HOMEWORK 4 – Q2

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2. You are given a 2D map consisting of an $R \times C$ grid of squares; in each square there is a number representing the elevation of the terrain at that square. Find a path going from square $(1, R)$ which is the top left corner of the map to square $(C, 1)$ in the lower right corner which from every square goes only to the square immediately below or the square immediately to the right so that the number of moves from lower elevation to higher elevation along such a path is as small as possible. (20 pts)

Solution:

Subproblems: what is the smallest number of moves from lower elevation to higher elevation we need to get arriving at the square at row i and column j

Build-up order:

Solve the subproblems in the order:

$$\begin{gathered} opt(1,R), opt(1,R-1) \dots opt(1,1) \\ opt(2,R), opt(2,R-1) \dots opt(2,1) \\ \vdots \\ opt(C,R), opt(C,R-1) \dots opt(C,1) \end{gathered}$$

Recursion:

$$\begin{gathered} opt(i,j) = \min(opt[i-1][j] + E((i-1,j),(i,j)), \\ opt[i][j-1] + E((i,j-1),(i,j))) \end{gathered}$$

$$E\big((i1,j1),(i,j)\big) = \begin{cases} 1 & if\ (i1,j1)\ is\ lower, and\ (i,j)\ is\ higher \\ 0 & otherwise \end{cases}$$

Base case:

$$opt(1,R) = 0$$

Final solution:

$$The\ final\ solution\ is\ given\ by\ opt(C,1)$$

Time complexity:

The complexity is $O(RC)$, we go through each row and each column to determine the next step.

$opt[1][R] = 0$

minPath(map):

for i in range(R,0,-1):

if ($map[0][i]$ is higher and $map[0][i-1]$ is lower):

$opt[0][i] = 1$

$else$:

$opt[0][i] = 0$

for j in range(C):

if ($map[j][R]$ is higher and $map[j-1][R]$ is lower):

$opt[j][R] = 1$

$else$:

$opt[j][R] = 0$

for i in range(R,0,-1):

for j in range(1, C):

$opt[i][j] = \min(opt[i-1][j] + (opt[i-1][j] < opt[i][j]), opt[i][j-1] + (opt[i][j-1] < opt[i][j]))$

return $opt[R-1][C-1]$