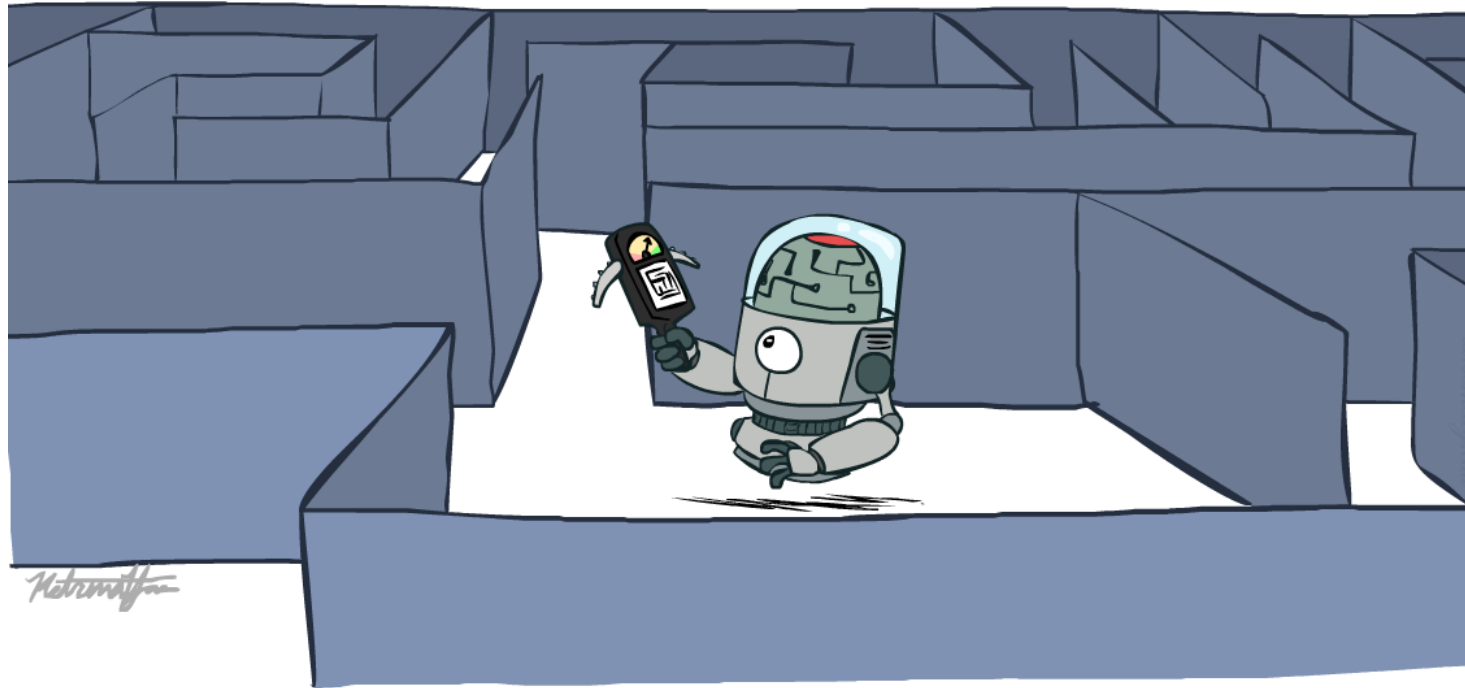


CS 480: Introduction to Artificial Intelligence

Search Continued

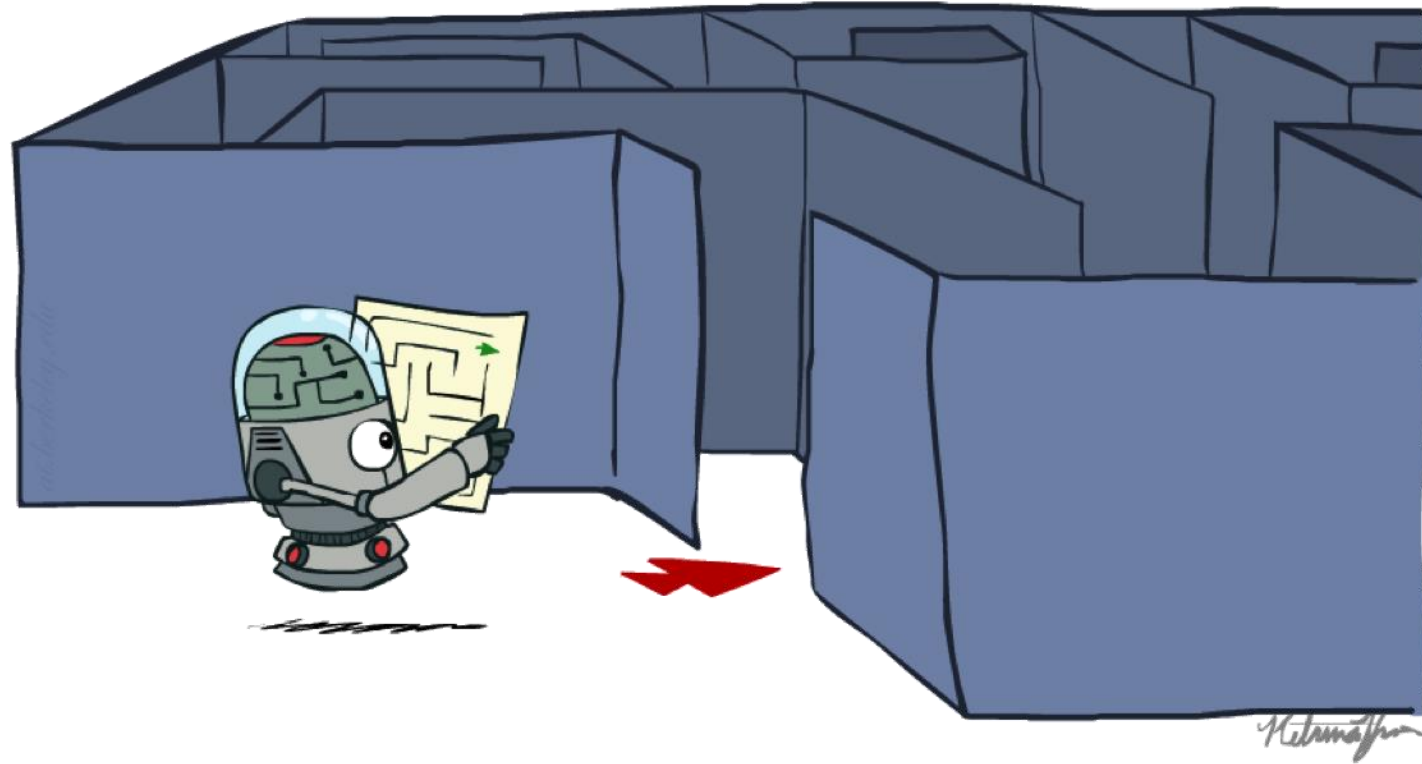


Instructors: Zoran Duric

George Mason University

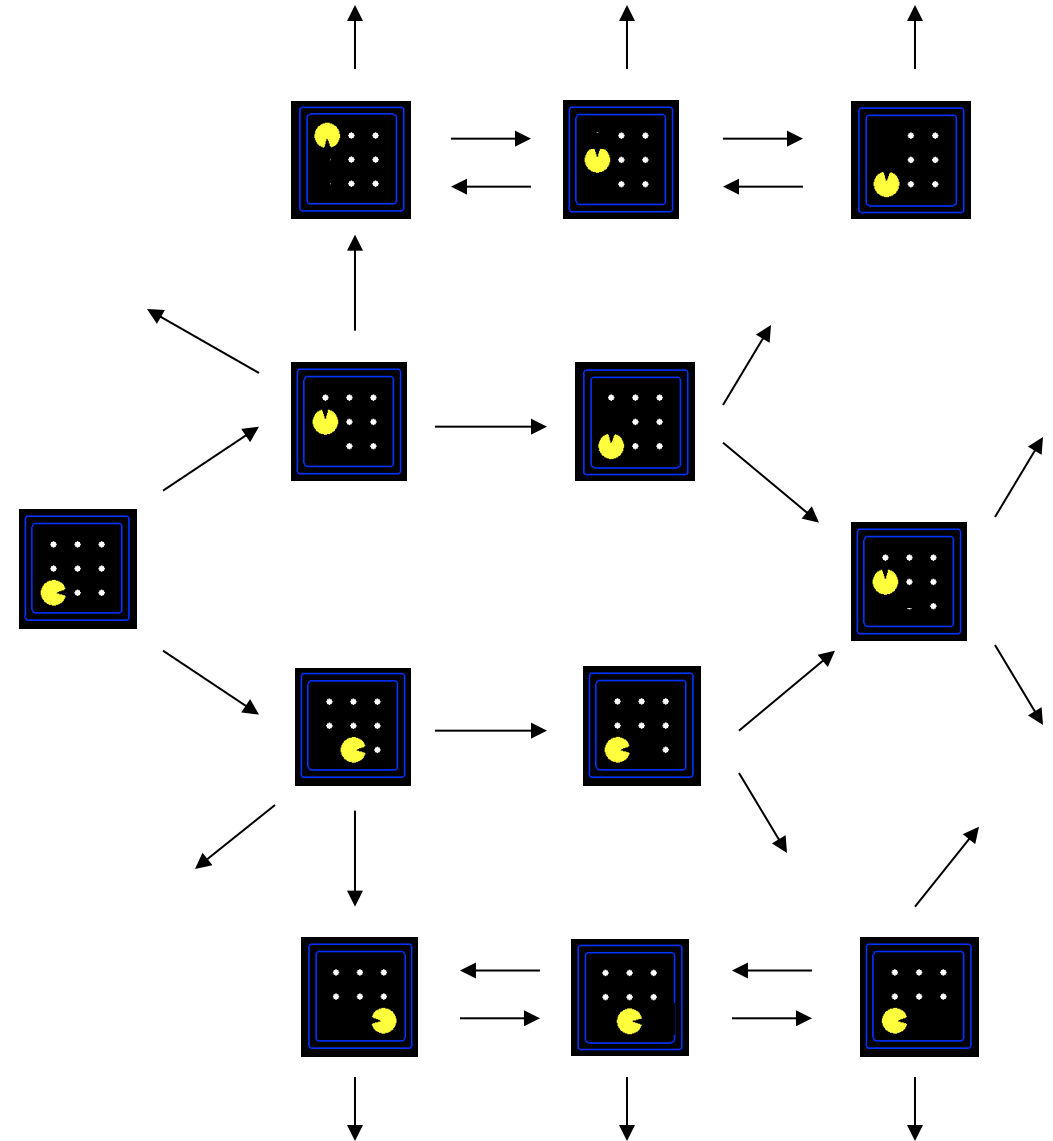
[These slides adapted from Dan Klein and Pieter Abbeel; ai.berkeley.edu]

Recap: Search



Search

- Search problem:
 - States (abstraction of the world)
 - Actions (and costs)
 - Successor function (world dynamics):
 - $\{s' \mid s, a \rightarrow s'\}$
 - Start state and goal test



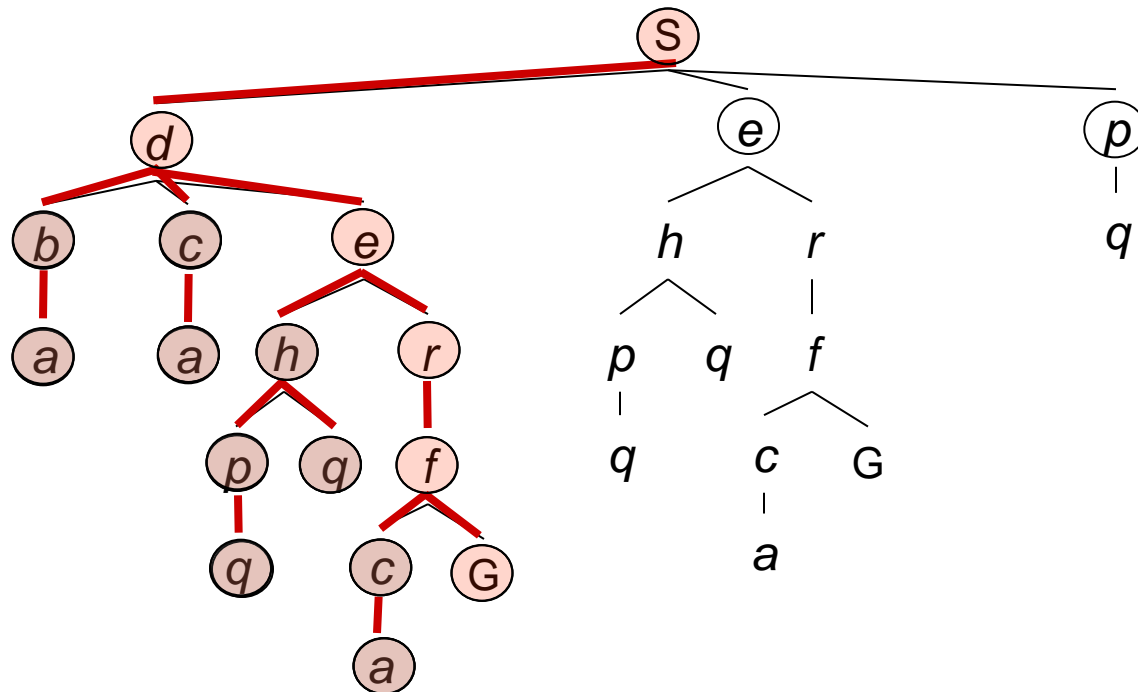
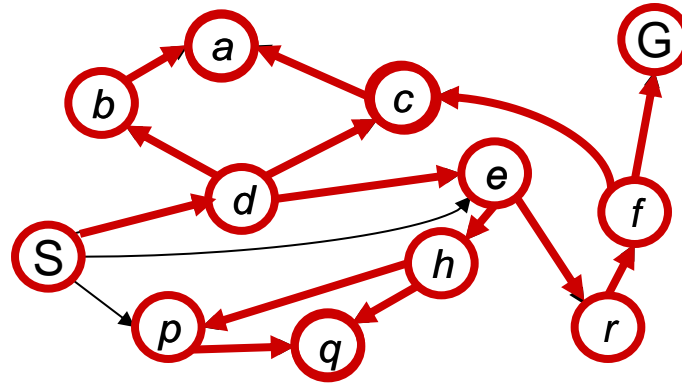
Depth-First Search



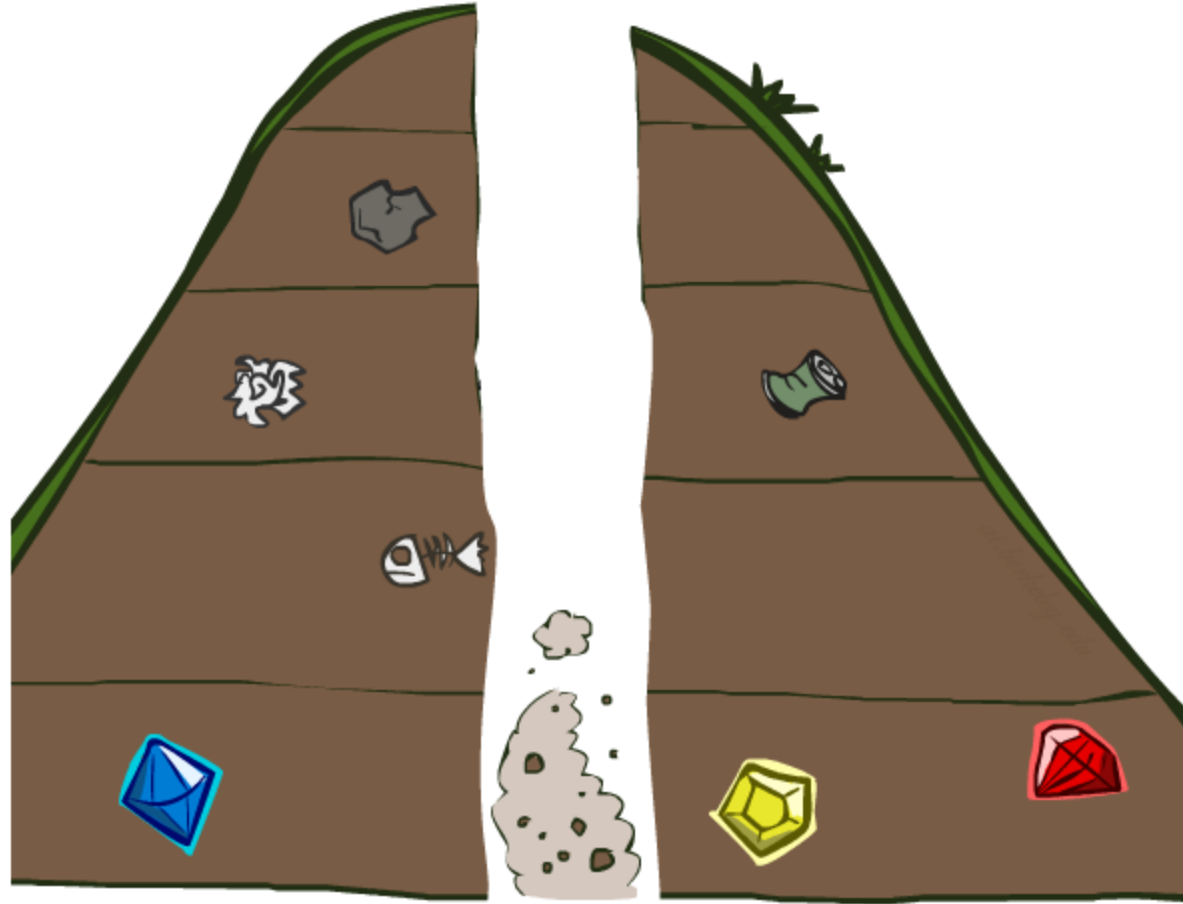
Depth-First Search

*Strategy: expand a
deepest node first*

*Implementation:
Fringe is a LIFO stack*



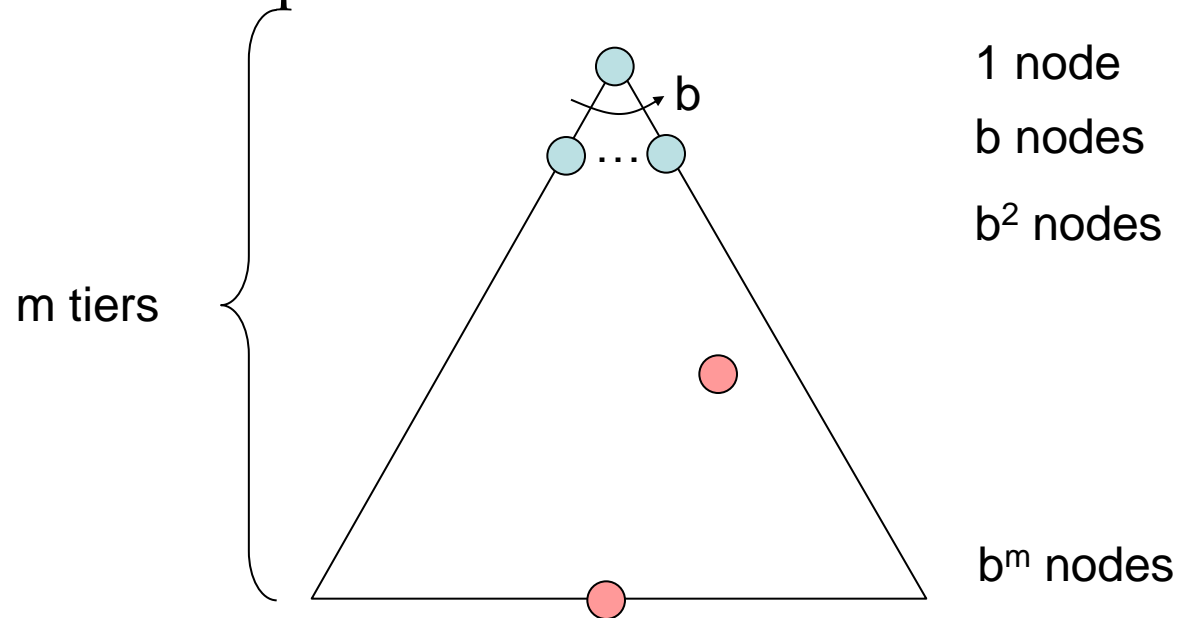
Search Algorithm Properties



Search Algorithm Properties

- Complete: Guaranteed to find a solution if one exists?
 - Return in finite time if not?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?

