A Muon's lifetime

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Abstract

Using a Teach Spin Scintillator, mean muon lifetime was measured by histogram decay half-time measurements in this experiment. The muon proper lifetime was determined to be 2.19 μs. The rate of measured muon flux was compared with known values to verify relativistic time-dilation effects and we examined briefly the Fermi Constant.

1 Introduction

Muons are a negatively charged lepton similar to the electron except with 200 times the mass. It is most frequently found in cosmic radiation. The first time the muon decay was measure was in 1941 when F. Rasetti showed that muons have a finite lifetime now known to be 2.19 s [?]. Today thanks to the particle data group we know the value to actually be 2.196981 0.000002 s [?]. The muon, along with its positively charged counterpart the antimuon are the make up the second lepton group. Their mean production altitude is 15km above sealevel [?] and assuming they travel normal to Earth's surface we expect the travel time to reach earth is then: $15km/c = 50\mu s$. Much larger than their mean lifetime. Muons are often the decay product of negative pions that decay them selves from decay of free protons in the atmosphere. Figure below is has sketch of these primary decays of cosmic ray radiation, which naturally occurs in Earths upper atmosphere due to primary cosmic rays interactions with air molecules. Cosmic rays include protons, neutrons, pions, kaons, delta-ons, Helium, and other particles from the air is not understood well [?]; suffices to know that pions are released which spontaneously decay into a muon and neutrino via the processes below [?] and as shown on the figure: In turn muons naturally decay is:

$$\begin{aligned} \pi^- &\rightarrow \mu^- \bar{\nu}_\mu \\ \pi^- &\rightarrow \mu^- \bar{\nu}_\mu \end{aligned} \tag{1}$$

As shown by Fermi diagram, this decay is mediated by the W boson. In fact, the fundamental constant of interaction of the weak electric force, called the Fermi constant $G_F ermi$, is best measured using the muon half-life and they are related

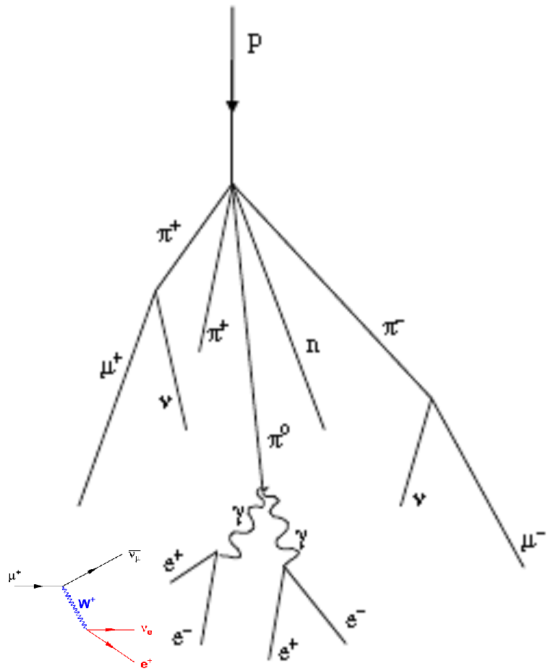

Figure 1: Labratory Setup showing the scintillator and other devices

by the following equation:

$$\Rightarrow \frac{G_F}{(\hbar c)^3} = \sqrt{\frac{\hbar}{\tau_\mu} \cdot \frac{192\pi^3}{(m_\mu c^2)^5}} \tag{2}$$

Relativistically, the muons produced travel at speeds near 99 percent the speed of light [?]; however, our detector only detects those that are going .8c [?]. Due to the vast number of muons have been detected on Earth's surface shows that they cannot have a mean lifetime of $2\mu s$ and travel time of $50\mu s$, as calculated from the the non relativistic speed. This is due o this as evidence for time dilation. Instead muons experience a time dilation of $t' = t_0/\gamma$, with $\gamma = (1-\beta^2)^{-1}$. With the velocity given above, we then expect the particles to experience a travel time 500 times less than previously predicted.To truly calculate the travel time in the muon's reference frame, we must also account for the speed loss as the muon travels through a medium. The energy loss for a muon as it travels through a constant density fluid as given by T.E. Coan and J. Ye [?] is:

$$\Delta E = 2MeV/g/cm^2 \cdot H \cdot \rho \tag{3}$$

Where ρ is the fluid density and H is the height traveled through. We then note that $dE = \rho C_0 dh$, and from Einstein's relation we have $E = \gamma mc^2, dE = mc^2 d\gamma$ which gives:

$$dh = \frac{mc^2}{\rho C_0} d\gamma \tag{4}$$

Noting that the travel time in the particle's rest frame is given by $dt' = dh/\left(c\beta\gamma\right)$ then gives:

$$t' = \frac{mc}{\rho C_0} \int_{\gamma_1}^{\gamma_2} \frac{d\gamma}{\beta\gamma} = \frac{mc}{\rho C_0} \int_{\gamma_1}^{\gamma_2} \frac{d\gamma}{\sqrt{\gamma^2 - 1}} \tag{5}$$

$$t' = \frac{mc}{\rho C_0} \log\left(\sqrt{\gamma^2 - 1} + \gamma\right)\Big|_{\gamma_1}^{\gamma_2} \tag{6}$$

Thus, muons reach the Earth's surface with a kinetic enery near 4GeV corresponding to $\gamma_1 = 38$. Using ΔE from Eq. 3, Therefore, the the upper atmosphere to be $\gamma_2 = 54$. Using these values as the integral bounds gives a reasonable $t' = 1.18\mu s$ in agreement with the first order calculation. A table of these calculations are give in the table at the end of this report. Given hat muons decay at a constant rate in any medium, we have $dN = \lambda dt$. This gives the familiar equation, we shall use to find the muon life time:

$$N = N_0 exp(-\lambda t) \tag{7}$$

Where λ is used to define the mean lifetime as $\lambda = 1/\tau$. This can now be used to determine a height difference necessary for an experiment. Suppose we wished to measure and compare muon counts at two different elevations. Use For a noticeable difference between the two locations, we will say the time difference for the muon between the two locations should be $t' \approx 0.16 \cdot \tau$ which gives $1 - e^{-0.1} \approx 10\%$ difference in muon counts between the two locations.

By using $t' = 0.16\tau$ we find $\Delta\gamma \approx 15 = \Delta E/mc^2$. Which is nearly the same change experienced from the height of the atmosphere to sea-level. To truly perform a elevation varying experiment, very sensitive equipment should be used and a difference in muon counts less than 1% must be able to be accurately measured.

In this experiment, we will use Eq. 8 to measure the mean proper lifetime of muons. First note that the probability distribution is exponential, a memoryless probability distribution. This means the muons we may detect in the laboratory will decay under the same distribution despite having already traveled through the atmosphere. By fitting an exponential to measured decay times, we may measure the proper mean lifetime.

2 Method

In order to detect muon decays, we will utilize a plastic scintillator. When a charged particle enters the scintillator it will begin to slow, emitting light as it looses energy. A photodetector then begins a timer when a flash indicates a particle has entered. If a muon enters with sufficiently low kinetic energy, it will stop inside the scintillator and decay. When it decays, it emits photons which tells the photodetector to stop the timer. In this way, the time needed for a muon to decay is measured. If no secondary flash is detected within a brief period, the particle is assumed to have passed through without decaying, or is not a muon and not recorded as an event.

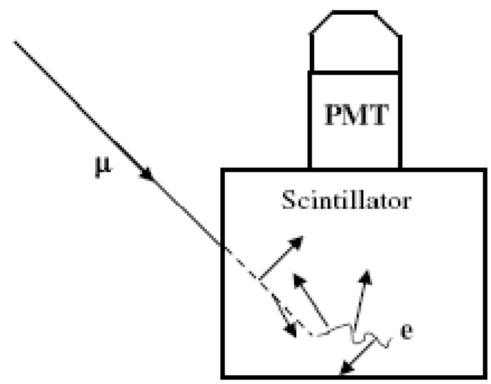

Figure 2: Photon emission used in determining the lifetime of a muon

We recorded data from 3:00pm on June 5th, 2012 to 6:00pm June 19th, 2012. The figure above shows the labratory setup with the black box being the scintillator. The scintillator was connected to a computer which recorded the time of each event, and the time (in nanoseconds) between the initial flash and decay flash of a given event. Inside the scintillator is a photomultiplier tube, or detector that converts photon emitted either by muon decay or electron collisions, into

