Sub-Atomic Particles from Outer Space!

PHY 143, Fall 2007

1 Why You Are Doing This Lab

References to muons and other elementary particles are abundant in a variety of places, ranging from scientific journals to science fiction novels. Regardless of the setting, these sub-atomic particles are often portrayed in a mystical, mysterious, and sometimes even a humorous light. In the September 2007 issue of *The Scientific American*, for example, one can find an article titled "Muons for Peace"[1] (which, interestingly enough, now also serves as the name of a new Facebook group for Physics 143 students[2]). Do you ever find yourself wondering what the hype is all about? Well here is your chance to find out. While completing this lab you will:

- measure the lifetime of an elementary particle (the muon),
- gain familiarity with the electronics and techniques used to detect showers from cosmic rays,
- witness for yourself the effect of time dilation that is necessary to understand the behavior of these relativistic particles,
- and take a more in depth look at probability distributions.[3]

2 What is a Muon and Where Can I Get One?

The muon (denoted μ) is one of nature's fundamental building blocks of matter and acts in many ways as if it were an unstable heavy electron, for reasons no one fully understands. It was discovered in 1937 by C.W. Anderson and S.H. Neddermeyer when they exposed a cloud chamber to cosmic rays. In 1941 F. Rasetti demonstrated that it had a finite lifetime, which we now know to be approximately 2.197 μs. Other fun facts about the muon include: it is negatively charged spin-1/2 particle, has a positively charged antimatter pair (the antimuon), has a mass of 105.7 MeV/c^2, is classified as a lepton, and is unusually penetrative of ordinary matter. In the rest of this document we will broaden the use of the term muon to refer to both muons and antimuons, which differ only in their charge.

[1]See the TA if you would like a copy of this article.
[2]See www.facebook.com if you would like to join.
[3]Sorry, I couldn't find a way to spice this one up.

So what are these so-called cosmic rays that produce muons? Essentially, they are high energy particles produced in other parts of the universe (by mechanisms that are again not fully understood) that bombard the top of the earth's atmosphere. The composition of these "primary cosmic rays" is somewhat energy dependent, but a useful approximation is that 98% of these particles are protons or heavier nuclei and 2% are electrons. Of the protons and nuclei, about 87% are protons, 12% helium nuclei and the balance are still heavier nuclei that are the end products of stellar nucleosynthesis.

The primary cosmic rays collide with the nuclei of air molecules and produce a shower of particles that include protons, neutrons, pions (both charged and neutral), kaons, photons, electrons, and positrons. These secondary particles then undergo electromagnetic and nuclear interactions to produce yet additional particles in a cascade process. Figure 1 indicates the general idea. Of particular interest is the fate of the charged pions produced in the cascade. Some of these will interact via the strong force with air molecule nuclei, but others will spontaneously decay via the weak force into a muon plus a neutrino or antineutrino.

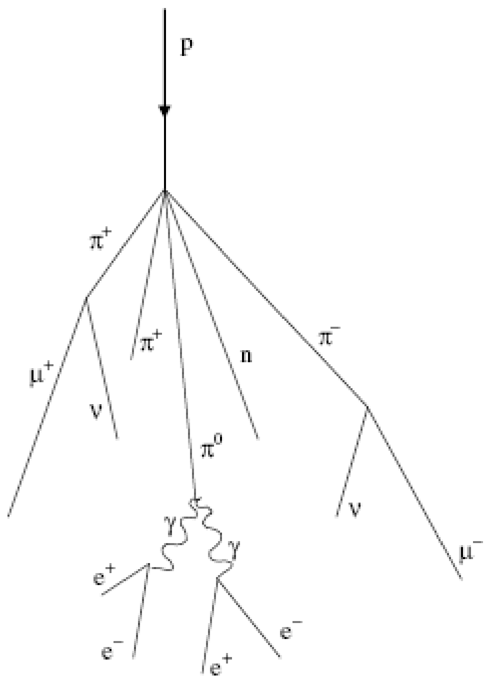

Figure 1: Cosmic ray cascade induced by a cosmic ray proton striking an air molecule nucleus.

