The Franck-Hertz Experiment

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We experimentally reproduce the key elements of the historic Franck-Hertz experiment. Electrons are accelerated through monatomic mercury vapor in an electric field. Inelastic scattering between the electrons and target atoms resulted in quantized excitation. We measured the excitation value to be 4.93 ± 0.06eV. A qualitative framework for understanding the temperature and electron-density dependence of the signal is presented. We also discuss the parameters under which a blue-white plasma is observed for our experimental geometry.

At the turn of the twentieth century, the physics community had just begun to tackle the problem of quantization as observed in the Millikan oil drop experiment, atomic spectra and the photoelectric effect. The Franck-Hertz experiment, first conducted in 1914, is a historic experiment that showed quantized internal energy excitation in atoms. The experiment provided confirmation for Bohr's theory of the hydrogen atom [1].

In the Franck-Hertz experiment, electrons are accelerated by an electric field through a monatomic vapor. The electrons traveling through the vapor collide with the atoms. These collisions are completely elastic unless the electron has more energy than the first excited state of the atom. In such an elastic collision, the energy transfered to the atom by the electron is found from basic nonrelativistic kinetics to be

$$\Delta K = \frac{4mM}{(m+M)^2} K_0 \tag{1}$$

where m is the mass of the electron, M is the mass of the atom and K_0 is the initial electron energy. For collisions with atoms, the coefficient $\frac{4mM}{(m+M)^2}$ is small as $M \gg m$. In this scenario, $\Delta K \approx 4\frac{m}{M}K_0$ and the energy that the electron loses (by transferring it to the atom) in a single elastic collision is negligible.

When the energy of the incident electrons is greater than the energy necessary to excite the atom, inelastic collisions can take the atom to one of its low-lying excited states. The electron loses this energy. After the collision, subsequent acceleration of the electron may bring it once again into an energy regime where it can inelastically collide with the atoms in the vapor.

To measure the energy of electrons after they have traveled some distance in the atomic vapor, an electric field that decelerates the electrons is introduced between a grid and an anode [Fig 1]. The potential is chosen so that electrons with relatively high kinetic energies will pass between the grid and anode, resulting in a measurable current I_{ag} between the anode and the grid. Less energetic electrons will pass through the grid and then return to it instead of reaching the anode. These electrons do not contribute to I_{ag}. Nicoletopoulos has pointed out that using this grid technique for measuring the current is intrusive and effects the experimental results [2].

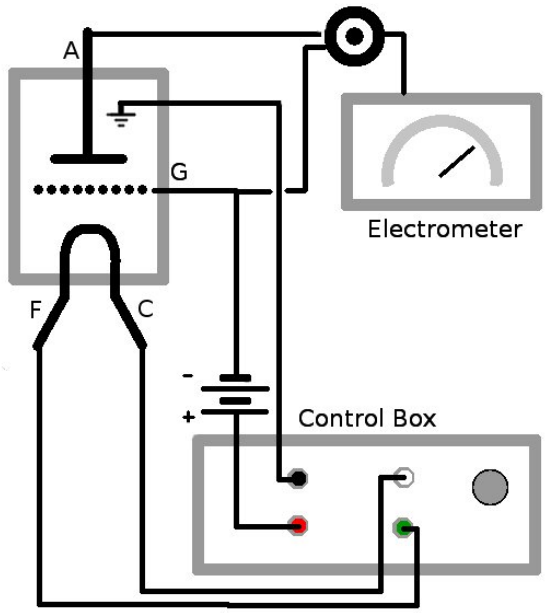

FIG. 1: Experimental setup and wiring. In the commercially-available Franck-Hertz experiment apparatus produced by Klinger S.A. Corporation, a glass tube with a small quantity of liquid mercury is heated so that some of the mercury goes into the gas phase. Electrons are liberated within this tube by a heated filament (F) and accelerated through the gas by an applied voltage (V_{acc}) between the cathode (H) and the grid (G). A retarding electric field produced by a 1.5V battery is applied between the grid and the anode (A). The resulting current flow between the grid and anode (I_{ag}) is measured by a precision electrometer (model 610B produced by Keithley Instruments). V_{acc} and V_f are controlled through custom electronics interfacing with a computer through a data acquisition card produced by National Instruments. A LabView program was used to vary V_{acc} and record I_{ag}.