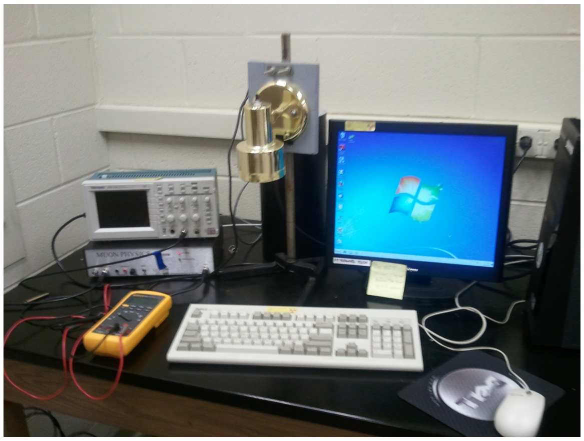

Figure 3: Labratory Setup showing the scintillator and other devices

electrons and where the electrons are captured in current then transformed into a digital signal. A Photomultipler Tube consists of a photocathode and several diodes in a vacuum. Typically, photomultipler tubes are build with whatever collaminator or attenuator already attached to [?] . In the PMT there is a special photocathode that converts photons to electrons, or photoelectrons, via the photoelectric effect. Through, using the secondary emission effect these electrons hit the various diodes increasing the current of going through the device. In modern day times, this current is then translated, via a chip into a digital signal that is readable by a computer. The formulas for time of flight and example of how they could detect are added. As one may notice the time of flight depends on the Lorentz factor (and thus the velocity) of the original particle by an inverse relationship as expected. Recall that as the orginal particle, in our case the muon, gets higher and higher Energy it moves faster due to $E = mc^2 E_light = mc^2$ witch 1. Due =1 parts of formula for time of flight are basically washed out, therefore photomultiplier tubes cannot really detect high-energy particles, very well and differentiate between muons and electrons. Additionally most of the time the light need to be collaminated or attenuated for a photomultiplier tube to work properly?

3 Analysis

In the experiment we only actually measured the time between entering and decaying in nanoseconds and the time of the measurement. These results were plotted on a histogram and then used . The initial histogram is shown below.

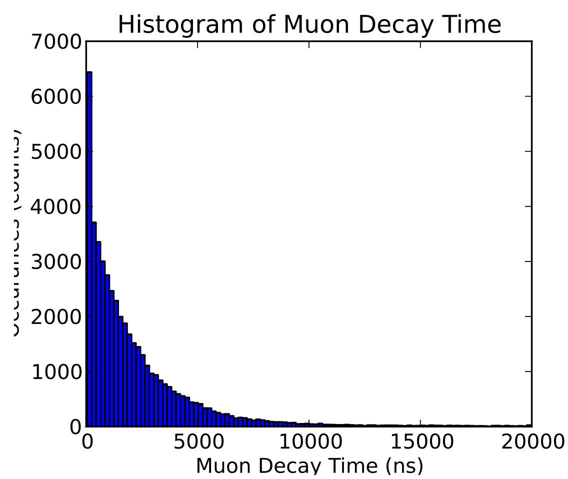

Figure 4: Raw Histogram

With some foresight that the proper lifetime is near $2\mu s$, we mas safely assume that counts above $10\mu s$ are noise and interference and should not be included in the analysis. Therefore, to calculate proper lifetime, we take the logarithm of Eq. 8 to obtain:

$$\log(N) = \frac{-t}{\tau} + \log(N_0) \tag{8}$$

We fit the data on a semilog plot where The slope of the line gives the decay rate. In order to get an accurate value the background noise is then subtracted out and the fit calculation redone. Using this method we found the slope of the fit line to be -0.4550 indicating $\tau = 2.19 + / - .01$ in agreement with known values /citePDG. We compared this to seperate trials of the muon data for each run we did.

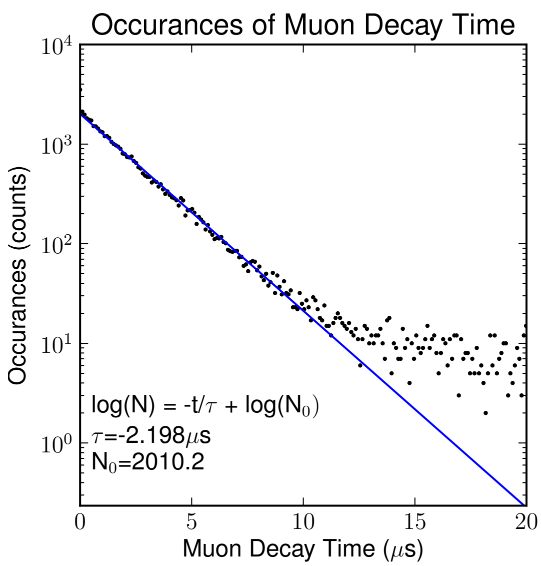

Figure 5: Histogram with fit equation

4 Error Analysis

5 Error Analysis

This experiment is very prone to false positives because a muon decay is only recognized by suf-

