Demographic Modeling: A Primer

Matthew Aiello-Lammens
Pace University

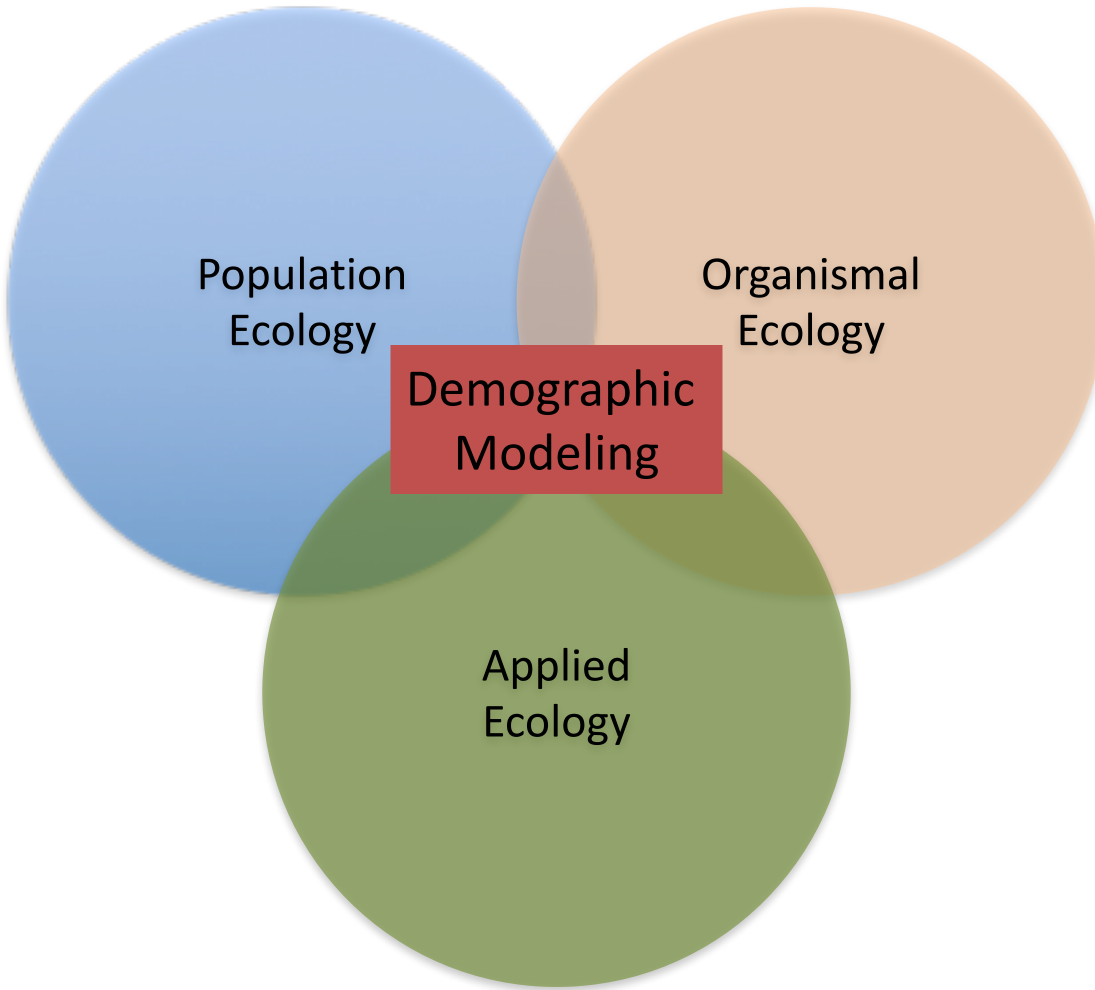
What is demographic modeling?

Populations Ecology

How populations change over time?

How populations change over space?

What is the internal structure of a population?



Autecology – ecology of inviduals

Life-history (e.g., reproduction, phenology)

Behavior affects?

Physiological response to environment?

Integration of ecological knowledge and a little math to answer questions in applied ecology

Why model?

Assess vulnerability

Evaluate management options

Assess human impacts

Assess invasion risk

Lots of ‘assessing’ – but ultimately we model to help us understand ecological processes and inform decisions

Different strokes for different folks
- there are different types of models

Individual Based

Very Flexible

Can Incorporate Many Realistic Factors (e.g. Genetics, Behavior)

Why they're good

Different strokes for different folks
- there are different types of models

Individual Based

Data Intensive!

Very sensitive to assumptions

Easy to make numerical and logical mistakes

Why they're not so good

Different strokes for different folks
- there are different types of models

Age- or Stage-Based

Flexible and realistic

Few implicit assumptions

Why they're good

Different strokes for different folks
- there are different types of models

Age- or Stage-Based

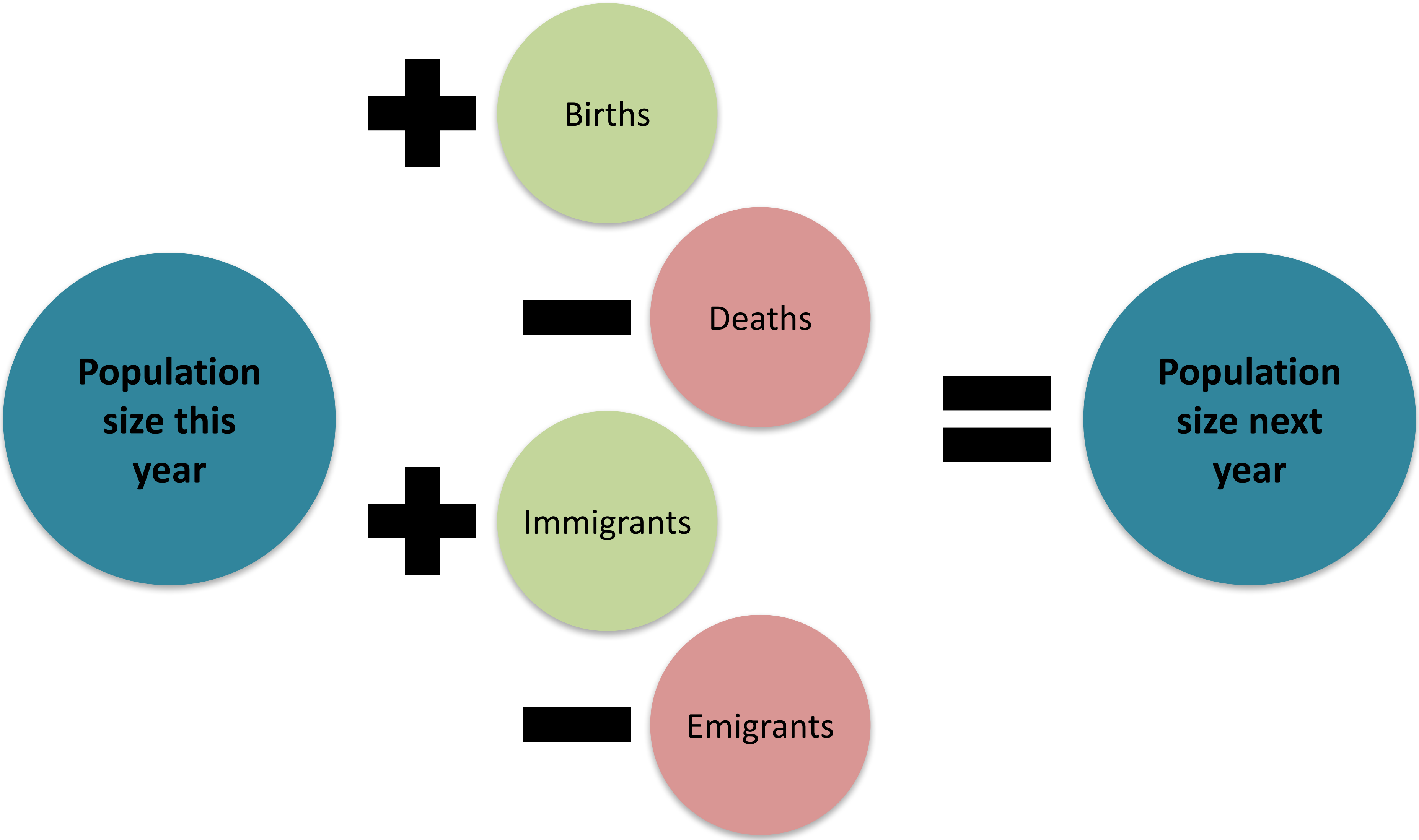
Date intensive

Numerical errors possible

Difficult to add individual characteristics (e.g. genetics or behavior)