Nonetheless, this historical method is sufficient to observe the desired quantization.

The physical picture presented above makes many idealistic assumptions. For example, it ignores the elastic collisions, electron-atom interactions in the region of space between the grid and anode, and velocity distributions of atoms and electrons. Refinements to this explanation of the experiment taking into account modern kinetic theory are described in Refs [3–5].

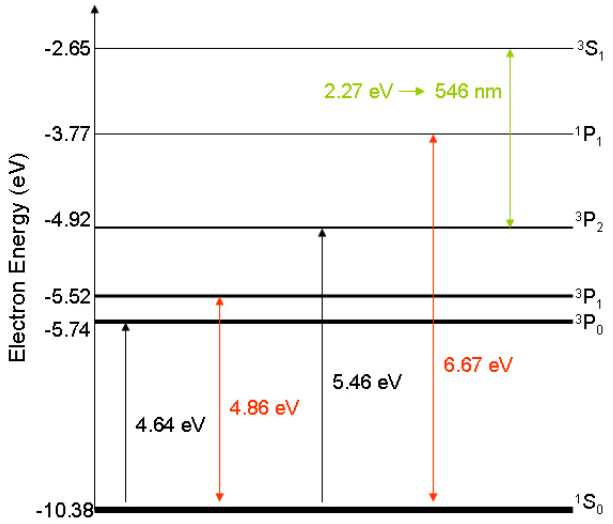

FIG. 2: Relevant energy levels of atomic mercury (Hg). Direct excitation of the $^3P_0$, $^3P_1$, $^3P_2$ and $^1P_1$ states is possible from the $^1S_0$ ground state through inelastic collisions with electrons. Transitions from the $^3P_0$ and $^3P_2$ to the $^1S_0$ ground state are forbidden. Also pictured is the 2.27eV transition $^3S_1 \rightarrow^3 P_2$, which is probably responsible for green light observed during the experiment. Quoted energies are from [6].

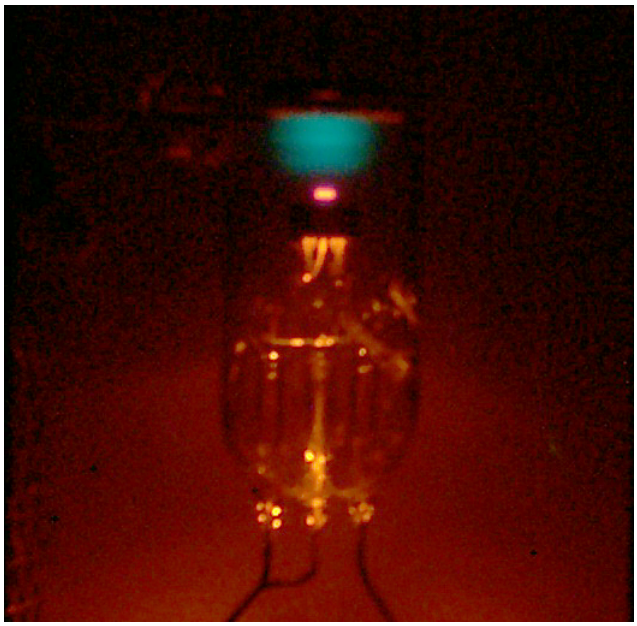

FIG. 3: Image of the mercury tube during experimentation. Note the three bands that appear in the areas where green light is observed. Based on the color, this light is most likely the 546.0750nm emission line $^3S_1 \rightarrow^3 P_2$ of mercury [7].

In order to conduct the experiment, we used a commercially available Franck-Hertz apparatus from the Klinger S.A. Corp. In this apparatus, mercury vapor was heated to temperatures between 130 and 180°C in a glass tube. Electrons liberated from a heated filament by means of an applied voltage, V_f, were accelerated through the mercury vapor by an applied electric field [Fig 1].