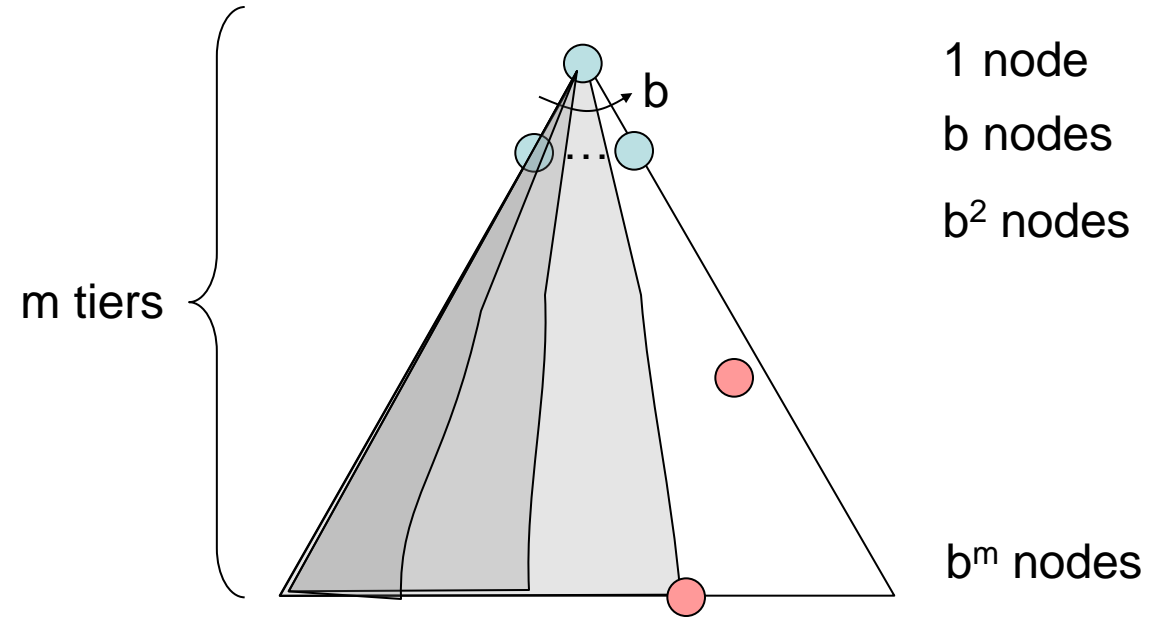
- Cartoon of search tree:
 - b is the branching factor
 - m is the maximum depth
 - solutions at various depths



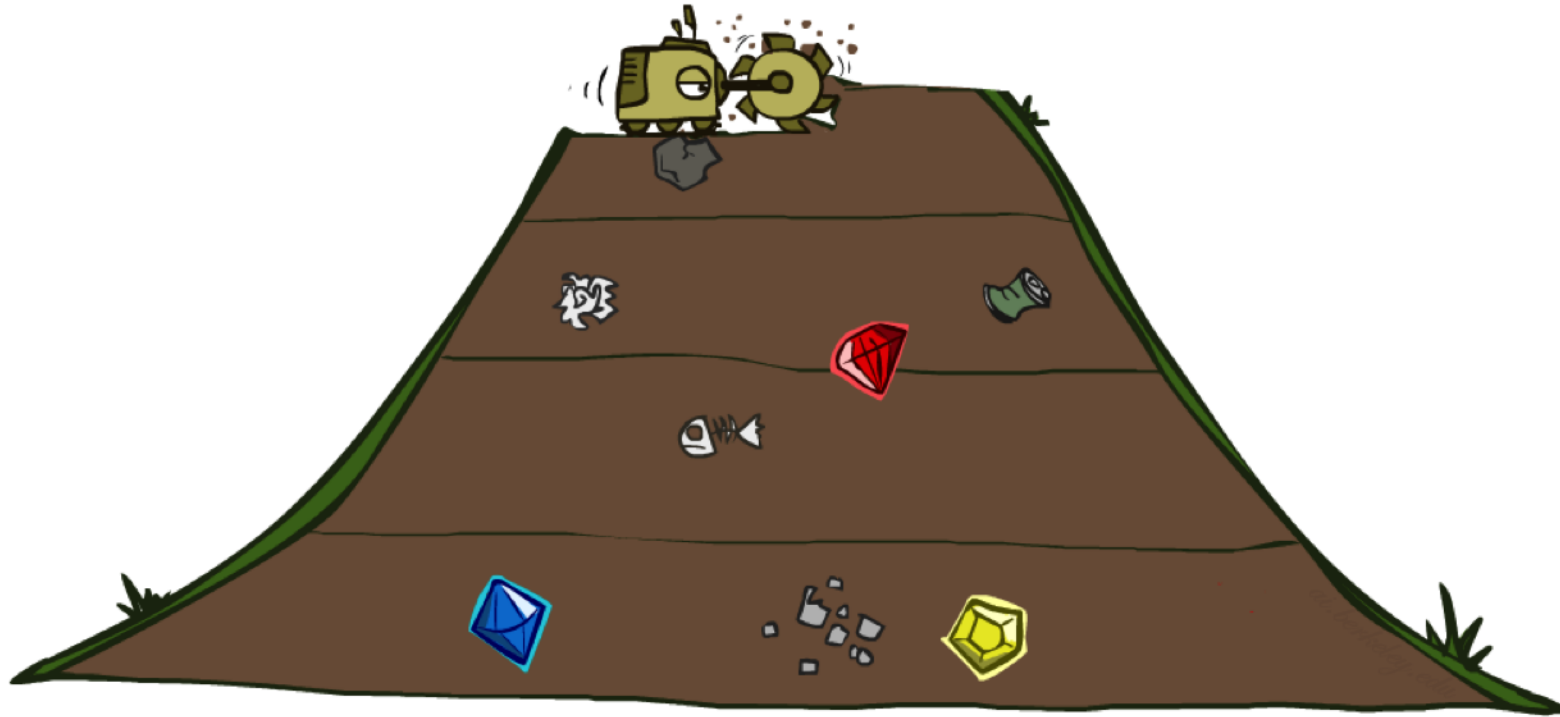
- Number of nodes in entire tree?
 - $1 + b + b^2 + \dots + b^m = O(b^m)$

Depth-First Search (DFS) Properties

- What nodes DFS expand?
 - Some left prefix of the tree.
 - Could process the whole tree!
 - If m is finite, takes time $O(b^m)$
- How much space does the fringe take?
 - Only has siblings on path to root, so $O(bm)$
- Is it complete?
 - m could be infinite, so only if we prevent cycles (more later)
- Is it optimal?
 - No, it finds the “leftmost” solution, regardless of depth or cost



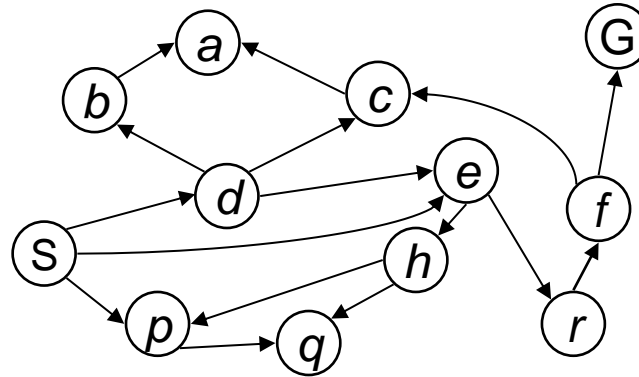
Breadth-First Search



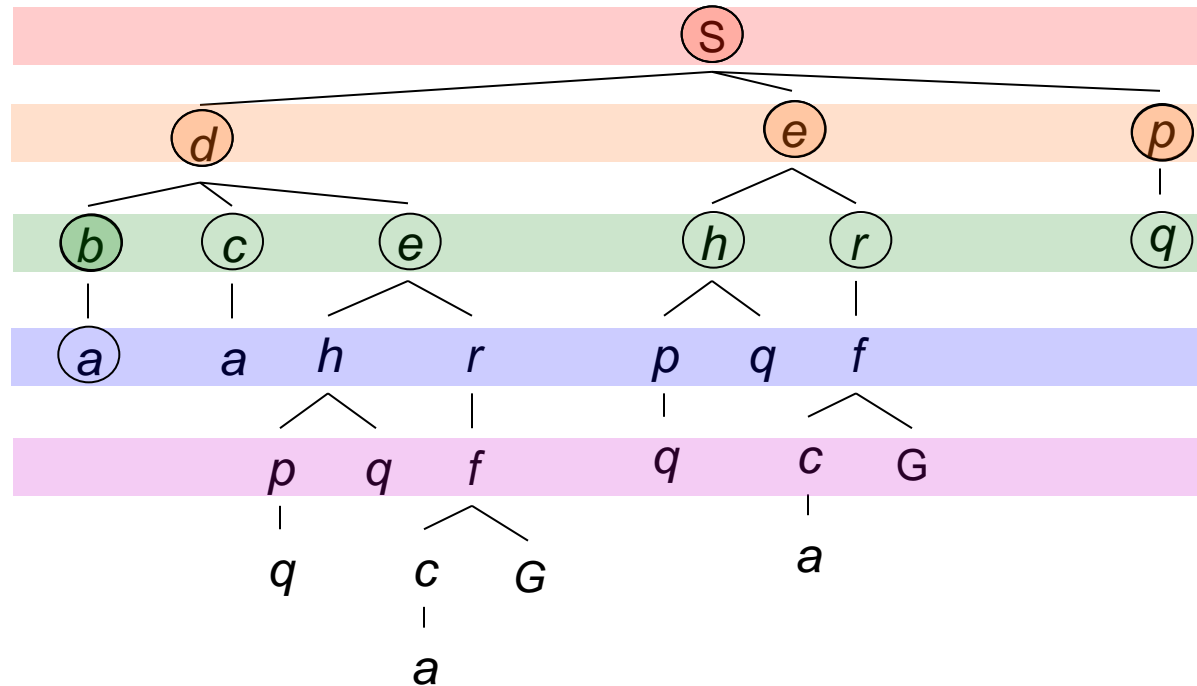
Breadth-First Search

Strategy: expand a shallowest node first

Implementation: Fringe is a FIFO queue

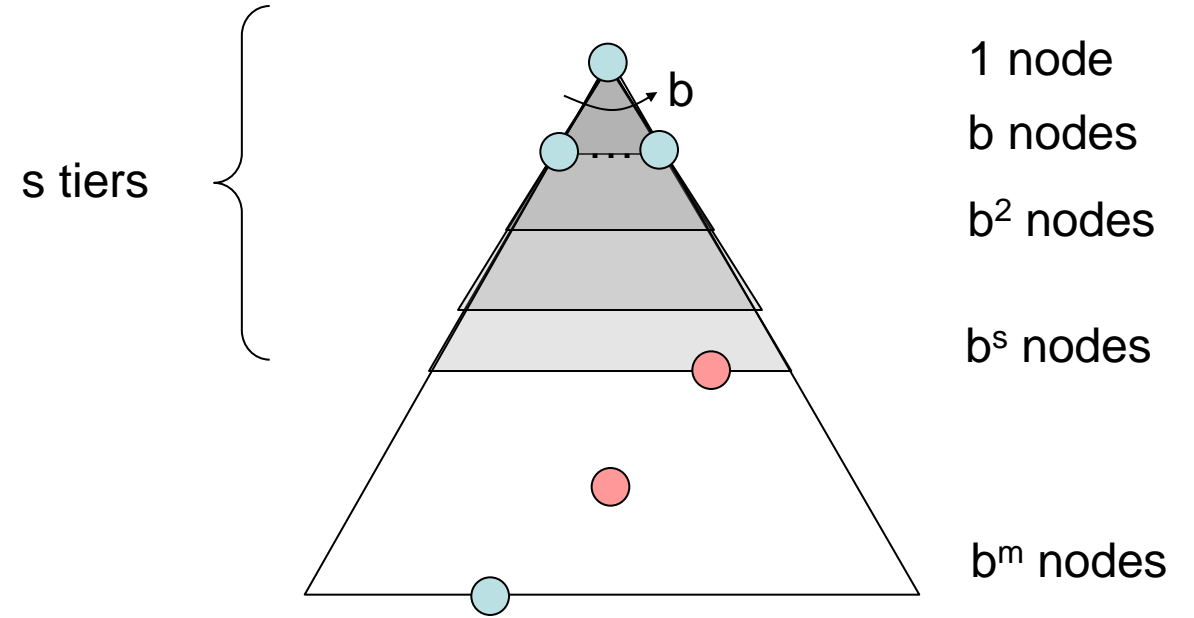


Search
Tiers

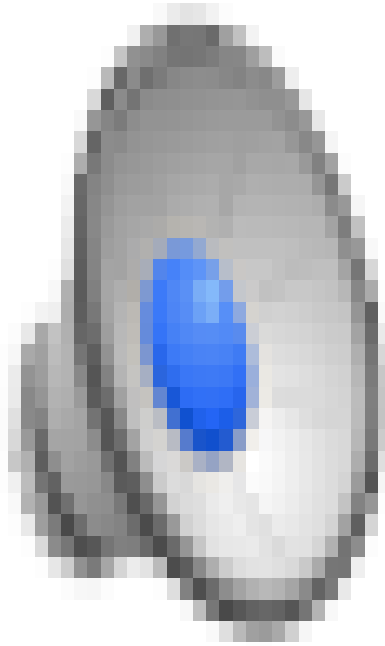


Breadth-First Search (BFS) Properties

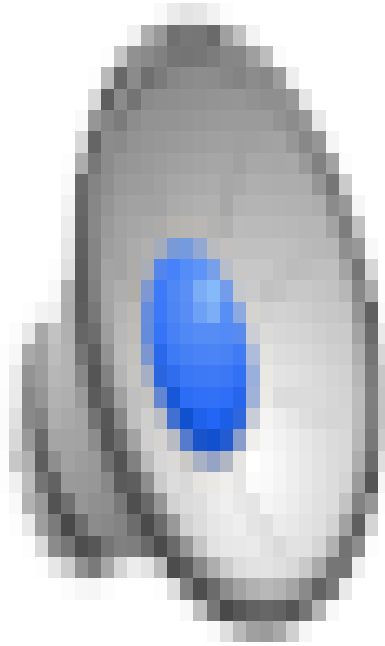
- What nodes does BFS expand?
 - Processes all nodes above shallowest solution
 - Let depth of shallowest solution be s
 - Search takes time $O(b^s)$
- How much space does the fringe take?
 - Has roughly the last tier, so $O(b^s)$
- Is it complete?
 - s must be finite if a solution exists, so yes!
(if no solution, still need depth $\neq \infty$)
- Is it optimal?
 - Only if costs are all 1 (more on costs later)



Video of Demo Maze Water DFS/BFS (part 1)

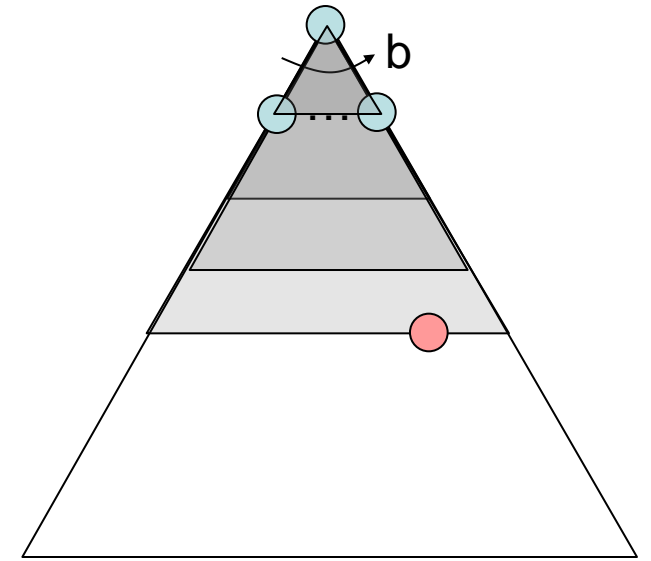


Video of Demo Maze Water DFS/BFS (part 2)

