Muons

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Abstract

In this experiment the mean muon lifetime was measured by muon decay probability measurements. The muon proper lifetime was determined to be 2.198 μs. The rate of measured muon flux was compared with known values to verify relativistic time-dilation effects.

1 Introduction

Muons are a negatively charged elementary particle similar to a heavy electron. In 1941, F. Rasetti demonstrated that muon's have a finite "lifetime" now known to be 2.197μs [1]. Where lifetime is related to the half-life decay rate. The muon, along with its positively charged counterpart the anti-muon, is classified as a lepton.

Muons are often created from the decay of pions, which naturally occurs in Earth's upper atmosphere due to primary cosmic rays interactions with air molecules. The complete process in which primary cosmic rays liberate protons, neutrons, pions, kaons, and other particles from the air is not understood well [2]; however, it suffices to know that pions are released which spontaneously decay into a muon and neutrino via the processes below [2].

$$\begin{aligned} \pi^+ &\to \mu^+ \nu_\mu \\ \pi^- &\to \mu^- \bar{\nu}_\mu \end{aligned} \tag{1}$$

The muons produced travel at speeds near 99.9% the speed of light [3]. Takeing their production altitude is 15km above sealevel [3] and assuming they travel normal to Earth's surface we expect the travel time to reach earth is then: $15km/c = 50\mu s$. Much larger than their mean lifetime.

The fact that any significant number of muons have been detected on Earth's surface indicates that they cannot have a mean lifetime of $2.2\mu s$ and travel time of $50\mu s$. We take this as evidence for time dilation.

Instead muons experience a time dilation given by $t' = t_0/\gamma$, with $\gamma = (1-\beta^2)^{-1}$. With the speed given above, we then expect muons to experience a travel time 500times less than previously predicted. A much more reasonable estimate.

To truly calculate the travel time in the muon's reference frame, we must also account for the speed loss as the muon travels through a medium. The energy loss for a muon as it travels through a constant density fluid as given by T.E. Coan and J. Ye [2] is:

$$\Delta E = 2MeV/g/cm^2 \cdot H \cdot \rho \tag{2}$$

Where ρ is the fluid density and H is the height traveled through. We then note that $dE = \rho C_0 dh$, and from Einstein's relation we have $E = \gamma mc^2, dE = mc^2 d\gamma$ which gives:

$$dh = \frac{mc^2}{\rho C_0} d\gamma \tag{3}$$

Noting that the travel time in the particle's rest frame is given by $dt' = dh/(c\beta\gamma)$ then gives:

$$t' = \frac{mc}{\rho C_0} \int_{\gamma_1}^{\gamma_2} \frac{d\gamma}{\beta\gamma} = \frac{mc}{\rho C_0} \int_{\gamma_1}^{\gamma_2} \frac{d\gamma}{\sqrt{\gamma^2 - 1}} \tag{4}$$

$$t' = \frac{mc}{\rho C_0} \log\left(\sqrt{\gamma^2 - 1} + \gamma\right)\Big|_{\gamma_1}^{\gamma_2} \tag{5}$$

This is useful for a number of simple calculations. Firstly, muons reach the Earth's surface with a kinetic enery near 4GeV corresponding to $\gamma_1 = 38$. Using ΔE from Eq. 2, we find the energy and gamma factor in the upper atmosphere to be $\gamma_2 = 54$. Using these values as the integral bounds gives a reasonable $t' = 1.18\mu s$ in agreement with the first order calculation.

Given also that muons decay at a constant rate in any medium, we have $dN = \lambda dt$. This gives the familiar equation:

$$N = N_0 exp(-\lambda t) \tag{6}$$

Where λ is used to define the mean lifetime as $\lambda = 1/\tau$. This can now be used to determine a height difference necessary for an experiment. Suppose we wished to measure and compare muon counts at two different elevations. For a noticable difference between the two locations, we will say the time difference for the muon between the two locations should be $t' \approx 0.10 \cdot \tau$ which gives $1 - e^{-0.1} \approx 10\%$ difference in muon counts between the two locations.

By using $t' = 0.1\tau$ we find $\Delta\gamma \approx 15 = \Delta E/mc^2$. Which is nearly the same change experienced from the height of the atmosphere to sea-level. To truly perform a elevation varying experiment, very sensitive equipment should be used and a difference in muon counts less than 1% must be able to be accurately measured.

In this experiment, we will use Eq. 7 to measure the mean proper lifetime of muons. First note that the probability distribution is exponential, a memoryless probability distribution. This means the muons we may detect in the laboratory will decay under the same distribution despite having already traveled through the atmosphere. By fitting an exponential to measured decay times, we may measure the proper mean lifetime.

2 Method

In order to detect muon decays, we will utilizie a plastic scintillator. When a charged particle enters the scintillator it will begin to slow, emitting light as it looses energy. A photodetector then begins a timer when a flash indicates a particle has entered. If a muon enters with sufficiently low kinetic energy, it will stop inside the scintillator and decay. When it decays, it emits photons which tells the photodetector to stop the timer. In this way, the time needed for a muon to decay is measured. If no secondary flash is detected within a brief period, the particle is assumed to have passed through without decaying, or is not a muon and not recorded as an event.

