HOMEWORK 2 – Q4

MINGLANG XIE

z5228006

4. (a) Compute the convolution $\langle 1, \underbrace{0, 0, \ldots, 0}_{k}, 1\rangle * \langle 1, \underbrace{0, 0, \ldots, 0}_{k}, 1\rangle$ (10 pts)

   (b) Compute the DFT of the sequence $\langle 1, \underbrace{0, 0, \ldots, 0}_{k}, 1\rangle$ (10 pts)

Solution:

(a) Let $A = \langle 1,0,0,\ldots,0,1\rangle$, the corresponding polynomial is $P_A(x) = 1 + x^{k+1}$. Then the convolution $A * A = A^2 = (P_A(x))^2$, which is $(1 + x^{k+1})^2$.

$(1 + x^{k+1})^2 = 1 + 2x^{k+1} + x^{2k+2}$

Therefore, we $A * A = \langle 1, k\ zeros, 2, k\ zeros, 1\rangle$

(b) $DFT(A) = \langle P_A(w_{k+2}^0), P_A(w_{k+2}^1), \ldots, P_A(w_{k+2}^{k+1})\rangle$

$= \langle 1 + w_{k+2}^{0*(k+1)}, 1 + w_{k+2}^{1*(k+1)}, \ldots, 1 + w_{k+2}^{(k+1)*(k+1)}\rangle$

$= \langle 2, 1 + w_{k+2}^{(k+1)}, \ldots, 1 + w_{k+2}^{(k+1)^2}\rangle$