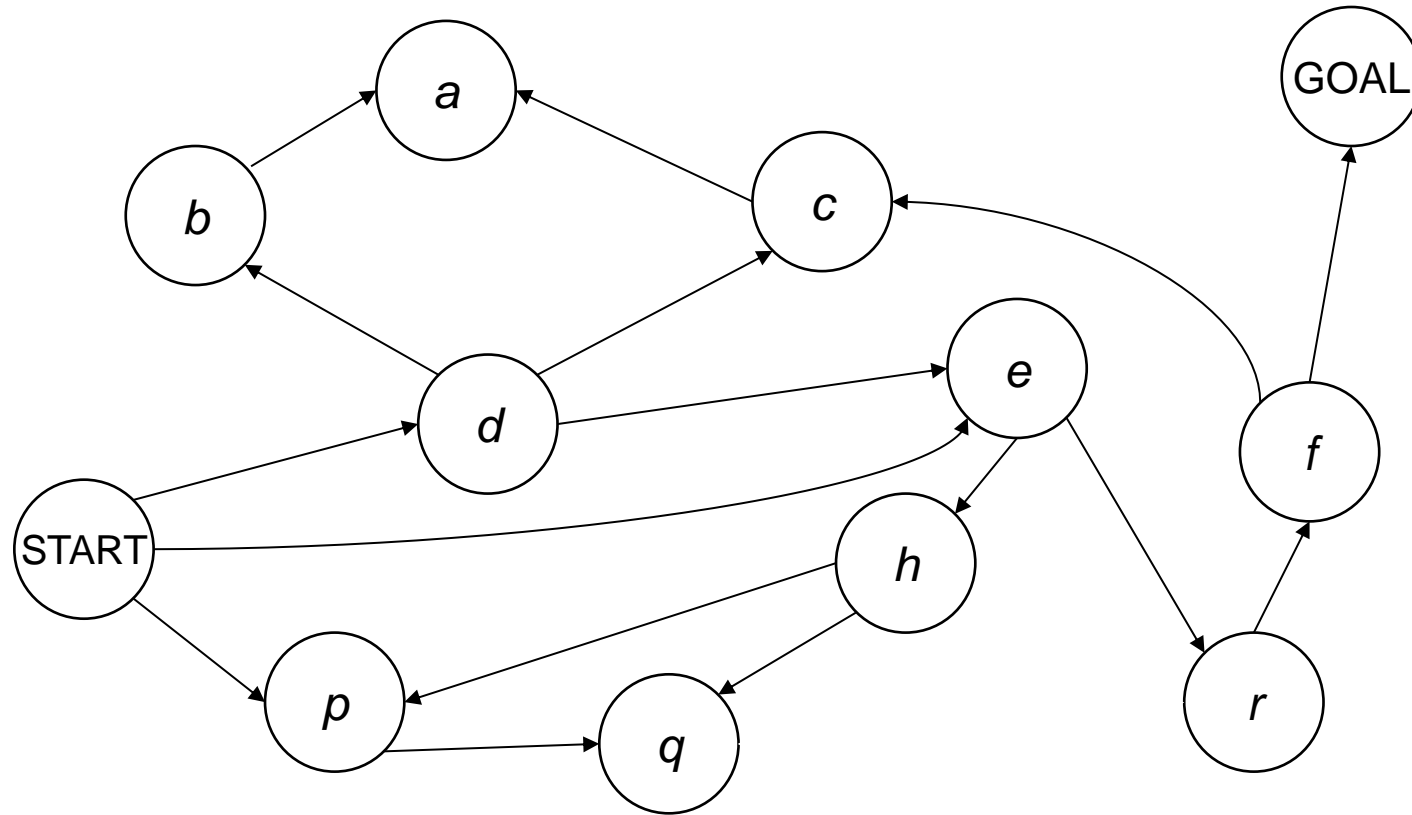


Iterative Deepening

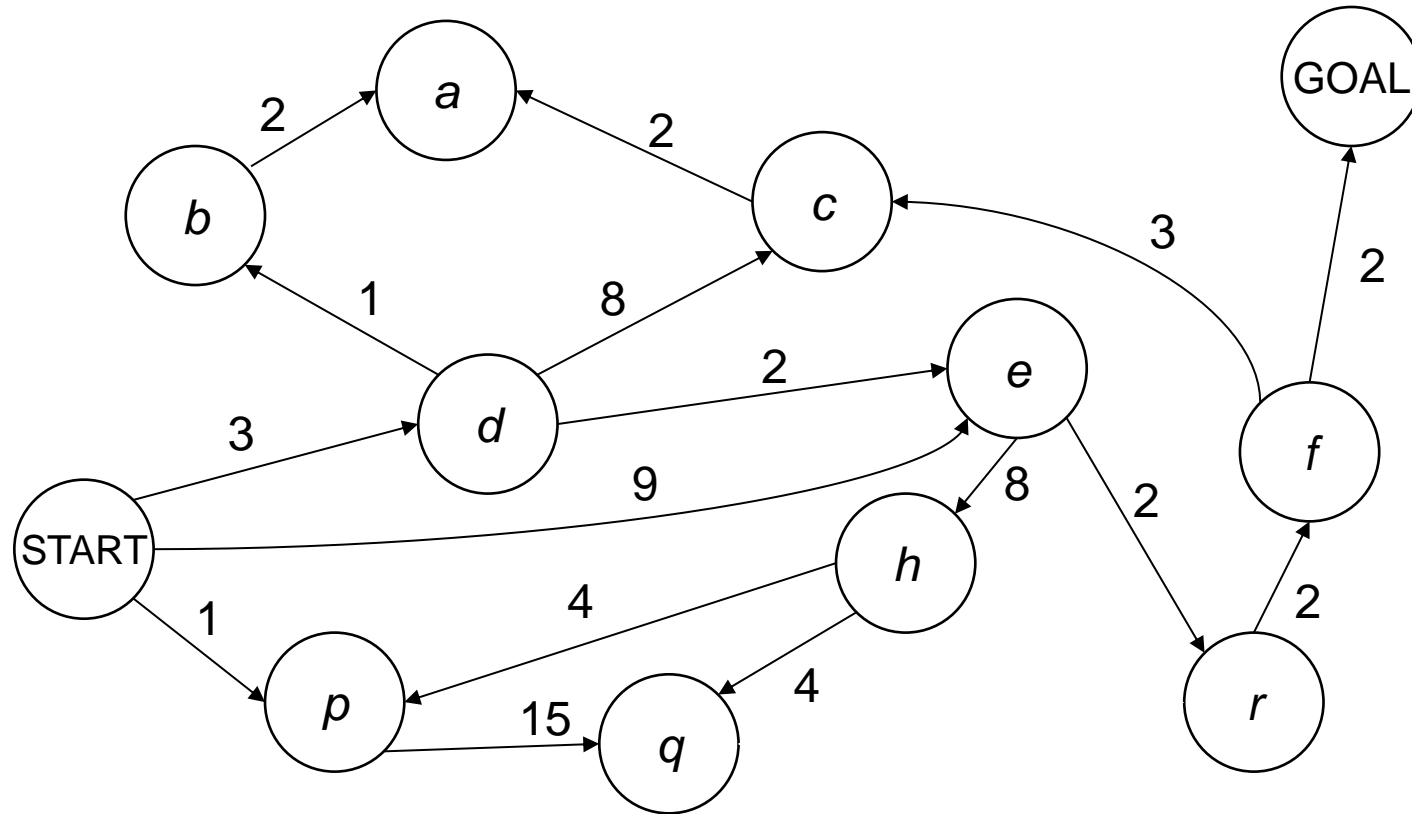
- Idea: get DFS's space advantage with BFS's time / shallow-solution advantages
 - Run a DFS with depth limit 1. If no solution...
 - Run a DFS with depth limit 2. If no solution...
 - Run a DFS with depth limit 3.
- Isn't that wastefully redundant?
 - Generally most work happens in the lowest level searched, so not so bad!



Cost-Sensitive Search



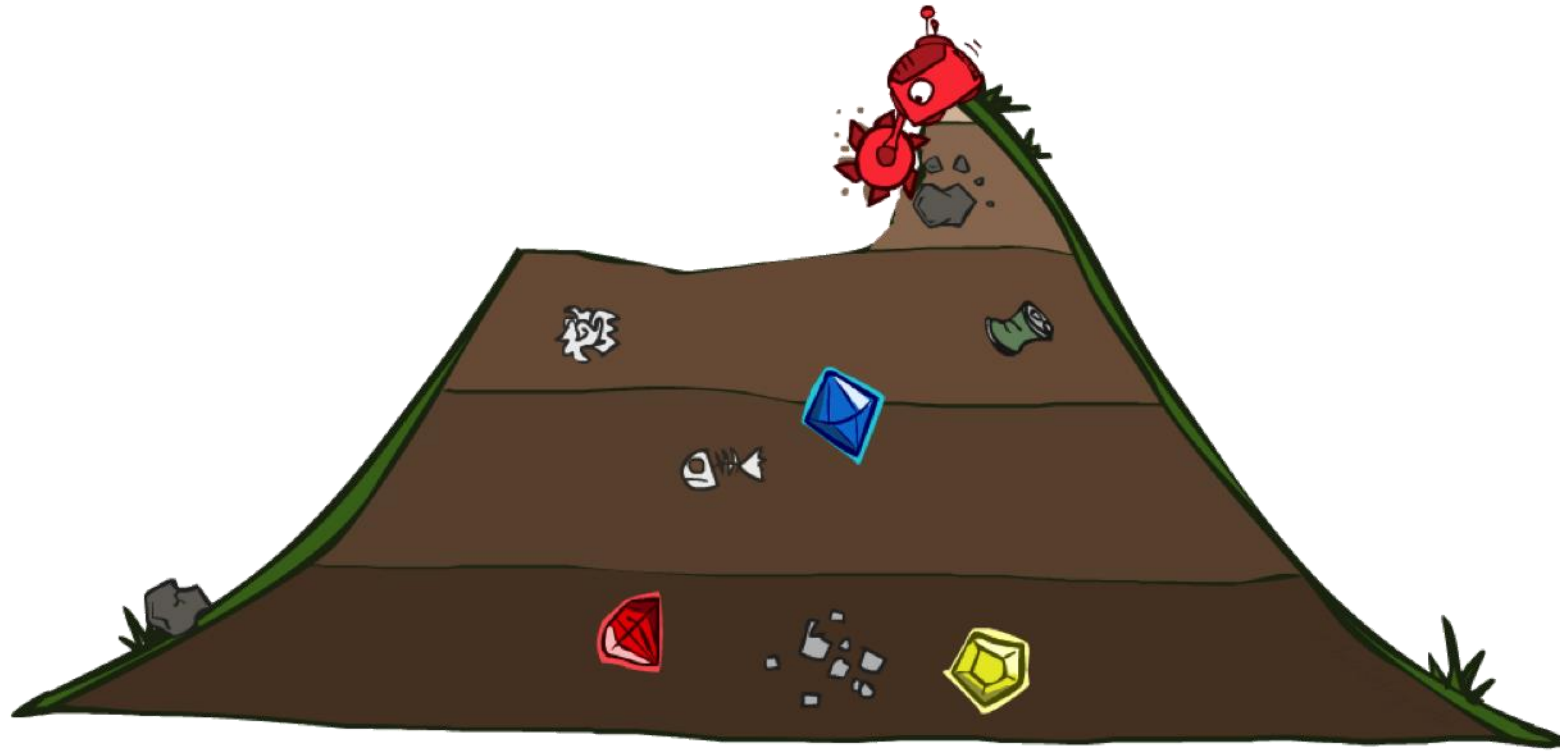
Cost-Sensitive Search



BFS finds the shortest path in terms of number of actions.
It does not find the least-cost path. We will now cover
a similar algorithm which does find the least-cost path.

How?

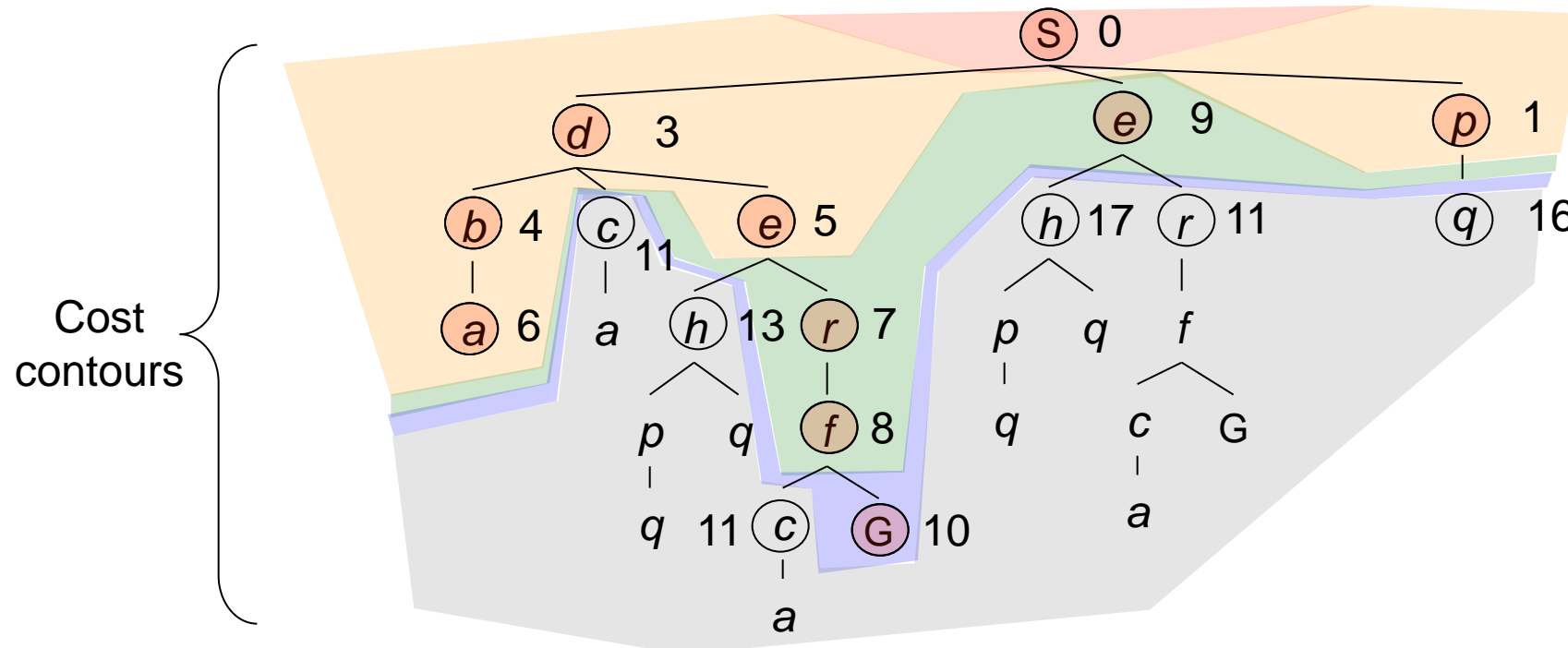
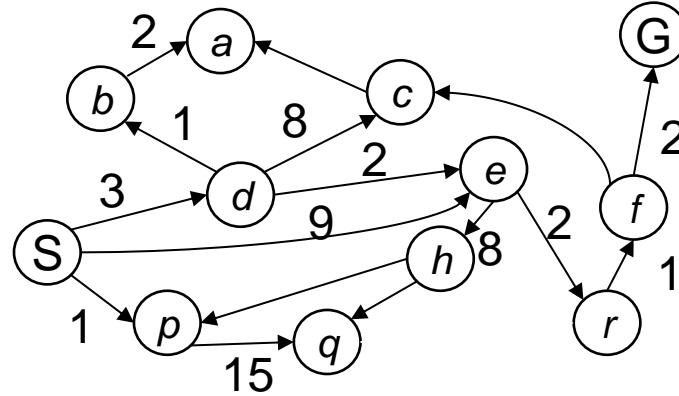
Uniform Cost Search



Uniform Cost Search

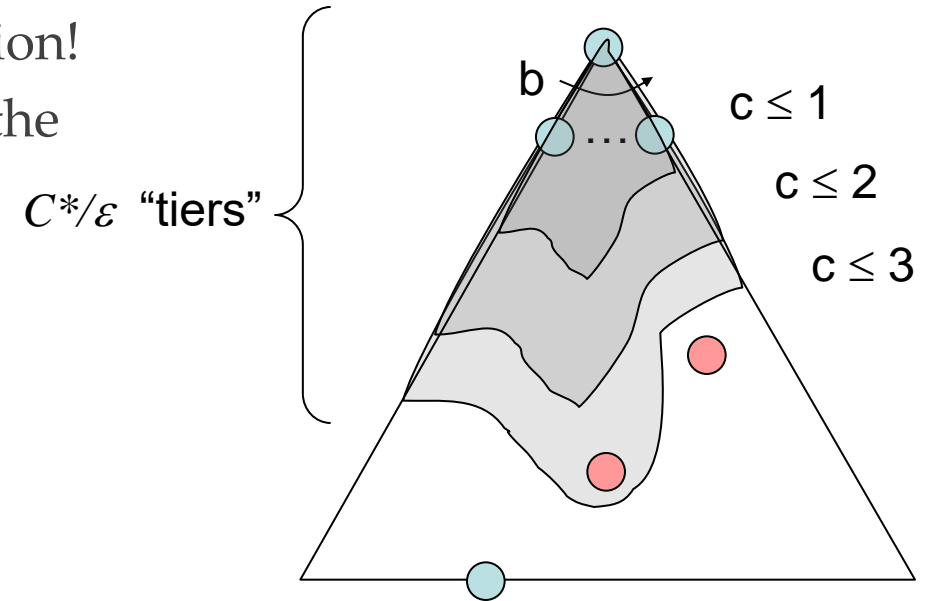
Strategy: expand a
cheapest node first:

Fringe is a priority queue
(priority: cumulative cost)



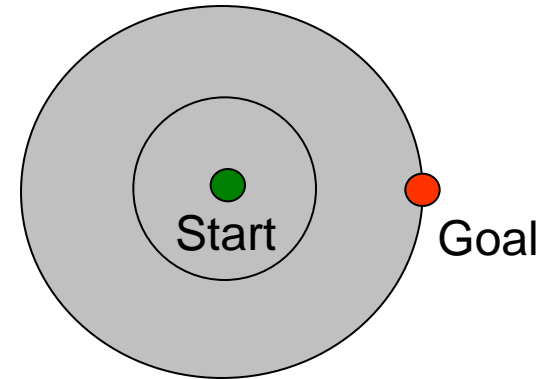
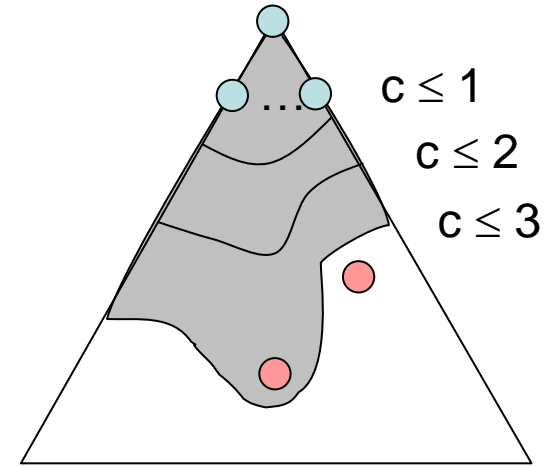
Uniform Cost Search (UCS) Properties

- What nodes does UCS expand?
 - Processes all nodes with cost less than cheapest solution!
 - If that solution costs C^* and arcs cost at least ε , then the “effective depth” is roughly C^*/ε
 - Takes time $O(b^{C^*/\varepsilon})$ (exponential in effective depth)
- How much space does the fringe take?
 - Has roughly the last tier, so $O(b^{C^*/\varepsilon})$
- Is it complete?
 - Assuming best solution has a finite cost and minimum arc cost is positive, yes! (if no solution, still need depth $!= \infty$)
- Is it optimal?
 - Yes! (Proof via A^*)



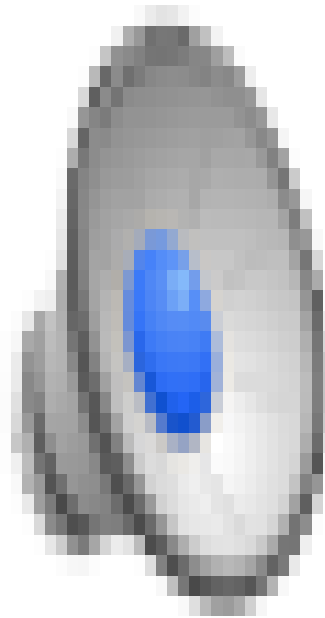
Uniform Cost Issues

- Remember: UCS explores increasing cost contours
- The good: UCS is complete and optimal!
- The bad:
 - Explores options in every “direction”
 - No information about goal location
- We'll fix that soon!

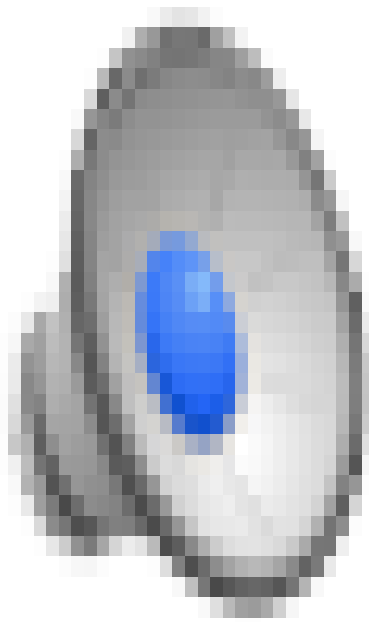


[Demo: empty grid UCS (L2D5)]
[Demo: maze with deep/shallow water DFS/BFS/UCS (L2D7)]

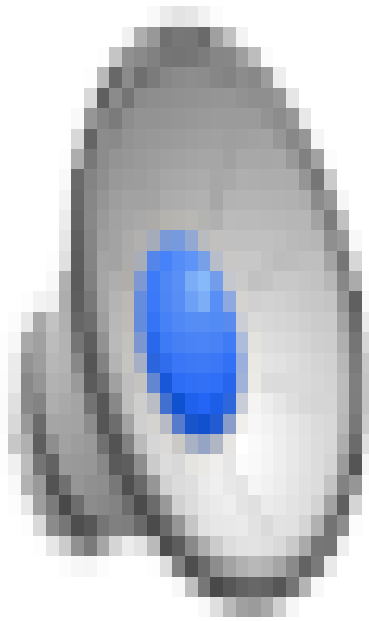
Video of Demo Empty UCS



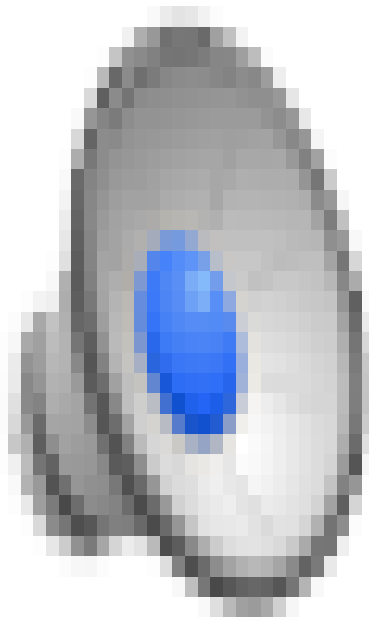
Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 1)



Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 2)

