Quantum physics for beginners

1981 Phys. Educ. 16 88

(http://iopscience.iop.org/0031-9120/16/2/306)

Phys. Educ., Vol 16, 1981. Printed in Great Britain.

Quantum physics for beginners

J Strnad

Inherent difficulties occur in teaching quantum mechanics in secondary school, including students' unsatisfactory background knowledge of mathematics and classical physics, and a limited amount of time. So it is not surprising that less has been done to improve teaching at this level than at introductory university level (e.g. Feynman *et al* 1965, Wichmann 1971, Gillespie 1973, Rüdiger 1976, Santos 1976). Nevertheless, it seems that there is room for improvement. In this article the traditional approach based on the concept of matter waves and on the naive photon concept is scrutinised and a new approach is proposed.

The photoelectric effect is traditionally considered first, with varying degrees of sophistication and elaboration (e.g. PSSC 1965, Nuffield 1970, 1972). Experiments show the failure of classical electrodynamics and it is argued that they imply:

$$\text{electromagnetic wages} \rightarrow \text{particles (photons)}.$$

A photon has the energy ($W = h\nu$) and momentum ($p = W/c = h\nu/c = h/\lambda$) of a particle with zero rest mass moving with the velocity of light *in vacuo* c. The Planck constant h is typical for quantum mechanics, ν being the frequency and $\lambda = c/\nu$ the wavelength.

Symmetry considerations lead to the de Broglie hypothesis:

$$\text{matter waves} \leftarrow \textit{particles} \text{ (e.g. electrons).}$$

Matter waves are associated with electrons and the same relation as for photons is applied to the wavelength, but the momentum is $p = mv$, m being the mass and v the particle velocity. The hypothesis is verified by interference experiments with electrons. Later the wave–particle duality of electrons is discussed. Electrons are described either as particles or as waves, both concepts being familiar from classical mechanics. Further arguments based on standing matter waves lead to discrete energy levels and to the time independent Schrödinger equation.

Both parts of the symmetry

$$\begin{array}{cc} \text{electromagnetic waves} & \text{photons} \\ \updownarrow & \updownarrow \\ \text{matter waves} & \text{electrons} \end{array}$$

are, however, open to serious objections. The state function (or the quantum mechanical wavefunction) $\Psi(x,t)$ describing an electron is not a classical wavefunction $E(x,t)$ describing electromagnetic waves (E being the electric field). A form of the equation of motion for electrons, the Schrödinger equation

$$-(\hbar^2/2m)\nabla^2\Psi + V(x)\Psi = \mathrm{i}\hbar\partial\Psi/\partial t$$

($\hbar = h/2\pi$ and $V(x)$ is the potential energy), differs from the classical wave equation

$$\nabla^2 E = c^{-2}\partial^2 E/\partial t^2.$$

The first contains the imaginary unit i and the first time derivative, while the second is real and contains the second time derivative. The state function is complex, whereas the classical wavefunction is real. It is true that classical wavefunctions can be written in complex form to shorten calculations, but only their real (or imaginary) part is significant. The real function $E(x,t)$ is in principle accessible to measurement. The time dependence of it at a fixed point can be observed on an oscilloscope if the frequency, in the microwave region, is not too high. (The frequency can be directly measured far into the infrared region.) On the contrary, neither the real nor the imaginery part of a state function is directly accessible to measurement, only the norm

$$\Psi^*\Psi = (\mathrm{Re}\Psi)^2 + (\mathrm{Im}\Psi)^2$$

being observable in principle.

