HOMEWORK 2 – Q3

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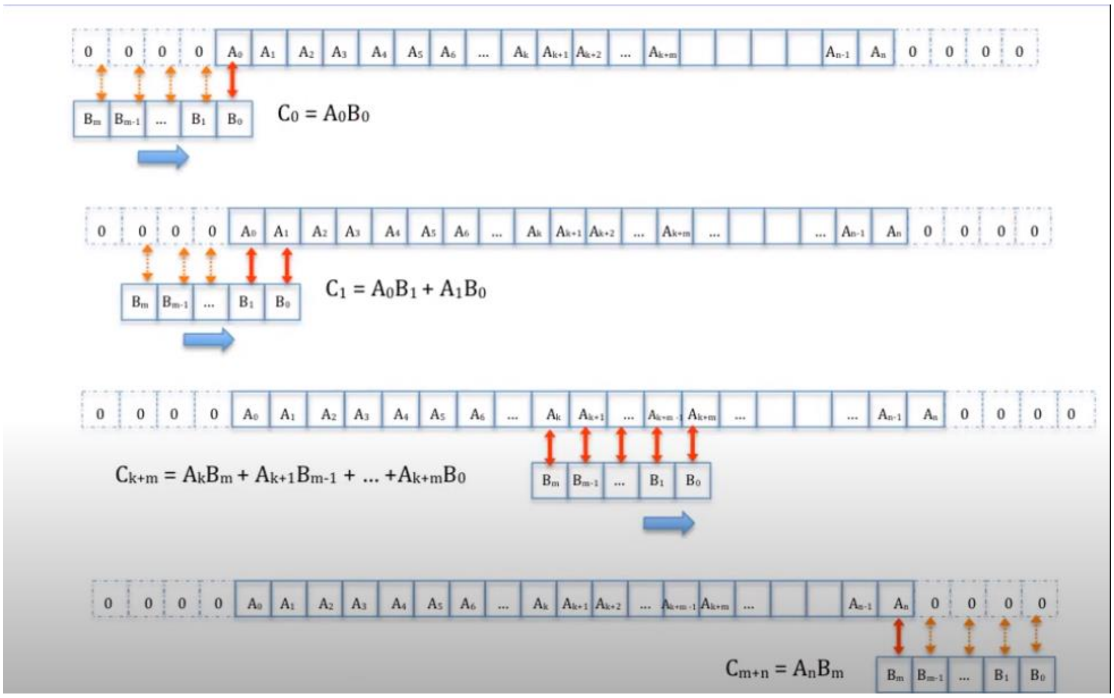
3. Assume you are given a map of a straight sea shore of length $100n$ meters as a sequence on $100n$ numbers such that A_i is the number of fish between i^{th} meter of the shore and $(i+1)^{th}$ meter, $0 \leq i \leq 100n - 1$. You also have a net of length n meters but unfortunately it has holes in it. Such a net is described as a sequence N of n ones and zeros, where 0's denote where the holes are. If you throw such a net starting at meter k and ending at meter $k + n$, then you will catch only the fish in one meter stretches of the shore where the corresponding bit of the net is 1. Find the spot where you should place the left end of your net in order to catch the largest possible number of fish using an algorithm which runs in time $O(nlog(n))$. (30 pts)

Solution:

First, we let $N^{'}$ be the net sequence N in the reverse order, and compute $N^{'}$ into a polynomial such as $P_N(x) = N_0 + N_1x + N_2x^2 ... + N_nx^{n-1}$. Then, we do the same thing for the sea shore A, let $P_A(x) = A_0 + A_1x + A_2x^2 ... + A_{100n-1}x^{100n-1}$. Once we got this, we can compute the sequence $A * N^{'}$ use FFT, and we have

$$
\begin{aligned}
A * N' = P_C(x) & \\
&= A_0N_0 + (A_0N_1 + A_1N_0)x \\
&+ (A_0N_2 + N_2A_0 + A_1N_1 \\
&+ N_1A_1)x^2 \dots A_{100n-1}N_{n-1}x^{101n-2}
\end{aligned}
$$

this will run in time $O(n\log_2 n)$.



Thus, we have a sequence of result $\langle A_0N_0, (A_0N_1 + A_1N_0), (A_0N_2 + N_2A_0 + A_1N_1 + N_1A_1), \dots, A_{100n-1}N_{n-1}\rangle$.

Then we just go through the sequence and find out which one have the largest value in $O(n)$ time.