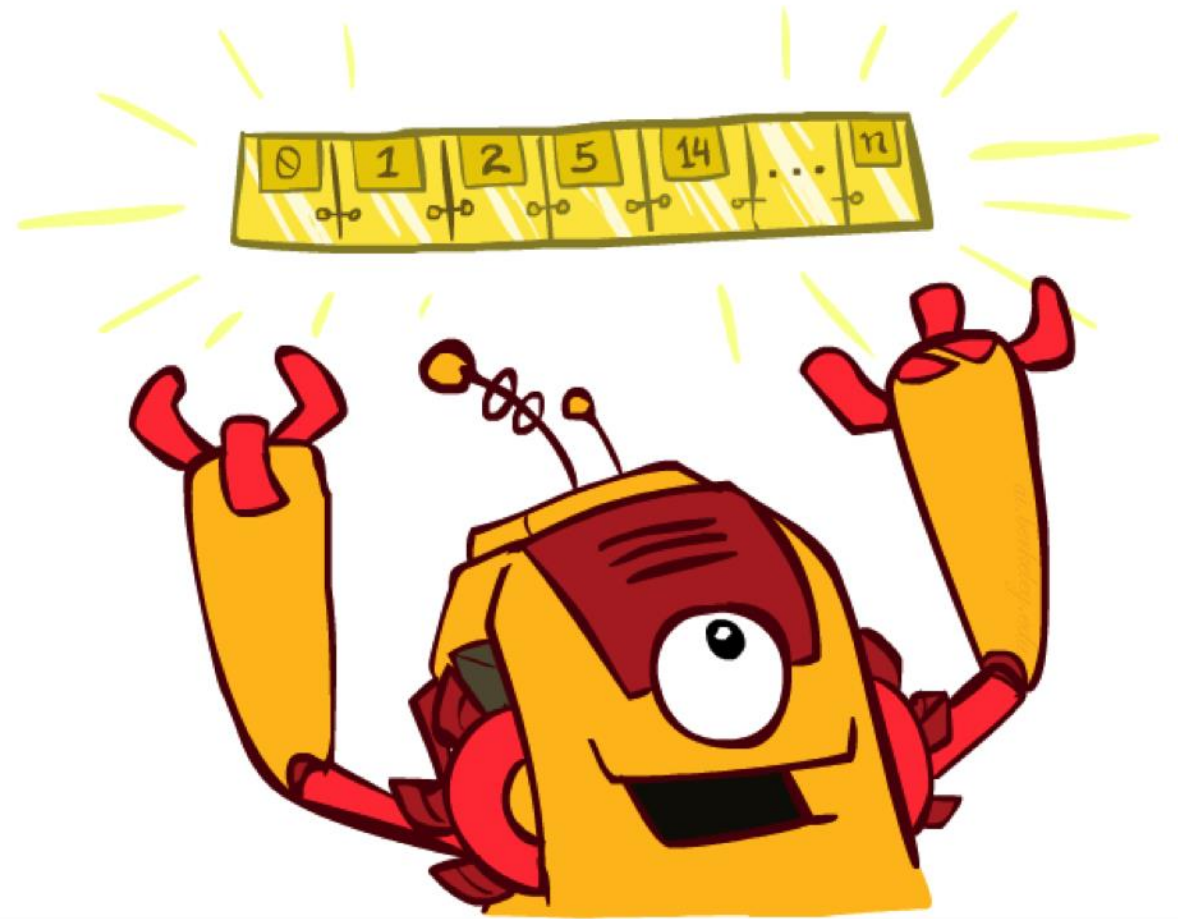


Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 3)



The One Queue

- All these search algorithms are the same except for fringe strategies
 - Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
 - Practically, for DFS and BFS, you can avoid the $\log(n)$ overhead from an actual priority queue, by using stacks and queues
 - Can even code one implementation that takes a variable queuing object



Up next: Informed Search

- Uninformed Search

- DFS
- BFS
- UCS



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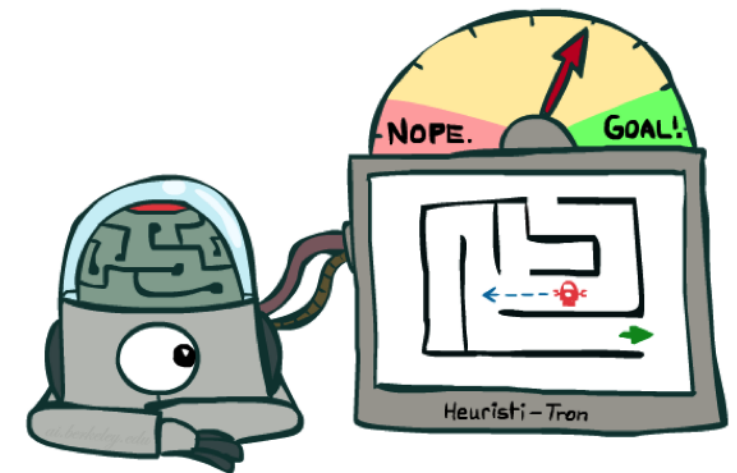
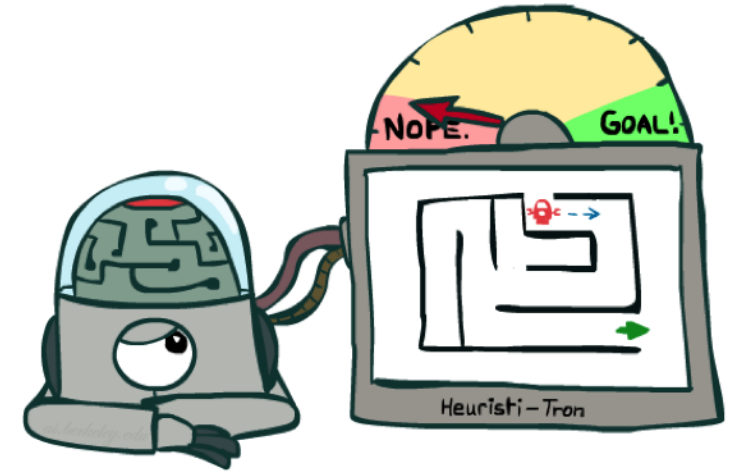
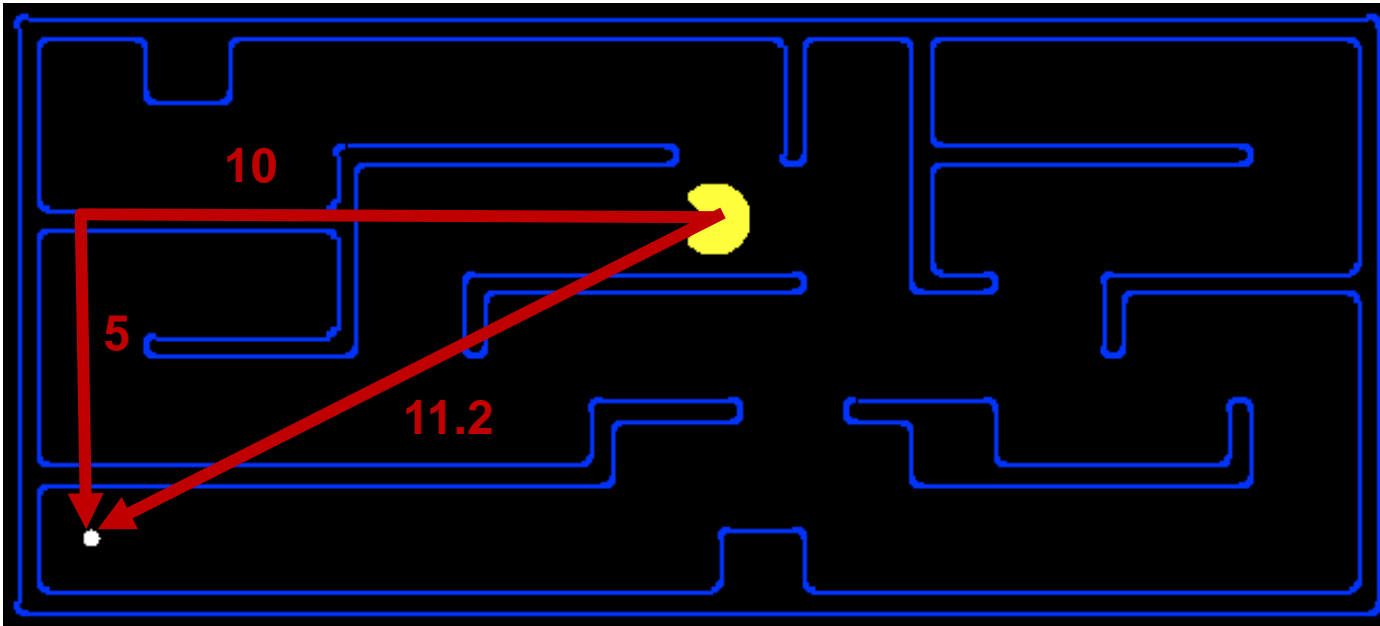
- Informed Search

- Heuristics
- Greedy Search
- A* Search
- Graph Search

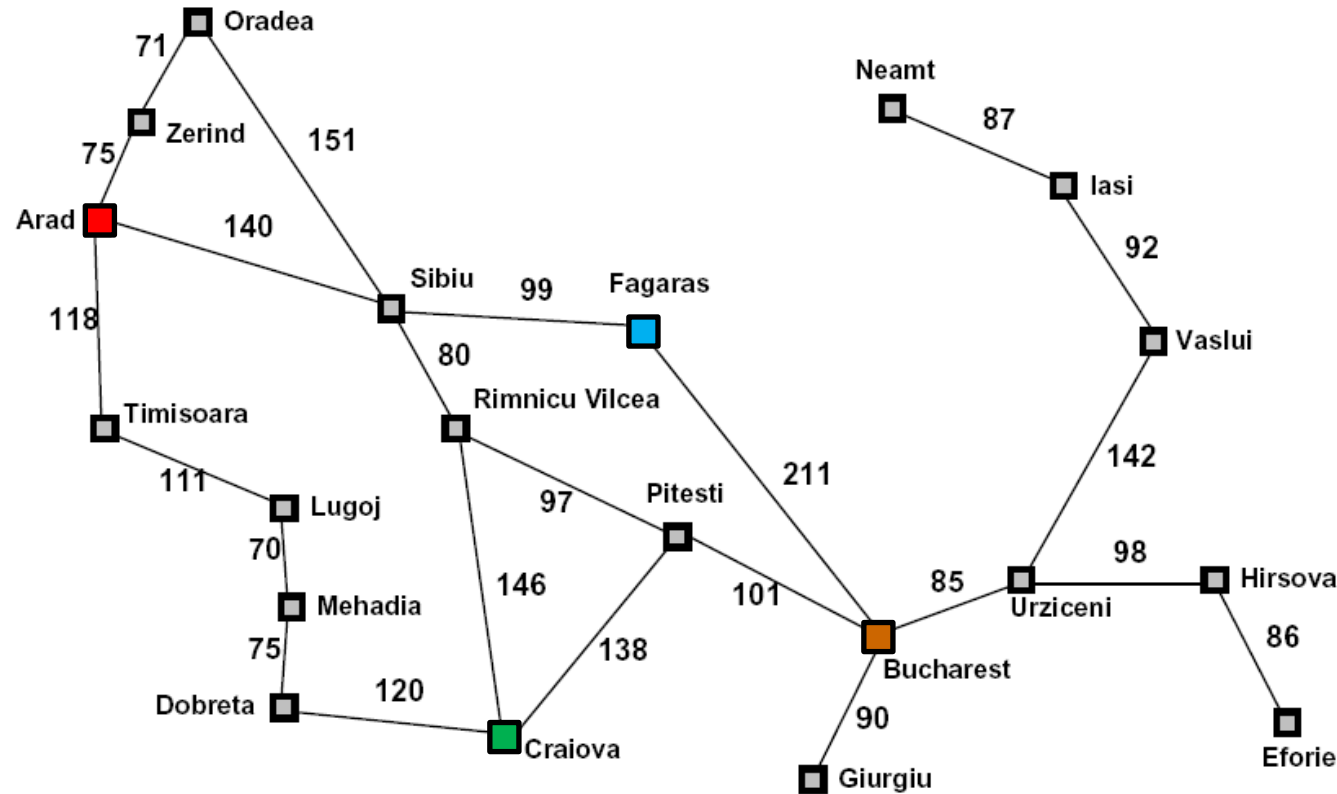


Search Heuristics

- A heuristic is:
 - A function that *estimates* how close a state is to a goal
 - Designed for a particular search problem
 - **Pathing?**
 - Examples: Manhattan distance, Euclidean distance for pathing



Example: Heuristic Function



Straight-line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

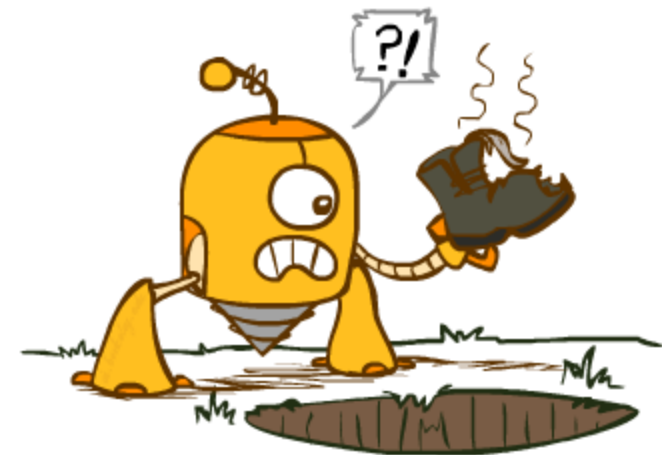
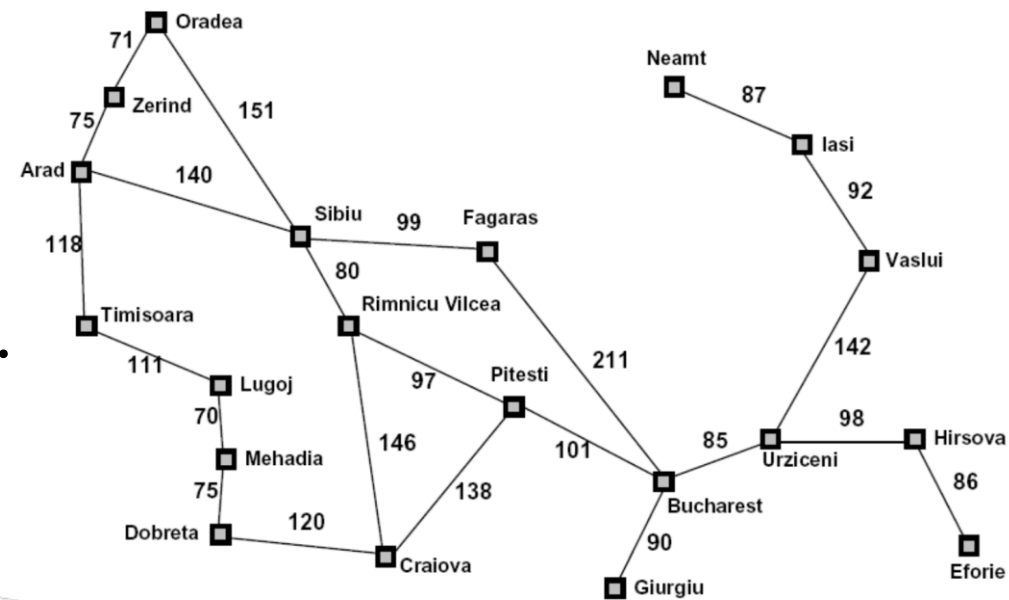
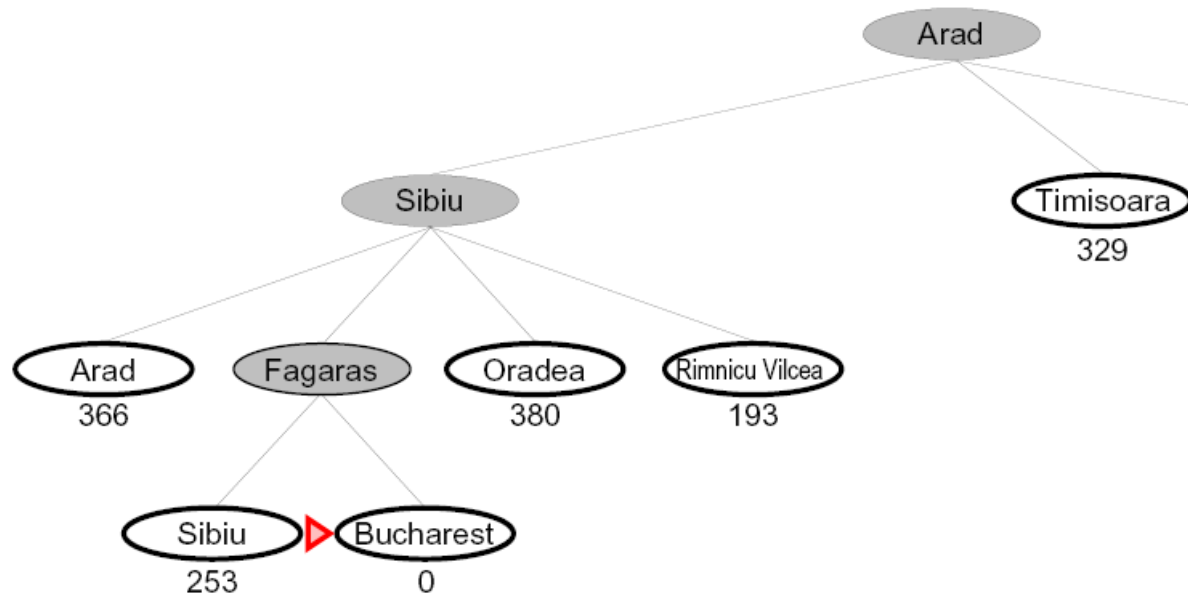
$h(x)$

Greedy Search



Greedy Search

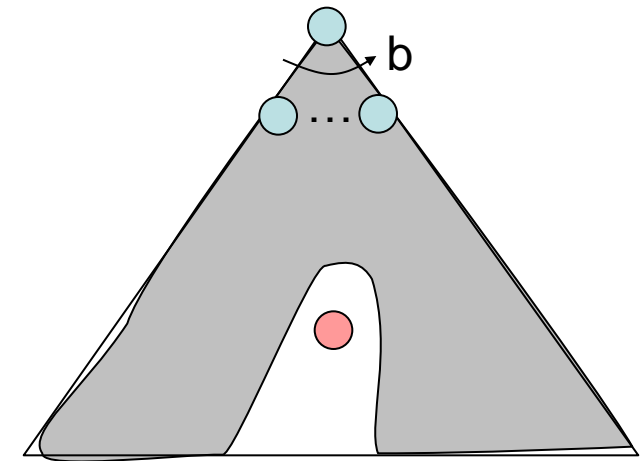
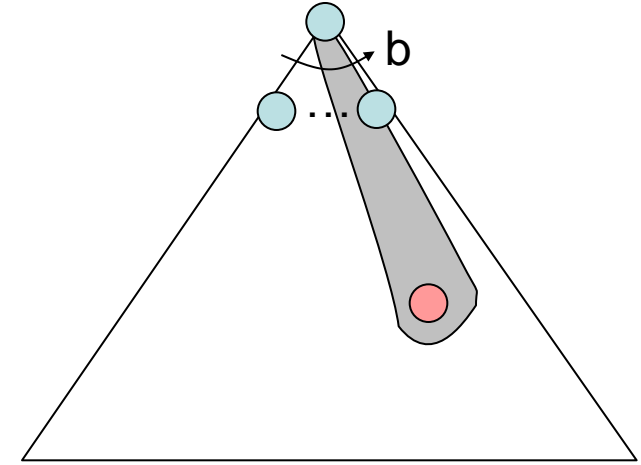
- Expand the node that seems closest...



- Is it optimal?
 - No. Resulting path to Bucharest is not the shortest.