At this point it is instructive to mention Landé's paradox. If the de Broglie wavelength $\lambda = h/p$ were a true spatial period in a nonrelativistic wave, i.e. a length, it should be invariant under a Galilean transformation: $\lambda' = \lambda$ and thus $p' = p$. At the same time the momentum of an electron is transformed as $p' = p - mv_0$ if the second inertial reference frame is moving with velocity v_0 with respect to the first in the direction of the momentum $p = mv$ of the electron. This contradiction is due to the unjustified demand

Janez Strnad *is professor of physics at the University of Ljubljana where he is the editor of a local journal for mathematics and physics teachers. He has been a member of the theoretical group at the J Stefan Institute since 1958. His research interests include nuclear physics, relativity and physics education and he is the author of a number of books and articles in these fields. During 1961 an IAEA fellowship enabled him to attend the Institute of Theoretical Physics, University of Heidelberg and he held a DAAD fellowship at the Institute of Physics Education, University of Giessen, in 1978.*

0031-9120/81/020088+05 $01.50

that under a Galilean transformation the phase of a state function is invariant. Indeed, the phase of a nonrelativistic wavefunction is invariant. But the state function under a Galilean transformation acquires an additional phase factor and only its norm is invariant. By demanding form invariance of the Schrödinger equation of a free particle, it can be shown that the appropriate transformation is $h/\lambda' = h/\lambda - mv_0$ (Levy-Leblond 1976).

Not only students but also teachers and textbook writers sometimes have difficulty in distinguishing a state function from a classical wavefunction. As an example let us quote the textbooks of Halliday and Resnick. In the 1966 edition of parts I and II it is asserted on p1207 that the matter wave is described, in strict analogy with a classical wavefunction

$$y = (2y_m \sin(n\pi x/l))\cos\omega t, \quad n = 1,2,3\ldots$$

by

$$\Psi = (\psi_m \sin(n\pi x/l))\cos\omega t, \quad n = 1,2,3,\ldots$$

In the 1978 edition of part 2 on p1124 the hydrogen ground state is described by

$$\Psi = (\pi a^3)^{-1/2} e^{-r/a} \cos\omega t.$$

It is obvious that both functions are not solutions of the Schrödinger equation.

One could add some further distinctions between a state function and a classical wavefunction (Rüdiger 1976). But let us, instead, note a real analogy. The time independent Schrödinger equation and the electromagnetic amplitude equation have the same form:

$$\nabla^2\psi + [2m(W' - V(x))/\hbar^2]\psi = 0$$
$$\nabla^2\zeta + (\omega/c)^2\zeta = 0.$$

Thereby $\Psi = \psi\exp(-iW't/h)$ and $E = \zeta\cos\omega t$. A position dependent potential $V(x)$ corresponds to an inhomogeneous medium with position dependent refractive index $n(x)$, since

$$\omega/c = (2\pi/\lambda_0)n(x)$$

where λ_0 is the wavelength *in vacuo*. For bound states, i.e. for boundary conditions of the type

$$\psi(x \to \pm\infty) = \zeta(x \to \pm\infty) = 0,$$

(real) solutions of both equations may have the same form.

A photon moving with the velocity of light does not belong to the realm of nonrelativistic quantum mechanics, which is based on classical mechanics. In nonrelativistic quantum mechanics, Heisenberg's indeterminacy relation $\delta x \delta p \gtrsim h$ is valid, δp being the indeterminacy of a momentum component and δx the indeterminacy of the corresponding coordinate at a simultaneous measurement of both variables. So either the position of a particle ($\delta x = 0, \delta p \to \infty$) or its momentum ($\delta p = 0, \delta x \to \infty$) can be determined exactly. In a relativistic theory the situation is completely different. The indeterminacy of position is (Landau and Peierls 1931)

$$\delta x \gtrsim hc/W$$

irrespective of the indeterminacy of momentum, W being the total energy. This gives for a photon with $W = h\nu$,

$$\delta x \gtrsim h/p = \lambda.$$

So speaking of the position of a photon has no meaning outside the domain of geometrical optics. On the other hand, the indeterminacy of the momentum is

$$\delta p \gtrsim h/c\tau,$$

irrespective of the indeterminacy of position. So the momentum can be exactly determined only if the measuring time τ is unlimited, i.e. if the relativistic particle is free.

