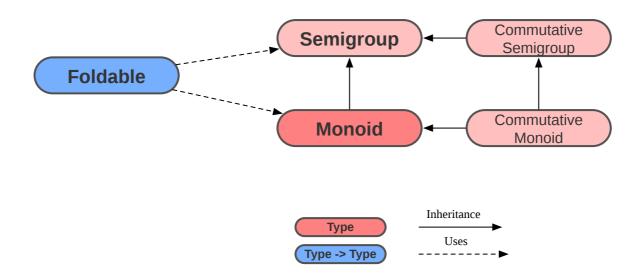
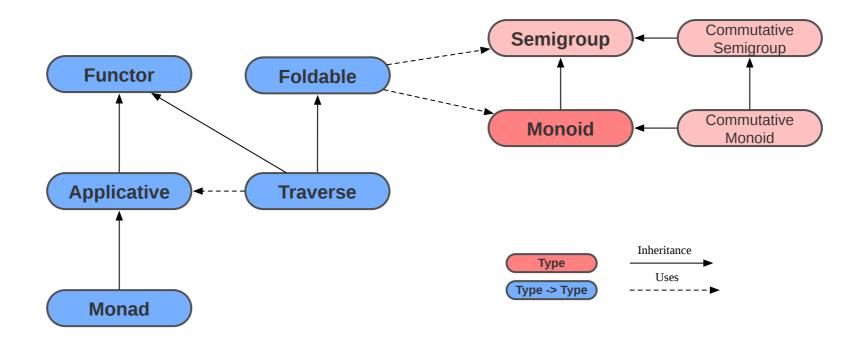


# Previously





# Plan





# Why Functors? Where are they used?



#### Where are Functors used?

Data structures: List, Vector, Map, Set, Graph

Error handling: Option, Either, Validated

Async: IO, Future

Parsing, Serialisation, Configuration, etc ...



## Plan

- Functor hierarchy
- 20+ generic functions
- Variance
- Typeclass coherence





```
trait Functor[F[_]] {
  def map[A, B](fa: F[A])(f: A => B): F[B]
}
```



```
trait Functor[F[_]] {
  def map[A, B](fa: F[A])(f: A => B): F[B]
}
```

#### Functor is a higher kinded typeclass

- It applies to **type constructor** with a **single** hole
- But **types** like Int, String **cannot** be a Functor

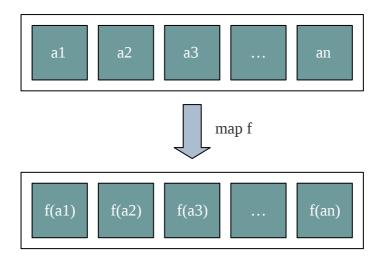


# Which F[\_] is a Functor?



#### F is a container

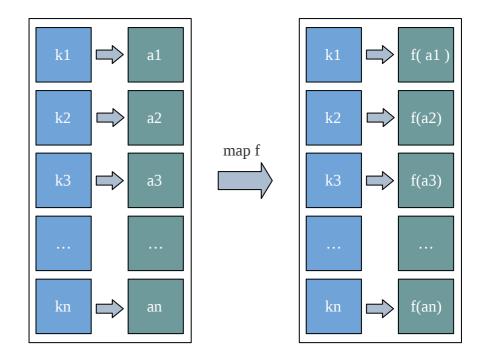
- List, Vector, Stream
- Option, Try





#### F is a container

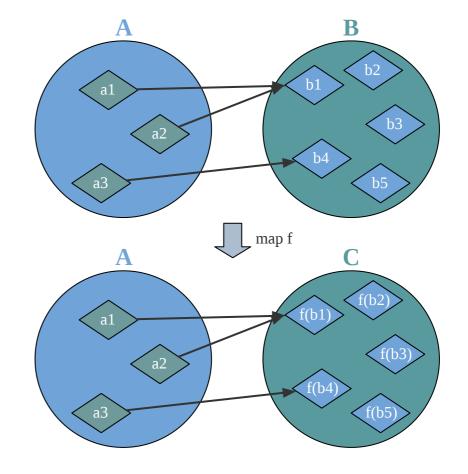
- List, Vector, Stream
- Option, Try
- Map[K, ?], Either[E, ?]





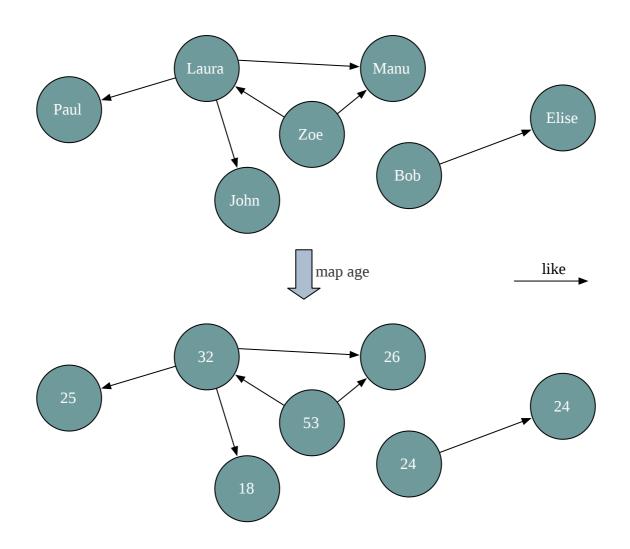
#### F is a container

- List, Vector, Stream
- Option, Try
- Map[K, ?], Either[E, ?]
- A => ?