Greedy Search

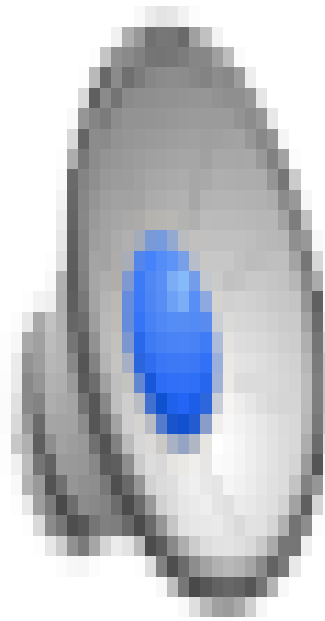
- Strategy: expand a node that you think is closest to a goal state
 - Heuristic: estimate of distance to nearest goal for each state
- A common case:
 - Best-first takes you straight to the (wrong) goal
- Worst-case: like a badly-guided DFS



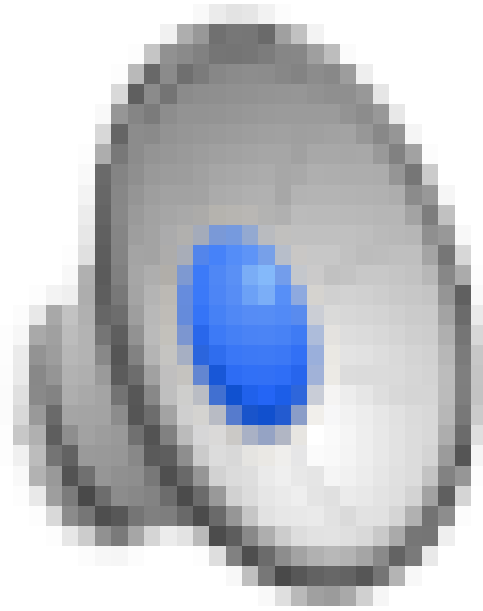
[Demo: contours greedy empty (L3D1)]

[Demo: contours greedy pacman small maze (L3D4)]

Video of Demo Contours Greedy (Empty)



Video of Demo Contours Greedy (Pacman Small Maze)



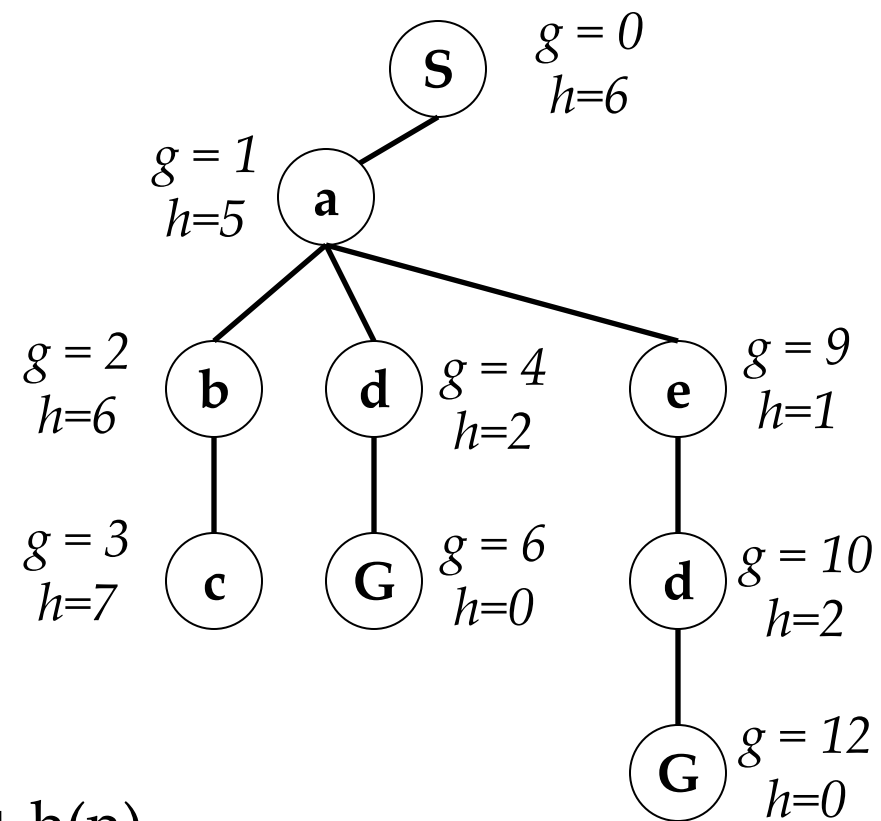
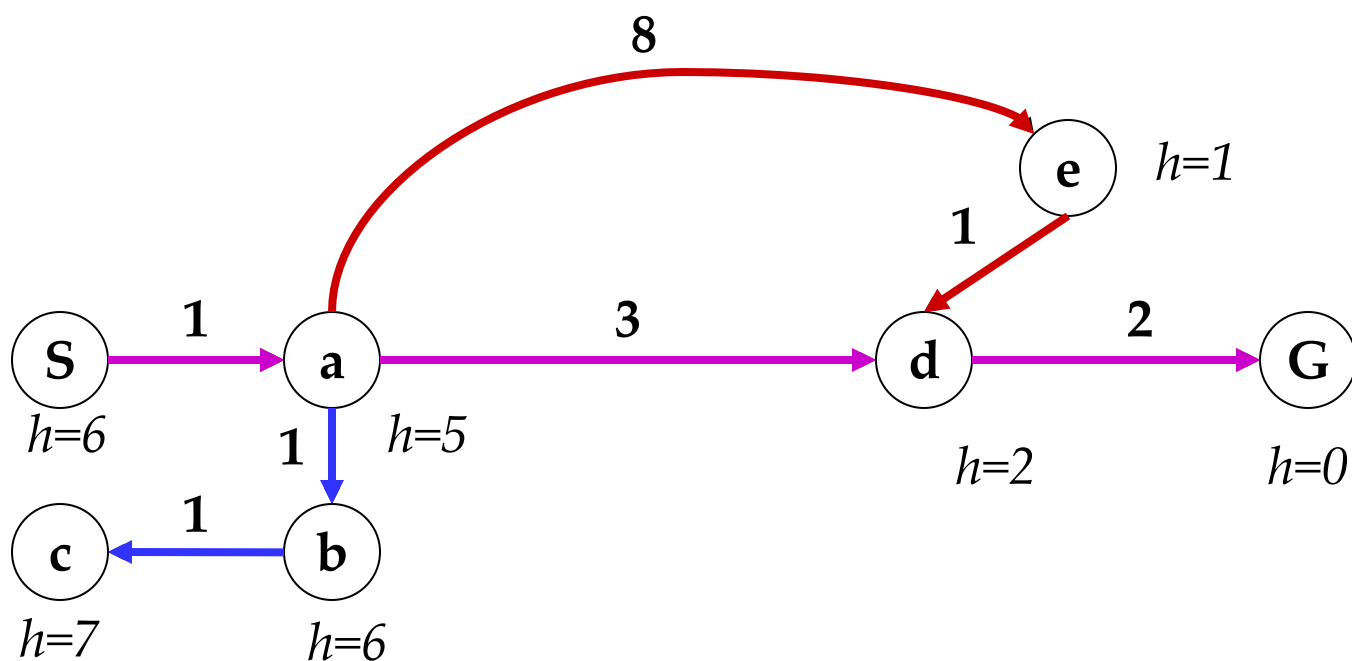
A* Search



A* Search

Combining UCS and Greedy

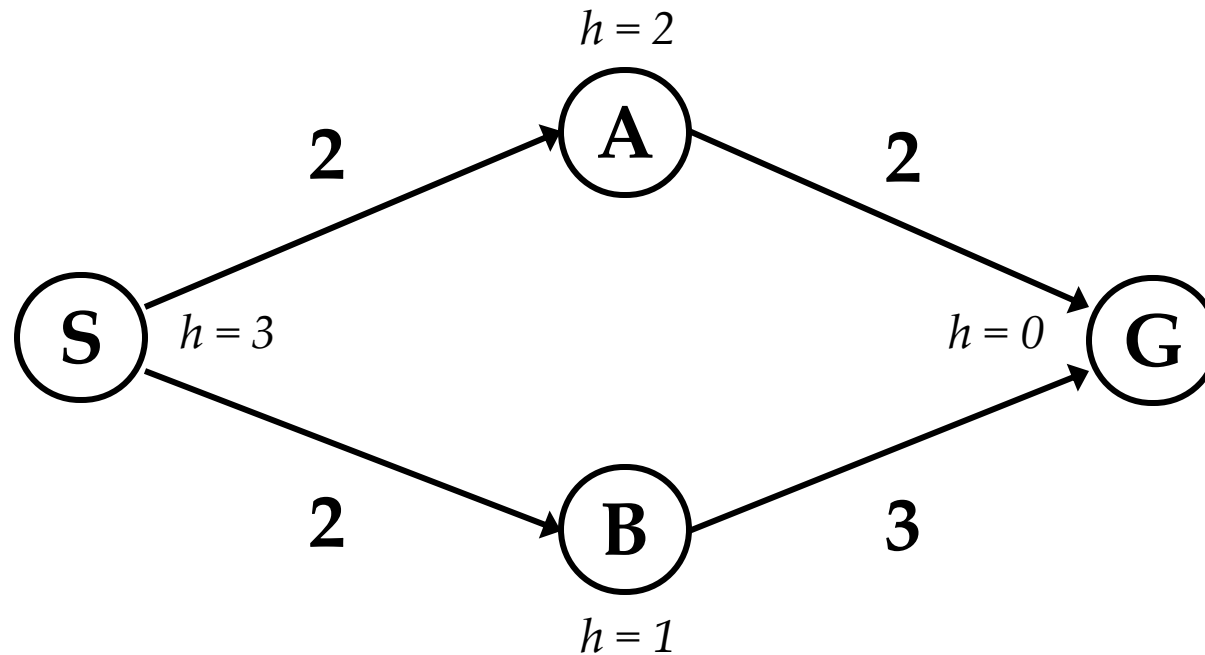
- **Uniform-cost** orders by path cost, or *backward cost* $g(n)$
- **Greedy** orders by goal proximity, or *forward cost* $h(n)$



- **A* Search** orders by the sum: $f(n) = g(n) + h(n)$

When should A* terminate?

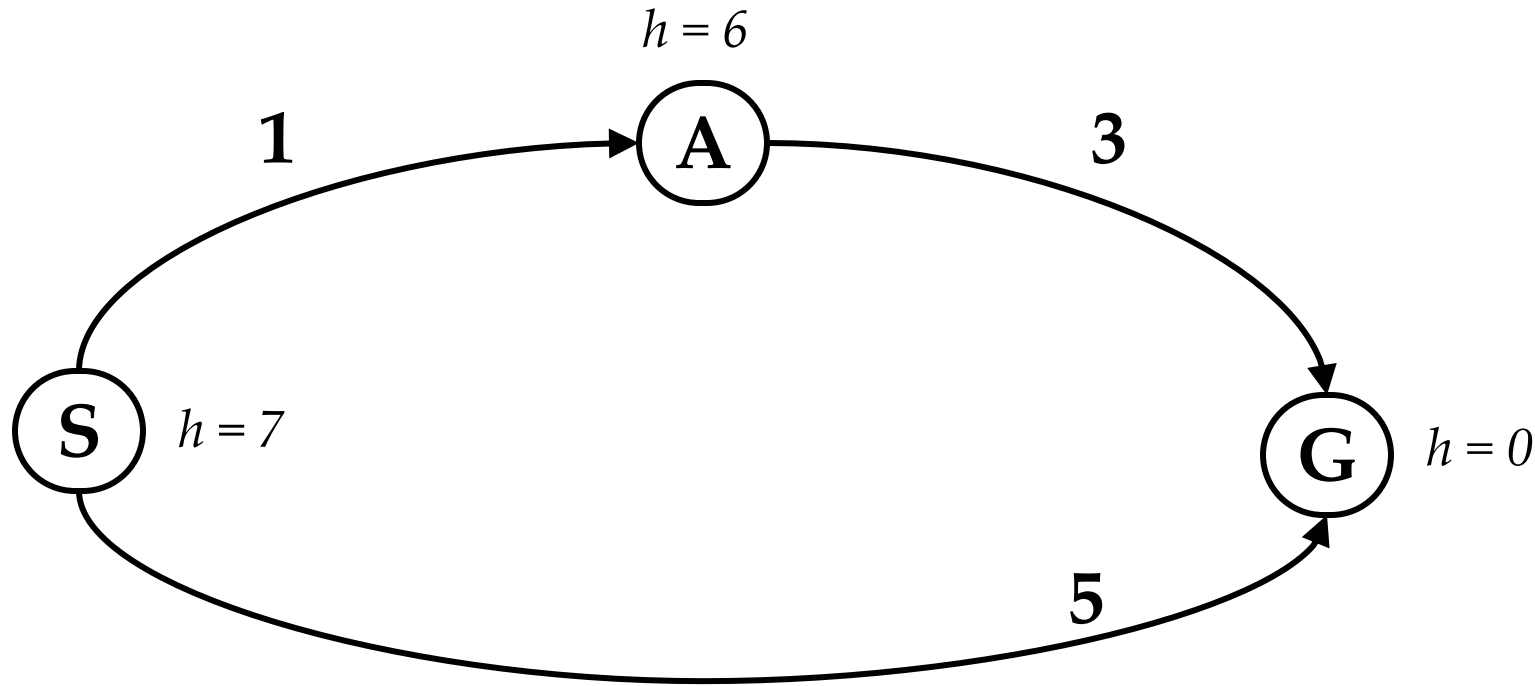
- Should we stop when we enqueue a goal?



	g	h	+
S	0	3	3
S → A	2	2	4
S → B	2	1	3
S → B → G	5	0	5
S → A → G	4	0	4

- No: only stop when we dequeue a goal

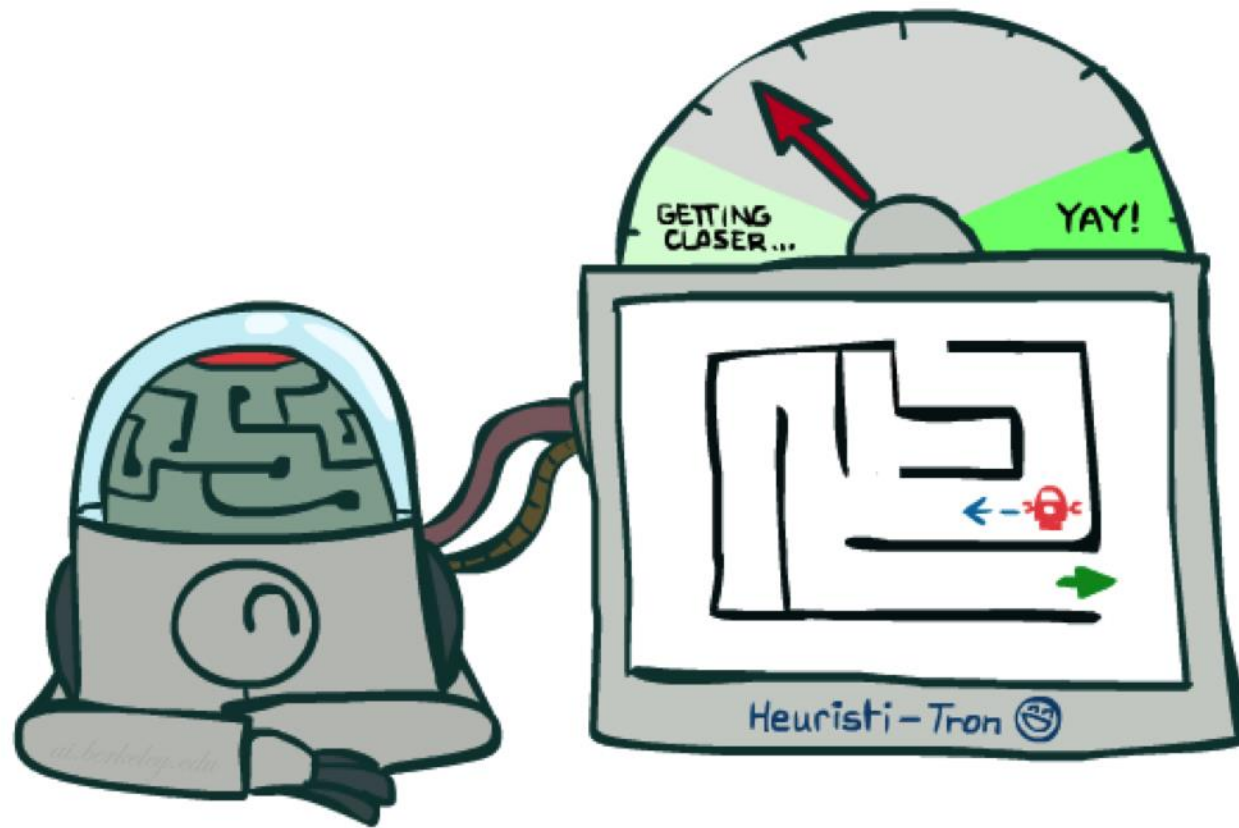
Is A* Optimal?



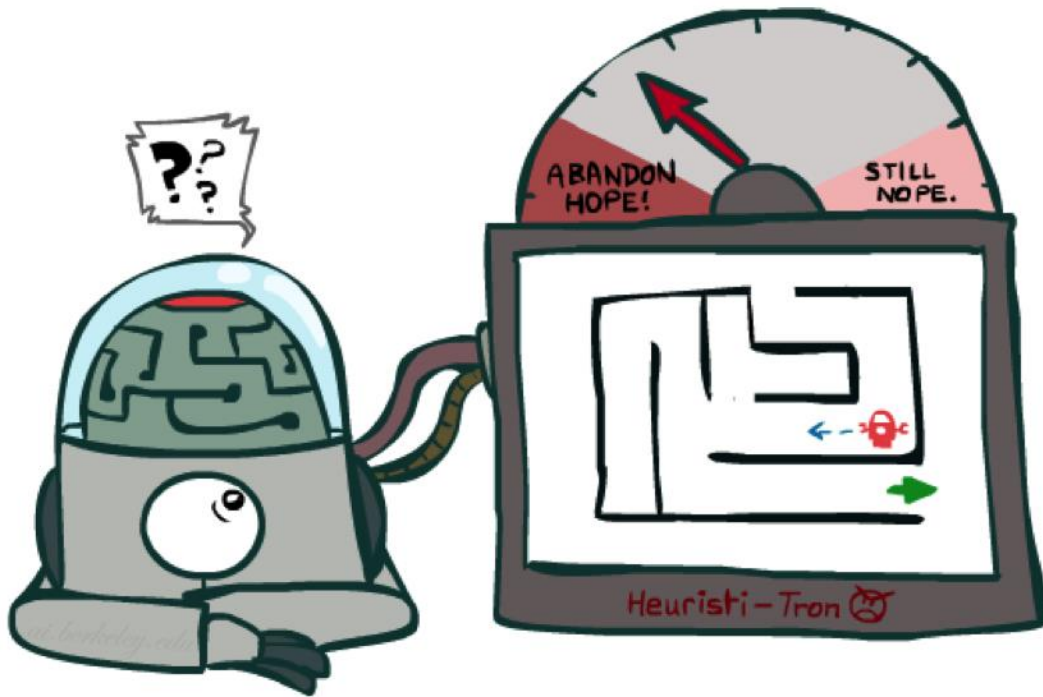
	g	h	+
S	0	7	7
S->A	1	6	7
S->G	5	0	5

- What went wrong?
- Actual bad goal cost < estimated good goal cost
- We need estimates to be less than actual costs!