Why they're not so good

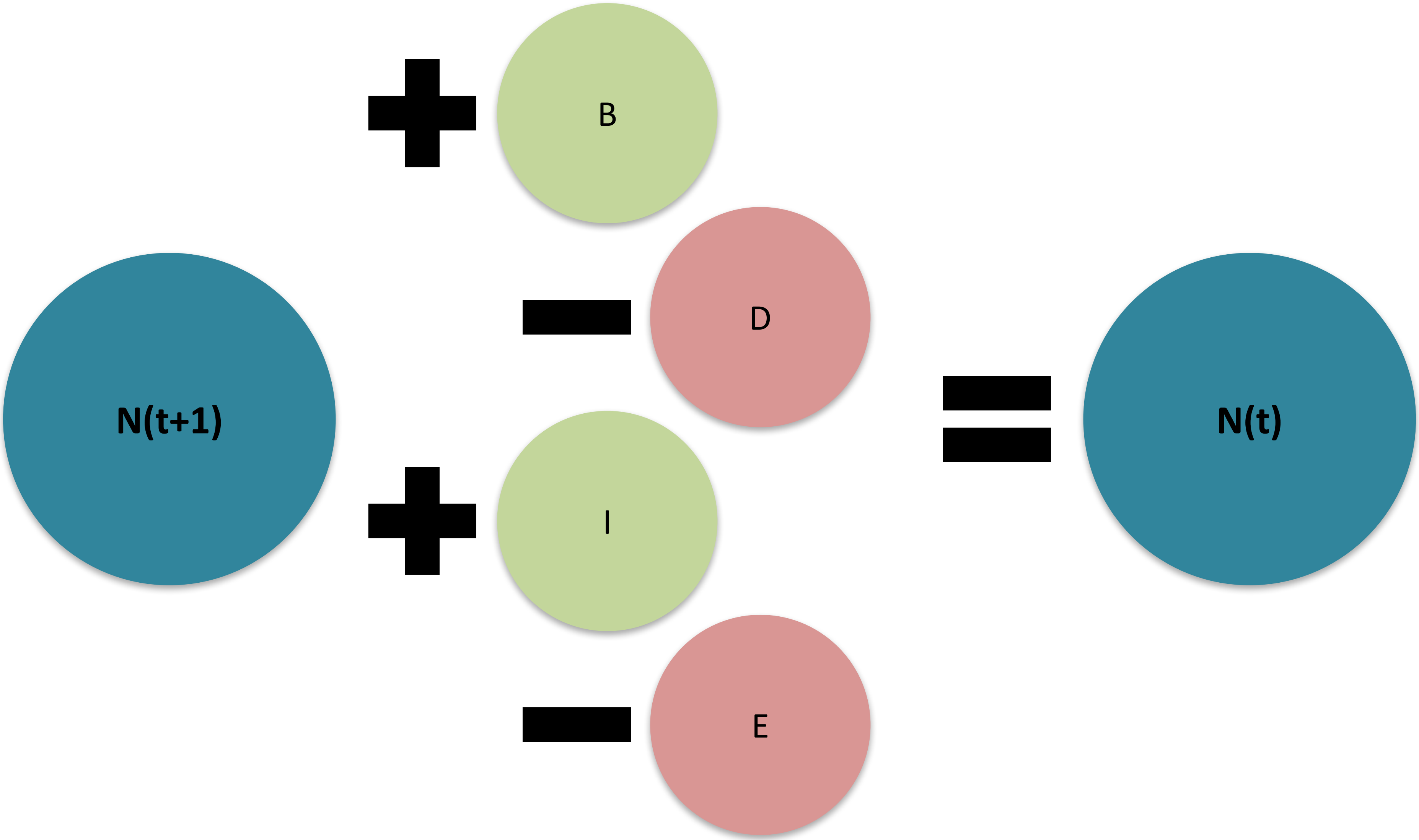
A simple model of population growth



Written as:

$$N(t+1) = N(t) + B - D + I - E$$

A simple model of population growth



Written as:

$$N(t+1) = N(t) + B - D + I - E$$

A *very* simple model of population growth

Popul
size
ye

\+ Births

− Deaths

rants

− Emigrants

= Population size next year

Annual plant species

Written as:

$$N(t+1) = B$$

A *very* simple model of population growth

Births

Each individual produces ***f*** offspring during it's life, so

Births = N * f

and

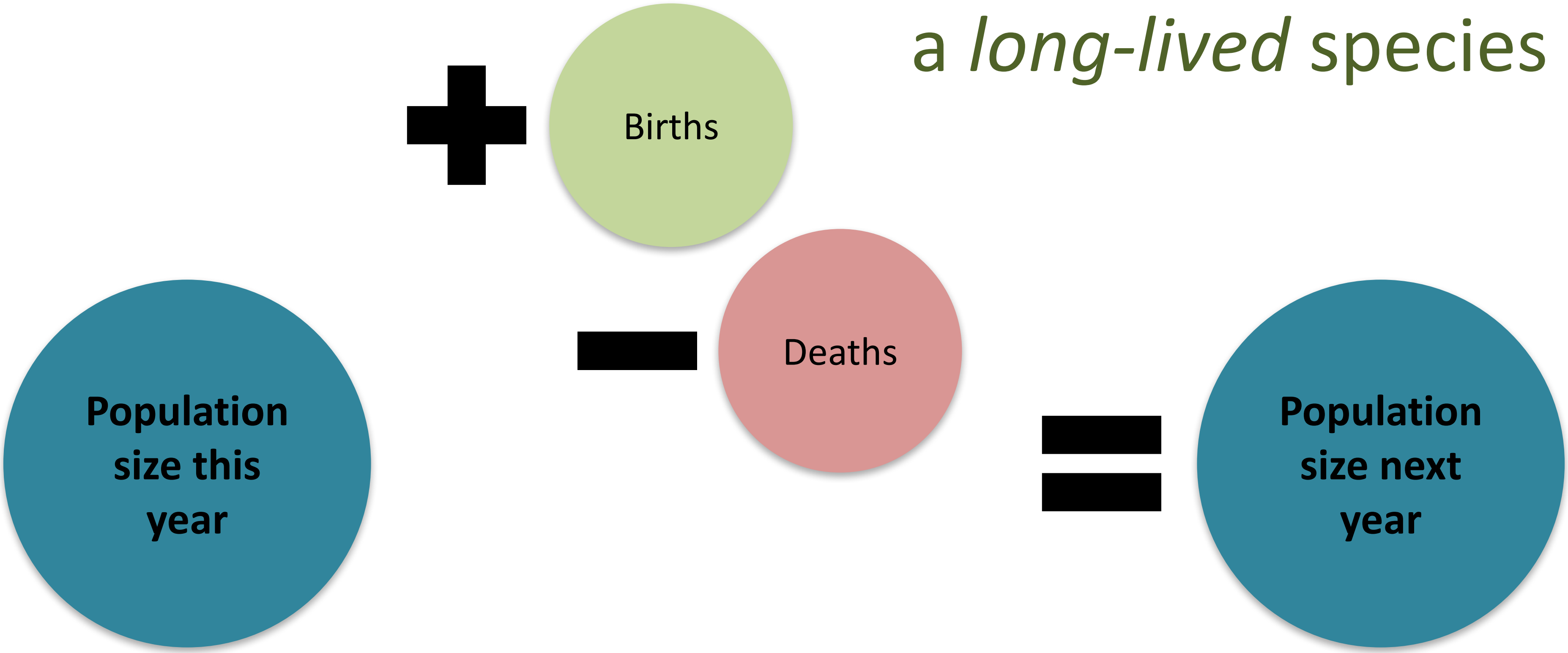
Pop. Size Next Year = N * f

Population size next year

Written as:

N(t+1) = N(t) * f

A little more complicated – a *long-lived* species



A little more complicated – a *long-lived* species

\+ Births

\- Deaths

Population size this year

= Population size next year

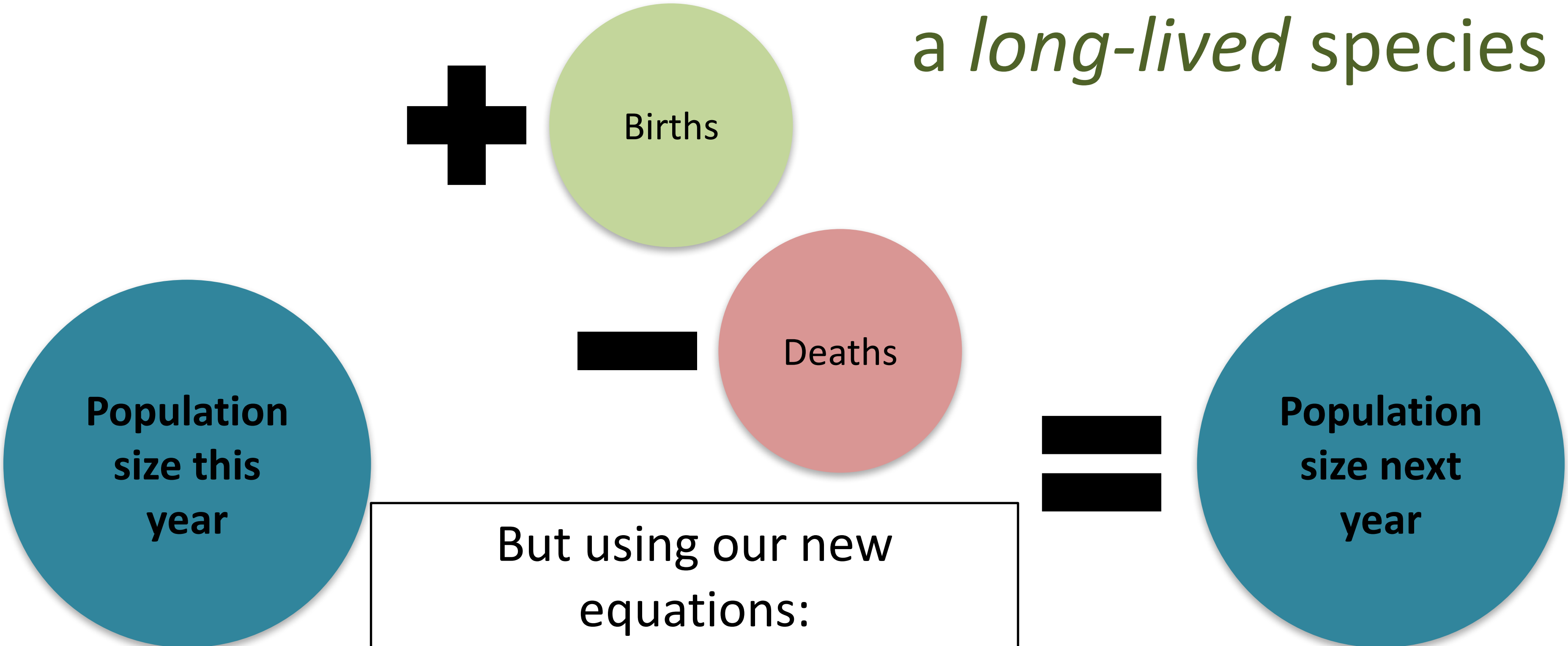
Those that didn't survive.
Then, if ***s*** is probability of surviving:

Death = (1-s)*N(t)

Written as:

$$N(t+1) = N(t) + B - D$$

A little more complicated – a *long-lived* species



But using our new equations:

$$B = N(t) * f$$

and:

$$D = (1-s)*N(t)$$

Written as:

$$N(t+1) = N(t) + N(t)*f - (1-s)*N(t)$$

A slide of algebra

Pop. Next Year = N + B - D

Pop. Next Year = N + f*N – (1-s)*N

Pop. Next Year = N + f*N – N + s*N

Pop. Nex Year = (f+s)*N

Another slide of algebra

Pop. Nex Year = (f+s)*N

↓

Let R = (f+s)

↓

Put in time indexes

↓

N(t+1) = R*N(t)

N(t+1) = R*N(t) is the basis for geometric growth
and we call ***R*** the '***replacement***', or growth rate

Multiple time steps – geometric growth

$$N(t=0) \xrightarrow{R} N(t=1) \xrightarrow{R} N(t=2) \xrightarrow{R} N(t=3) \xrightarrow{R} N(t=4)$$

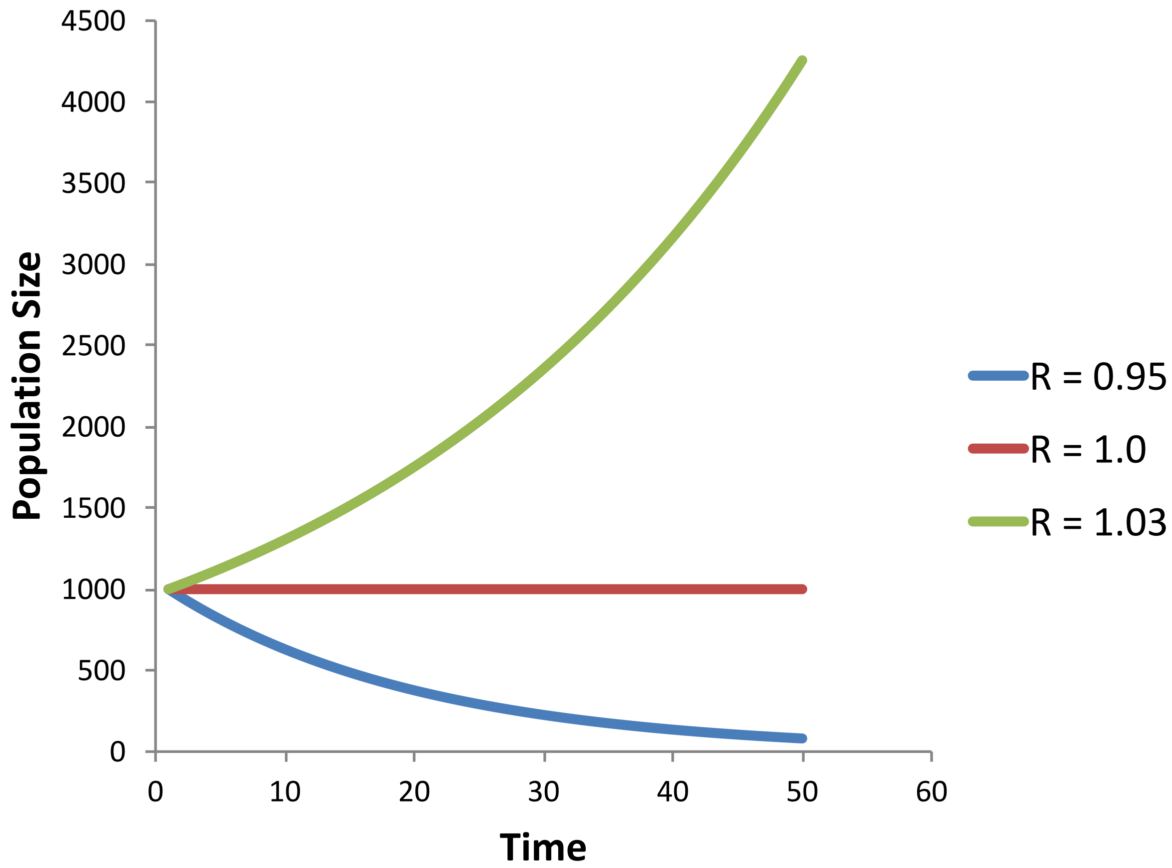
$$N(t=1) = R*N(t=0)$$

$$N(t=2) = R*\{R*N(t=0)\}$$

This recursive relationship yields the following general relationship: $N(t) = R^t N(0)$

Multiple time steps

N(0) = 1000



Plot of geometric growth: $N(t) = R^t N(0)$

Exponential growth

$N(t+1) = N(t) + B - D$

$\Delta N = N(t+1) - N(t) = b*N - d*N$

Let ΔN be *very* small

$dN/dt = (b - d)*N = r*N$

Solve the differential equation

$N(t) = N(0)*e^{rt}$

Why go over this?

Exponential growth is a basic demographic model, and usually a good place to start!

Estimating *R*

Often our data is in annual time steps

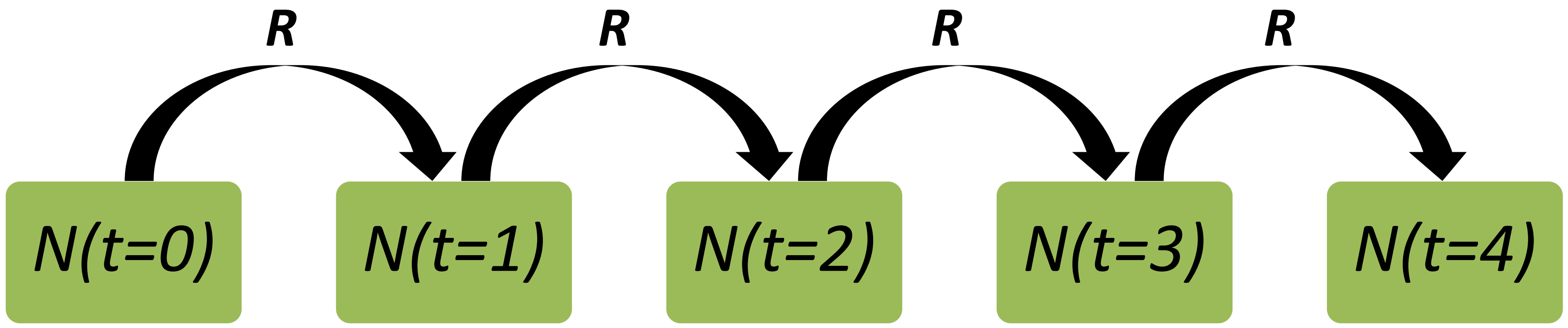
Many plants have a discrete growing season

Many plants have a discrete reproduction period (e.g. fruiting)