The muon does not interact with matter via the strong force but only through the weak and electromagnetic forces. It travels a relatively long distance while losing its kinetic energy and decays by the weak force into an electron plus a neutrino and antineutrino. We will detect the decays of some of the muons produced in the cascade. (Our detection efficiency for the neutrinos and antineutrinos is utterly negligible.)

Not all of the particles produced in the cascade in the upper atmosphere survive down to sea-level due to their interaction with atmospheric nuclei and their own spontaneous decay. The flux of sea-level muons is approximately one per minute per square centimeter with a mean kinetic energy of about 4 GeV.

Careful study shows that the mean production height in the atmosphere of the muons detected at sea-level is approximately 15 km. Travelling at the speed of light, the transit time from production point to sea-level is then 50 μs. Since the lifetime of at-rest muons is more than a factor of 20 smaller, the appearance of an appreciable sea-level muon flux is qualitative evidence for the time dilation effect of special relativity.

3 Muon Math

The decay times for muons are easily described mathematically. Suppose at some time t we have $N(t)$ muons. If the probability that a muon decays in some small interval dt is λdt, where λ is a constant "decay rate" that characterizes how rapidly a muon decays, then the change dN in our population of muons is just

$$dN = -N(t)\lambda dt, \tag{1}$$

or

$$dN/N(t) = -\lambda d. \tag{2}$$

Integrating, we have

$$N(t) = N_0 \exp(-\lambda t), \tag{3}$$

where $N(t)$ is the number of surviving muons at some time t and N_0 is the number of muons at $t = 0$. This simple exponential relation is typical of radioactive decay. What we refer to as the "lifetime" τ of a muon is just the reciprocal of λ. Note that this does not mean that each muon decays after a time τ, but instead if we average the decay times of a bunch of muons we will approach τ as we include more muons in our average.

Now, we do not have a single clump of muons whose surviving number we can easily measure. Instead, we detect muon decays from muons that enter our detector at essentially random times, typically one at a time. It is still the case that their decay time distribution has a simple exponential form of the type described above. By decay time distribution $D(t)$, we mean that the time-dependent probability that a muon decays in the time interval between t and $t + dt$ is given by $D(t)dt$. If we had started with N_0 muons, then the fraction $-dN/N_0$ that would on average decay in the time interval between t and $t + dt$ is just given by differentiating the above relation:

$$-dN = N_0 \lambda e^{-\lambda t} dt \tag{4}$$

$$-dN/N_0 = \lambda e^{-\lambda t} dt \tag{5}$$

The left-hand side of the last equation is nothing more than the decay probability we seek, so

$$D(t) = \lambda e^{-\lambda t}. \tag{6}$$

This is true regardless of the starting value of N_0. That is, the distribution of decay times for new muons entering our detector is also exponential with the very same exponent used to describe the surviving population of muons.

Because the muon decay time is exponentially distributed, it does not matter that the muons whose decays we detect are not born in the detector but somewhere above us in the atmosphere. An exponential function always "looks the same" in the sense that whether you examine it at early times or late times, its e-folding time is the same.

4 General Description of the Experiment

4.1 The Apparatus

The active volume of the detector is a plastic scintillator in the shape of a right circular cylinder of 15 cm diameter and 12.5 cm height placed at the bottom of the black anodized aluminum alloy tube. A charged particle passing through the scintillator will lose some of its kinetic energy by ionization and atomic excitation of the molecules that make up the plastic. Upon radiative de-excitation, light is emitted with a typical decay time of a few nanoseconds. A typical photon yield for a plastic scintillator is one optical photon emitted per 100 eV of deposited energy.