## A Functor does not alter the structure





### F is an effect

```
type Failure[A] = Option[A] or Either[String, A]

type Nondeterminism[A] = List[A] or Vector[A]

type SideEffect[A] = IO[A]

type MutableState[A] = State[Int, A]

type ImmutableState[A] = Reader[Int, A]

type Console[A] = ...

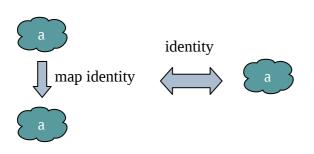
type Logger[A] = ...
```

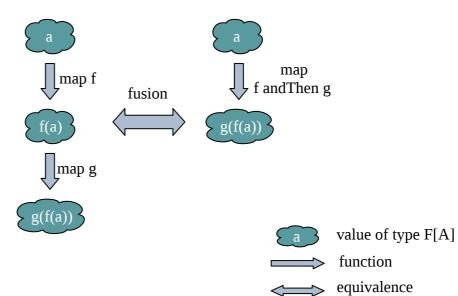
A Functor is an abstraction to update an effectful value **without** altering its effect



#### **Functor Laws**

```
forAll(fa: F[A] => fa.map(identity) == fa)
forAll((fa: F[A], f: A => B, g: B => C) =>
  fa.map(f).map(g) == fa.map(f andThen g)
)
```







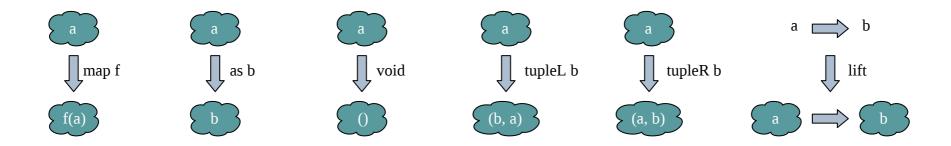
#### **Functor API**

```
trait Functor[F[_]] {
  def map[A, B](fa: F[A])(f: A => B): F[B]

  def as[A, B](fa: F[A])(value: B): F[B]
  def void[A] (fa: F[A]): F[Unit]

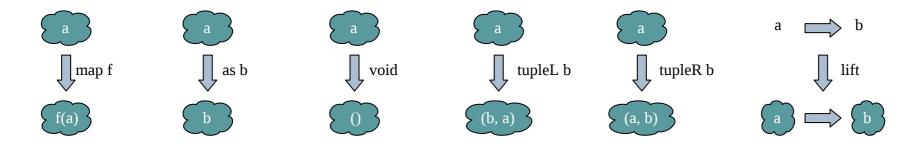
  def tupleL[A, B](fa: F[A])(value: B): F[(B, A)]
  def tupleR[A, B](fa: F[A])(value: B): F[(A, B)]

  def lift[A, B](f: A => B): F[A] => F[B]
}
```

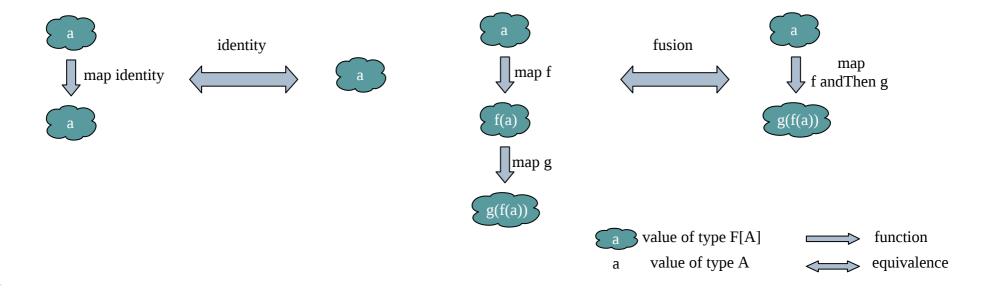




#### **API**



#### Laws



# Exercise 1



ADT

```
case class Foo[A](i: Int, a: A)

sealed trait Bar[A]
case class Bar1[A](i: Int , a: A) extends Bar[A]
case class Bar2[A](b: Boolean, a: A) extends Bar[A]
case class Bar3[A](s: String ) extends Bar[A]
```

☐ Function result

```
case class Producer[A](func: Int => A)
```



ADT

```
case class Foo[A](i: Int, a: A)

sealed trait Bar[A]
case class Bar1[A](i: Int , a: A) extends Bar[A]
case class Bar2[A](b: Boolean, a: A) extends Bar[A]
case class Bar3[A](s: String ) extends Bar[A]
```

☐ Function input

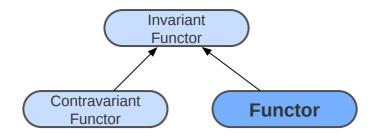
```
case class Predicate[A](func: A => Boolean)
```

☐ Function result

```
case class Producer[A](func: Int => A)
```



#### Functor variance



```
trait InvariantFunctor[F[_]]{
   def imap[A, B](fa: F[A])(f: A => B)(g: B => A): F[B]
}

trait ContravariantFunctor[F[_]] extends InvariantFunctor[F] {
   def contramap[A, B](fa: F[A])(f: B => A): F[B]
}

trait Functor[F[_]] extends InvariantFunctor[F] { // CovariantFunctor
   def map[A, B](fa: F[A])(f: A => B): F[B]
}
```



#### Functor variance

```
trait JsonDecoder[A]{
  def decode(value: Json): A
}

trait JsonEncoder[A]{
  def encode(value: A): Json
}

trait JsonCodec[A] extends JsonDecoder[A] with JsonEncoder[A]
```

What kind of Functor is JsonDecoder, JsonEncoder and JsonCodec?