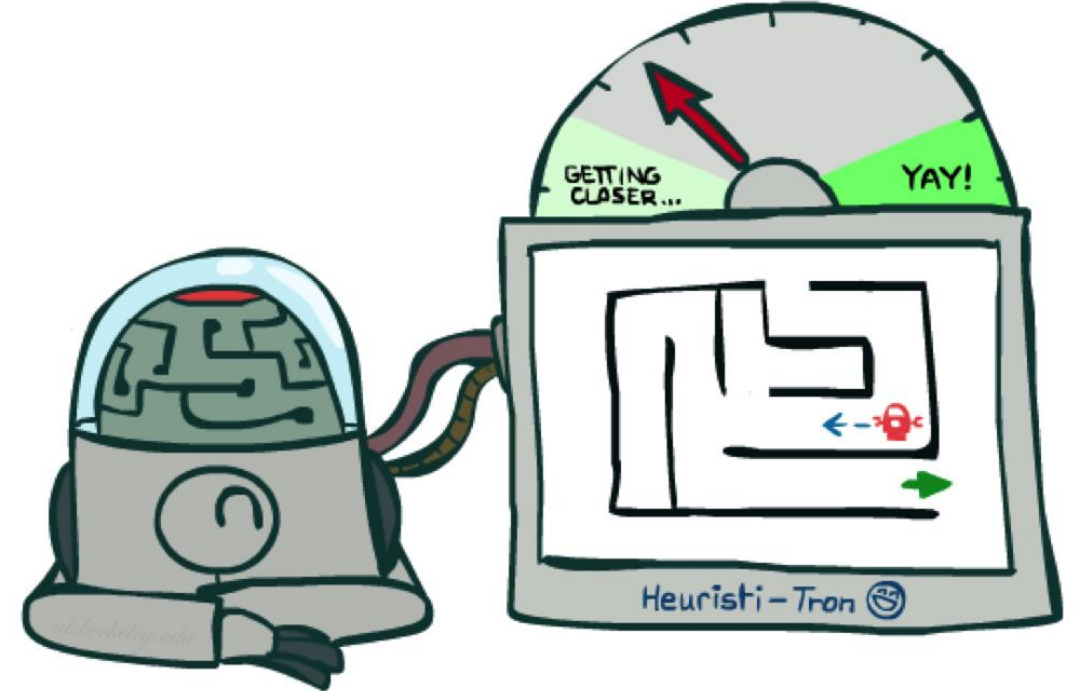
Admissible Heuristics



Idea: Admissibility



Inadmissible (pessimistic) heuristics
break optimality by trapping
good plans on the fringe



Admissible (optimistic) heuristics
slow down bad plans but
never outweigh true costs

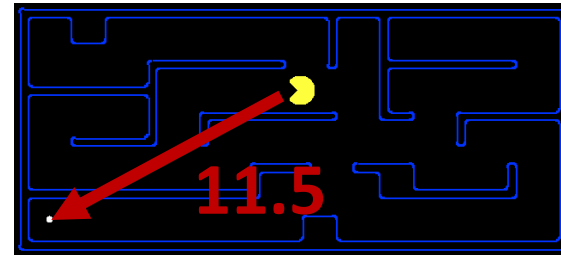
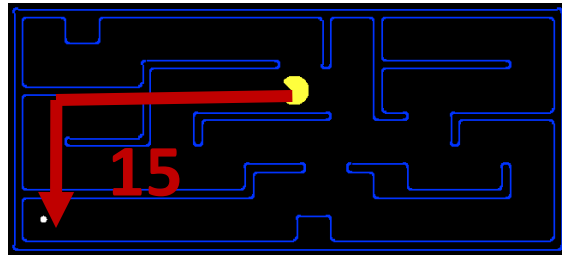
Admissible Heuristics

- A heuristic h is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost to a nearest goal

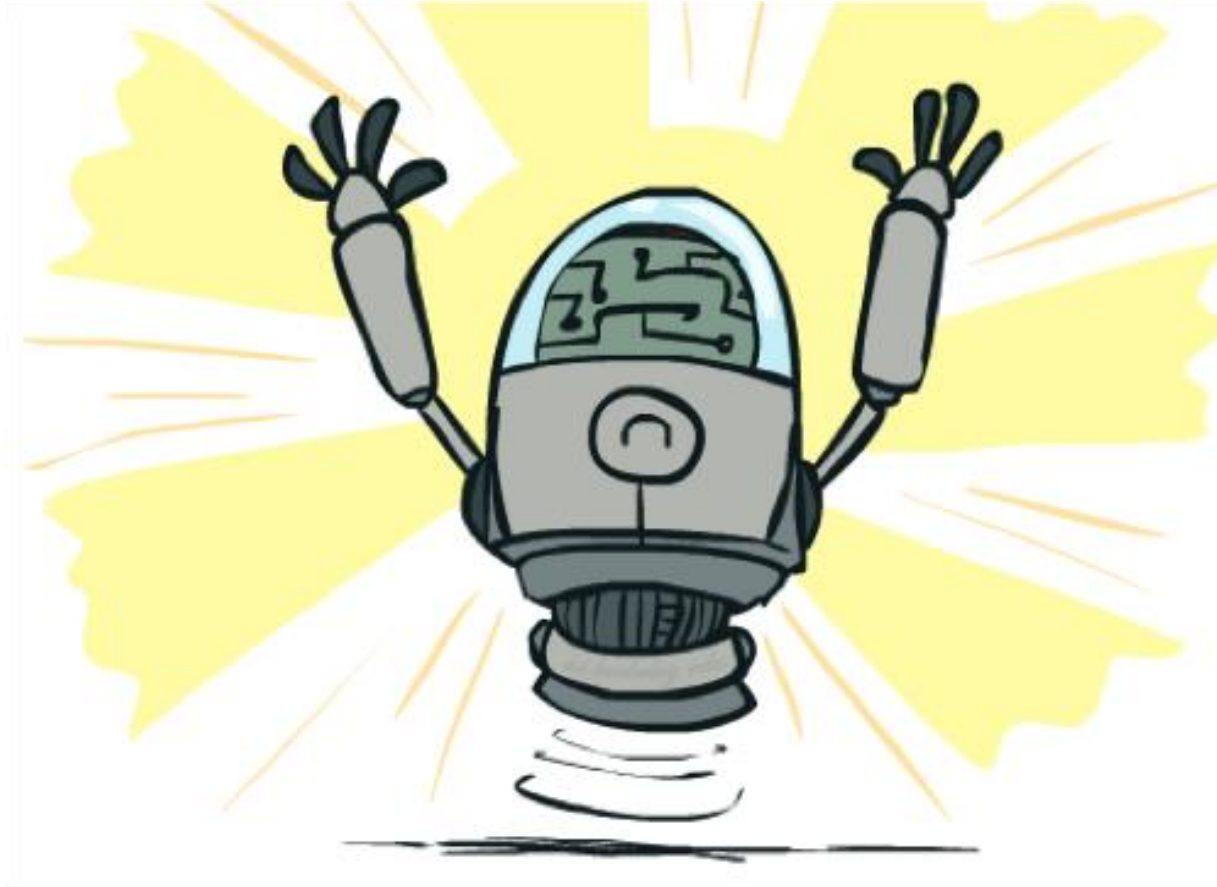
- Examples:



0.0

- Coming up with admissible heuristics is most of what's involved in using A^* in practice.

Optimality of A* Tree Search



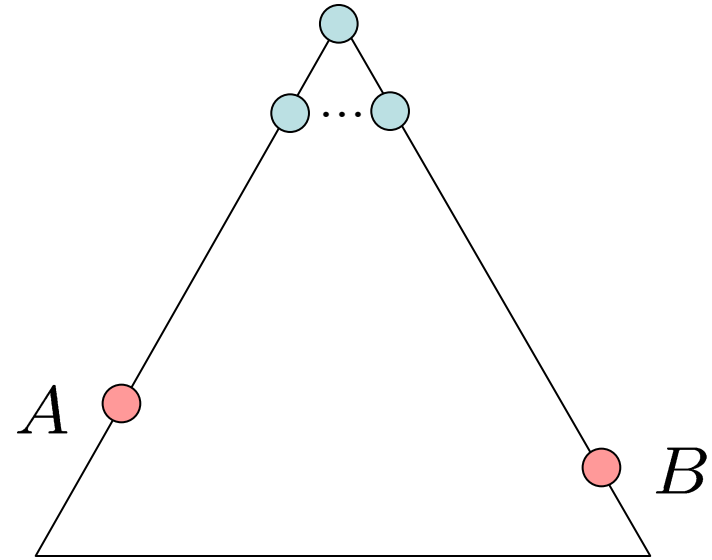
Optimality of A* Tree Search

Assume:

- A is an optimal goal node
- B is a suboptimal goal node
- h is admissible

Claim:

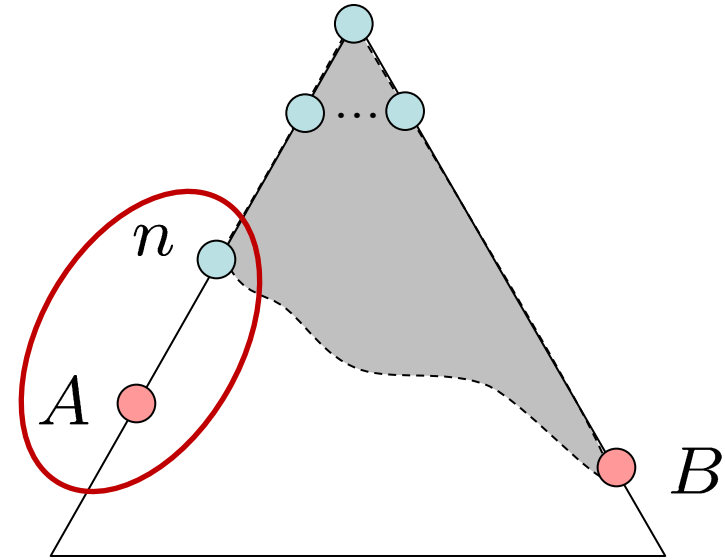
- A will exit the fringe before B



Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor n of A is on the fringe, too (maybe A!)
- Claim: n will be expanded before B
 1. $f(n)$ is less or equal to $f(A)$



$$f(n) = g(n) + h(n)$$

Definition of f-cost

$$f(n) \leq g(A)$$

Admissibility of h

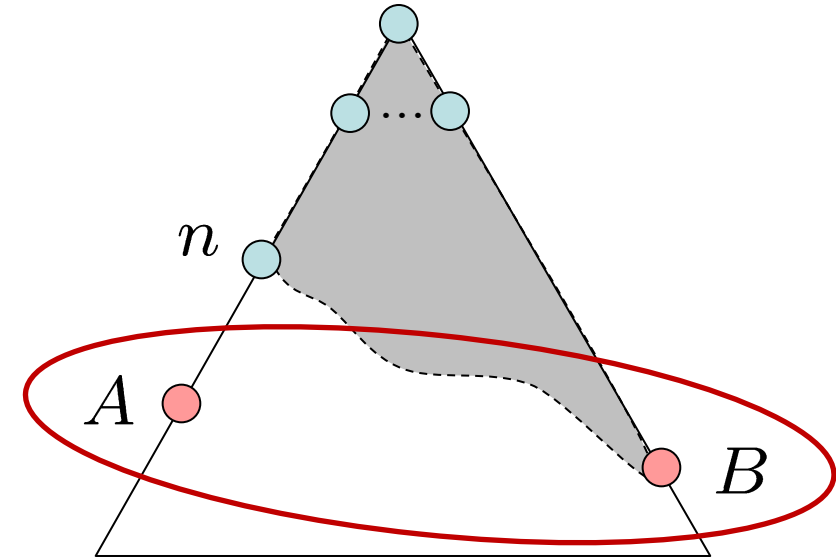
$$g(A) = f(A)$$

$h = 0$ at a goal

Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor n of A is on the fringe, too (maybe A!)
- Claim: n will be expanded before B
 1. $f(n)$ is less or equal to $f(A)$
 2. $f(A)$ is less than $f(B)$



$$g(A) < g(B)$$

$$f(A) < f(B)$$

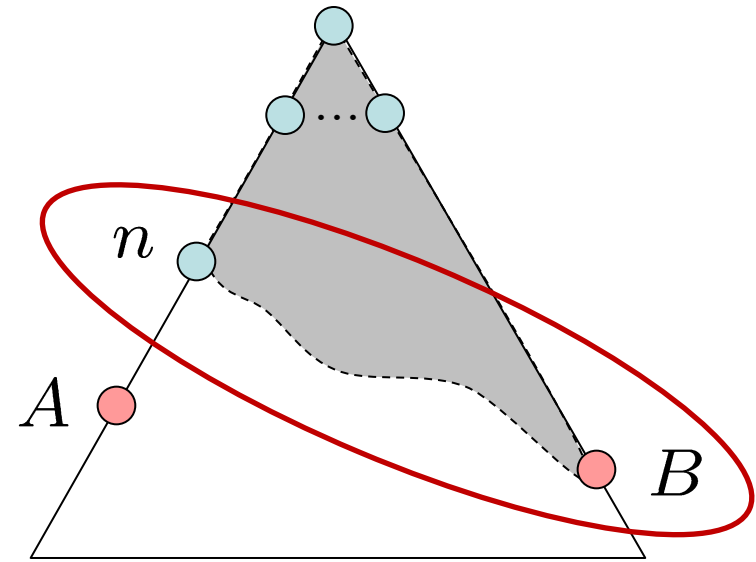
B is suboptimal

$h = 0$ at a goal

Optimality of A* Tree Search: Blocking

Proof:

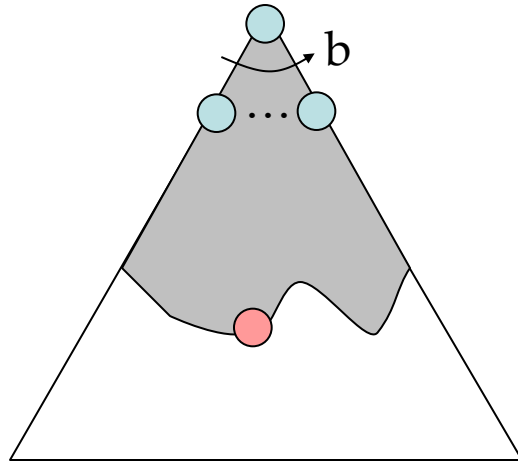
- Imagine B is on the fringe
- Some ancestor n of A is on the fringe, too (maybe A!)
- Claim: n will be expanded before B
 1. $f(n)$ is less or equal to $f(A)$
 2. $f(A)$ is less than $f(B)$
 3. n expands before B
- All ancestors of A expand before B
- A expands before B
- A* search is optimal



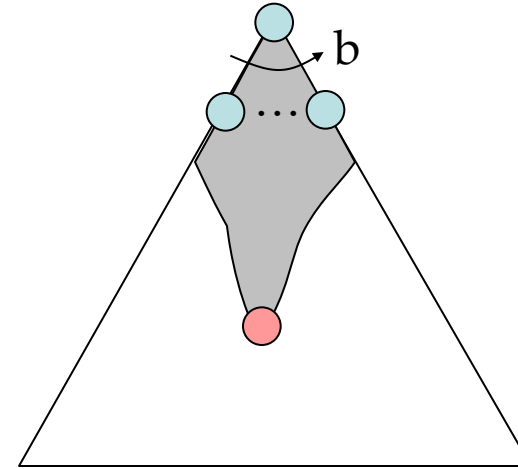
$$f(n) \leq f(A) < f(B)$$

Properties of A^*

Uniform-Cost

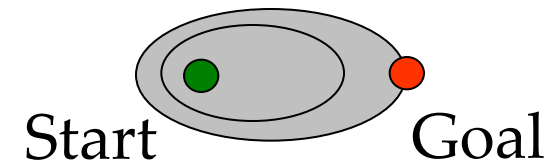
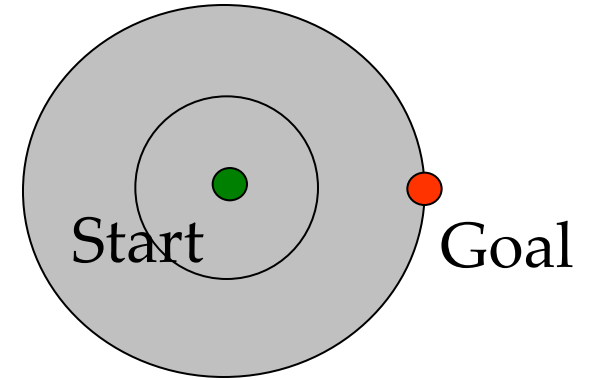


A^*



UCS vs A* Contours

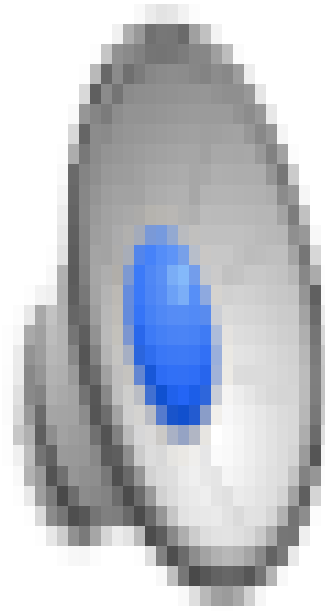
- Uniform-cost expands equally in all “directions”
- A* expands mainly toward the goal, but does hedge its bets to ensure optimality



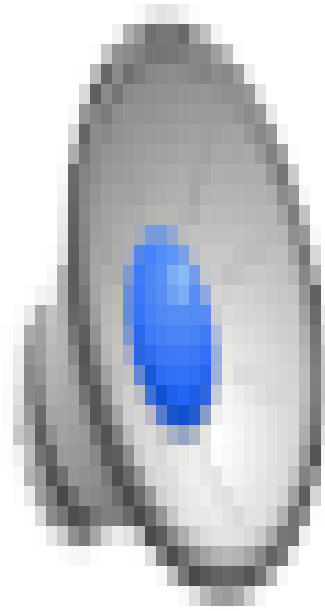
[Demo: contours UCS / greedy / A* empty (L3D1)]

[Demo: contours A* pacman small maze (L3D5)]

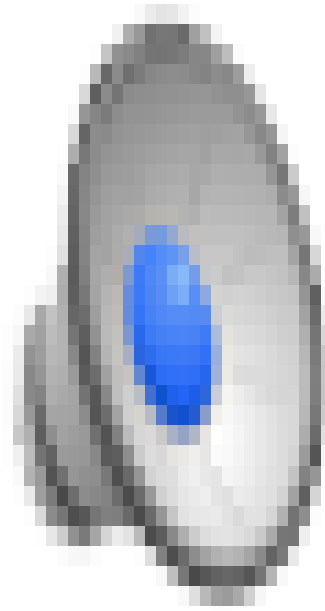
Video of Demo Contours (Empty) -- UCS



Video of Demo Contours (Empty) -- Greedy

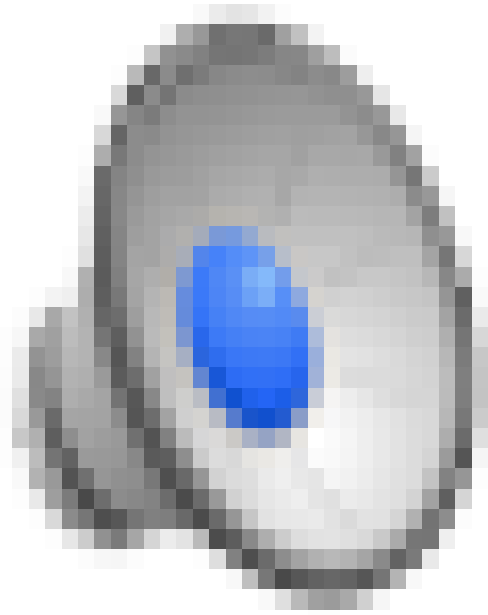


Video of Demo Contours (Empty) – A^*



Video of Demo Contours (Pacman Small Maze)