4.2 The Detection Process

To measure the muon's lifetime, we are interested in only those muons that enter, slow, stop and then decay instide the plastic scintillator. Figure 2 summarizes this process. Such muons have a total energy of only about 160 MeV as they enter the tube. As a muon slows to a stop, the excited scintillator emits light that is detected by a photomultiplier tube (PMT), eventually producing a logic signal that triggers a timing clock. A stopped muon, after a bit, decays (usually) into an electron, a neutrino, and an anti-neutrino. Since the electron mass is so much smaller than the muon mass, $m_\mu/m_e \approx 210$, the electron tends to be very energetic and to produce scintillator light essentially all along its pathlength. (The neutrino and anti-neutrino also share some of the muon's total energy, but they entirely escape detection.) This second burst of scintillator light is also seen by the PMT and used to trigger the timing clock. The distribution of time intervals between successive clock triggers for a set of muon decays is the physically interesting quantity used to measure the muon lifetime.

The detector responds to any particle that produces enough scintillation light to trigger its readout electronics. These particles can be either charged, like electrons or muons, or neutral, like photons, that produce charged particles when they interact inside the scintillator. Now, the detector has no knowledge of whether a penetrating particle stops or not inside the scintillator and so has no way of distinguishing between light produced by muons that stop and decay inside the detector, from light produced by a pair of through-going muons that occur one right after the other. This important source of background events can be dealt

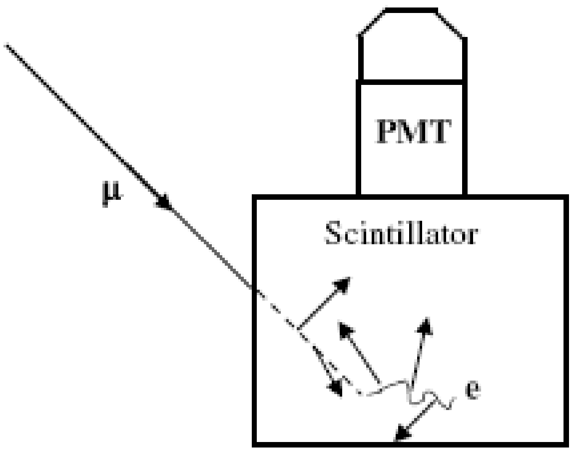

Figure 2: Schematic showing the generation of the two light pulses (short arrows) used in determining the muon lifetime. One light pulse is from the slowing muon (dotted line) and the other is from its decay into an electron or positron (wavey line).

with in two ways. First, we can restrict the time interval during which we look for the two successive flashes of scintillator light characteristic of muon decay events. Secondly, we can estimate the background level by looking at large times in the decay time histogram where we expect few events from genuine muon decay.

4.3 The Effect of Time Dilation

In addition to the fact that we can detect muons with a lifetime much less than the transit time needed for them to travel from their birth place to earth (as measured in our frame of reference), we can also use this device to probe the effects of time dilation another way. The muon stopping rate, $S(h)$[4], has been measured using a detector of the same design at two elevations much higher than the one where our apparatus sits.[5] Due to the finite size of the detector, only muons with a typical total energy of about 160 MeV will stop inside the plastic scintillator. $S(h)$ is therefore proportional to the flux of muons with a total energy of about 160 MeV, and this flux decreases with diminishing altitude as the muons descend and decay in the atmosphere. We will find that the ratio of the stopping rates is $R = S(h_{low})/S(h_{high}) = R_0 e^{-time/\tau}$, where R_0 is a correction factor that will be defined later. We can thus make two predictions for the stopping rate ratio R: one with the effect of time dilation ($time = t'$) and one without ($time = t$). Since we are given $S(h_{high})$, we can measure the stopping rate in our lab ($S(h_{low})$) and use it to experimentally determine $R \pm \delta R$. A comparison between our experiment and the two competing theories should tell

[4]That is, the number of muons that stop in the detector at height h per unit time. For measurement purposes, we can say that this is approximately equal to the decay rate that we measure with our detector (since muons that stop will decay about 2 μs later on average).

[5]see the AJP paper on the lab website

us whether or not this “time is relative” business should be taken seriously.

