HOMEWORK 5 – Q2

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2. You are given a usual $n \times n$ chess board with k white bishops on the board at the given cells (a_i, b_i), $(1 \le a_i, b_i \le n,\ 1 \le i \le k)$. You have to determine the largest number of black rooks you can place on the board so that no two rooks are in the same row or in the same column and are not under the attack of any of the k bishops (recall that bishops go diagonally).(20 pts)

Solution:

Make a bipartite graph with all the columns C_j as vertices on the left-hand side and all the rows R_i as vertices on the right-hand side. Each square s_{ij} can now be represented by edge from column j to row i. For each column C_j connect it to all rows R_i and of capacity each equal 1. Then for each white bishop on the board at the given cells $(a_i, b_i), (1 \le a_i, b_i \le n, 1 \le i \le k)$ delete all edges:

$$\begin{cases} from\ column\ b_i - d\ to\ row\ a_i - d\ (for\ 0 \le d \le n)\ where\ 1 \le b_i - d \le n,\ 1 \le a_i - d \le n \\ from\ column\ b_i + d\ to\ row\ a_i + d\ (for\ 0 \le d \le n)\ where\ 1 \le b_i + d \le n,\ 1 \le a_i + d \le n \\ from\ column\ b_i + d\ to\ row\ a_i - d\ (for\ 0 \le d \le n)\ where\ 1 \le b_i + d \le n,\ 1 \le a_i - d \le n \\ from\ column\ b_i - d\ to\ row\ a_i + d\ (for\ 0 \le d \le n)\ where\ 1 \le b_i - d \le n,\ 1 \le a_i + d \le n \end{cases}$$

This will delete all square that k white bishops can go.

Introduce a super source and a super sink, connect each column C_j on the left side with the super source by a directed edge of capacity equal to 1. Connect each row R_i on the right side with the super sink by a directed edge of capacity equal to 1.
Finally, use max flow algorithm, and look at occupied edges to determine the largest number of black rooks you can place on the board.