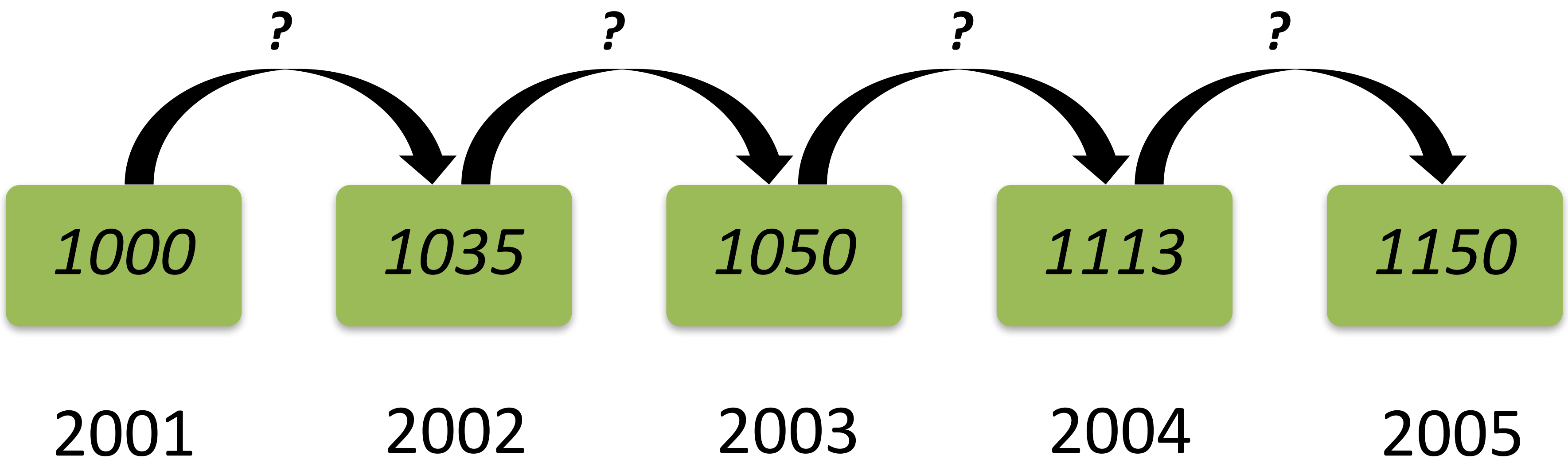
Assume discrete annual growth

Thus, we focus on ***R*** (and not *r*)

Estimating R



Estimating *R*



Five seasons of census data

Estimating *R*

$$N(t+1) = R*N(t)$$

$$R = N(t+1) \,/\, N(t)$$

1035/1000

R = 1.035

R = 1.014

R = 1.060

R = 1.033

1000	*1035*	*1050*	*1113*	*1150*
2001	2002	2003	2004	2005

Estimating *R*

Year	Estimate for R
2001 - 2002	1.035
2002 - 2003	1.014
2003 - 2004	1.060
2004 - 2005	1.033
Geometric Average	1.036
Standard Deviation	0.019

Because discrete-time population growth is a multiplicative process, use *Geometric Mean*

Estimating *R*

N(t+1) = R^t*N(0)

$$R = \sqrt[4]{\frac{1150}{1000}}$$

$$R = 1.036$$

But no St.Dev. Estimate!

What if I don't have consecutive years?

1000	*1035*	*1050*	*1113*	*1150*
2001	2002	2003	2004	2005

Why is standard deviation important?

Individual Survival Probability

Individual Fecundity Rate

Demographic Stochasticity

Standard Deviation

Uncertainty

Measurement Error

Stochasticity

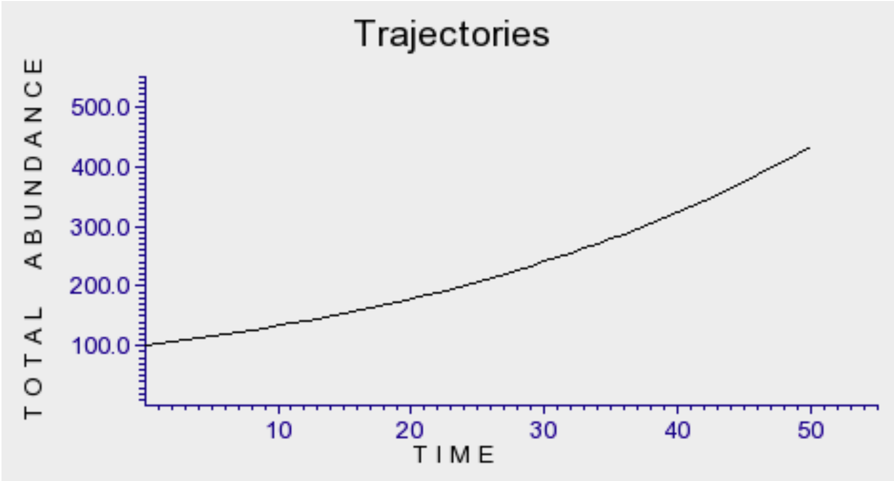
Fluctuations in Temp/Precip

Catastrophes

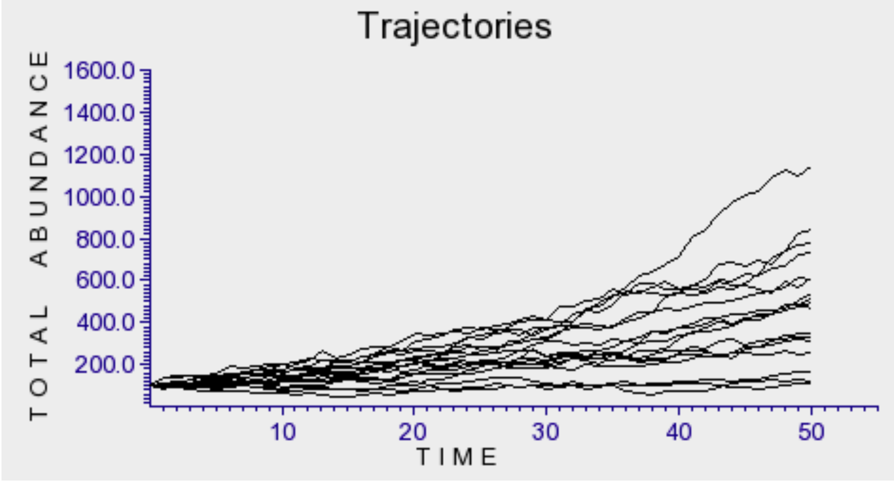
Variability can have great impacts on **population dynamics**

Why is standard deviation important?

Without Demographic Stochasticity
R = 1.03



With Demographic Stochasticity
R = 1.03

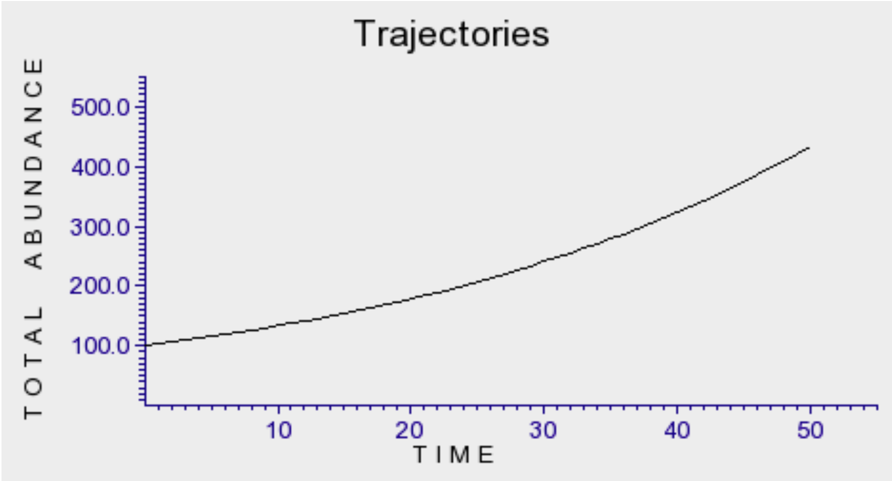


Variability can have great impacts on **population dynamics**

Why is standard deviation important?