4.4 The Statistics

Since muons enter our detector at random times but with an average rate, the probability of a muon entering our detector in a given time interval should be well described by the Poisson distribution:

$$P(\nu) = e^{-\mu}\frac{\mu^{\nu}}{\nu!}, \tag{7}$$

were ν is the number of events observed in some time interval T. This distribution has the properties that μ is the expected average number of events in time T and $\sqrt{\mu}$ is the standard deviation of the observed number ν.

Before you begin the ‘B’ period you should take the time to read about the Poisson distribution in Chapter 11 of John Taylor’s book on error analysis. (If there are not enough copies of the book available for everyone in your group, ask the TA to make copies.)

5 Procedure

Caution:

- You cannot break anything unless you drop the detector on the floor or do something equally dramatic.
- All inputs and outputs have an impedance of 50Ω so make sure that the input impedance of your scope matches or your signals will be distorted.

5.1 Tinkering

A good way to start any experiment is to spend some time tinkering with the apparatus. First turn on the power to the electronics box. The red LED power light should be steadily shining and the green LED may or may not be flashing. Figure out how to set the High Voltage (HV) to the Photo Multiplier Tube (PMT) somewhere between -1100 and -1200 Volts using the knob at the top of the detector tube. The exact setting is not critical for now and the voltage can be monitored by using the multimeter probe connectors at the top of the detector tube. *Hint: You are only measuring a fraction of the actual voltage.*

You can look directly at the output of the PMT using the *PMT Output* on the detector tube and a digital oscilloscope. As previously mentioned, be certain to terminate the scope input at 50Ω or your signal will be distorted. **What do you see? Make note of the sign and approximate shape of the signal.**

Next, connect the BNC cable between *PMT Output* on the detector and *PMT Input* on the box. Adjust the discriminator setting on the electronics box so that it is in the range 180-220 mV. The green LED on the box front panel should now be flashing, with each flash (or beep if the sound is turned on) corresponding to a "detected muon." Observe the output of the amplifier and discriminator on your digital scope. **What do you see? Make note of the signs and approximate shapes of the signals.** *Hint: In order to avoid drawing too much current, there are attenuation resistors in between the amplifier output and the front panel connector. So the voltage you measure with your scope is a factor of 21 less than the actual output of the amplifier.*

The apparatus also comes with software to help control the instrument and to record and process the raw data. The program you will use is called "muon" and is located in the folder "muon_data." This is the principle data acquisition program used to collect real data. This program stores its data in one of two files: the file named "data" or a file named with the date on which the data was collected. Open the program and you will see an interface with allows you to set port setting on your PC, observe various data rates, control how data is displayed on the PC, and to fit the collected data.

Select the configure sub-menu and specify port 3. Make sure that the apparatus is connected to the computer with the USB cable and then hit *Start* in the main menu. The data acquisition should begin and you should immediately see data begin to be displayed on the screen. Take a minute or two to figure out what information is being displayed by which graphs.

Next you will want to figure out the basics of what is going on inside the detector and electronics box. By adjusting (or misadjusting) both the discriminator threshold and HV (but probably not at the same time), try to determine what impact each has on the detection process. The block diagram in Figure 3 along with the answers to the questions below may help.

- **Under what conditions do you see both an amplifier pulse and a discriminator pulse?**
- **Under what conditions do you see an amplifier pulse but no discriminator pulse?**
- **How do the values for the HV and discriminator threshold affect the rate at which muons are detected?**

Once you feel that you understand the basics of what the apparatus is doing, you are ready to take some actual data. Set the HV to -1150 V and the discriminator threshold to 200 mV and observe the various rates displayed on the screen.