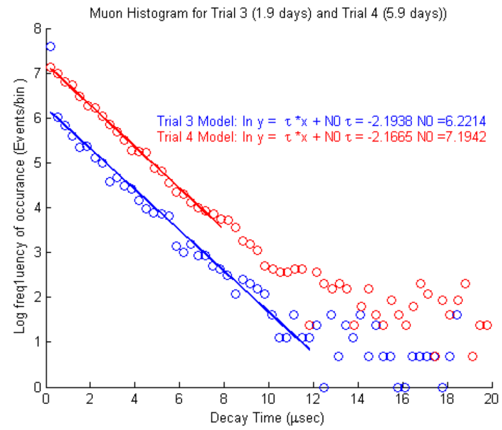

Figure 6: Histogram Trial 1 and 2 with fit equation

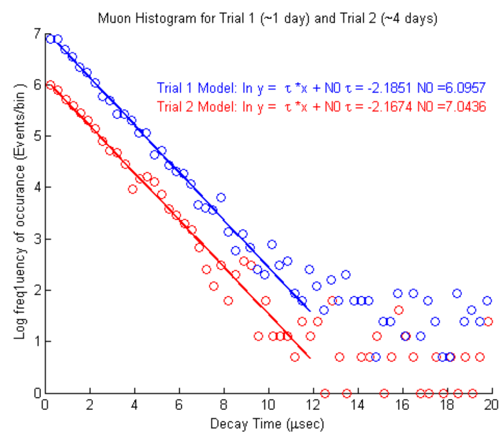

Figure 7: Histogram Trial 3 and 4 with fit equation

ficiently quick flashes within the scintillator. It is very likely that many recorded events are not a decay, but two particles passing through in the same time period. It is assumed however, that any "coincidental events" will not occur preferentially to a specific time range. They shift the entire distribution vertically upward and not affect the shape of the graph.

To compensate for this, we may take an initial estimate of the lifetime and perform a fit as shown in Fig. 4. Notice the fit holds for small times, but above 5τ noise becomes significant. Therefore we take the background noise to be a constant number of counts for each bin of the histogram and calculate it by averaging the deviation from measured and fit for values of $t > 5\tau = 11.8\mu s$.

To obtain a confidence estimate for our measurement of τ, we look to the coefficient of determination of the linear regression. The fit performed has: $r^2 = 0.992$ indicating the linear fit is well formed in the low-t region. We will say that a reasonable fit requires $r^2 > 0.98$, and found that a 5% variation in τ caused r^2 to drop below the threshold value. Therefore, we assert that $\tau = 2.2 \pm 5\%$.

References

[1] R. Clay and B.Dawson. Cosmic bullets, high energy particle in astrophysics. 1997.

[2] T.E. Coan and J. Ye. Muon physcs. 2003.

[3] J.Beringer et al. (Particle Data Group). Charged lepton particle properties. *Physical Review*, D86, 010001, 2012.