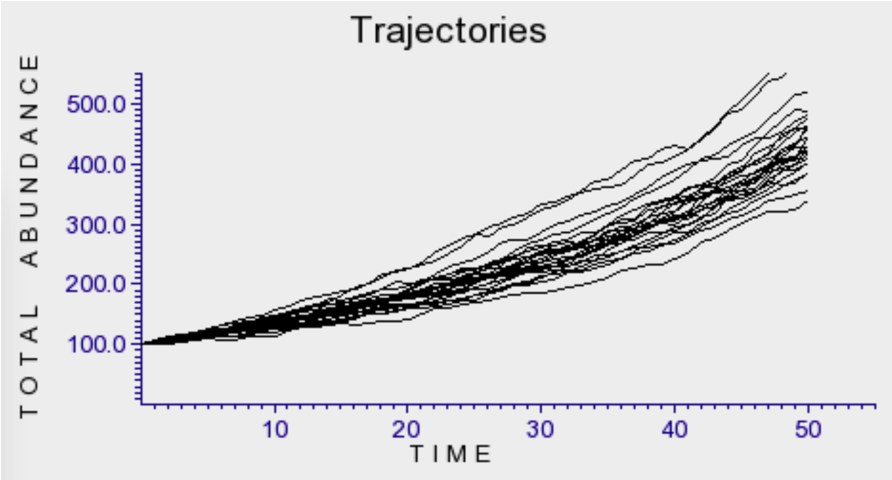
Without Environmental Stochasticity

R = 1.03



With Environmental Stochasticity

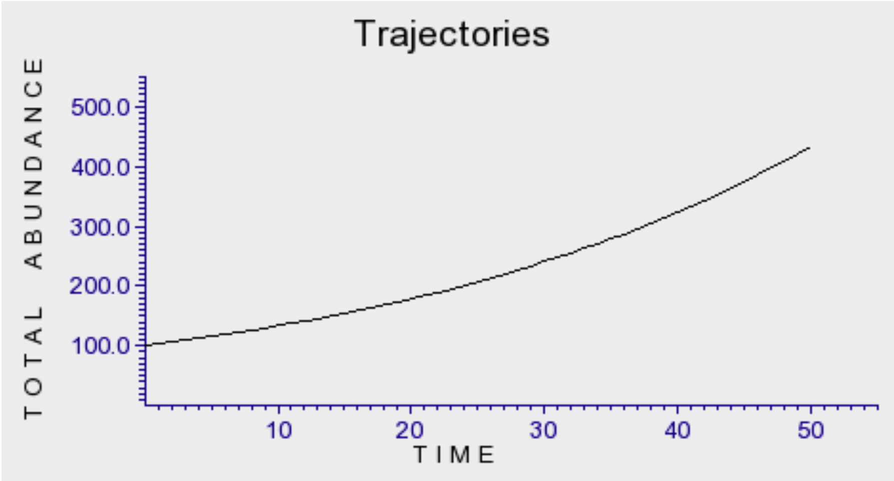
R = 1.03 (SD = 0.019)



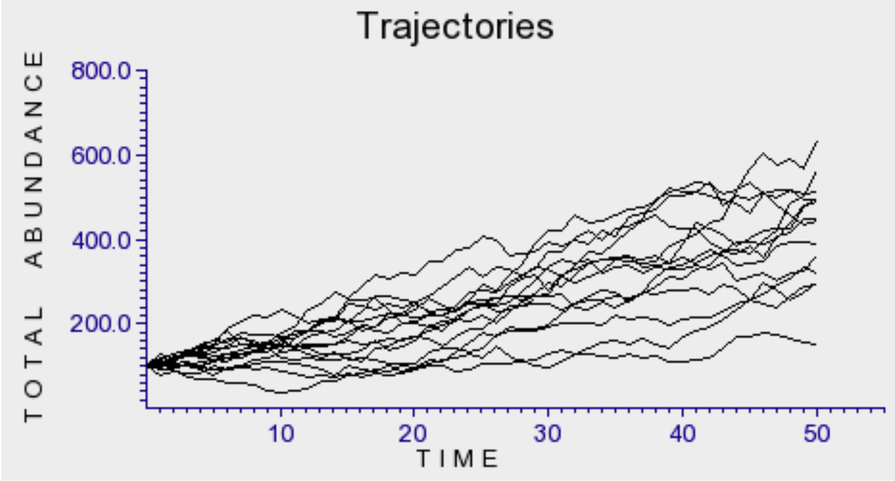
Variability can have great impacts on **population dynamics**

Why is standard deviation important?

Without
Stochasticity
R = 1.03



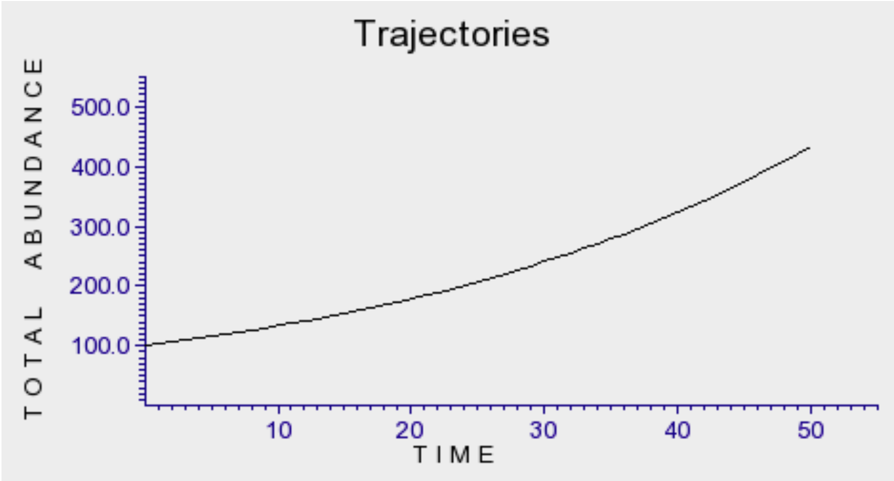
With Both Environmental
and Demographic
Stochasticity
R = 1.03 (SD = 0.019)



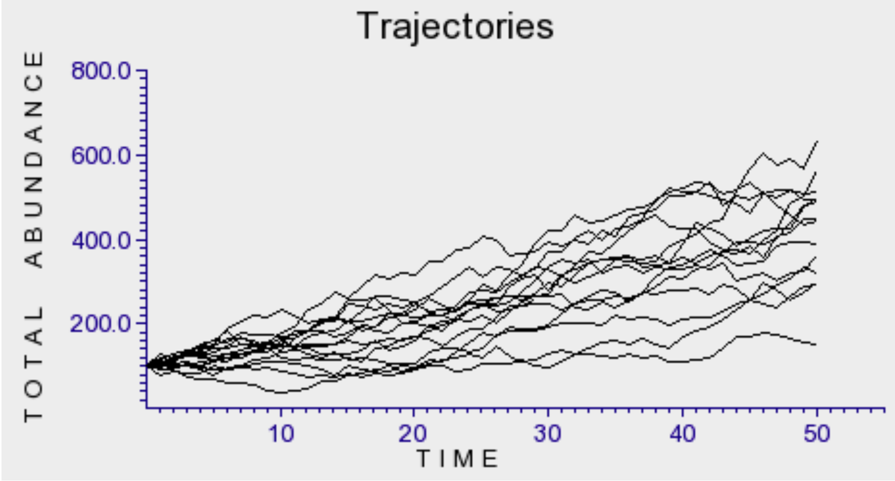
Variability can have great impacts on **population dynamics**

Why is standard deviation important?

Deterministic



Stochastic



Variability can have great impacts on **population dynamics**

Why is standard deviation important?

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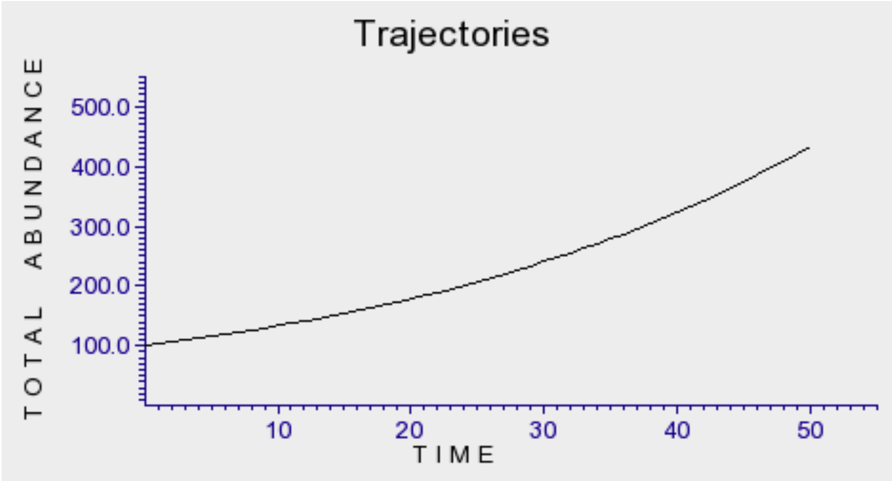

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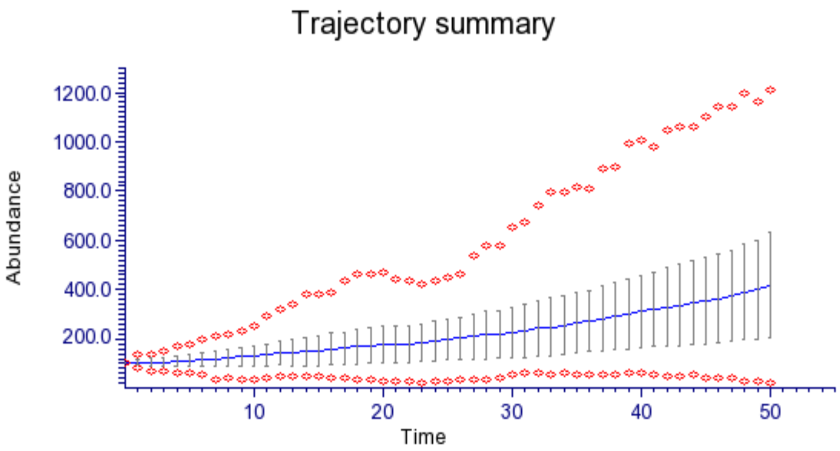
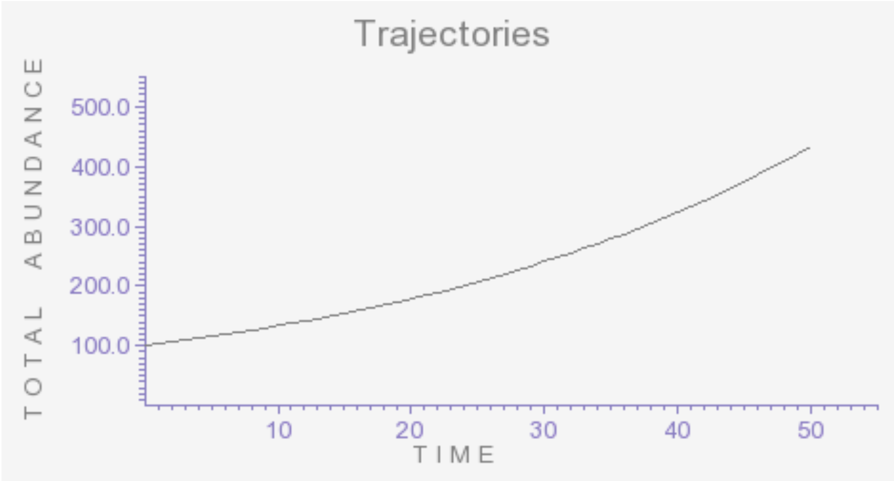