5.2 Lifetime/Stopping Rate Measurement

For the actual measurement we found that you can get better data upstairs in the High Energy Physics clean room (room 216). Carefully move the apparatus upstairs and set up the instrument for a muon lifetime measurement (which will simultaneously serve as a low elevation stopping rate measurement). With the HV set to -1150 V and the discriminator threshold to 200 mV, start and observe the decay time spectrum. The average rate of muons

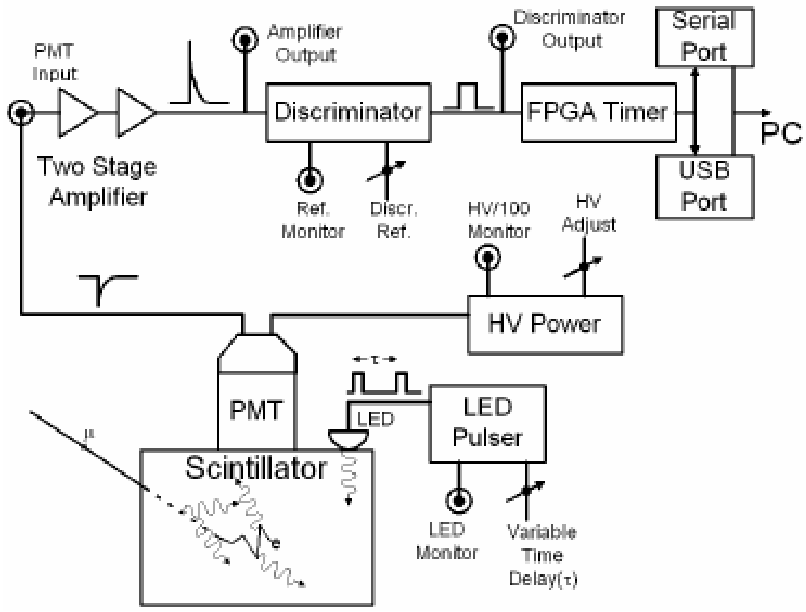

Figure 3: Block diagram of the readout electronics.

through the detector should be slightly less than in the sub-basement, which at first glance seems counter-intuitive.[6]

You will leave the experiment running for an amount of time determined by your group (I recommend at least 24 hours). When you are ready, restart the program, select com 3, and a bin size that is appropriate for the length of your experimental run. The program will make a histogram of the time between a muon entering the detector and its subsequent decay, and we will use this to determine the lifetime of the muon. **Note, however, that the muons whose decays we observe are born outside the detector and therefore spend some unknown portion of their lifetime outside the detector. So, we never measure the actual lifetime of any muon. Yet, we claim we are measuring the lifetime of muons. How can this be?**

When your experimental run is completed hit the *Pause* button, followed by the *Fit* button. You will be prompted to enter a password, which was cleverly chosen to be *muon*. Record all of the necessary information off the screen in your lab notebook (you may want to save an image of the screen as well). **What do you measure the lifetime of the muon to be? How well does it agree with the accepted value, and how good is the fit?** *Hint: There is a discussion of the χ^2 test for a probability distribution in Chapter 12 of Taylor.*

For those who are interested, there is a discussion of the muon lifetime fitter in the Appendix.

[6]If you are curious as to how this could happen, ask the TA.

5.3 Stopping Rate Ratio Calculations

A prediction of the muon stopping rate ratio between Durham and the elevation of your choice should depend on the vertical transit time between the two heights, variations with energy in the shape of the muon energy spectrum, and the varying zenith angles of the muons that stop in the detector.

Let's tackle the transit time first. Like all charged particles, a muon loses energy through coulombic interactions with the matter it traverses. Its velocity will therefore change as descends through the atmosphere and loses energy. The average energy loss rate in the matter for singly charged particles traveling close to the speed of light is approximately 2 MeV/g/cm^2, where we measure the effective thickness s of the matter in units of g/cm^2. Here $s = \rho x$, where ρ is the mass density of the material through which the particle is passing, measured in g/cm^3, and the x is the particle's pathlength, measured in cm.[7] A simple estimate of the energy lost ΔE by a muon as it travels a vertical distance h is $\Delta E = 2$ MeV/g/cm$^2 \times h \times \rho_{air}$, where ρ_{air} is the density of air, possibly averaged over h.