– A^*



Comparison



Greedy



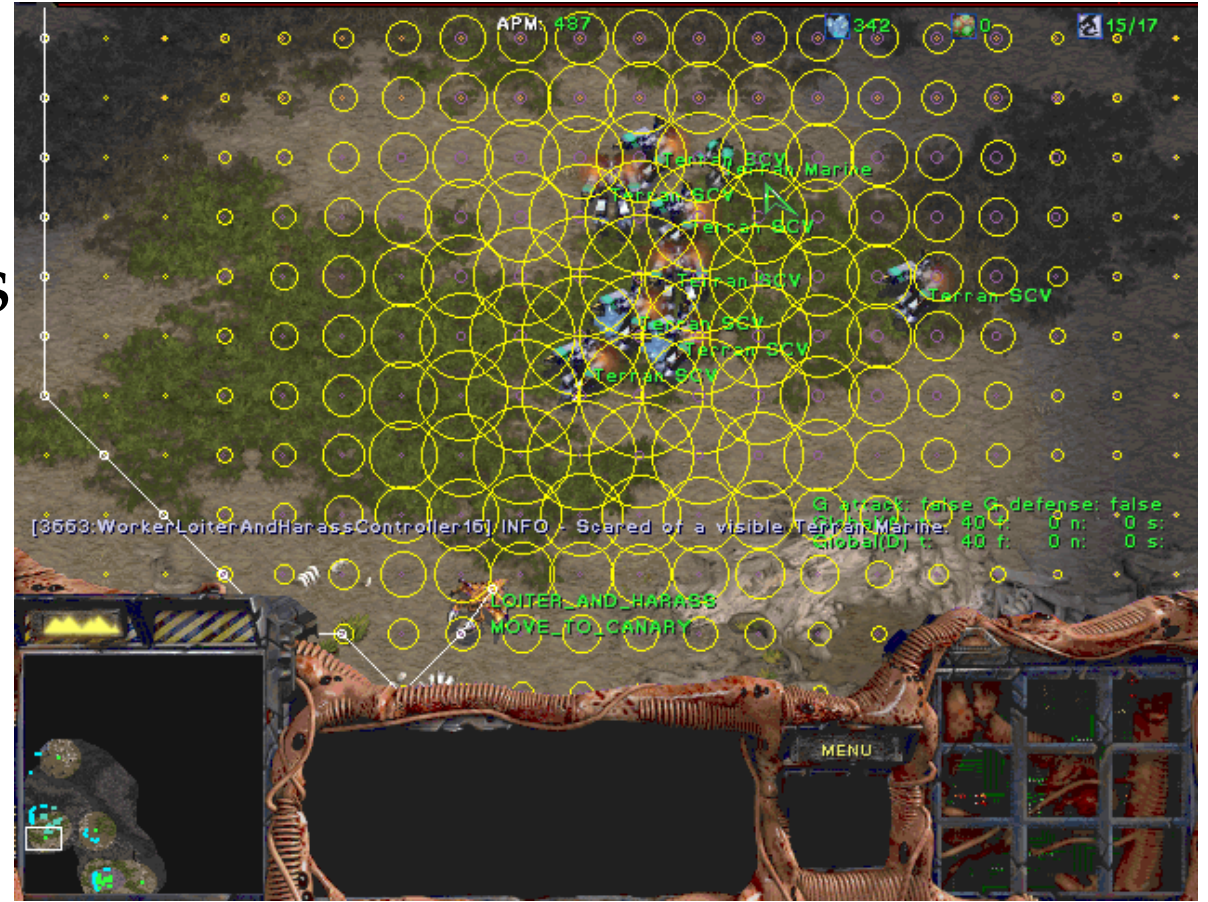
Uniform Cost



A*

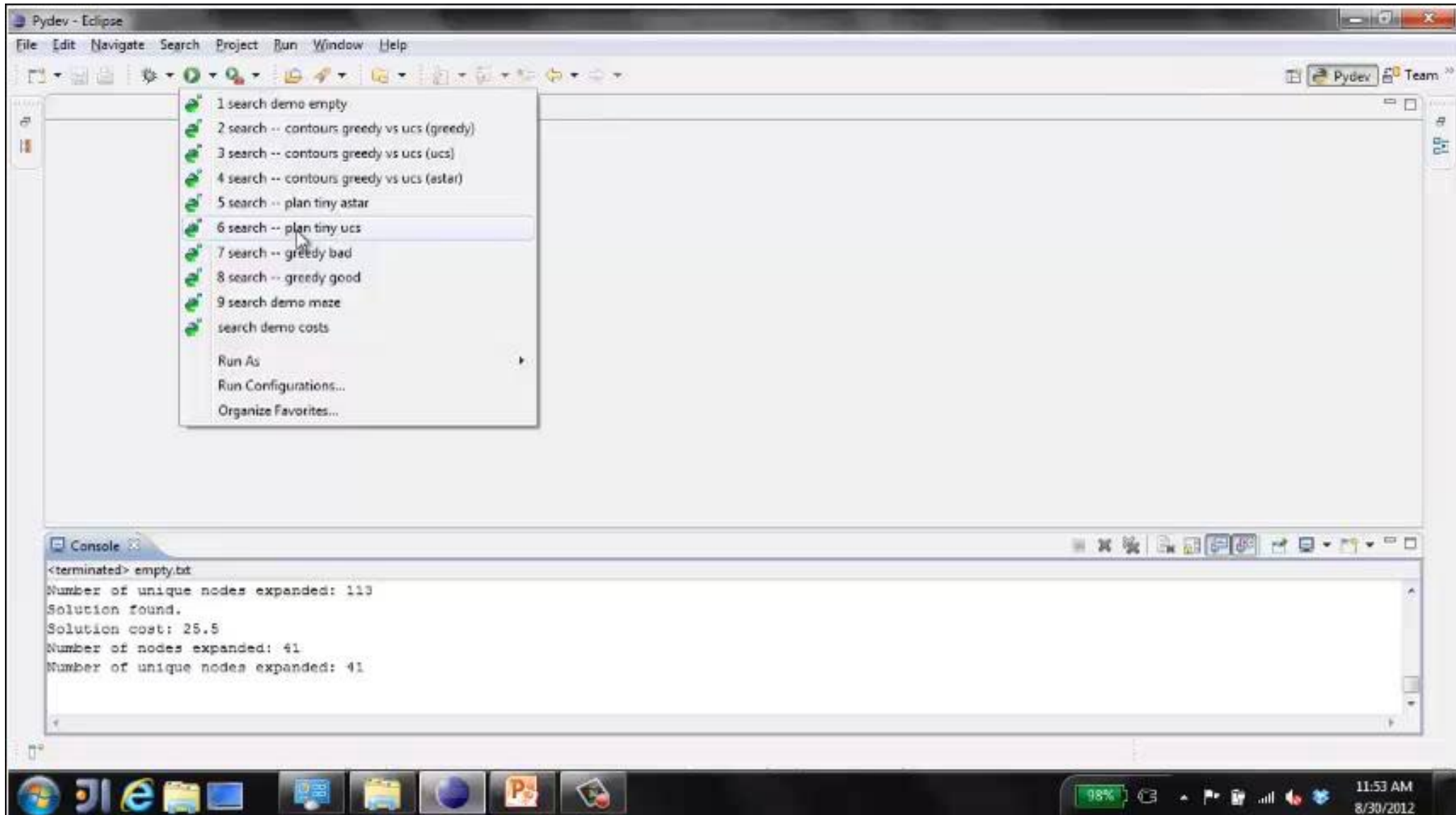
A* Applications

- Video games
- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- ...

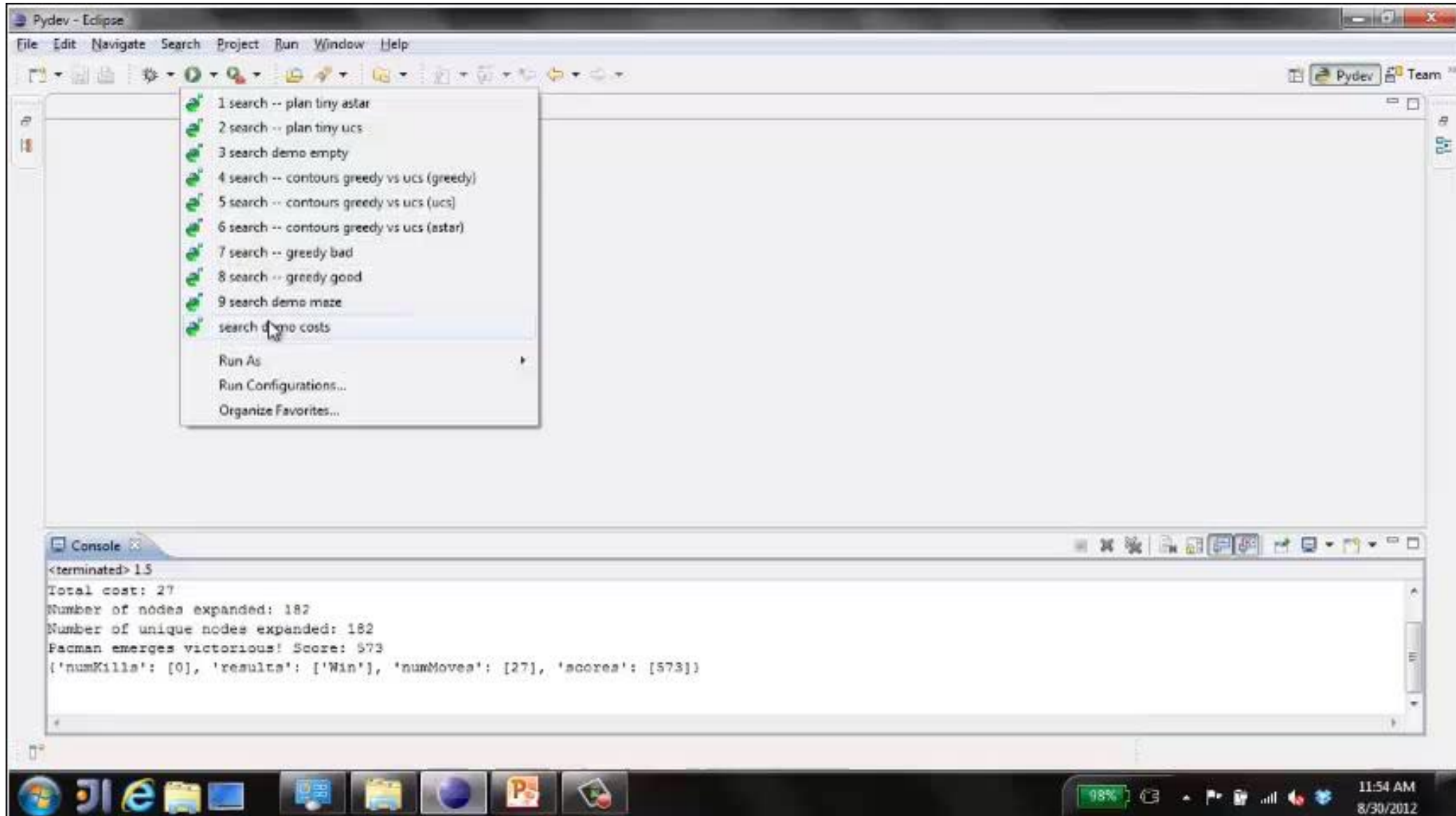


[Demo: UCS / A* pacman tiny maze (L3D6,L3D7)]
[Demo: guess algorithm Empty Shallow/Deep (L3D8)]

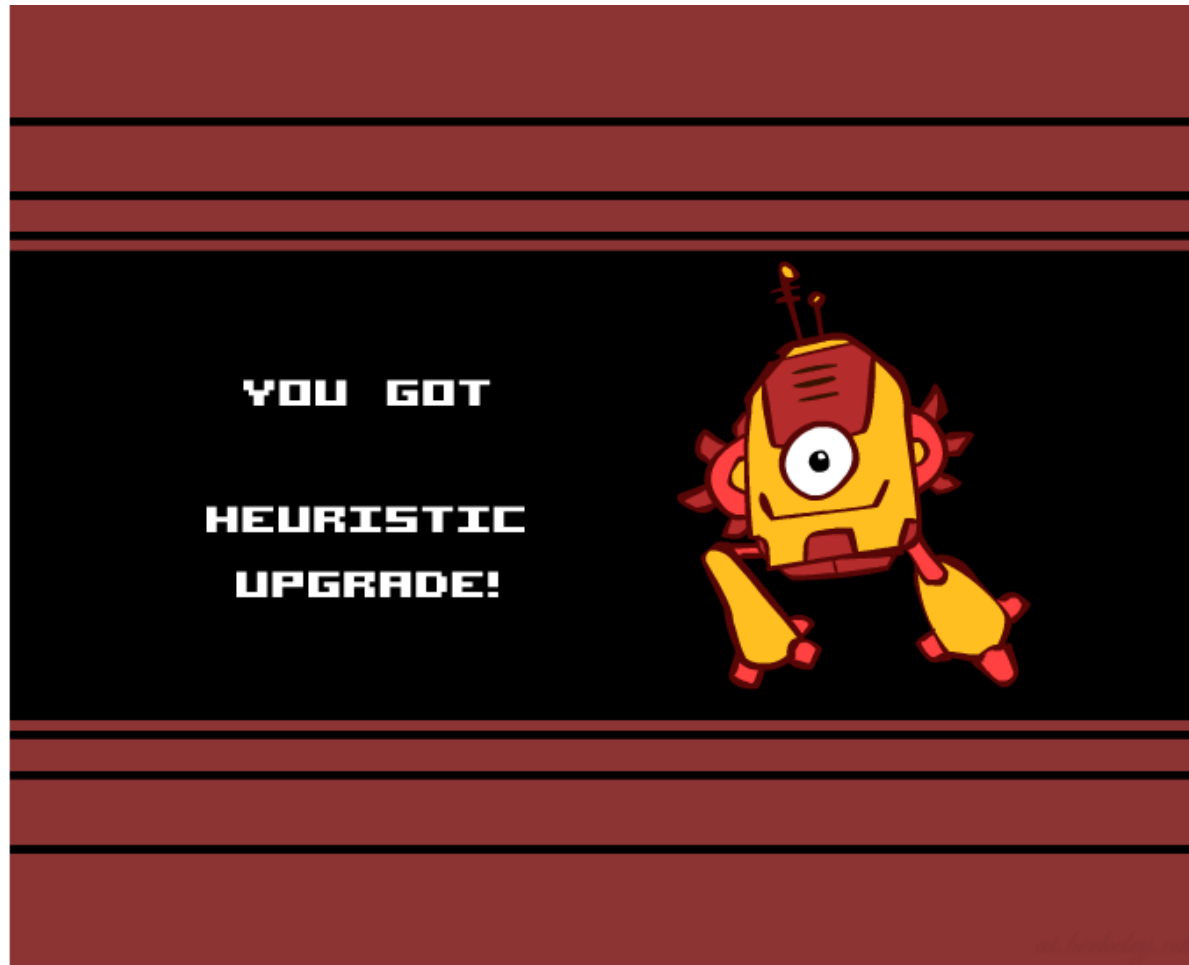
Video of Demo Pacman (Tiny Maze) – UCS / A*



Video of Demo Empty Water Shallow/Deep – Guess Algorithm

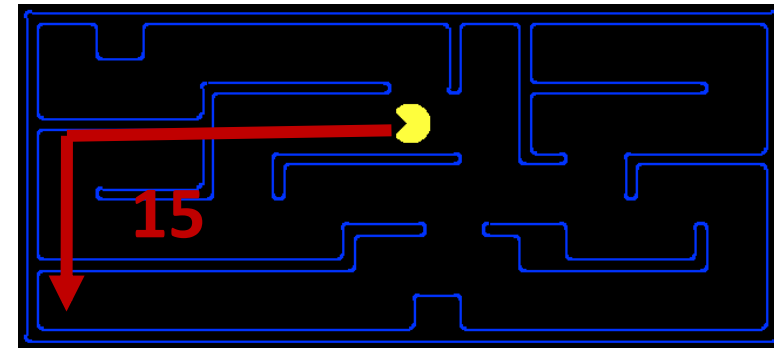
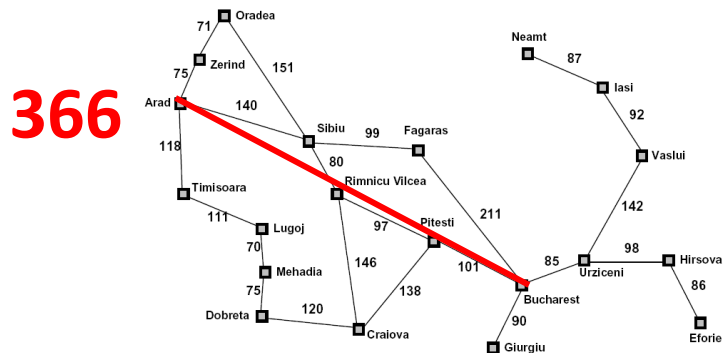


Creating Heuristics



Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics
- Often, admissible heuristics are solutions to *relaxed problems*, where new actions are available

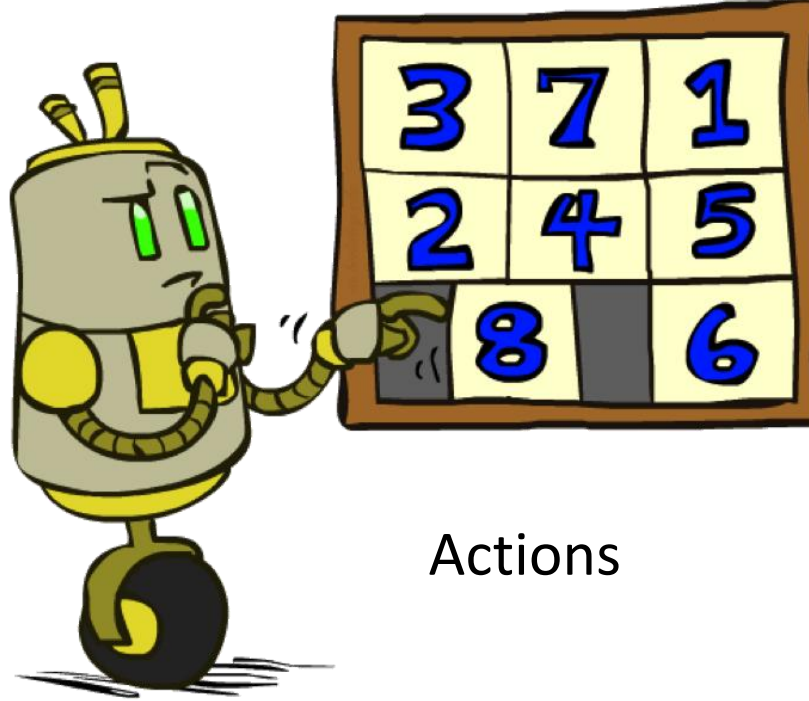


- Inadmissible heuristics are often useful too

Example: 8 Puzzle

7	2	4
5		6
8	3	1

Start State



Actions

	1	2
3	4	5
6	7	8

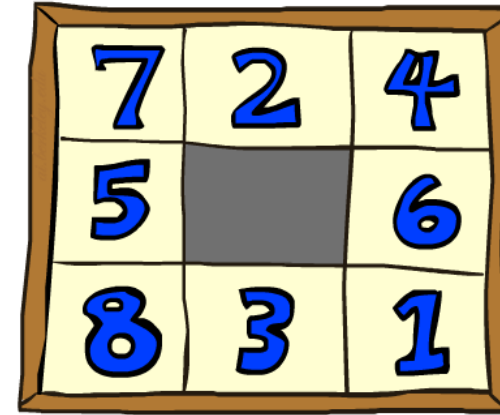
Goal State

- What are the states?
- How many states?
- What are the actions?
- How many successors from the start state?
- What should the costs be?

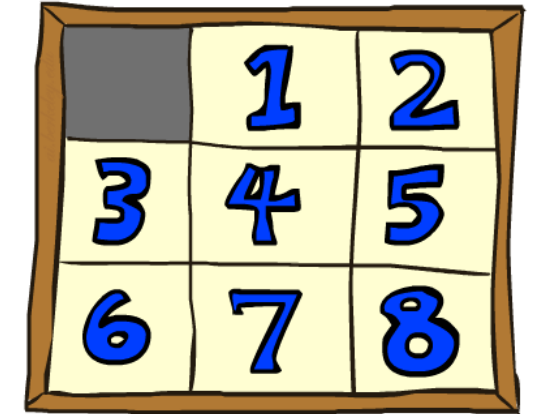
Admissible
heuristics?