Variability can have great impacts on **population dynamics**

Why is standard deviation important?

Deterministic



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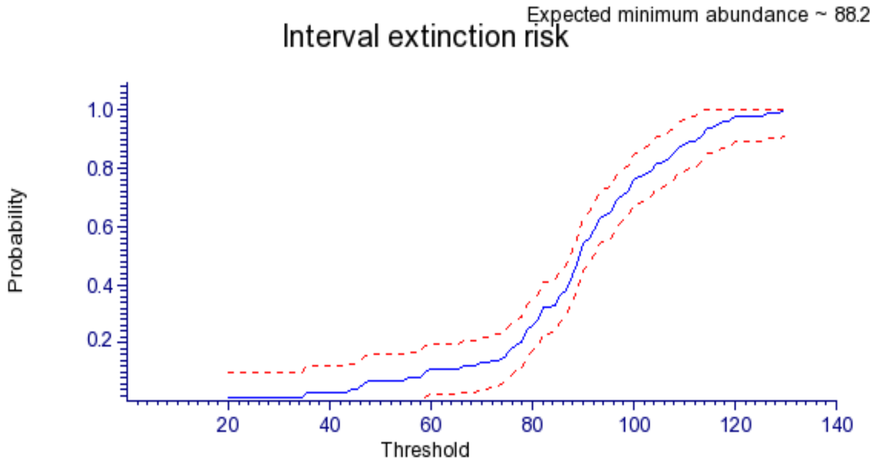
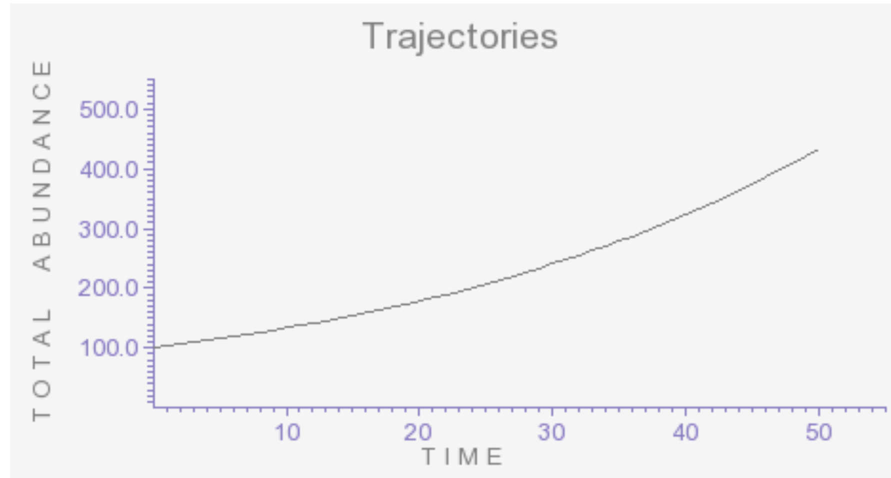


Variability can have great impacts on **population dynamics**

Why is standard deviation important?

Deterministic



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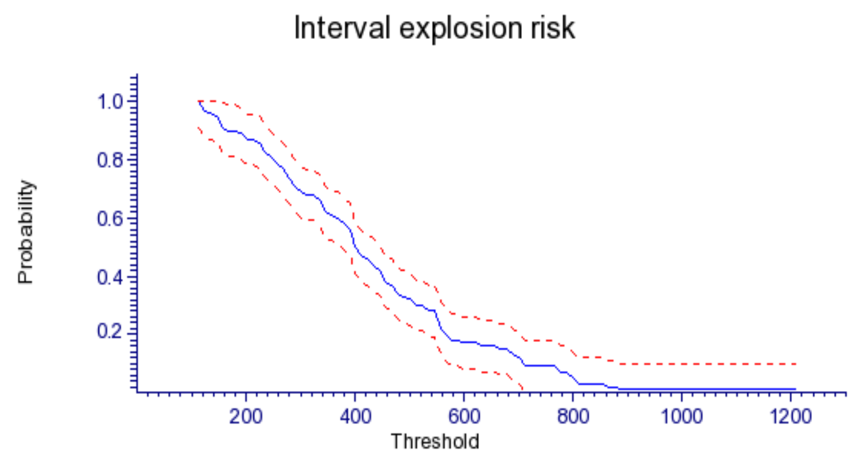


Variability can have great impacts on **population dynamics**

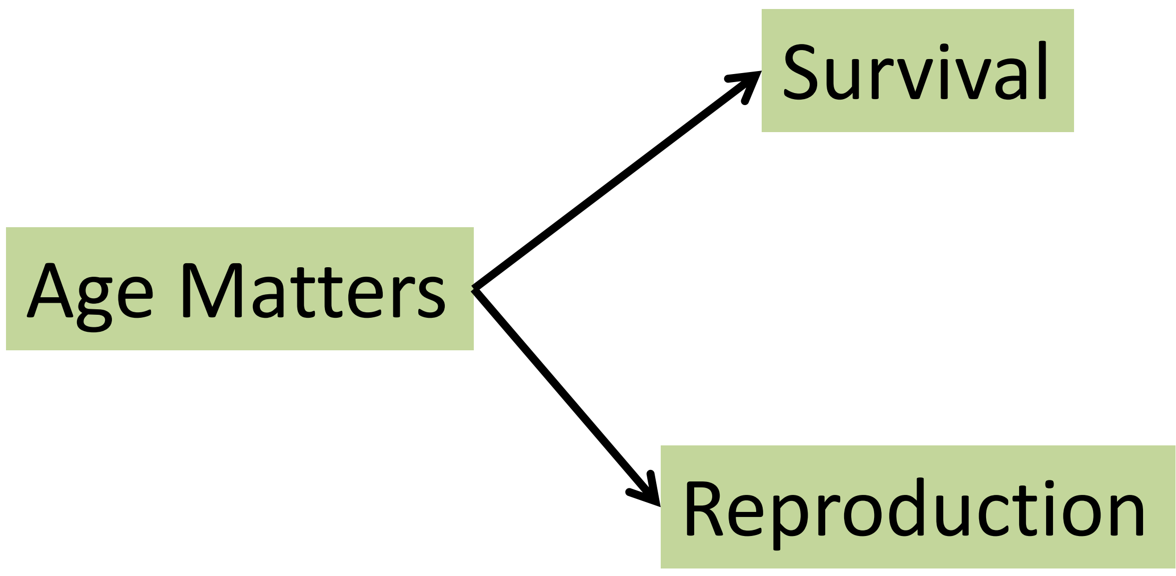
In case we missed that!

