HOMEWORK 5 – Q3

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3. There are N computers in a network, labelled $\{1, 2, 3, \ldots, N\}$. There are M one-directional links which connect pairs of computers. Computer 1 is trying to send a virus to computer N. This can happen as long as there is a path of links from computer 1 to computer N. To prevent this, you've decided to remove some of the links from the network so that the two computers are no longer connected. For each link, you've calculated the cost of removing it. What is the minimum total cost to disconnect the computers as required, and which edges should be removed to achieve this minimum cost? (20 pts)

Solution:

We first construct a corresponding flow network with the computers as vertices and computer 1 as the source and computer N as the sink. If there is a link l between two computers i and j, we connect i and j with a directed edge and of capacity each equal to the cost of removing link l. We now find max flow (such as Edmond-Karp's algorithm) in such a network, max flow ($|f_{max}|$) is the minimum total cost to disconnect the computers as required, and links that need to be removed are those that cross the corresponding min cut.

This completes the algorithm, the complexity of the algorithm will be the same as Edmond-Karp's algorithm, which is $O(|V||E|^2)$ for V vertices and E edges.