Power Delivered to Food in a Microwave

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To find the power delivered to food inside of a microwave you would have to consider both the electric ($\vec{E}$) and magnetic ($\vec{B}$) fields, how they influence the Poynting vector, and how that relates to the surface of the food defined. To code this I will be creating a 3D simulation of a microwave by simplifying food geometry to the simplest possible shapes, rectangular prisms and cubes. I will display how a microwave unevenly heats food by showing that certain spots on the surface of the food will receive more energy than others. I would then look into how placing the food in different places affects the total power it receives.

I. INTRODUCTION

Microwaves are everyday tools used to very quickly heat up food. It is taken for granted how much planning goes into determining how the food is heated up. To simplify a microwave it could be treated as a waveguide with perfect conductors on either open side, or a 'wavebox,' This means that every face on the so called wavebox should be perfect conductors. I will also have to consider the inside of the microwave to be a vaccuum in order to simplify some of the derivation.

Microwaves have a source for electric and magnetic fields called a magnetron. I won't go into how it produces either field. The oscillating fields inside the microwave are bouncing off all the walls just as they would a waveguide. All of these reflected fields can be summed as a standing wave in the wavebox. If the standing wave could be defined it would be fairly simple to show what the inside of a microwave would look like.[1] Because it is simplified to a standing wave there are nodes and antinodes. The spots in the microwave where the nodes and antinodes are create hotspots and cool spots, this is why putting food in a microwave can result in certain areas receiving more energy and thus not heating evenly like they would in an oven. In an oven the energy is slowly being transferred into the food so it is able to easily dissipate throughout, however in a microwave the energy comes directly from the EM wave to the food and because it is so quick there is no chance for the energy to dissipate.

II. PHYSICS OF A MICROWAVE

From Maxwell's equations we know that:

$E^{\parallel} = 0$

$B^{\perp} = 0$

This allows for the construction of boundary conditions for both the electric and magnetic fields. Of course each field will need to be broken into components so that a parallel or perpendicular component can be defined.

$\vec{E} = E_x\hat{x} + E_y\hat{y} + E_z\hat{z}$

$\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$

For a microwave with the dimensions L_x, L_y, L_z the boundary conditions for each will be:

$E_x(x,0,z,t) = E_x(x,L_y,z,t) = E_x(x,y,0,t) = E_x(x,y,L_z,t) = 0$

$E_y(0,y,z,t) = E_y(L_x,y,z,t) = E_y(x,y,0,t) = E_y(x,y,L_z,t) = 0$

$E_z(0,y,z,t) = E_z(L_x,y,z,t) = E_z(x,0,z,t) = E_z(x,L_y,z,t) = 0$

$B_x(0,y,z,t) = B_x(L_x,y,z,t) = 0$

$B_y(x,0,z,t) = B_y(x,L_y,z,t) = 0$

$B_z(x,y,0,t) = B_z(x,y,L_z,t) = 0$

An additional boundary condition that can be imposed at each boundary:

$\nabla \cdot \vec{E} = 0$

Lastly the solution should also satisfy the wave equation

$\nabla^2\vec{E} = \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$

One assumption we will make is that the solution for one component is separable so the solution should be expected to have the form.

$E_x(x,y,z,t) = X(x)Y(y)Z(z)T(t)$

We have defined boundary conditions for each of the separable functions that can be easily solved. A cosine term satisfies the derivative of the electric field being zero at $x = 0$ and $x = L_x$. Two sine terms satisfy being zero at $y = 0, y = L_y, z = 0, z = L_z$. Lastly an exponential will satisfy the wave equation giving a solution of the form.[2]

$E_x(x,y,z,t) = cos(k_x x)sin(k_y y)sin(k_z z)e^{-i\omega t}$

the three k_i terms are defined by the dimensions of the microwave to form the standing wave inside.