## What kind of Functor is JsonDecoder?

```
trait JsonDecoder[A]{
  def decode(value: Json): A
}
```



#### What kind of Functor is JsonDecoder?

```
trait JsonDecoder[A]{
  def decode(value: Json): A
}
```

#### is equivalent to

```
case class JsonDecoder[A](decode: Json => A)
```

#### hence

```
implicit val functor: Functor [JsonDecoder] = ???
```



## What kind of Functor is JsonEncoder?

```
trait JsonEncoder[A]{
  def encode(value: A): Json
}
```



#### What kind of Functor is JsonEncoder?

```
trait JsonEncoder[A]{
  def encode(value: A): Json
}
```

#### is equivalent to

```
case class JsonEncoder[A](encode: A => Json)
```

#### hence

```
implicit val functor: ContravariantFunctor [JsonEncoder] = ???
```



## What kind of Functor is JsonCodec?

trait JsonCodec[A] extends JsonDecoder[A] with JsonEncoder[A]



#### What kind of Functor is JsonCodec?

```
trait JsonCodec[A] extends JsonDecoder[A] with JsonEncoder[A]
```

#### is equivalent to

```
case class JsonCodec[A](
  decode: Json => A,
  encode: A => Json
)
```

#### hence

```
implicit val functor: InvariantFunctor [JsonCodec] = ???
```



# Variance

Туре	Example	A	В
Product	(A, B)	Covariant	Covariant
Sum	Either[A, B]	Covariant	Covariant
Function	A => B	Contravariant	Covariant
Endo Function	A => A	Invariant	N/A



## Variance

Туре	A	В	С
(A => B) => C	Covariant	Contravariant	Covariant

Thinking with types by Sandy Maguire



# **Generalising Functor**

See <u>Functor-Of</u> from Vladimir Ciobanu and @Iceland\_jack



# A Functor is an abstraction to update an effectful value without altering its effect



## What if we have more than 1 effect?



How can we combine effects? e.g. F[A] and F[B]

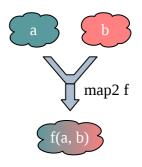


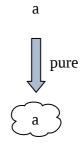
# **Applicative**



# **Applicative**

```
trait Applicative[F[_]] extends Functor[F] {
    def map2[A, B, C](fa: F[A], fb: F[B])(f: (A, B) => C): F[C]
    def pure[A](a: A): F[A]
}
```

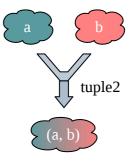






# Applicative alternative encoding

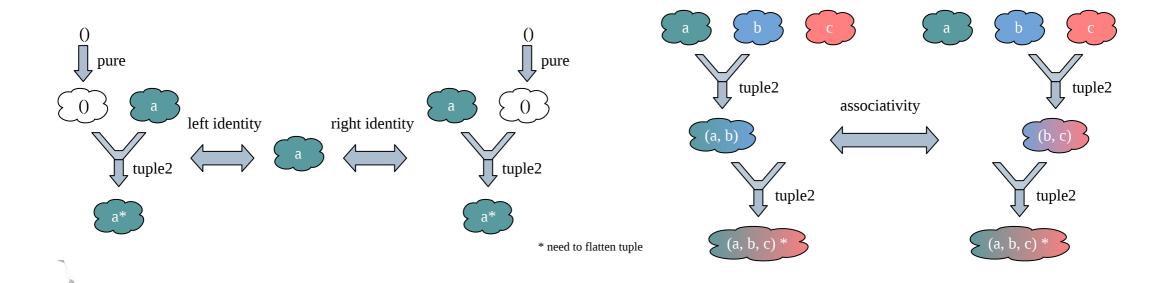
```
trait Applicative[F[_]] extends Functor[F] {
  def tuple2[A, B](fa: F[A], fb: F[B]): F[(A, B)]
  def unit: F[Unit]
}
```







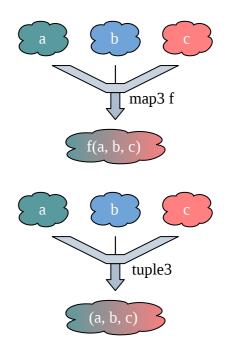
# Applicative laws



# An Applicative combines 2 effects associatively and lift pure values into NOOP effect



# **Applicative API**



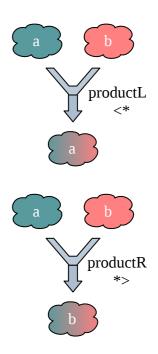


# **Applicative API**

```
trait Applicative[F[_]] extends Functor[F] {
  def pure[A](a: A): F[A]
  def map2[A, B, C](fa: F[A], fb: F[B])(f: (A, B) => C): F[C]

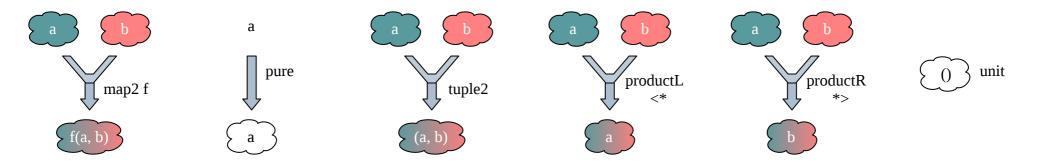
// alias to <*
  def productL[A, B](fa: F[A], fb: F[B]): F[A]

// alias to *>
  def productR[A, B](fa: F[A], fb: F[B]): F[B]
}
```