Finally, there is no combination of the electric and magnetic field that would transform as probability density should (Peierls 1979)†. Thus a photon cannot be described in a similar way to an electron. A photon cannot be considered a point-like particle‡. Also, the fuzzy ball picture of the photon can lead to difficulties (Scully and Sargent 1972, Sargent *et al* 1974). A straightforward introduction of the photon is accomplished only through field quantisation in quantum electrodynamics. In this case the correspondence principle does not lead to a classical limit. The expectation value of the field in a state with a given number of photons vanishes even in the limit of very large photon number. A classical limit for this expectation value is obtained only for a superposition of photon states with a Poisson distribution, known as the coherent state.

Historical background

The historical background of the traditional approach seems to warrant motivation for each new idea introduced. But a closer look reveals its quasi-historical character (Whitaker 1979). Evolution was by no means as simple and straightforward as the approach implies. Quanta were reluctantly introduced by Planck in 1901 as bundles of energy exchanged by oscillators in the walls of a cavity filled with electromagnetic radiation. At first he considered emission and absorption, but later excluded absorption since he could not reconcile a spreading wavefront with the idea of a discrete process. Einstein

† The electromagnetic radiation field is described by a second rank tensor. It cannot be formed into a quadratic expression to transform as a time component of a four-vector (Landau and Peierls 1930).

‡ It is instructive to study how far the analogy can be taken. A Schrödinger equation for photons with mass $h\nu/c^2$ can be formulated for eigenfunctions of energy, i.e. for free photons with completely indeterminate position. This device is not very useful since only elastic scattering can be described (Young 1976). See also Joyce and Joyce (1976).

(1905) introduced quanta of the electromagnetic field proposing (ter Haar 1967) that 'energy . . . consists of a finite number of energy quanta, localised [in the original: at points] in space'. Planck's and Einstein's ideas were not taken seriously by the majority of physicists even after Millikan's experimental verification of the photoelectric equation in 1916. The theory and measurement of the Compton effect not earlier than 1922 made the ideas more acceptable. The word photon was coined by Lewis in 1926.

Landau and Peierls were among the first to envisage in 1931 that the photon cannot be treated on the same footing as the electron. Gradually it became evident that the naive photon concept is not necessary. Absorption, stimulated emission, the photoelectric effect, the Compton effect and bremsstrahlung could be treated within the semi-classical approximation in which particles are described quantum mechanically whereas radiation is accounted for classically. Field quantisation is necessary to account for vacuum fluctuations and, in an advanced treatment, spontaneous emission, the anomalous magnetic moment of the electron and the Lamb shift.

A detailed analysis of de Broglie's original conjecture in 1924 has shown that he relied on an unjustified identification of two quite different relations (MacKinnon 1976). It is not at all evident that the relationship between the velocity of a particle and the relativistic phase wave associated with it can be identified with the relationship between the group velocity of a wave packet and the velocity of individual waves. The conjecture was applicable to strictly relativistic situations, whereas the prediction first tested by the Davisson–Germer experiment was the nonrelativistic relation $\lambda = h/mv$. MacKinnon gives a tentative explanation as to why the formula proposed by de Broglie was successful in spite of the nontransparent derivation.

It is true that the (real) results for the interference of 'matter waves' are the same as for light, but the analogy cannot be followed further. The idea of a classical oscillating quantity at a point is not easy to reconcile with probability. Wave–particle duality takes the student completely aback, since he is not used to answers of the either–or type. Bearing all this in mind one wonders why 'matter waves' and the naive photon concept have remained the cornerstones of quantum mechanics teaching at secondary school level.

In teaching quantum mechanics at the introductory university level both concepts are not so critical. The limitations of the analogy between interference experiments with electrons and with light can be shown with a few arguments, as presented here. The relation $\delta x \gtrsim \lambda$ can be discussed to reveal the limitations of the photon concept. Even field quantisation may be hinted at by means of equations for the harmonic oscillator. As it is understood that only the real parts of the results for interference of electrons and light have the same form and that classical mechanics is a limiting case of quantum mechanics, a discussion of wave–particle duality does not lead to confusion.