Mercury was used for this experiment because its vapor is monatomic. Monatomic vapors are preferred over molecular gases such as hydrogen for this experiment. Molecular gases have closely-spaced rovibrational states that effectively wash out the quantized states in the excitation spectrum. Furthermore, mercury's vapor density is variable with temperature, an easily controlled experimental variable. Treating mercury vapor as an ideal gas, the molar density n in $\frac{mol}{m^3}$ of mercury is given by

$$n = \frac{133.3224 \times 10^{-3204/(T+273.15)+8.009}\ \text{Pa}}{R(T+273.15)} \qquad (2)$$

where T is the temperature in degrees Celsius and R is the gas constant, kN_A [1]. For the temperatures used in this experiment, we had mercury vapor densities between 0.04583 and 0.3071 $\frac{mol}{m^3}$. Also, it is valid in a first-order approximation to ignore the energy transferred from electrons to mercury atoms in elastic collisions as the coefficient $\frac{4mM}{(m+M)^2}$ that appears in Eq. 1 (which describes the energy an electron transfers to an atom in an elastic collision) is approximately 1.094×10^{-5}.

The lowest lying states of mercury that can be accessed by inelastic collisions are the transitions from the $^1S_0$ ground state to the $^3P_0$, $^3P_1$, $^3P_2$ and $^1P_1$ states [Fig 2]. All of these transitions are equally likely to be excited; however, this experimental setup favors the $^1S_0 \rightarrow^3 P_1$ 4.89eV transition [8].

The preference for the $^1S_0 \rightarrow^3 P_1$ transition can also be explained as follows. In our idealized description, electron energy increases linearly in the electric field between the filament and the anode until it reaches an energy (which is proportional to distance) at which it can excite the mercury atoms. The $^3P_0$ state is the first to be excited as it has the lowest lying energy. However, the $^3P_0$ state is metastable and does not decay quickly. The vapor is moving slowly enough that the majority of the mercury atoms at the proper distance for excitation to the $^3P_0$ state are transferred to this state. With the saturation at this distance scale, electrons travel through the vapor unaffected by elastic collisions with these excited atoms until they have enough energy to excite the $^3P_1$ state. The transition $^3P_1 \rightarrow^1 S_0$ is allowed, so the transition rate is fast enough that the atoms at the proper distance do not become saturated with excited atoms. So, the observed current spikes can be accounted for by $^1S_0 \rightarrow^3 P_1$ transitions. Not all electrons lose energy in these transitions, so some will excite the higher energy transitions to $^3P_2$ and $^1P_1$. Transitions at these other energies (i.e. to the $^3P_0$, $^3P_2$ and $^1P_1$ states) account for some of the smoothing in the observed current signal.

While V_{acc} is changed, green light is observed coming from the mercury chamber. Based on our knowledge of the mercury spectrum, this light is most likely coming

from the decay of the $^3S_1$ to the $^3P_2$ state. This reasonably strong transition involves the emission of a 2.27eV photon. The model described is based on the assumption that atoms remain in a ground state configuration; however, to get to the $^3S_1$ state, the atoms must be in higher energy state. It is interesting to note that, for the first few excitations, a banded structure can be observed in this light [Fig 3]. These bands can be explained by the space-dependence of the electron energies. As discussed above, electron energy is roughly proportional to distance. After traveling the proper distance to excite the $^3S_1$ state, an electron can inelastically collide with a local mercury atom, exciting the $^3S_1$ state. The electron's energy is reduced by this inelastic collision. The electron then again must travel some distance in the electric field to increase its energy to a level at which it can again excite a mercury atom. This rough distance-dependence of electron energy sets up bands of atoms that are excited to the $^3S_1$ state. These atoms can decay to the $^3P_2$ state, emitting the observed green light.

For the conducted experiment, we measured the periodicity of the electron energy at the grid as follows. Using LabView, anode current I_{ag} was recorded while V_{acc} was increased from 0.0 to 45.0V in increments of 0.05V with a scan rate of 10 scans per second. For each run, I_{ag} and V_{acc} were stored in a text file. Each file was then analyzed using Microsoft Excel 2003, looking at the dependence of I_{ag} on V_{acc} [Fig 4]. Electrons that reached the anode were registered as negative current. Because electrons were decelerated between the grid and anode, this current is a measure of the electron energy. We have reason to believe that the observed -2 nA offset visible in Fig 4 and all other data sets is a nonphysical product of our data acquisition system.