Variability can have great impacts on **population dynamics**

This does *not* mean that models are inherently uncertain

Plant ecology complicates model outcomes

Stage (Age) structured models
- adding ecological realism



Incorporating the effects of age can increase ecological **realism** of our models

Stage (Age) structured models
- adding ecological realism

Age Matters

$S_{Seedling}$

$F_{seedling} = 0$

S_{Adult}

$F_{adult} = 22.5$

Incorporating the effects of age can increase ecological **realism** of our models

Stage (Age) structured models
- adding ecological realism

Age Matters

<u>Age Distribution</u>
The number of individuals in in each age category can vary (e.g. seeds vs. seedlings vs. adults)

Incorporating the effects of age can increase ecological **realism** of our models

Stage (Age) structured models
- adding ecological realism

Age → Stage → Size

Incorporating the effects of **stage** can increase ecological **realism** of our models

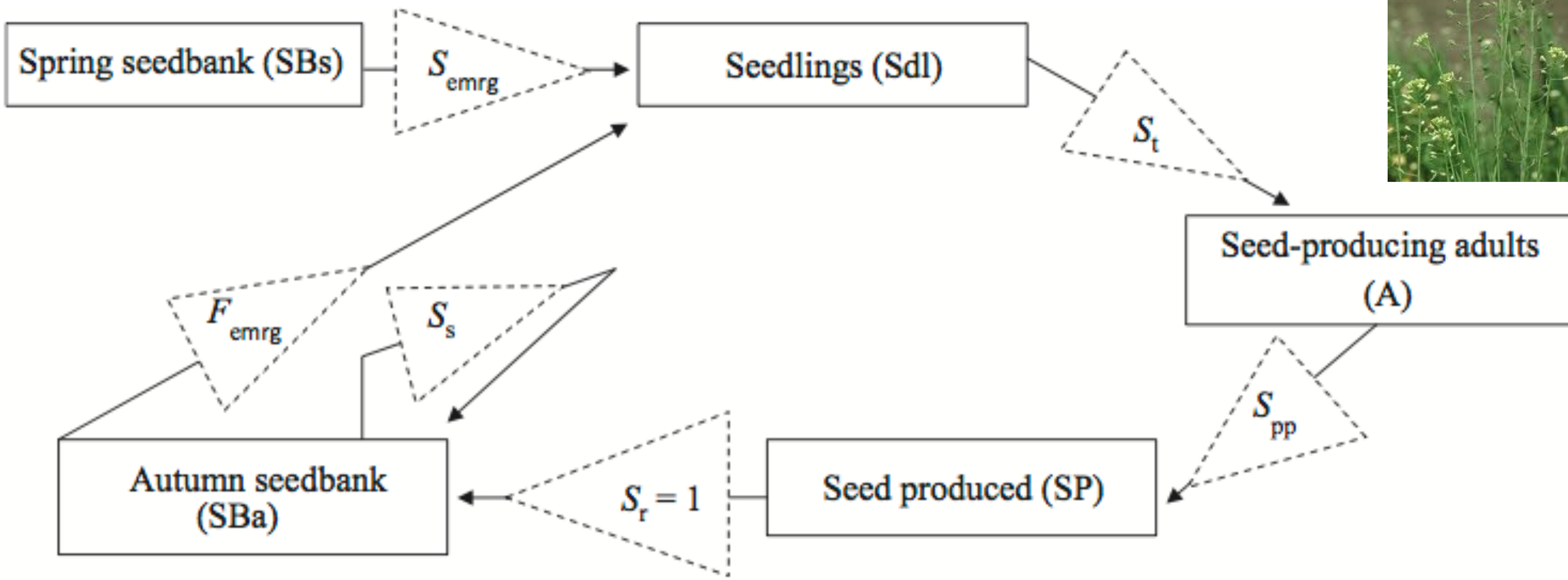
Stage structured models
- adding ecological realism

OK, what does one of these models look like?

Incorporating the effects of **stage** can increase ecological **realism** of our models

Stage structured models
- adding ecological realism

Stage structured model for *Camelina sativa*



Davis et al. 2011 J. Appl. Ecol.

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Stage structured models
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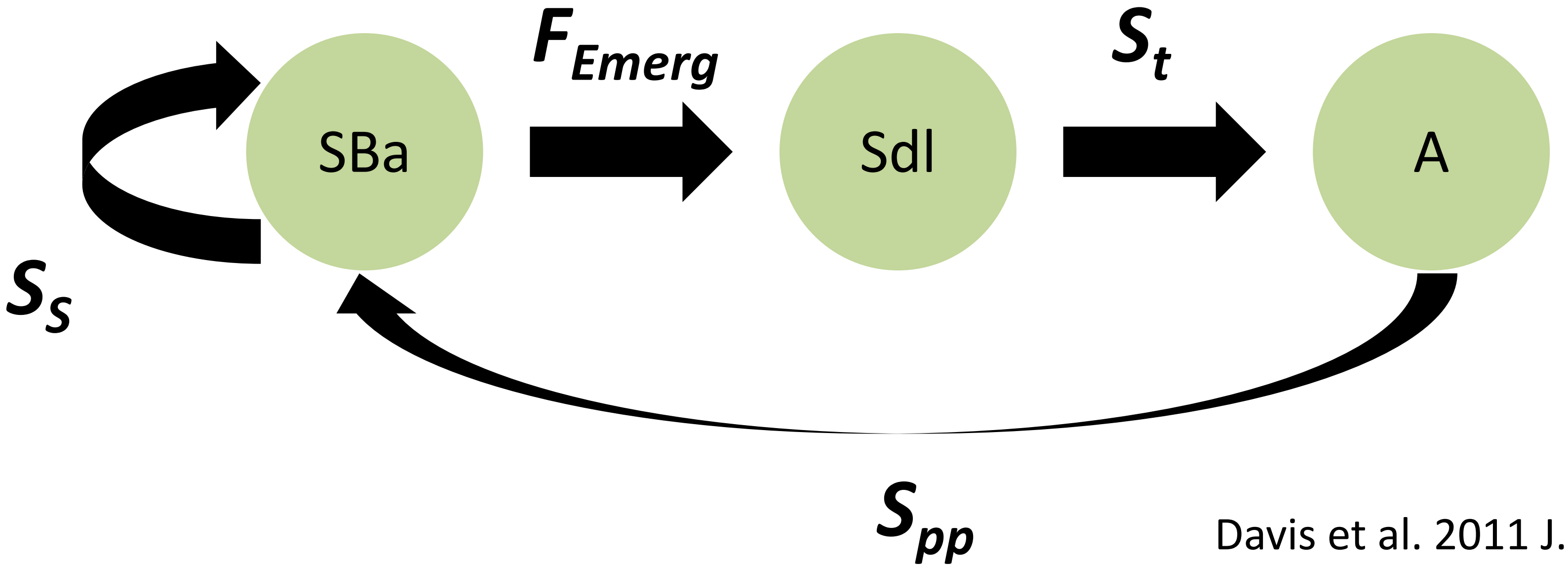


Davis et al. 2011 J. Appl. Ecol.

Incorporating the effects of **stage** can increase ecological **realism** of our models

Stage structured models
- adding ecological realism

Stage structured model for *Camelina sativa*
Parameter Estimation

Parameter	Meaning	Data	Estimate
F_{Emerg}	Seed emergence from Fall (Autumn) seedbank	Counts of emergence from marked plots (Fig 2,3)	0.024 ± 0.02

Incorporating the effects of **stage** can increase ecological **realism** of our models

Stage structured models
- adding ecological realism

Stage structured model for *Camelina sativa*
Calculations and Simulation

$$\text{Total Population Size} = \begin{bmatrix} \text{Seed Bank (SBa)} \\ \text{Seedlings (Sdl)} \\ \text{Adults (A)} \end{bmatrix}$$

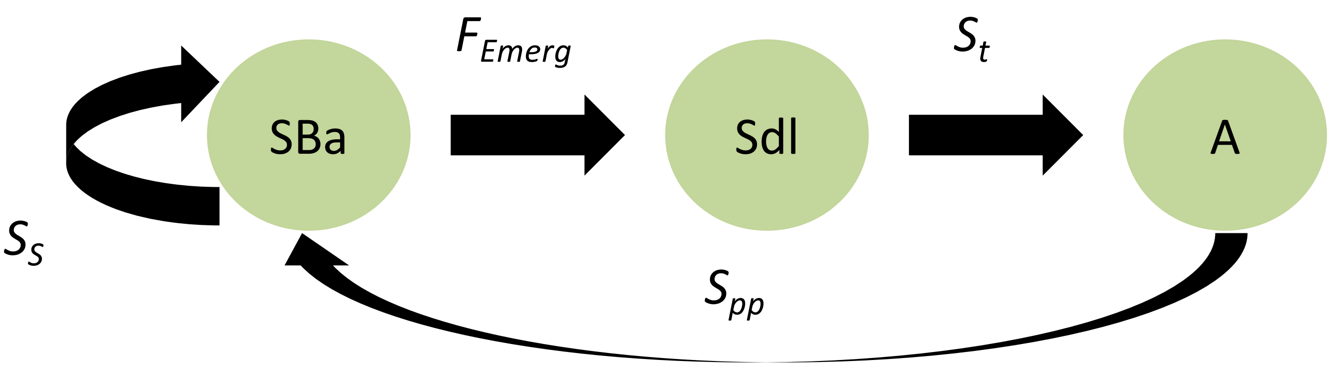
Incorporating the effects of **stage** can increase ecological **realism** of our models

