HOMEWORK 2 – Q5

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5. Find the sequence x satisfying $x * \langle 1,1-1\rangle = \langle 1,0,-1,2,-1\rangle$.(20 pts)

Solution:

Clearly, x is a sequence of length 5+1-3=3, write it as $\langle a,b,c\rangle$, the corresponding polynomial is $P_x(y) = a + by + cy^2$. For $A = \langle 1,1,-1\rangle$, the corresponding polynomial is $P_A(y) = 1 + y - y^2$. Then we have:

$$x * \langle 1,1-1\rangle = (a + by + cy^2) *(1 + y - y^2)$$

$$= a + ay - ay^2 + by + by^2 - by^3 + cy^2 + cy^3 - cy\text{^}4$$

$$= a + (a+b)y + (b+c-a)y^2 + (c-b)y^3 - cy^4$$

We know that $x * \langle 1,1-1\rangle = \langle 1,0,-1,2,-1\rangle$ and $x * \langle 1,1,-1\rangle = a + (a+b)y + (b+c-a)y^2 + (c-b)y^3 - cy^4$

Combine these two polynomials, we have:

$$\begin{cases} a = 1; \\ a + b = 0; \\ b + c - a = -1; \\ c - b = 2; \\ -c = -1; \end{cases}$$

By solving this, we can get $\begin{cases} a = 1 \\ b = -1. \\ c = 1 \end{cases}$

Therefore $x = \langle 1,-1,1\rangle$.