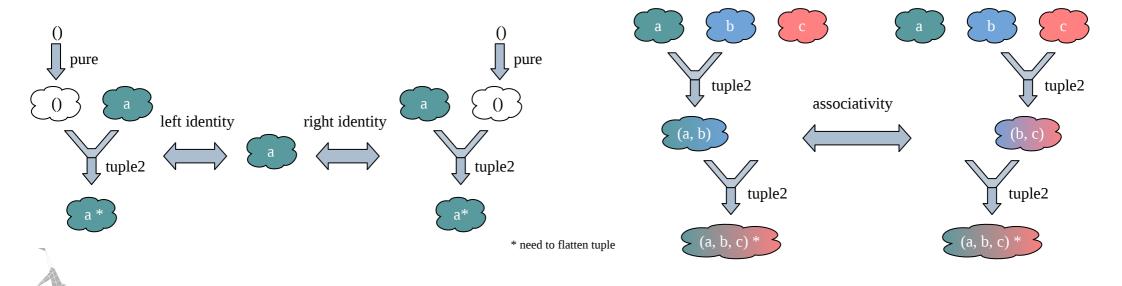




#### **API**



### Laws



# Applicative combine effects: Failure

```
def right[A, E](a: A): Either[E, A] = Right(a)
def left [A, E](e: E): Either[E, A] = Left(e)

scala> (right(1), right("hello")).tuple2
res0: Either[Nothing,(Int, String)] = Right((1,hello))

scala> (right(1), left("an error")).tuple2
res1: Either[String,(Int, Nothing)] = Left(an error)

scala> (left("oops"), right(2)).tuple2
res2: Either[String,(Nothing, Int)] = Left(oops)

scala> (left("oops"), left("an error")).tuple2
res3: Either[String,(Nothing, Nothing)] = Left(oops)
```



# Applicative combine effects: Failure

Invalid(NonEmptyList(oops, an error))

```
scala> println((validNel(1), validNel("hello")).tuple2)
Valid((1,hello))
scala> println((validNel(1), invalidNel("an error")).tuple2)
Invalid(NonEmptyList(an error))
scala> println((invalidNel("oops"), validNel(2)).tuple2)
Invalid(NonEmptyList(oops))
scala> println((invalidNel("oops"), invalidNel("an error")).tuple2)
```



### Applicative combine effects: Nondeterminism

```
scala> (List(1,2,3), List("foo", "bar")).tuple2
res8: List[(Int, String)] = List((1,foo), (1,bar), (2,foo), (2,bar), (3,foo), (3,bar))
scala> (List(1,2,3), Nil).tuple2
res9: List[(Int, Nothing)] = List()
```



# Applicative combine effects: Dependency injection

```
case class State(userName: String, useLargeList: Boolean)

def greet(state: State): String = s"Welcome ${state.userName}"

def listSize(state: State): Int = if(state.useLargeList) 100 else 10

val combined: State => (String, Int) = (greet _, listSize _).tuple2

scala> combined(State("John Doe", true))
res10: (String, Int) = (Welcome John Doe,100)

scala> combined(State("Lena Doe", false))
res11: (String, Int) = (Welcome Lena Doe,10)
```



# Applicative combine effects: Side Effect

```
import cats.effect.IO

val combined = (IO(println("hello")), IO(println("I love FP!"))).tuple2

scala> combined.unsafeRunSync
hello
I love FP!
res12: (Unit, Unit) = ((),())
```



# Exercise 2

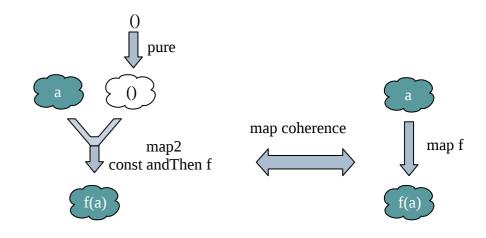


# Applicative is a Functor

```
trait Applicative[F[_]] extends Functor[F] {
  def pure[A](a: A): F[A]
  def map2[A, B, C](fa: F[A], fb: F[B])(f: (A, B) => C): F[C]
}
```

#### **Coherence Law**

```
forAll((fa: F[A], f: A => B) =>
  (fa, pure(())).map2((a, _) => f(a)) == fa.map(f)
)
```





### Applicative is a Monoidal Functor

```
combine :: A => A => A
tuple2 :: F[A] => F[B] => F[(A, B)]
empty :: A
pure :: A => F[A]
```

#### **Laws**

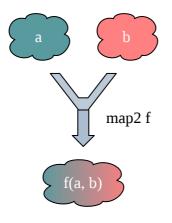
```
forAll(a: A => combine(empty, a) == a)
forAll(fa: F[A] => tuple2(unit , fa) == fa) // equalities hold if we flatten tuples

forAll(a: A => combine(a, empty) == a)
forAll(fa: F[A] => tuple2(fa, unit ) == fa)

forAll(( a: A , b: B , c: C) => combine(combine(a, b), c) == combine(a, combine(b, c)))
forAll((fa: F[A], fb: F[A], fc: F[C]) => tuple2(tuple2(fa, fb), fc) == tuple2(fa, tuple2(fb, fc)))
```



# Applicative combines effects

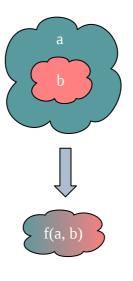




# What if one effect depends on another one? e.g. F[B] depends on A

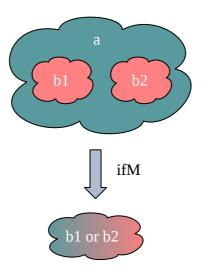


# **Nested effects**





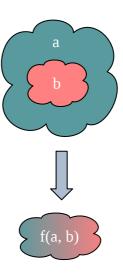
# Nested effects: ifM





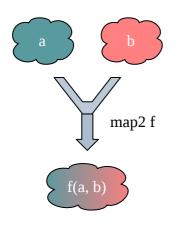
### **Nested effects**

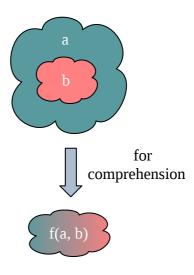
```
for {
    a <- fa
    b <- fb(a)
} yield f(a, b)</pre>
```





# Independent vs dependent effects







# For comprehension

```
val res: F[D] =
  for {
    a: A <- fa: F[A]
    b: B <- fb: F[B]
    c: C <- fb: F[C]
} yield f(a,b,c): D</pre>
```



