HOMEWORK 3 – Q2

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2. Rock. Paper. Scissors. The rules are simple. The game is contested by two people over N rounds. During each round, you and your opponent simultaneously throw either Rock, Paper or Scissors. Rock beats Scissors, Scissors beats Paper, and Paper beats Rock. If your throw beats your opponent's, you gain one point. Conversely, if their throw beats yours, you lose one point. Your opponent is very predictable. You know that they will throw Rock in the first R_a rounds, throw Paper in the next P_a rounds, then finally throw Scissors in the last S_a rounds, where $R_a + P_a + S_a = N$. You have to throw Rock in R_b rounds, Paper in P_b rounds, and Scissors in S_b rounds, where $R_b + P_b + S_b = N$. However, as you are an experienced player, you may throw these in any order you like. At the beginning of the game, you start with 0 points. How should you play to maximise the number of points you can finish with?

Solution:

First, you know your opponent will throw Rock in the first R_a rounds, throw Paper in the next P_a rounds, then finally throw Scissors in the last S_a rounds, and you have to throw Rock in R_b rounds, Paper in P_b rounds, and Scissors in S_b rounds.

Second, you know Rock beats Scissors, Scissors beats Paper, and Paper beats Rock. If you throw beats your opponent's, you gain one point. Conversely, if their throw beats yours, you lose one point. If you throw the same as your opponent's, you tie for this round (not gain, no lose).

Then, if you want to maximize the number of points you can finish with, you will want to win every round, but it's impossible unless you don't have same number of Rock as your opponent's Scissors, same number of Scissors as your opponent's Paper, and same number of Paper as your opponent's Rock. So, to be able to maximize the number of points, you will want to tie a round instead of lose a round.

Therefore, you can first throw the same number of Paper as your opponent's Rock to win all R_a round. If there is not enough Paper, you need to throw a Rock to make a tie unless there is not enough Rock, then you must throw Scissor though will lose a round. If there are some Papers left after R_a round, then leave these Papers to P_a round in order to make some tie round; If there are some Rocks left after R_a round, then leave these Rocks to S_a rounds in order to make some winning round.

After your opponent throw all Rock, and start to throw Paper, you then throw your Scissor to beats your opponent's Paper. If there is not enough Scissor, you need to throw a Paper to make a tie unless there is not enough Paper, then you

must throw Rock though will lose a round. If there are some Paper left after P_a round, then there will be no way to make a tie or a win for these Papers, which means you have to lose some points that are the same as the number of Papers; If there are some Rocks left after P_a round, then leave these Rocks to S_a rounds in order to make some winning round.

Finally, your opponent throws all Rock, Paper, and start to throw Scissor, you then throw your Rock to beats your opponent's Paper. If there is not enough Rock, you need to throw a Scissor to make a tie unless there is not enough Scissor, then you must throw Paper though will lose a round.

Assume there is a strategy S which is better than this strategy T. For example, your opponent throws three rocks first, then two Papers, finally two Scissors, and you must throw three Rocks, two Papers, and two Scissors. You throw two Papers to beat two Rocks first, then you throw a Scissor which lose a round, then you throw a Scissor beats Paper, then you only have three Rocks left, which make you lose a round in P_a round, and win S_a round. Finally, you are getting 3 points. However, by using strategy T, you will be able to get 6 points which is more than 3 points getting from strategy S. Therefore, this strategy T is optimal.