The excitation potential for the favored $^1S_0 \rightarrow^3 P_1$ transition was determined by multiplying the elementary electron charge with the measured voltage difference (for V_{acc}) between the current peaks in a run. Data points representing the current peaks were selected by visually inspection of the graphs produced by Microsoft Excel. For each run, the voltage difference between adjacent current peaks was calculated. The mean and standard deviation of these measurements were calculated on a run-by-run basis for all runs, including runs with varying oven temperatures and filament voltages. The final excitation potential was calculated by taking the weighted average of these measured values.

The excitation potential was measured to be 4.93 $\pm$ 0.06 eV. This value agrees with the expected value of 4.86 eV corresponding to the $^1S_0 \rightarrow^3 P_1$ transition [7].

This determination of the excitation potential of mercury also gives us an estimate for Planck's constant, h. We know

$$hc/\lambda = eV_0 \tag{3}$$

where λ is the wavelength of the emitted light, c is the speed of light, e is the charge of an electron and V_0 is the excitation potential for the $^1S_0 \rightarrow^3 P_1$ transition.

Solving equation 3 for h and using the known spectral value of for the wavelength of the first excited state of mercury, 253.6521nm [7], we find an experimental value of h to be $6.68 \pm 0.08 \times 10^{-34} J \cdot s$. This agrees with the accepted value of $6.626\ 068\ 96 \pm 33 \times 10^{-34} J \cdot s$ [9].

We measured the excitation potential in a range of temperatures (10°C increments between 130 and 180°C) and electron densities (as determined by V_f, which was varied between 4.0 and 6.3V). Seven runs were performed at 180°C with different filament voltages. Below a filament voltage of 5V, the tube current was not large enough to observe over the background noise. For V_f above 5V, the data showed clear current peaks. Higher values of V_f resulted in progressively better-defined and larger-amplitude current peaks. This increased data quality was a result of the the increased tube current. Altering the filament voltage did not result in any statistically significant change to the calculated excitation potential.

Six runs were performed at temperatures between 130°C to 180°C. These runs were all taken with a V_f of 6.3V. Altering the temperature did not result in any statistically significant change to the calculated excitation. However, data analysis showed that, for higher temperatures, the current peaks began at a slightly lower voltage. The maximum offset observed was 0.8V. This phenomenon was a result of the initial kinetic energy distribution of the electrons. For higher temperature, the kinetic energy distribution is greater than that in a low temperature environment. An electron with a large initial kinetic energy required slightly less acceleration to

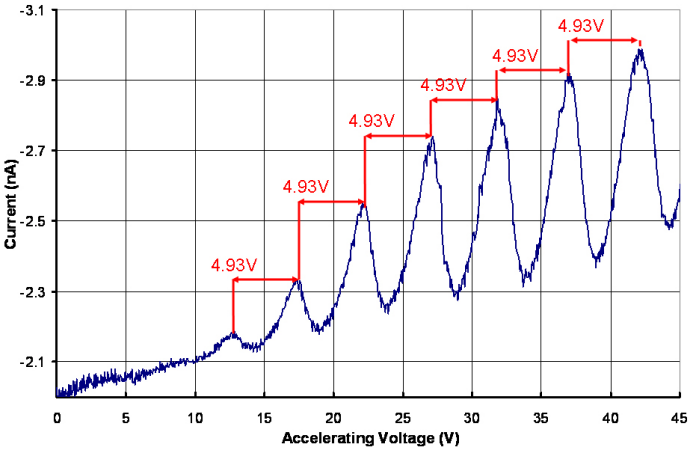

FIG. 4: Data from a run taken at 180°C with V_f at 6.3V. For this run, the observed excitation potential was calculated to be 4.89 ± 0.26eV. The arrows indicate the calculated value for the excitation potential from all 6 data sets, 4.93 $\pm$ 0.06eV, showing qualitative matching of the calculated value with this typical run.