# For comprehension

```
val res: F[D] =
  for {
    a: A <- fa: F[A]
    b: B <- fb: F[B]
    c: C <- fb: F[C]
} yield f(a,b,c): D</pre>
```

### **Using Monad**

```
val res =
  fa.flatMap(a =>
    fb.flatMap(b =>
      fc.flatMap(c =>
            f(a,b,c).pure[F]
      )))
```

### Using FlatMap

```
val res =
  fa.flatMap(a =>
    fb.flatMap(b =>
      fc.map(c =>
          f(a,b,c)
      )))
```



# For comprehension is for a single Monad

```
for {
    a: A <- fa: F[A]
    b: B <- fb: G[B] []
    c: C <- fb: H[C] []
} yield f(a,b,c)</pre>
```

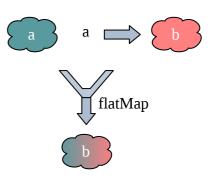


# Monad



### Monad

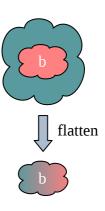
```
trait Monad[F[_]] extends Applicative[F] {
  def flatMap[A, B](fa: F[A])(f: A => F[B]): F[B]
}
```





# Monad alternative encoding

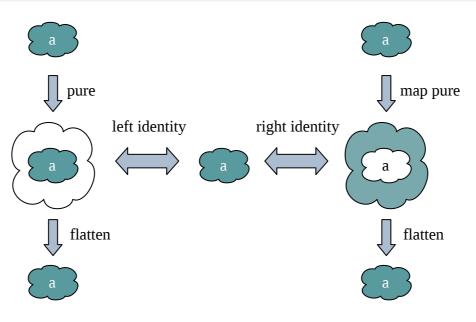
```
trait Monad[F[_]] extends Applicative[F] {
  def flatten[A](ffa: F[F[A]]): F[A]
}
```





# Monad identity laws

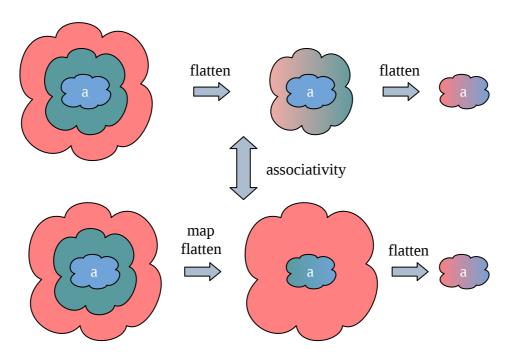
```
forAll(fa: F[A] => fa.pure[F].flatten == fa)
forAll(fa: F[A] => fa.map(_.pure[F]).flatten == fa)
```





# Monad associativity law

```
forAll(fffa: F[F[F[A]]] =>
  fa.flatten.flatten == fa.map(_.flatten).flatten
)
```



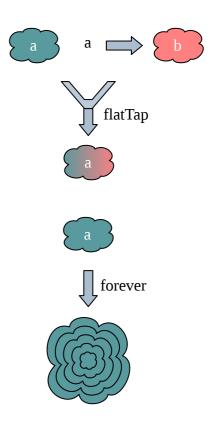


### Monad API

```
trait Monad[F[_]] extends Applicative[F] {
  def flatMap[A, B](fa: F[A])(f: A => F[B]): F[B]
  def flatten[A](ffa: F[F[A]]): F[A]

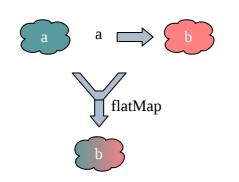
def flatTap[A, B](fa: F[A])(f: A => F[B]): F[A]

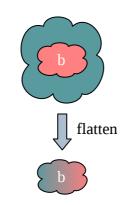
def forever[A](fa: F[A]): F[Nothing]
}
```

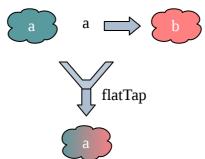


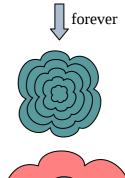


#### **API**

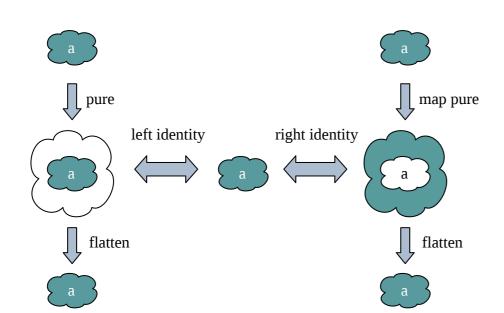


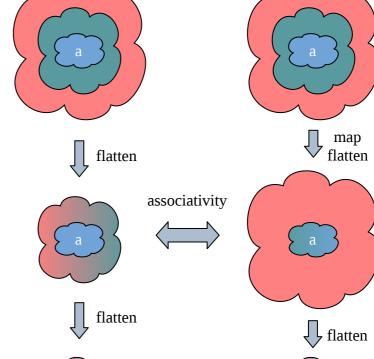






#### Laws







# Exercise 3



# Monad is an Applicative

```
trait Monad[F[_]] extends Applicative[F] {
  def flatMap[A, B](fa: F[A])(f: A => F[B]): F[B]
}
```

#### Coherence Law

```
forAll((fa: F[A], fb: F[B]) =>
  fa.flatMap(a => fb.map(b => (a, b))) == (fa, fb).tuple2
)
```



# Monad is an Applicative

```
trait Monad[F[_]] extends Applicative[F] {
  def flatMap[A, B](fa: F[A])(f: A => F[B]): F[B]
}
```

#### Coherence Law

```
forAll{ (fa: F[A], fb: F[B]) =>

val combinedFor = for {
    a <- fa
    b <- fb
} yield (a, b)

combinedFor == (fa, fb).tuple2
}</pre>
```



