HOMEWORK 4 – Q4

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4. Given a weighted **directed** graph $G(V, E)$, find a path in G (possibly self-intersecting) of length exactly K that has the maximum total weight. The path can visit a vertex multiple times and can traverse an edge also multiple times. It can also start and end at arbitrary vertices or even start and end at the same vertex. (30 pts)

Solution:

Setup: Assume every vertex have a unit length to the other vertex.

Subproblems: For every node i and every $1 \leq k \leq K$ find the maximum weight path of length exactly k which ends at i.

Build-up order:

Solve the subproblems in the order $opt(i, 1), opt(i, 2) \ldots opt(i, k)$

Recursion:

$$opt(i, k) = \max_{p \in V}(opt(p, k-1) + w(e(p, i)))$$

$w(e(p, i))$ is the weight of the edge $e(p, i)$ from vertex p to vertex i

If $w(e(p, i)) = 0$ means there is no edge from vertex p to vertex i, then just skip this vertex p, try another vertex p.

Base case:

$$opt(i, 1) = \max_{p \in V}(w(e(p, i)))$$

Final solution:

$$The\ final\ solution\ is\ given\ by\ \max_{i \in V}(opt(i, K))$$

Time complexity:

The time complexity is $O(|V|^2 * |E|)$, we need to go through all vertex, and for each vertex, check whether other vertex is connected to this vertex.