If the transit time for a particle to travel vertically from some height h_1 down to height h_2, all measured in the lab frame, is denoted by t, then the corresponding time in the particle's rest frame, t', is given by

$$t' = \int_{h_1}^{h_2} \frac{dh}{c\beta(h)\gamma(h)} \tag{8}$$

Here β and γ have their usual relativistic meanings for the projectile and are measured in the lab frame. Since relativistic muons lose energy at essentially a constant rate when travelling through a medium of mass density ρ, $dE/dx = C_0$, so we have $dE = \rho C_0 dh$, with $C_0 = 2\text{MeV}/(\text{g/cm}^2)$. Also, from the Einstein relation, $E = \gamma mc^2$, $dE = mc^2 d\gamma$, so $dh = (mc^2/\rho C_0)d\gamma$. Hence,

$$t' = \frac{mc}{\rho C_0}\int_{\gamma_1}^{\gamma_2} \frac{d\gamma}{\beta\gamma} = \frac{mc}{\rho C_0}\int_{\gamma_1}^{\gamma_2} \frac{d\gamma}{\sqrt{\gamma^2 - 1}}. \tag{9}$$

Here γ_1 is the muon's gamma factor at height h_1 and γ_2 is its gamma factor just before it enters the scintillator at height h_2. We can take $\gamma_2 = 1.5$ since we want muons that stop in the scintillator and assume that on average stopped muons travel halfway into the scintillator, corresponding to a distance $s = 10$ g/cm^2. The entrance muon momentum is then taken from range-momentum graphs at the Particle Data Group website and the corresponding γ_2 computed. The lower limit of integration is given by $\gamma_1 = E_1/mc^2$, where $E_1 = E_2 + \Delta E$, with $E_2 = 160$ MeV. The integral can be evaluated numerically, or looked up in a mathematical handbook.

Hence, the ratio R of muon stopping rates for the same detector at two different positions separated by a vertical distance $h_1 - h_2$, and ignoring for the moment any variations in the shape of the energy spectrum of muons, is just $R = exp(-t'/\tau)$, where τ is the muon proper lifetime.

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[7] This way of measuring material thickness in units of g/cm^2 allows us to compare effective thicknesses of two materials that might have very different mass densities.

This is not the full story, however. We also need to account for variations in the energy spectrum. Here is why: Since the detector stops only low energy muons, the stopped muons detected by the low altitude detector will, at the elevation of the higher altitude detector, necessarily have a greater energy. This energy difference $\Delta E(h_1 - h_2)$ will clearly depend on the pathlength between the two detector positions. If the shape of the muon energy spectrum changes significantly with energy, then the relative muon stopping rates at the two different altitudes will reflect this difference in the spectrum shape at the two different energies.[8] Indeed, the muon momentum spectrum does peak, at around $p = 500$ MeV/c or so, although the precise shape is not known with high accuracy.

We therefore need a way to correct for variations in the shape of the muon energy spectrum in the region from about 160 MeV - 800 MeV. The authors of the AJP paper did this by first measuring the muon stopping rate at two different elevations and then computing the ration R_{raw} of raw stopping rates. Next, using the above expression for the transit time between the two elevations, they computed the transit time in the muon's rest frame for vertically travelling muons and calculate the corresponding theoretical stopping rate ratio $R = exp(-t'/\tau)$. They then computed the double ratio $R_0 = R_{raw}/R = 1.5 + 0.2$ of the measured stopping rate ratio to this theoretical rate ratio and we can interpret this as a correction factor to account for the increase in muon flux between about $E = 160$ MeV and $E = 600$ MeV. This correction is to be used in all subsequent measurements for any pair of elevations.

To calculate the stopping rate ratio without the effects of time dilation, we would again need to know the velocity of the muons as they descended from h_1 to h_2. If we instead pretend that all of the muons travel at the speed of light for the entire trip we can come up with an (under)estimate for the transit time t in our frame and then use this to calculate a naive estimate for the stopping rate ratio without the effects of time dilation.