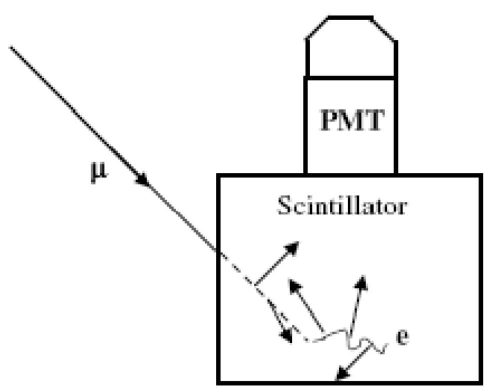

Figure 1: Photon emission used in determining the lifetime of a muon

The scintillator was connected to a computer which recorded the time of each event, and the time (in nanoseconds) between the initial flash and decay flash of a given event. The events were set to record beginning at 3:00pm on June 5th, 2012 and were collected every Tuesday and Thursday until data collection was halted at 6:00pm June 19th, 2012.

3 Analysis

The data collected only consisted of the time between entering and decaying in nanoseconds. This was converted into a probability distribution by creating a histogram from the data. The initial histogram is shown below.

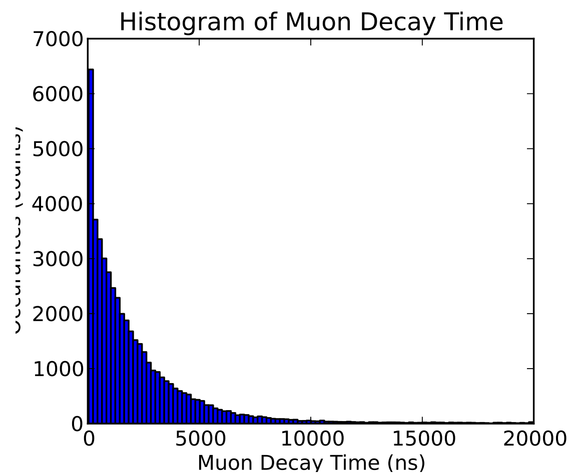

Figure 2: Raw Histogram

With some foresight that the proper lifetime is near $2\mu s$, we mas safely assume that counts above $10\mu s$ are noise and interference and should not be included in the analysis. Therefore, to calculate proper lifetime, we take the logarithm of Eq. 7 to obtain:

$\log(N) = \frac{-t}{\tau} + \log(N_0)$ (7)

The data is then plotted on semi-log axes and a line fit to the data. The slope of the line gives the decay rate while the intercept is related to the counts recieved. As will be discussed in the error analysis section, the initially calculated lifetime is used to estimate the background noise. The background noise is then subtracted out and the fit calculation redone. Using this method we found the slope of the fit line to be -0.4550 indicating $\tau = 2.1978$ in agreement with known values.

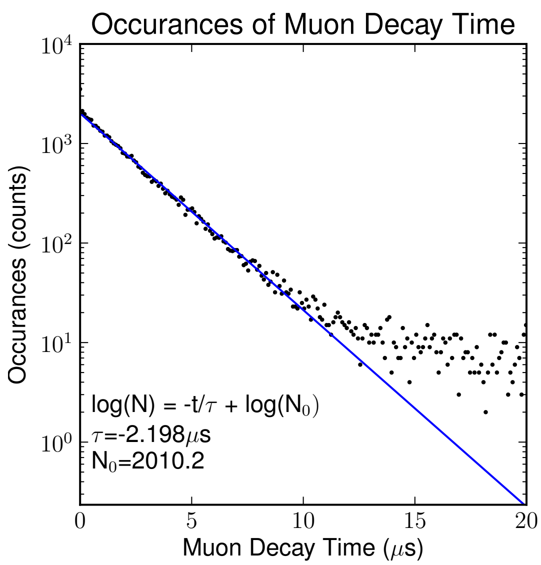

Figure 3: Histogram with fit equation

4 Error Analysis

This experiment is very prone to false positives because a muon decay is only recognized by sufficiently quick flashes within the scintillator. It is very likely that many recorded events are not a decay, but two particles passing through in the same time period. It is assumed however, that any "coincidental events" will not occur preferentially to a specific time range. They shift the entire distribution vertically upward and not affect the shape of the graph.

To compensate for this, we may take an initial estimate of the lifetime and perform a fit as shown in Fig. 3. Notice the fit holds for small times, but above 5τ noise becomes significant. Therefore we take the background noise to be a constant number of counts for each bin of the histogram and calculate it by averaging the deviation from measured and fit for values of $t > 5\tau$.

To obtain a confidence estimate for our measurement of τ, we look to the coefficient of determination of the linear regression. The fit performed has: $r^2 = 0.992$ indicating the linear fit is well formed in the low-t region. We will say that a reasonable fit requires $r^2 > 0.98$, and found that a 5% variation in τ caused r^2 to drop below the threshold value. Therefore, we assert that $\tau = 2.2 \pm 5\%$.

References

[1] R. Clay and B.Dawson. Cosmic bullets, high energy particle in astrophysics. 1997.

[2] T.E. Coan and J. Ye. Muon physcs. 2003.

[3] J.Beringer et al. (Particle Data Group). Charged lepton particle properties. *Physical Review*, D86, 010001, 2012.