Quantum physics project

From the standpoint described a project entitled Quantum Physics for Beginners has been initiated to search for an alternative approach. At this level one cannot teach a simple but meaningful version of non-relativistic quantum mechanics in the q representation of the Schrödinger picture (e.g. Gillespie 1973). Electromagnetic radiation must be included. All measurements of the Planck constant were carried out using electromagnetic waves, from the first with black body radiation to recent measurements using the Josephson effect. So it is reasonable to combine nonrelativistic quantum mechanics and classical electrodynamics in the framework of the semiclassical approximation.

It may be worthwhile to describe briefly some characteristic steps of the proposed alternative approach (table 1).

Quantum physics

(I.1) *(Corresponding to entries in table 1.)* Atoms of helium gas in a container behave quite differently from steel balls in a box shaken for a short time.

(I.2) Inelastic collisions with electrons in the Teltron tube TEL 535 show that helium atoms can accept discrete amounts of energy. The Franck–Hertz experiment shows the same for mercury atoms. Atoms have states with discrete energies W_a', W_b', ... In their energy spectra there are discrete states, at variance with macroscopic systems. Atoms, electrons, etc, are quantum particles for which classical mechanics does not apply. Classical particles (macroparticles), for which the old classical mechanics is valid, are consequently distinguished from quantum particles (microparticles), for which a new quantum mechanics is valid. (Thus, electrons are neither classical particles nor waves but quantum particles.)

(I.3) Line spectra of gases indicate that transitions occur between states with discrete energy through emission of electromagnetic radiation. The energy of an emitted wave packet equals the energy difference of the atom: $W_{rad} = W_b' = W_a', \ldots$

(I.4) The empirically established Ritz combination principle gives evidence that the energy W_{rad} is proportional to the frequency. The original form of the Franck–Hertz experiment (ter Haar 1967) enables us to estimate the Planck constant. The equation $W_{rad} = h\nu$ is a supplementary condition to classical electro-dynamics.

Table 1 *Quantum physics for beginners*

Part I Quantum physics
1 Failure of classical mechanics
2 Energy of the helium atom and other atoms
3 Line spectra
4 The Planck constant
5 Spontaneous emission, absorption and stimulated emission
6 States of the hydrogen atom
7 Energy levels of electrons in atoms
8 The periodic table
9 Energy bands in crystals
10 Isolators, semiconductors and conductors
11 Extrinsic conduction in semiconductors
12 Semiconductor diodes and transistors
13 Lasers
14 The photoelectric effect
15 Bremsstrahlung and x-rays
16 The Compton effect
17 Photons
18 The indeterminacy relation
Part II Quantum mechanics
1 Quantum particles, description of motion
2 State functions and probability density
3 Interference experiments with quantum particles
4 'Matter waves'
5 Perturbation at measurement
6 Stationary equation of motion (time independent Schrödinger equation)
7 The hydrogen atom
8 The equation of motion (time dependent Schrödinger equation)
Part III Historical survey

(I.6) Data from electron collisions and spectroscopic data for hydrogen are reproduced by the equation $W_n' = W_1'/n^2$. This gives rise to the Balmer formula. The continuum part of the energy spectrum is discussed as well.

(I.7) Energy levels of electrons in many-electron atoms are considered on the basis of spectroscopic data for alkaline vapours.

(I.8) The exclusion principle is introduced through consideration of the periodic table.

Items I.9–I.13 are discussed on a qualitative basis.

(I.14) The photoelectric effect is described as the absorption of a wave packet giving rise to the transition of an electron from the conduction band to a level in the continuum.

(I.15) Bremsstrahlung corresponds to the transition of an electron from a level in the continuum to a lower level in the continuum in the electric field of a nucleus. The energy difference is carried off by the wave packet. Bremsstrahlung gives rise to the continuous x-ray spectrum. X-ray lines result, however, from electron transitions from an upper discrete level to a hole in a lower discrete level.