$k_x = \frac{n_x\pi}{L_x}$,

$k_y = \frac{n_y\pi}{L_y}$,

$k_z = \frac{n_z\pi}{L_z}$

Where n_x, n_y, n_z are positive integers. These are also used to solve the wave equation giving the frequency of the standing wave as:

$\frac{\omega^2}{c^2} = k_x^2 + k_y^2 + k_z^2$, $\omega = c\sqrt{k_x^2 + k_y^2 + k_z^2}$

The y and z components of the electric field will have a similar form. The electric field will then be:

$\vec{E}(x,y,z,t) = [cos(k_x x)sin(k_y y)sin(k_z z)\hat{x}$

$+sin(k_x x)cos(k_y y)sin(k_z z)\hat{y}$

$+sin(k_x x)sin(k_y y)cos(k_z z)\hat{z}]e^{-i\omega t}$

Next for the magnetic field ($\vec{B}$) which can be found much more easily. $\vec{B}$ shoudl also solve the wave equation and thus should have the same exponential term as the electric field. One of Maxwell's equations can be used to find the magnetic field:

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -i\omega \vec{B}$
$\vec{B} = -\frac{i}{\omega}(\nabla \times \vec{E})$

Skipping the tedious math brings the equation for the magnetic field:

$\vec{B}(x,y,z,t) = -\frac{i}{\omega}[[(k_y - k_z)sin(k_x x)cos(k_y y)cos(k_z z)]\hat{x}$
$+[(k_z - k_x)cos(k_x x)sin(k_y y)cos(k_z z)]\hat{y}$
$+[(k_x - k_y)cos(k_x x)cos(k_y y)sin(k_z z)]\hat{z}]e^{-i\omega t}$

Once this is actually plotted only the real parts should be considered the real parts of $\vec{E}$ and $\vec{B}$ are:

$Re[\vec{E}(x,y,z,t)] = [cos(k_x x)sin(k_y y)sin(k_z z)\hat{x}$
$+sin(k_x x)cos(k_y y)sin(k_z z)\hat{y}$
$+sin(k_x x)sin(k_y y)cos(k_z z)\hat{z}]cos(\omega t)$
$Re[\vec{B}(x,y,z,t)] = \frac{1}{\omega}[[(k_y - k_z)sin(k_x x)cos(k_y y)cos(k_z z)]\hat{x}$
$+[(k_z - k_x)cos(k_x x)sin(k_y y)cos(k_z z)]\hat{y}$
$+[(k_x - k_y)cos(k_x x)cos(k_y y)sin(k_z z)]\hat{z}]sin(\omega t)$

III. POWER IN A MICROWAVE

To find the power first the Poynting vector needs to be defined.

$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$

As is apparent this is not a very fun calculation. However tedious it may be it is rather simple to actually do. Each component of the Poynting vector follows the same general pattern and shape so I will only show the x component.

$S_x = \frac{cos(\omega t)sin(\omega t}{\omega}[cos^2(k_y y)cos^2(k_z z)(k_y + k_z - 2k_x)$
$+cos^2(k_y y)(k_x - k_y) + cos^2(k_z z)(k_x - k_z)]$

This is not the prettiest component and because of that I am going to simplify the geometry in the problem to make calculations easier. To find the power on any surface of a shape the following integral is used.

$P = \int \vec{S} \cdot \vec{da}$

I will only use surfaces whose faces point in either the $\hat{x}, \hat{y}, \hat{z}$ direction this means I will only have to worry about one component of the Poynting vector for each calculation and can then sum the power on each surface of the shape.

IV. CODING

This is a preliminary plan for the coding part so I am not entirely sure what will actually end up working. To start the code I want to create a 3D space. This means I will treat the space inside the microwave as many little cubes stacked together. If I had a really powerful computer I would consider calculating the Poynting vector at each point in the 3D space but that doesn't seem feasible. What is likely to get done is that I calculate only the necessary component of the Poynting vector along the surface of the 'food' which might be a rectangular prism or cube. I would like to then take these calculated values and plot them to show what parts of the food received more or less energy. To dig a bit further I would like to look into first where in my simulated microwave are there hotzones and then depending on where I place the food how does the amount of power it receives change. If I center it on a hotzone will it receive more power than if I center it on a cold zone?

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[4] Vollmer, M. (2004). Physics of the microwave oven. Physics Education.

[5] HOUŠOVÁ, J., & HOKE, K. (2002). Microwave Heating – the Influence of Oven and Load Parameters on the Power Absorbed in the Heated Load. Czech J. Food Sci, 20(3).