6 appendix

Table of Relativstic Calculations									**Transit**
x (m)	h =elevation (m)	ρ (g/cm3)	H (cm)	ρ/A * c	ΔE (MeV)	E2 (MeV)	E1 (MeV)	γ1 (1/c^2)	t'_1
0.0	320	1.23E-03	1468000	6.96E+05	3600	160.0	3760	35.58742	6.71E-06
50.0	370	1.23E-03	1463000	7.00E+05	3609	160.0	3769	35.67409	6.68E-06
100.0	420	1.24E-03	1458000	7.04E+05	3618	160.0	3778	35.76059	6.64E-06
1000.0	1320	1.38E-03	1368000	7.84E+05	3779	160.0	3939	37.28057	6.02E-06
1050.0	1370	1.39E-03	1363000	7.88E+05	3788	160.0	3948	37.36259	5.98E-06
1100.0	1420	1.40E-03	1358000	7.93E+05	3796	160.0	3956	37.44432	5.95E-06
1150.0	1470	1.41E-03	1353000	7.98E+05	3805	160.0	3965	37.52575	5.92E-06
1200.0	1520	1.41E-03	1348000	8.03E+05	3813	160.0	3973	37.60687	5.89E-06
1250.0	1570	1.42E-03	1343000	8.07E+05	3822	160.0	3982	37.68767	5.85E-06
1300.0	1620	1.43E-03	1338000	8.12E+05	3831	160.0	3991	37.76815	5.82E-06
1350.0	1670	1.44E-03	1333000	8.17E+05	3839	160.0	3999	37.8483	5.79E-06
1400.0	1720	1.45E-03	1328000	8.22E+05	3847	160.0	4007	37.92812	5.76E-06
1450.0	1770	1.46E-03	1323000	8.27E+05	3856	160.0	4016	38.0076	5.73E-06
1500.0	1820	1.47E-03	1318000	8.32E+05	3864	160.0	4024	38.08673	5.70E-06
1550.0	1870	1.47E-03	1313000	8.37E+05	3872	160.0	4032	38.1655	5.66E-06
1600.0	1920	1.48E-03	1308000	8.42E+05	3881	160.0	4041	38.24391	5.63E-06
1650.0	1970	1.49E-03	1303000	8.47E+05	3889	160.0	4049	38.32195	5.60E-06
1700.0	2020	1.50E-03	1298000	8.52E+05	3897	160.0	4057	38.39961	5.57E-06
1750.0	2070	1.51E-03	1293000	8.57E+05	3905	160.0	4065	38.47688	5.54E-06
1800.0	2120	1.52E-03	1288000	8.62E+05	3914	160.0	4074	38.55377	5.51E-06
1850.0	2170	1.53E-03	1283000	8.67E+05	3922	160.0	4082	38.63025	5.48E-06
1900.0	2220	1.54E-03	1278000	8.72E+05	3930	160.0	4090	38.70633	5.45E-06
1950.0	2270	1.55E-03	1273000	8.78E+05	3938	160.0	4098	38.782	5.42E-06
2000.0	2320	1.56E-03	1268000	8.83E+05	3946	160.0	4106	38.85724	5.39E-06
2050.0	2370	1.57E-03	1263000	8.88E+05	3953	160.0	4113	38.93205	5.36E-06
2100.0	2420	1.57E-03	1258000	8.93E+05	3961	160.0	4121	39.00642	5.33E-06
2150.0	2470	1.58E-03	1253000	8.99E+05	3969	160.0	4129	39.08035	5.30E-06
2200.0	2520	1.59E-03	1248000	9.04E+05	3977	160.0	4137	39.15383	5.27E-06
2250.0	2570	1.60E-03	1243000	9.10E+05	3985	160.0	4145	39.22684	5.24E-06
2300.0	2620	1.61E-03	1238000	9.15E+05	3992	160.0	4152	39.29938	5.21E-06
2350.0	2670	1.62E-03	1233000	9.20E+05	4000	160.0	4160	39.37145	5.18E-06
2400.0	2720	1.63E-03	1228000	9.26E+05	4007	160.0	4167	39.44303	5.15E-06
2450.0	2770	1.64E-03	1223000	9.31E+05	4015	160.0	4175	39.51411	5.13E-06
2500.0	2820	1.65E-03	1218000	9.37E+05	4022	160.0	4182	39.58469	5.10E-06
2550.0	2870	1.66E-03	1213000	9.43E+05	4030	160.0	4190	39.65476	5.07E-06
2600.0	2920	1.67E-03	1208000	9.48E+05	4037	160.0	4197	39.72431	5.04E-06
2650.0	2970	1.68E-03	1203000	9.54E+05	4044	160.0	4204	39.79333	5.01E-06
2700.0	3020	1.69E-03	1198000	9.60E+05	4052	160.0	4212	39.86182	4.99E-06
2750.0	3070	1.70E-03	1193000	9.65E+05	4059	160.0	4219	39.92975	4.96E-06
2800.0	3120	1.71E-03	1188000	9.71E+05	4066	160.0	4226	39.99713	4.93E-06
2850.0	3170	1.72E-03	1183000	9.77E+05	4073	160.0	4233	40.06395	4.90E-06
2900.0	3220	1.73E-03	1178000	9.83E+05	4080	160.0	4240	40.13019	4.87E-06
2950.0	3270	1.74E-03	1173000	9.89E+05	4087	160.0	4247	40.19585	4.85E-06
3000.0	**3320**	**1.75E-03**	**1168000**	**9.95E+05**	**4094**	**160.0**	**4254**	**40.26092**	**4.82E-06**

^ bolded is for dallas texas

Table of Measured Values				
# of days	τ (μs)	Error (μs)	G_{fermi} =GeV/(ħc)^3	R (#/hr)
1.96	2.185	0.002	1.17E-05	63.1
4.99	2.160	0.003	1.18E-05	62.6
1.90	2.194	0.009	1.16E-05	97.2
5.95	2.160	0.001	1.18E-05	61.7
Overall	2.19	0.01	1.17E-05	62 ± 2

Figure 8: Table of Muon values