(I.16) The Compton effect is described as the scattering of an electromagnetic wave packet by approximately free electrons at rest. Energy and momentum are thereby conserved.

Electromagnetic waves that deliver an energy W to a perpendicular absorbing surface carry momentum W/c. The magnetic field of a plane wave exerts a force on the electric current driven in the surface by the electric field of the same wave.

(I.17) The term photon is introduced for a wave packet with energy $h\nu$ and momentum $h\nu/c$.

(I.18) The indeterminacy relation is derived using a Heisenberg x-ray microscope.

Quantum mechanics

(II.1) An experiment with electrons moving along the axis of a long vacuum tube is considered. The momentum of an electron is almost exactly determined, whereas its position is almost completely indeterminate. Electrons are detected in the vicinity of a given point with a small detector. All points along the axis of the tube have to be equivalent. This can be accomplished only by measuring in the vicinity of many points and treating the results statistically. In this way a probability density, which in this particular case does not depend on position, is introduced.

(II.2) An electron in the tube is described by a state function which is a continuous function of the coordinate and contains the momentum component as a constant parameter. To reconcile this with the constant probability density one starts with the sum of two functions always being constant:

$$A^2\cos^2\varphi + A^2\sin^2\varphi = A(\cos\varphi + \mathrm{i}\sin\varphi) \times A(\cos\varphi - \mathrm{i}\sin\varphi) = Ae^{\mathrm{i}\varphi}Ae^{-\mathrm{i}\varphi}.$$

The *ansatz* is made for the state function $\psi = Ae^{\mathrm{i}\varphi}$ and the norm $\psi^*\psi = A^2$ is interpreted as the probability density. The phase φ with dimensionality 1 should contain the coordinate x and the momentum p linearly and symmetrically. According to the indeterminacy relation an appropriate guess is $\varphi = 2\pi(xp/h) = xp/\hbar$.

(II.3) With the state function of the form $\psi = A\exp(\mathrm{i}xp/\hbar)$ the results of electron diffraction on a double slit (Jönsson 1974) and on carbon atoms in graphite in the Teltron tube TEL 555 are explained.

(II.4) Probability density corresponds formally to energy density in light diffraction. So the concept 'matter waves' used in the old quantum theory can be understood.

(II.5) A thought experiment in which additional detectors are placed at the slits in a double slit experiment with electrons, to find out through which slit a particular electron is moving, shows that in quantum mechanics the system is essentially perturbed by the measuring process.

(II.6) The time independent one-dimensional Schrödinger equation is given. The connection of it with the energy conservation of classical mechanics is discussed. Operators for position, momentum, kinetic and potential energy are introduced. The Schrödinger equation is recognised as the eigenvalue problem of the total energy operator.

(II.7) The time independent Schrödinger equation is generalised to three dimensions for the spherical symmetrical case with a Coulomb potential energy. The first three spherical symmetrical hydrogen state functions are tested and accepted as solutions of this equation. Hence the equation $W_n' = W_1'/n^2$ is verified.

(II.8) The time dependent one-dimensional Schrödinger equation is given. The operator of the time evolution of a system is recognised, being put equal to the operator of total energy. The time independent Schrödinger equation is a special case. The form of a state function for a stationary state is deduced. Finally, in the relativistic equation $W^2 = c^2p^2 + m^2c^4$ dynamical variables are replaced by corresponding operators. For $m = 0$ the electromagnetic wave equation is obtained and the distinction with respect to the Schrödinger equation is discussed.

Historical survey

(III) A short historical survey of the discoveries considered is given. In the first part the state is the central concept and the spectrum of energy levels of electrons the central device. Phenomena are described as transitions of electrons between energy levels, in accordance with the exclusion principle. This quantum phenomenology is based mostly on direct experimental evidence. Semiconductor devices and lasers, playing an important role in everyday life, are considered as soon as possible. This is also a characteristic of a few recent texts at introductory university level (e.g. Landshoff and Metherel 1979). The second part is considerably more sophisticated. In it the motion of quantum particles is studied to get deeper insight into some phenomena described in the first part.