Stage structured models
- adding ecological realism

Stage structured model for *Camelina sativa*

Calculations and Simulation

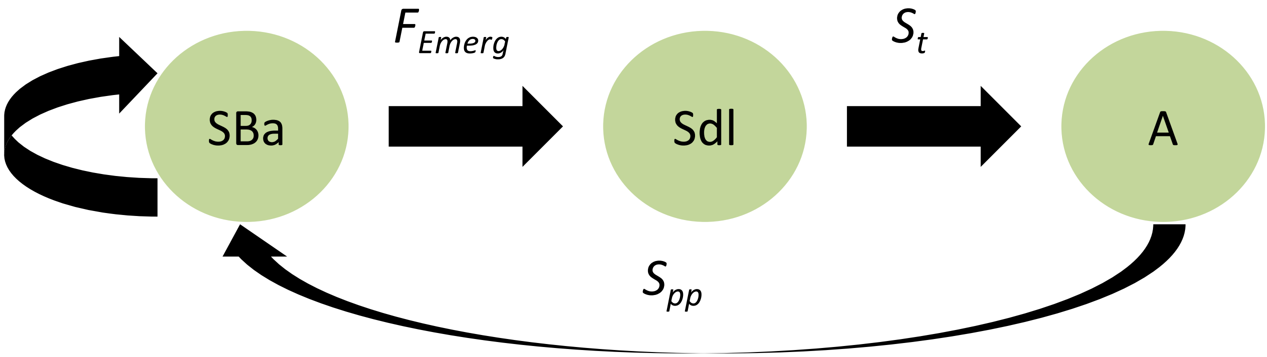
$$\begin{bmatrix} SBa(t+1) \\ Sdl(t+1) \\ A(t+1) \end{bmatrix} = \begin{bmatrix} \\ SBa(t) * F_{Emerg} \\ \\ \end{bmatrix}$$



Stage structured models
- adding ecological realism

Stage structured model for *Camelina sativa*
Calculations and Simulation

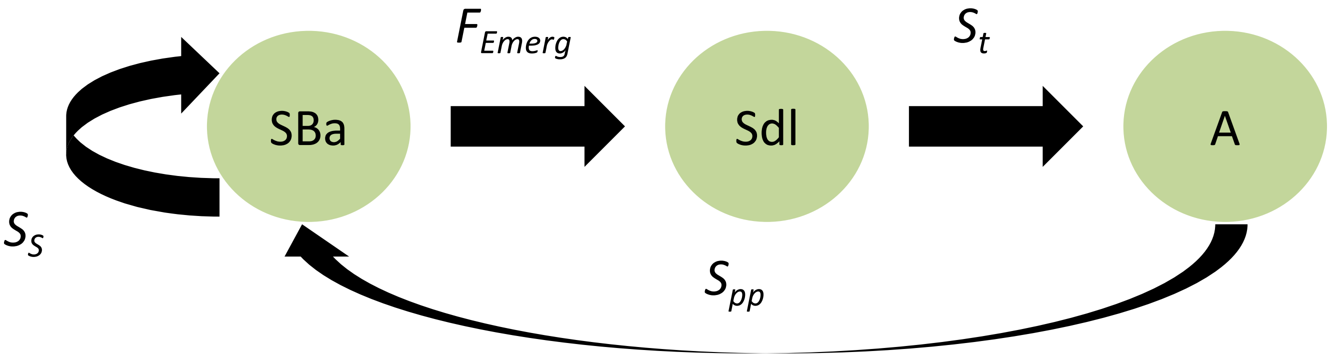
$$\begin{bmatrix} SBa(t+1) \\ Sdl(t+1) \\ A(t+1) \end{bmatrix} = \begin{bmatrix} \\ SBa(t) * F_{Emerg} \\ Sdl(t) * S_t \end{bmatrix}$$



Stage structured models
- adding ecological realism

Stage structured model for *Camelina sativa*
Calculations and Simulation

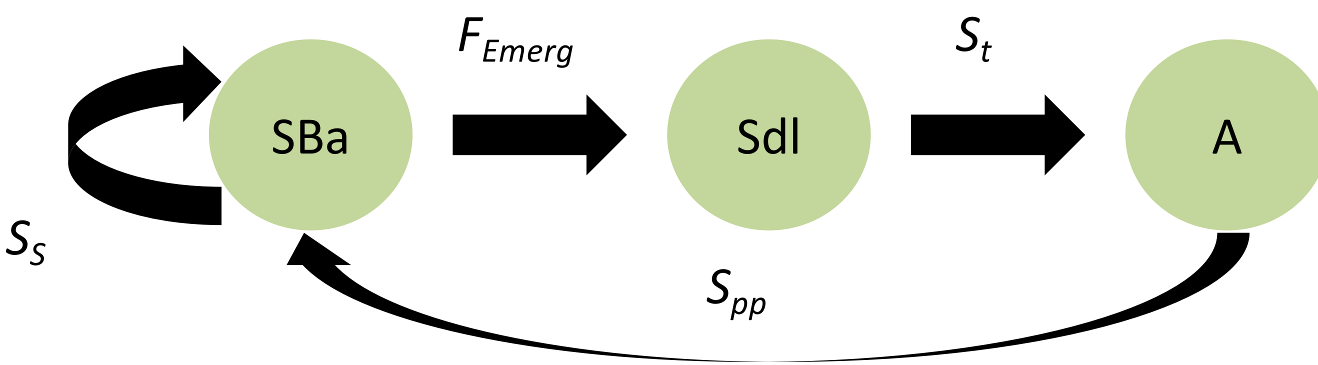
$$\begin{bmatrix} SBa(t+1) \\ Sdl(t+1) \\ A(t+1) \end{bmatrix} = \begin{bmatrix} A(t) * S_{pp} \\ SBa(t) * F_{Emerg} \\ Sdl(t) * S_t \end{bmatrix}$$



Stage structured models
- adding ecological realism

Stage structured model for *Camelina sativa*
Calculations and Simulation

$$\begin{bmatrix} SBa(t+1) \\ Sdl(t+1) \\ A(t+1) \end{bmatrix} = \begin{bmatrix} A(t) * S_{pp} + SBa(t) * (1-F_{emerg})S_S \\ SBa(t) * F_{Emerg} \\ Sdl(t) * S_t \end{bmatrix}$$



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Stage structured model for *Camelina sativa*
Calculations and Simulation

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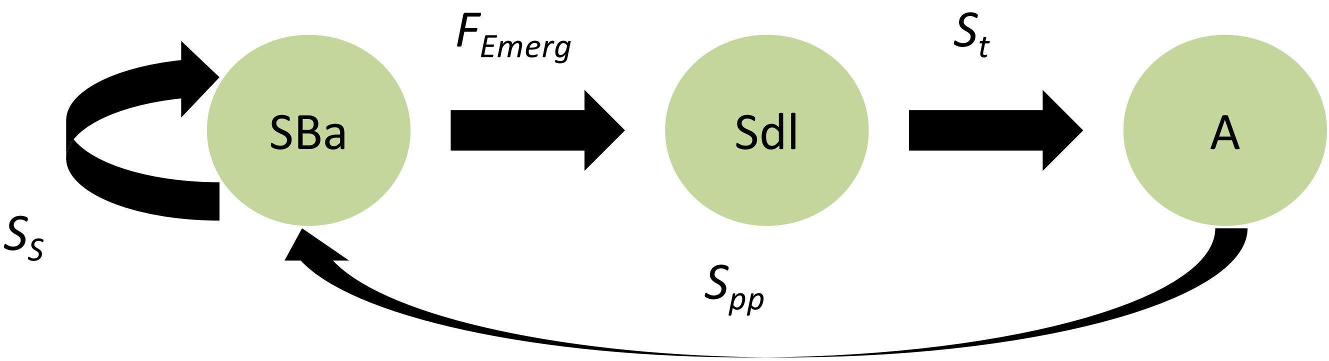

Stage structured models
- adding ecological realism

Stage structured model for *Camelina sativa*
Calculations and Simulation

$$\begin{bmatrix} SBa(t+1) \\ Sdl(t+1) \\ A(t+1) \end{bmatrix} = \boxed{\begin{bmatrix} S_s & 0 & S_{pp} \\ F_{emerg} & 0 & 0 \\ 0 & S_t & 0 \end{bmatrix}} \begin{bmatrix} SBa(t) \\ Sdl(t) \\ A(t) \end{bmatrix}$$

Stage Matrix

This type of demographic model is often called a ***matrix model***, or a ***matrix projection model***

Stage structured models
- adding ecological realism

Current year (t)

$$
\begin{array}{c} \text{Next year (t+1)} \\ SBa \\ Sdl \\ A \end{array}
\begin{array}{c} \begin{array}{ccc} \text{SBa} & \text{Sdl} & \text{A} \end{array} \\ \left[\begin{array}{ccc} S_s & 0 & S_{pp} \\ F_{emerg} & 0 & 0 \\ 0 & S_t & 0 \end{array}\right] \end{array}
$$

This type of demographic model is often called a ***matrix model***, or a ***matrix projection model***

Stage structured models
- adding ecological realism

Current year (t)

$$
\begin{array}{c|c}
 & \begin{array}{ccc} SBa & Sdl & A \end{array} \\
\text{Next year (t+1)} & \\
\begin{array}{c} SBa \\ Sdl \\ A \end{array} & \left[\begin{array}{ccc} \boxed{\begin{array}{c} S_s \\ F_{emerg} \\ 0 \end{array}} & \begin{array}{c} 0 \\ 0 \\ S_t \end{array} & \begin{array}{c} S_{pp} \\ 0 \\ 0 \end{array} \end{array}\right]
\end{array}
$$

This type of demographic model is often called a ***matrix model***, or a ***matrix projection model***