# Monad is for sequential composition of effects

```
val res =
  for {
    a <- fa
    b <- foo(a)
  } yield f(a,b)</pre>
```



# Monad is for sequential composition of effects

```
val res =
  for {
    a <- fa
    b <- foo(a)
  } yield f(a,b)</pre>
```



# Monad is for sequential composition &

Applicative must be coherent with Monad



# All Monadic effects must have a sequential Applicative



## Coherence implication

```
import exercises.errorhandling.{Country, User, Username}
import answers.functors.FunctorsAnswers.DefaultMonad
def validateUsername(x: String): ValidatedNel[String, Username] = {
 if(x.length > 5) valid(Username(x))
 else invalidNel(s"Username $x too short")
def validateCountry(x: String): ValidatedNel[String, Country] =
 x match {
   case "FRA" => valid(Country.France)
              => invalidNel(s"Unsupported country $x")
implicit def validatedMonad[E]: Monad[Validated[E, ?]] = new DefaultMonad[Validated[E, ?]] {
    def pure[A](a: A): Validated[E, A] = Valid(a)
    def flatMap[A, B](fa: Validated[E, A])(f: A => Validated[E, B]): Validated[E, B] =
     fa match {
        case Invalid(e) => Invalid(e)
        case Valid(a) => f(a)
```

# Coherence implication

```
val combinedFor: ValidatedNel[String, User] = for {
   username <- validateUsername("foo")
   country <- validateCountry("UK")
} yield User(username, country)

scala> println(combinedFor)
Invalid(NonEmptyList(Username foo too short))
```



## Coherence implication

```
val combinedFor: ValidatedNel[String, User] = for {
  username <- validateUsername("foo")</pre>
  country <- validateCountry("UK")</pre>
} yield User(username, country)
scala> println(combinedFor)
Invalid(NonEmptyList(Username foo too short))
def map2[E: Semigroup, A, B, C](fa: Validated[E, A], fb: Validated[E, B])(f: (A, B) => C): Validated[E, C] =
  (fa, fb) match {
    case (Valid(a), Valid(b)) => Valid(f(a, b))
case (Invalid(e), Valid(_)) => Invalid(e)
case (Valid(_), Invalid(e)) => Invalid(e)
    case (Invalid(e1), Invalid(e2)) => Invalid(e1 |+| e2)
val combinedMap2 = map2(validateUsername("foo"), validateCountry("UK"))(User( , ))
```





## Validated cannot be a Monad



# IO Applicative cannot combine in parallel (but IO.Par can)



## Composition

if F and G are a Functor then F[G[\_]] is a Functor

if F and G are an Applicative then F[G[\_]] is an Applicative



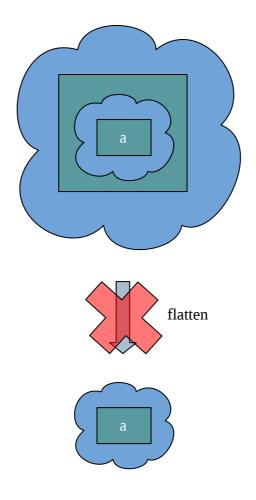
## Composition

if F and G are a Functor then F[G[\_]] is a Functor if F and G are an Applicative then F[G[\_]] is an Applicative

but if F and G are a Monad then F[G[\_]] is not necessarily a Monad



# Monad do not compose





If you can combine effects then ...



## Imperative code is Monadic

```
def getOrderTotal(userId: Int, orderId: Int): Double = {
  log.trace(s"User $userId get order total for order $orderId"): Unit
  val hasAccess = canUserCanAccessOrder(userId, orderId): Boolean
  if(! hasAccess)
    throw new Exception(s"User $userId does not have access to order $orderId")
  else {
    val order = getOrder(orderId): Order
    order.total: Double
  }
}
```



## Imperative code is Monadic

```
def getOrderTotal(userId: Int, orderId: Int): Double = {
  log.trace(s"User $userId get order total for order $orderId"): Unit
  val hasAccess = canUserCanAccessOrder(userId, orderId): Boolean
  if(! hasAccess)
    throw new Exception(s"User $userId does not have access to order $orderId")
  else {
    val order = getOrder(orderId): Order
    order.total: Double
  }
}
```

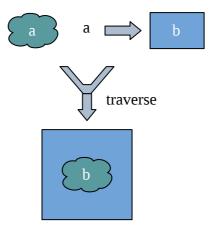


## Traverse



## Traverse

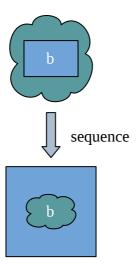
```
trait Traverse[F[_]] extends Functor[F] with Foldable[F] {
  def traverse[G[_]: Applicative, A, B](fa: F[A])(f: A => G[B]): G[F[B]]
}
```





## Traverse alternative encoding

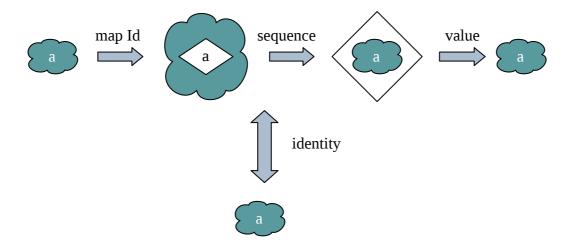
```
trait Traverse[F[_]] extends Functor[F] with Foldable[F] {
  def sequence[G[_]: Applicative, A](fga: F[G[A]]): G[F[A]]
}
```