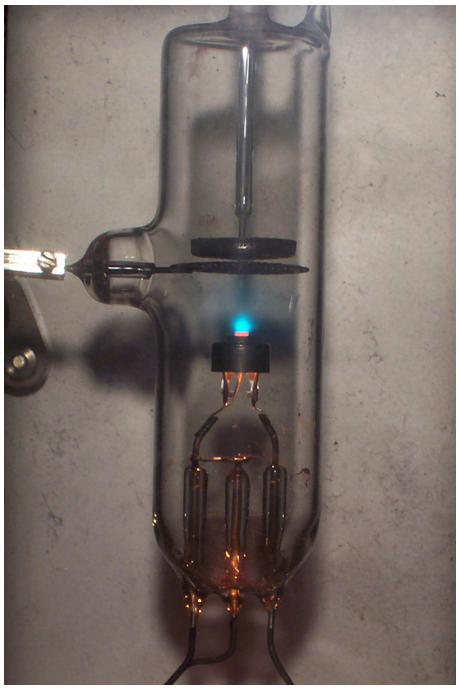

FIG. 5: A blue-white plasma formed at the filament (bottom of glass tube). This plasma was observed at a variety of values for temperature, V_{acc} and V_f. The image shown above was taken at a temperature of 140°C with V_{acc}=25V and V_f=6.2V.

reach the lowest lying excitation energies for mercury.

An interesting effect was observed for particular sets of V_f, V_{acc} and temperature. While the accelerating voltage was ramped up, a white-blue plasma formed near the filament in the tube. This plasma grew as the accelerating voltage was increased. When the plasma finally attached itself to the filament, the current readings no longer moved in a sinusoidal pattern and the glowing discharge in the expected range for mercury excitation stopped [Fig 5]. We call the value of V_{acc} at which the plasma attached itself to the filament the onset voltage, V_{onset}.

V_{onset} was observed to be dependent on both V_f and the temperature of the tube. While this phenomenon was not studied exhaustively, there seemed to be a negative, linear dependence on the temperature and a positive linear dependence on V_f (which, in turn, was roughly related to the electron density in the tube) [Fig 6]. Further study is warranted to characterize and understand this phenomenon.

In summary, experimental results of the historic Franck-Hertz experiment were reproduced, giving values in agreement with the expected values within one standard deviation. We explored the parameter space of temperature and filament voltage and determined that the measured excitation voltage was independent of these parameters. Finally, we described an interesting phenomenon in which a blue-white plasma was formed and attached itself to the filament under highly repeatable conditions.

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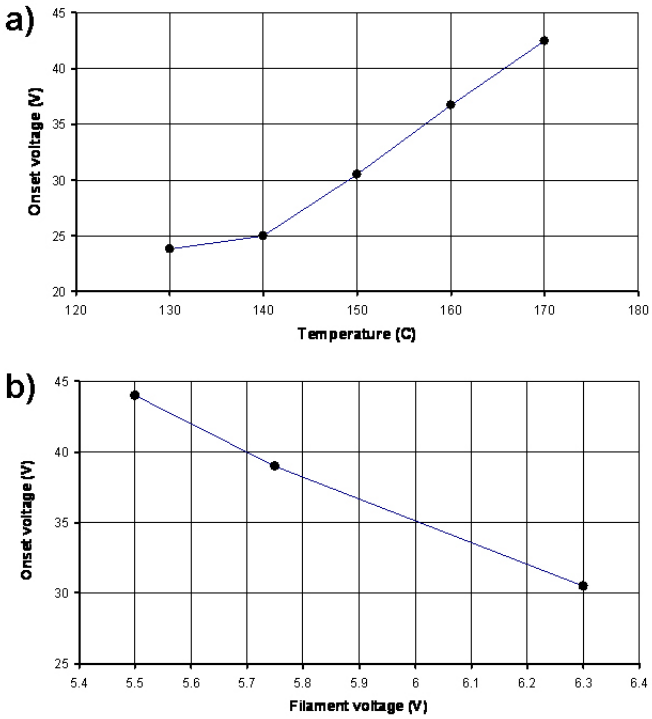

FIG. 6: The above graphs show how V_{onset}, the value of V_{acc} at which the blue-white plasma attaches itself to the filament, varies with temperature (a) and V_f (b). These graphs suggest that the relationship between V_{onset} and temperature is negative and linear, while the relationship between V_{onset} and V_f (and thus the electron density in the tube) is positive and linear.