Your job is to calculate R with and without the effect of time dilation, and then compare both of these to your experimental results. **How well do each of your calculations agree with the measured stopping rate ratio? Keeping in mind your assumptions and the statistical[9] error in your measurement, what can you conclude, if anything, from this comparison?** *Hint: You do not need to estimate the error in your calculation, only the error in your measurement.*

5.4 The Poisson Distribution

The Poisson distribution is used to model situations in which events occur at random times, but with a definite average rate. Our muon experiment is an example of one of these situations, since we can "see" muons hitting our detector at an average rate (for a fixed altitude) but cannot accurately predict when the next muon will arrive. Therefore the Poisson distribution, $P_\mu(\nu)$, should reasonably describe how many of these "events" (ν) we count in a given time interval:

$$P_\mu(\nu) = e^{-\mu}\frac{\mu^\nu}{\nu!}. \tag{10}$$

[8] To see this it may help to first pretend that muons do not decay at all.

[9] You may need to see the next section before you can calculate the statistical error.

With a little math (see Taylor) we can show that the average number of events that occur in this time interval is given by μ and the standard deviation is given by $\sqrt{\mu}$.

To give you some practice with the Poisson distribution, make several (≈ 200) measurements of the number of muons that hit the detector in a given time interval. (A convenient choice for the time interval is one second, since this number is displayed and updated each second.) Using a spreadsheet program, make frequency plots of your first 50 measurements and your entire data set. On each plot overlay the Poisson distribution parametrized by the average of the corresponding data set. **Describe what happens as you include more measurements in your data set.**

In certain situations the Poisson distribution is well approximated by the Gaussian distribution with the same mean and standard deviations. **Under what condition is this true, and is this condition satisfied in our experiment involving the number of muons incident on the detector?** Justify your answer with both a discussion and a visual representation.

The Poisson distribution governs not only the number of muons incident on the detector in a given time interval (as witnessed in the most recent experiment), but also the number of muons that are stopped in a given time interval. Thus, we can use the square-root rule to estimate our uncertainty in the number of stopped muons recorded during our experimental run, and use this to find the statistical error associated with the stopping rate. **What, therefore, is the error associated with the observed stopping rate? How could you minimize the error in your measurement for the stopping rate?** *Hint: Think carefully about what you should take the square root of.*

6 Closing Remarks

By the time you sit down to write your lab report you will have become acquainted with a sub-atomic particle from outer space, tinkered with high-speed electronics similar to those used to detect particles in laboratories around the world, seen for yourself that absolute time is not consistent with experiments where stuff moves really fast, and learned about the Poisson distribution.[10] To demonstrate all that you have learned and accomplished, your lab report should contain a brief discussion about muons, a description of the apparatus and how it functions, the results of each of your experiments, and the overall conclusions reached. And as with all of the labs, make sure the answers to the questions asked in the lab description (written in bold) appear somewhere in your lab report and that your results are presented clearly, concisely, and in a logical order.

7 Appendix: Muon Lifetime Fitter

The included muon lifetime fitter for the decay time histogram assumes that the distribution of times is the sum of an exponential distribution and a flat distribution. The exponential distribution is attributed to real muon decays while the flat distribution is attributed to

[10] Again, there is just no way to make this sound exciting.

background events. The philosophy of the fitter is to first estimate the flat background from the data at large nominal decay times and to then subtract this estimated background from the original distribution to produce a new distribution that can then be fit to a pure exponential.

The background estimation is a multi-step process. Starting with the raw distribution of decay times, the program fits the distribution with an exponential to produce a tentative lifetime τ'. It then fits the part of the raw distribution that has times greater than $5\tau'$ with a straight line of slope zero. The resulting number is the first estimate of the background. The program then subtracts this constant number from all bins of the original histogram to produce a new distribution of decay times. Again, it fits to produce a tentative lifetime τ'' and fits again the part of this new distribution that has times greater than $5\tau''$. The tentative background level is subtracted from the previous distribution to produce a new distribution and the whole process is repeated again for a total of 3 background subtraction steps.