## Traverse Law

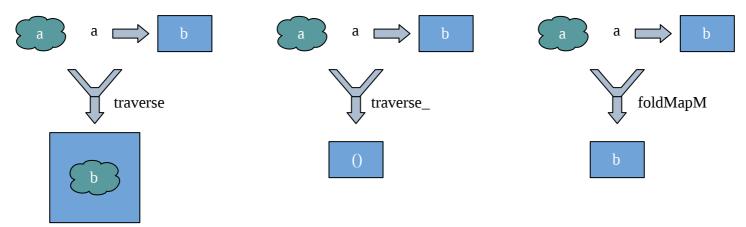
```
forAll(fa: F[A] => fa.map(Id(_)).sequence.value == fa)
```





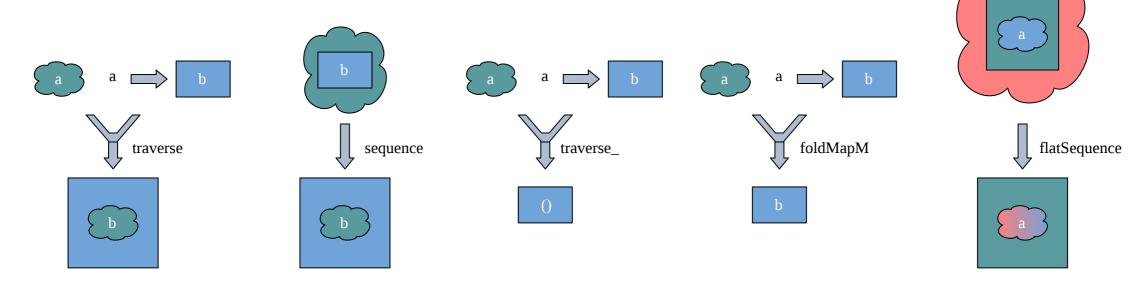
## Traverse API

```
trait Traverse[F[_]] extends Functor[F] with Foldable[F] {
   def traverse[G[_]: Applicative, A, B](fa: F[A])(f: A => G[B]): G[F[B]]
   def traverse_[G[_]: Applicative, A, B](fa: F[A])(f: A => G[B]): G[Unit]
   def foldMapM[G[_]: Applicative, A, B: Monoid](fa: F[A])(f: A => G[B]): G[B]
}
```

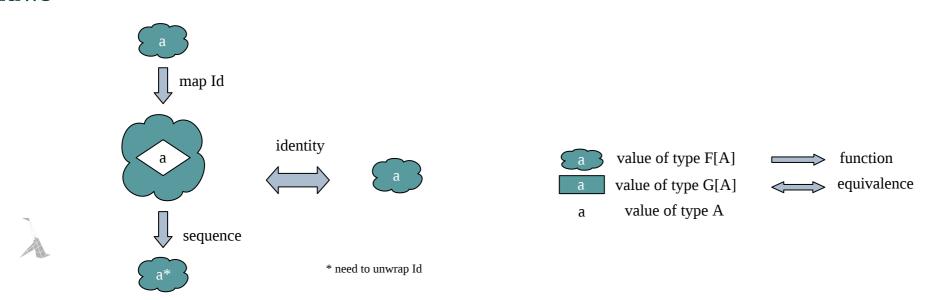




#### **API**



#### Laws



## Exercise 4



### Traverse is a Functor

```
trait Traverse[F[_]] extends Functor[F] with Foldable[F] {
  def traverse[G[_]: Applicative, A, B](fa: F[A])(f: A => G[B]): G[F[B]]
}
```

#### Coherence Law

```
forAll((fa: F[A], f: A \Rightarrow B) \Rightarrow fa.traverse(a \Rightarrow Id(f(a))).value == fa.map(f))
```



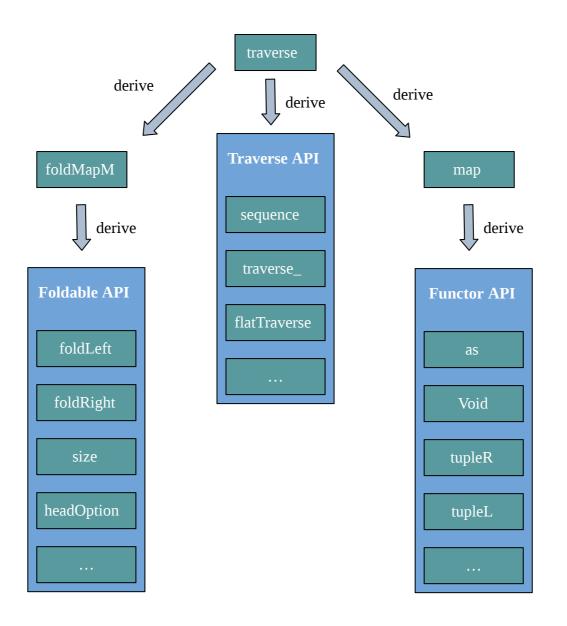
## Traverse is a Foldable

```
trait Traverse[F[_]] extends Functor[F] with Foldable[F] {
  def traverse[G[_]: Applicative, A, B](fa: F[A])(f: A => G[B]): G[F[B]]
}
```

#### Coherence Law

```
forAll((fa: F[A], f: A \Rightarrow B) \Rightarrow fa.traverse(a \Rightarrow Const(f(a))).getConst == fa.foldMap(f))
```







# Default implementation

```
traverse > foldMap > size

def sizeDefault[F[_]: Traverse, A](fa: F[A]): Int =
   fa.traverse(_ => Const(1)).getConst

sizeDefault = (Const(1) |+| Const(1) |+| ... |+| Const(1)).getConst // O(n)
```



## Default implementation

```
traverse > foldMap > size

def sizeDefault[F[_]: Traverse, A](fa: F[A]): Int =
    fa.traverse(_ => Const(1)).getConst

sizeDefault = (Const(1) |+| Const(1) |+| ... |+| Const(1)).getConst // O(n)

val vectorTraverse: Traverse[Vector] = new Traverse[Vector] {
    def traverse[G[_]: Applicative, A, B](fa: Vector[A])(f: A => G[B]): G[Vector[B]] = ???
    overload def size[A](fa: Vector[A]): Int = fa.size // O(1)
}
```



