
## Agile AV Platform

- 1 • We began with simple differential drive robots which did not capture realistic dynamics.
- Progressed from line following to map based navigation.
- 2 • Powerful non-holonomic robot has more realistic dynamics.
- Utilizes embedded GPU to run realistic planning, perception, and control algorithms.



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## Agile AV Platform



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## Demo

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