Possible drawbacks

Let us stress some drawbacks of the proposed approach. Electron scattering experiments, i.e. the experiment with the Teltron helium tube and the Franck–Hertz experiment, give pronounced minima in the current–voltage dependence, but they do not imply directly discrete states. The true evidence comes afterwards from line spectra. Some steps in the second part are rather abstract. A problem in semantics is the consequent use of terms like quantum physics, quantum particle, state function, etc.

The Quantum Physics for Beginners project is described in a booklet (Strnad 1980). Although not without appeal from the physics viewpoint, it is not yet in a suitable state for use in school. Obviously it is too extensive, but the second part could be dropped without harming the general idea of the approach. After all, the solar system is often taught without considering the orbits of the planets in detail.

Numerous discussions with physics teachers and students suggest that, at least in part, the approach may be acceptable from the didactical point of view. As a whole the proposal may be used, in a more quantitative version, even at introductory university level. In assessing the proposal students were more enthusiastic than physics teachers, particularly those who have been teaching for a long time. Nevertheless, even teachers who oppose the proposal may benefit by thinking over its basic ideas. It would be unrealistic to expect that 'matter waves' and the naive photon concept will follow the Bohr atomic model which has now almost disappeared from textbooks and curricula. However, the teaching of quantum mechanics may be improved by exploiting both concepts more critically.

References

Einstein A 1905 Ann. Physik **17** 132

Feynman R P, Leighton R B and Sands M 1965 'Quantum mechanics' *Feynman Lectures in Physics Vol III* (Reading, Mass.: Addison-Wesley)

Gillespie D T 1973 *A Quantum Mechanics Primer* (Aylesbury: International Textbook)

GIREP 1975 *Seminar on the Teaching of Physics in Schools* 2 ed A Loria and P Thomsen (Copenhagen: Gyldendal)

ter Haar D 1967 *The Old Quantum Theory* (Oxford: Pergamon)

Halliday D and Resnick R 1966 *Physics, Parts I and II* (New York: Wiley)

——1978 *Physics, Part* 2 (New York: Wiley)

Jönsson C 1974 *Am. J. Phys.* **42** 4

Joyce W B and Joyce A 1976 *J. Opt. Soc. Am.* **66** 1

Landau L and Peierls R 1930 *Z. Phys.* **62** 188

——1931 *Z. Phys.* **69** 59

Landshoff P and Metherel A 1979 *Simple Quantum Mechanics* (Cambridge: Cambridge University Press)

Levy-Leblond J-M 1976 *Am. J. Phys.* **44** 1130

MacKinnon E 1976 *Am. J. Phys.* **44** 1047

Nuffield Advanced Science 1972 *Physical Science Students' Workbook II* (London: Longman)

Nuffield Physics 1970 *Teachers' Guide V*(London: Longman)

Peierls R E 1979 *Surprises in Theoretical Physics* (Princeton: Princeton University Press)

PSSC 1965 *Physics* (Boston: Heath)

Rüdiger E 1976 *Am. J. Phys.* **44** 144

Santos E 1976 *Am. J. Phys.* **44** 278

Sargent M III, Scully M O and Lamb W E Jr 1974 *Laser Physics* (Reading Mass.: Addison-Wesley)

Scully M O and Sargent M III 1972 *Phys. Today* March p38

Strnad J 1980 *Quantum Physics for Beginners* (Ljubljana: DMFA) (in Slovenian)

Unesco 1972 *New Trends in Physics Teaching Vol II* (Paris: Unesco)

Whitaker E H 1979 *Phys. Educ.* **14** 108, 239

Wichmann E H 1971 *Quantum Physics, Berkeley Physics Course Vol IV*(New York: McGraw-Hill)

Young R A 1976 *Am. J. Phys.* **44** 1043