## Default implementation

```
traverse > foldMap > size

def sizeDefault[F[_]: Traverse, A](fa: F[A]): Int =
    fa.traverse(_ => Const(1)).getConst

sizeDefault = (Const(1) |+| Const(1) |+| ... |+| Const(1)).getConst // O(n)

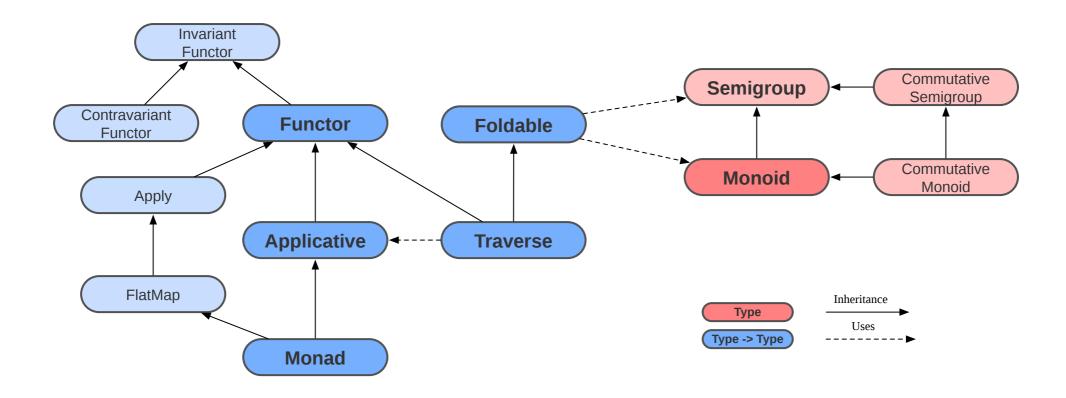
val vectorTraverse: Traverse[Vector] = new Traverse[Vector] {
    def traverse[G[_]: Applicative, A, B](fa: Vector[A])(f: A => G[B]): G[Vector[B]] = ???
    overload def size[A](fa: Vector[A]): Int = fa.size // O(1)
}
```

#### Coherence law

```
forAll(fa: F[A] => fa.size == sizeDefault(fa))
```



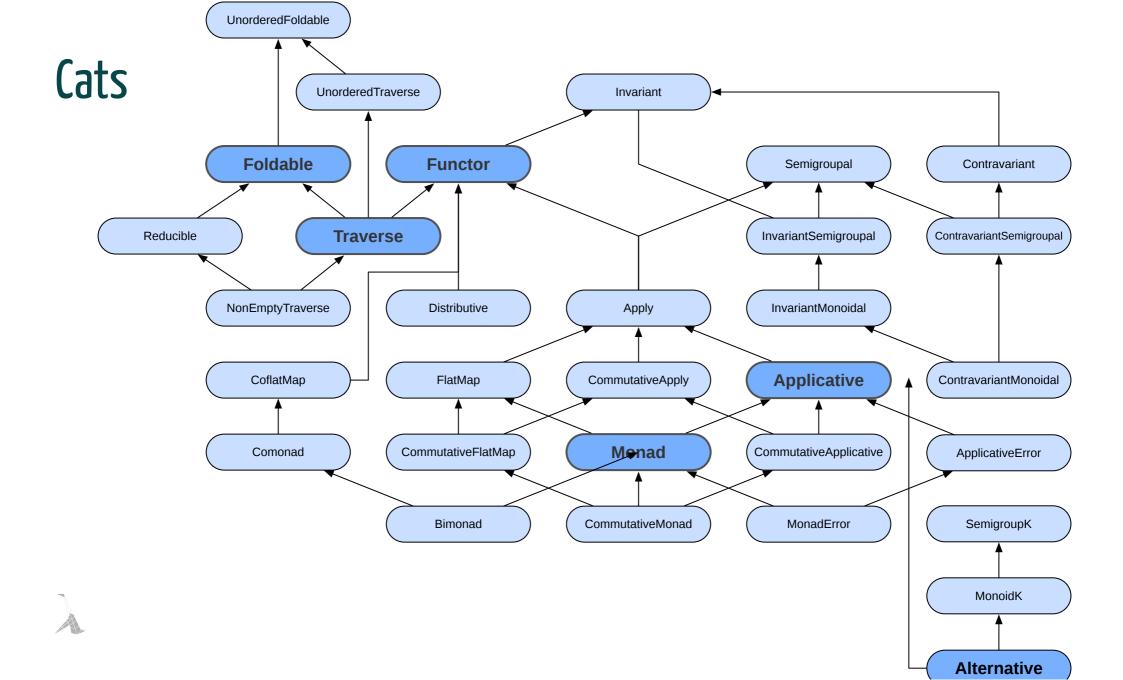
## Review





# Granularity





# Granularity

• There is not a single "correct" granularity



## Granularity

- There is not a single "correct" granularity
- Applicative was discovered in 2008
- Selective Applicative Functor in 2018!



## Recommendations

#### Define strongest typeclasses for data types

```
val instances: Monad[Option] with Traverse[Option] = ?
```

- Better defined behaviour
- More tests for free
- More powerful API

#### Use weakest typeclasses for functions

```
def program[F[_]: Console: Applicative]: F[String] = ?
```

- Principle of least power
- More reasoning
- Less tests to write



# Resources and further study

- <u>Cats infographic</u>: typeclass diagrams for cats
- Thinking with types: variance
- <u>Constraints Liberate</u>, <u>Liberties Constrain</u> from Runar Bjarnason
- Applicative paper <u>Applicative Programming with Effects</u>
- Selective Applicative Functor paper <u>Selective Applicative Functors</u>



# The End