8 Puzzle I

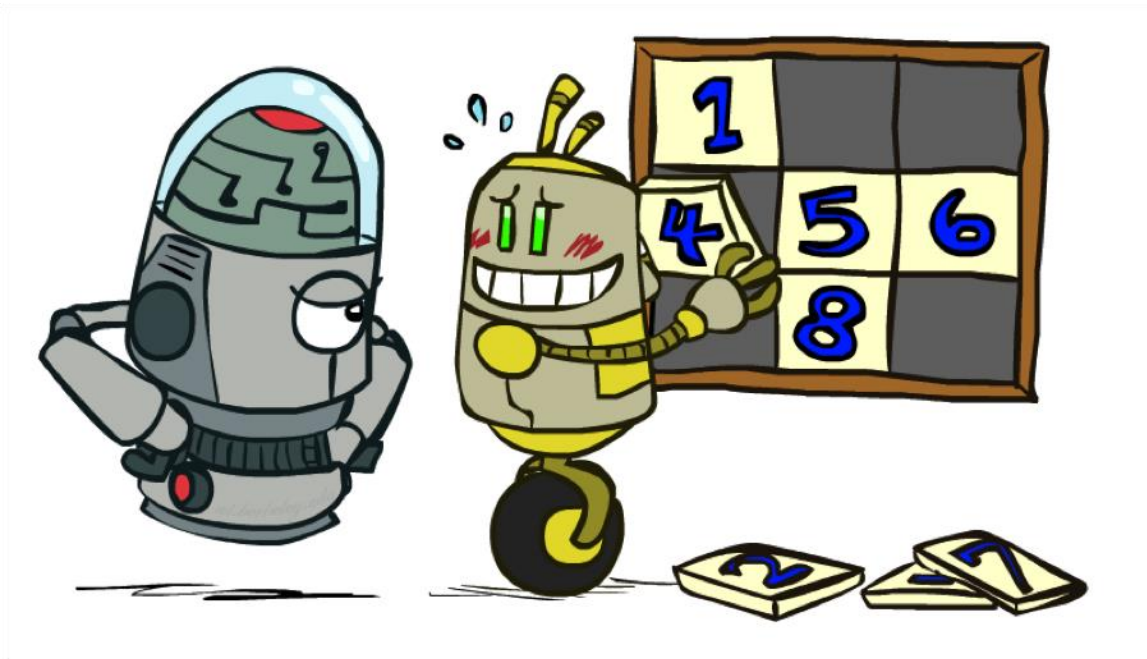
- Heuristic: Number of tiles misplaced
- Why is it admissible?
- $h(\text{start}) = 8$
- This is a *relaxed-problem* heuristic



Start State



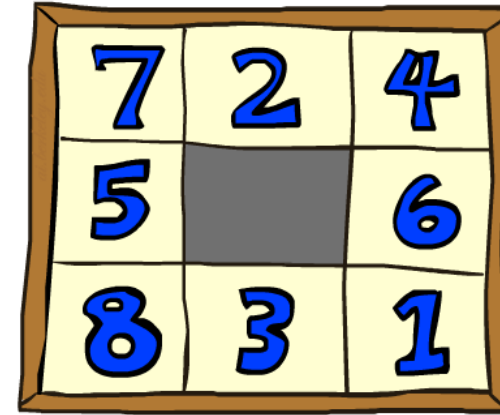
Goal State



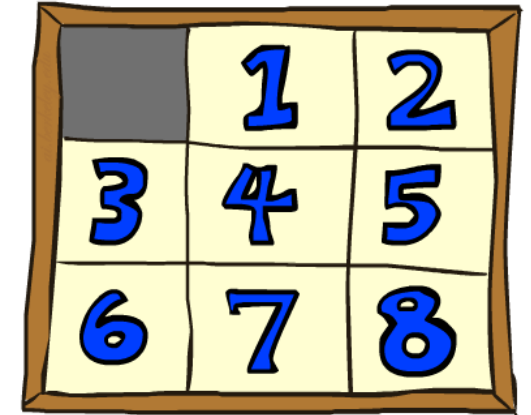
Average nodes expanded when the optimal path has...			
	...4 steps	...8 steps	...12 steps
UCS	112	6,300	3.6×10^6
TILES	13	39	227

8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?
- Total *Manhattan* distance
- Why is it admissible?
- $h(\text{start}) = 3 + 1 + 2 + \dots = 18$



Start State

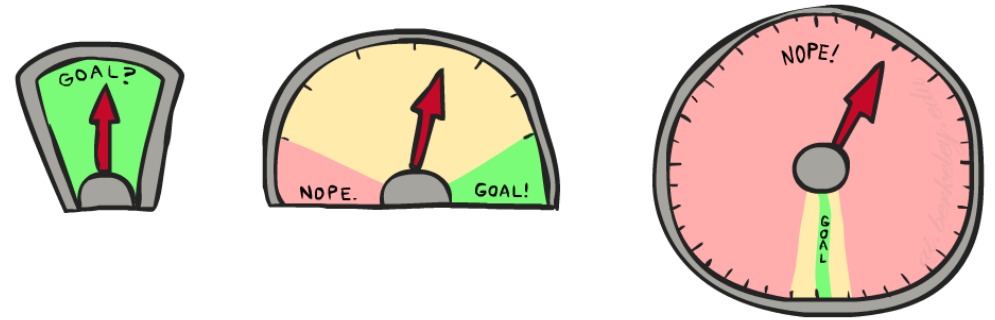


Goal State

Average nodes expanded when the optimal path has...			
	...4 steps	...8 steps	...12 steps
TILES	13	39	227
MANHATTAN	12	25	73

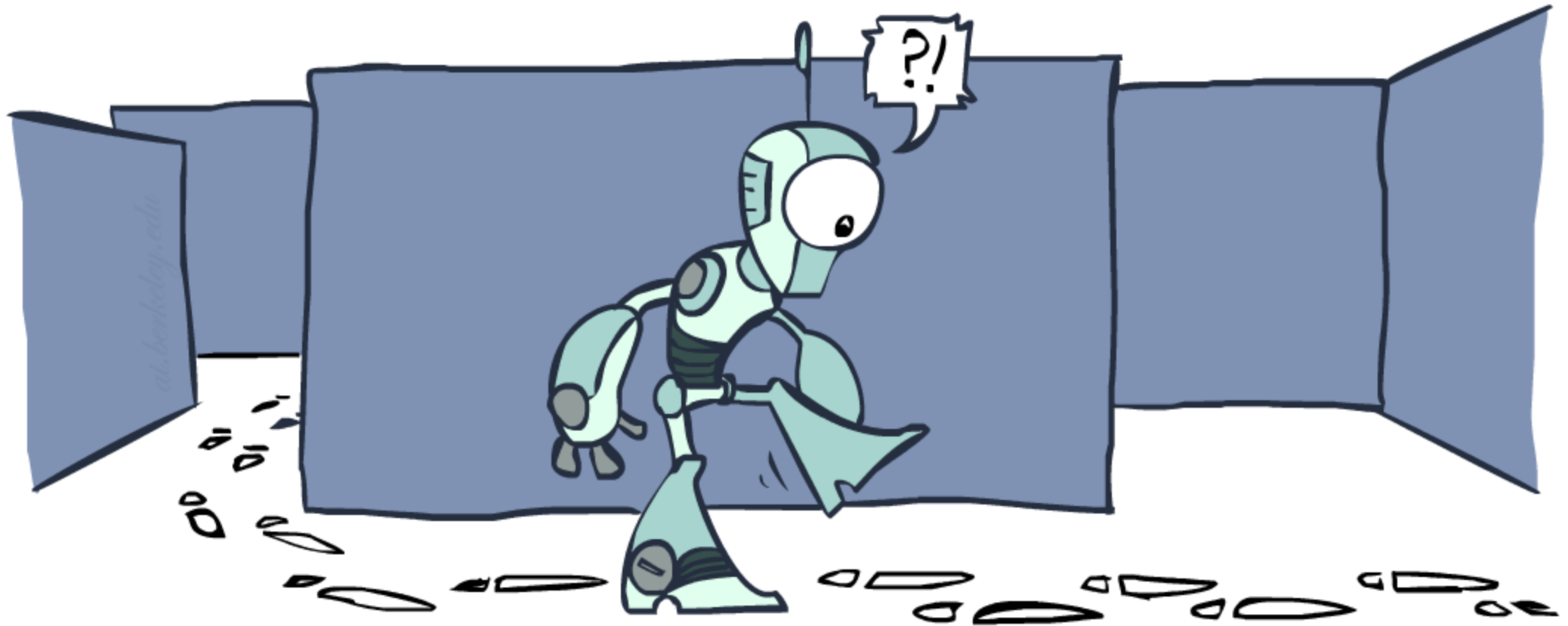
8 Puzzle III

- How about using the *actual cost* as a heuristic?
 - Would it be admissible?
 - Would we save on nodes expanded?
 - What's wrong with it?



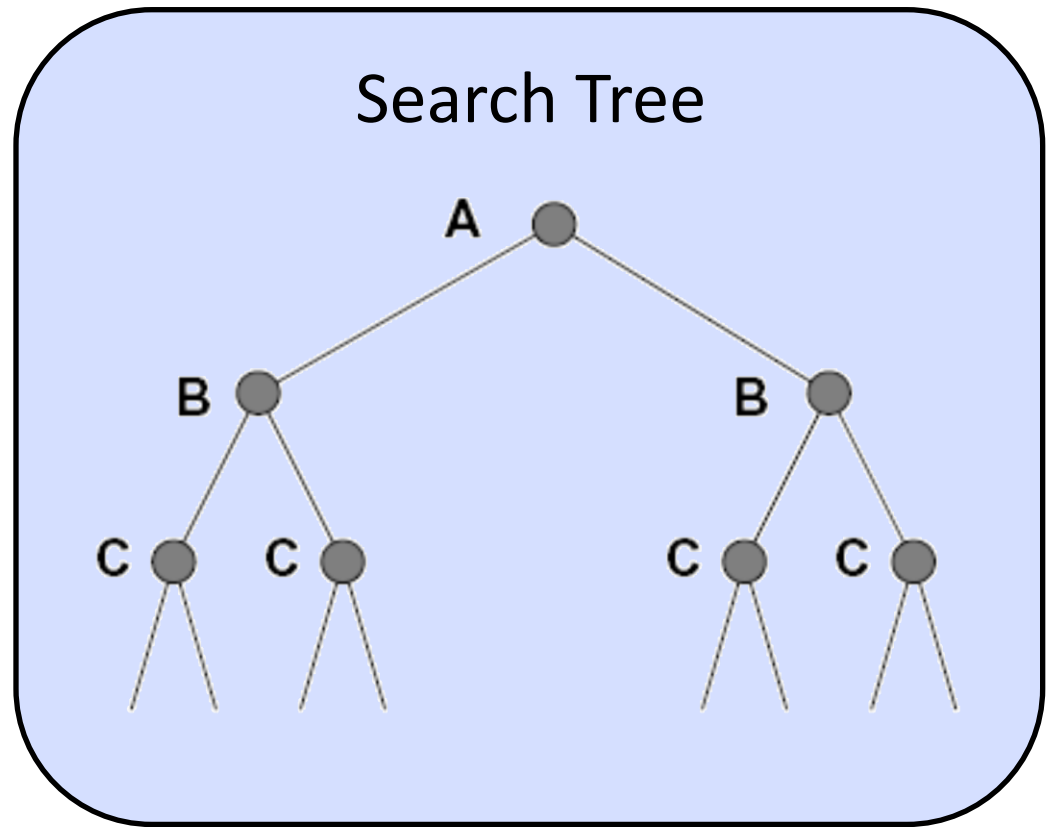
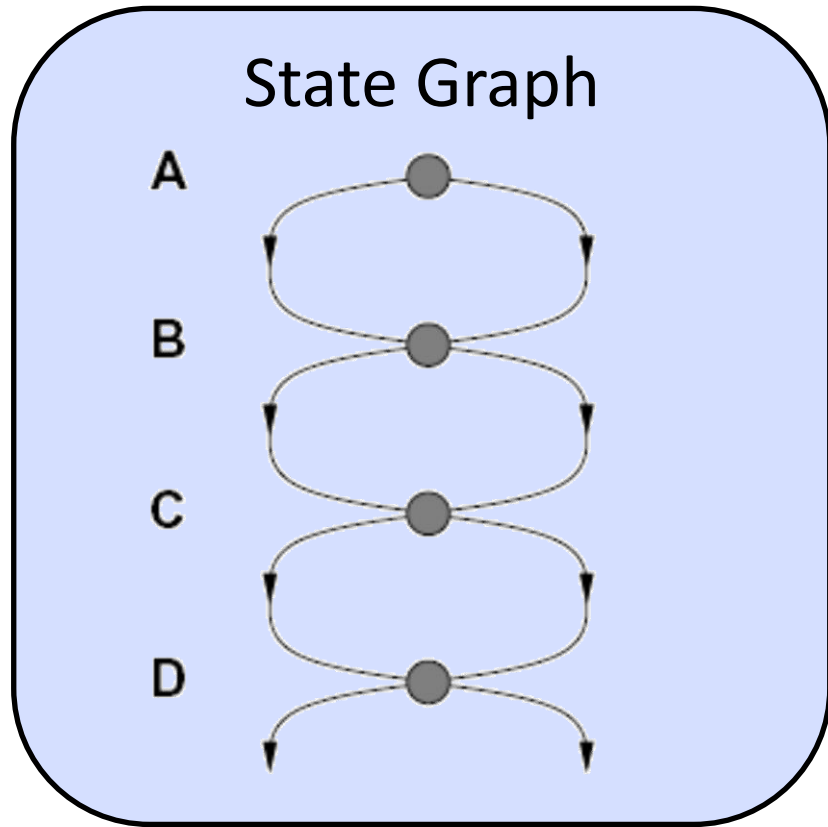
- With A^* : a trade-off between quality of estimate and work per node
 - As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself

Graph Search



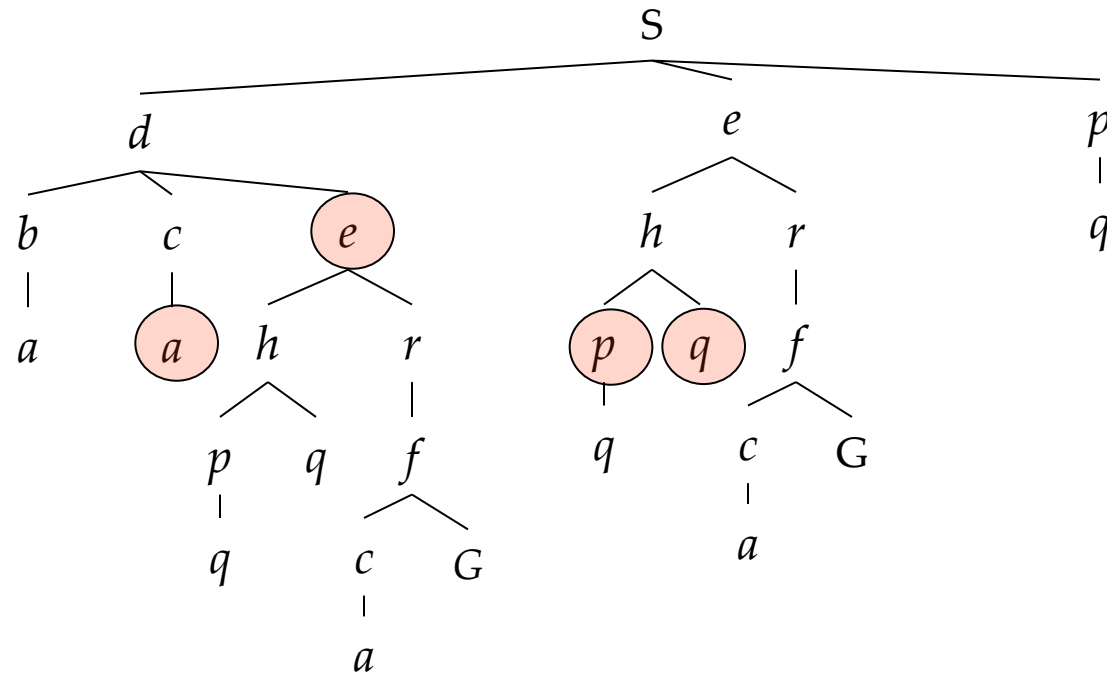
Tree Search: Extra Work!

- Failure to detect repeated states can cause exponentially more work.



Graph Search

- In BFS, for example, we shouldn't bother expanding the circled nodes (why?)

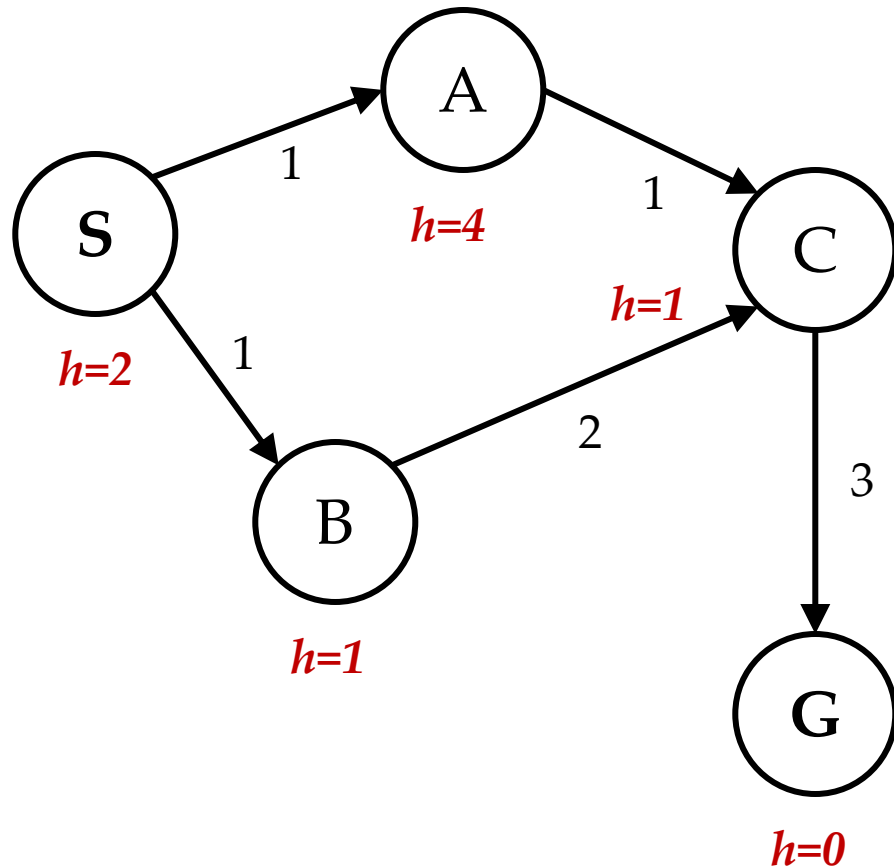


Graph Search

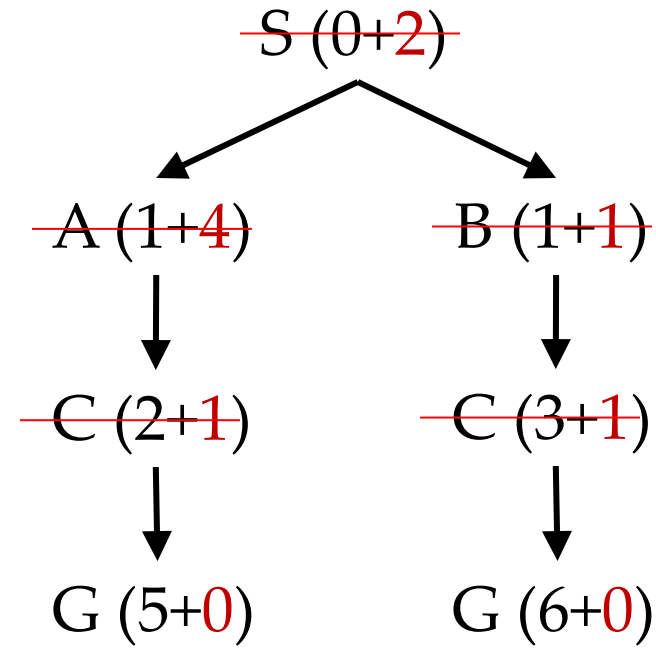
- Idea: never **expand** a state twice
- How to implement:
 - Tree search + set of expanded states (“closed set”)
 - Expand the search tree node-by-node, but...
 - Before expanding a node, check to make sure its state has never been expanded before
 - If not new, skip it, if new add to closed set
- Important: **store the closed set as a set**, not a list
- Can graph search wreck completeness? Why/why not?
- How about optimality?

A* Graph Search Gone Wrong?

State space graph

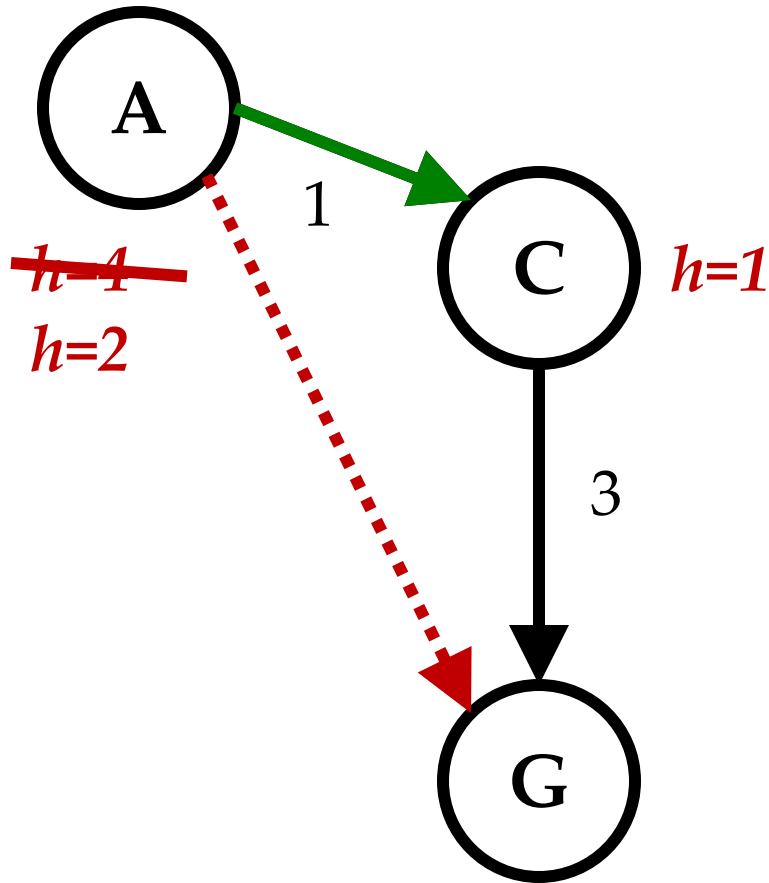


Search tree



Closed Set: S B C A

Consistency of Heuristics

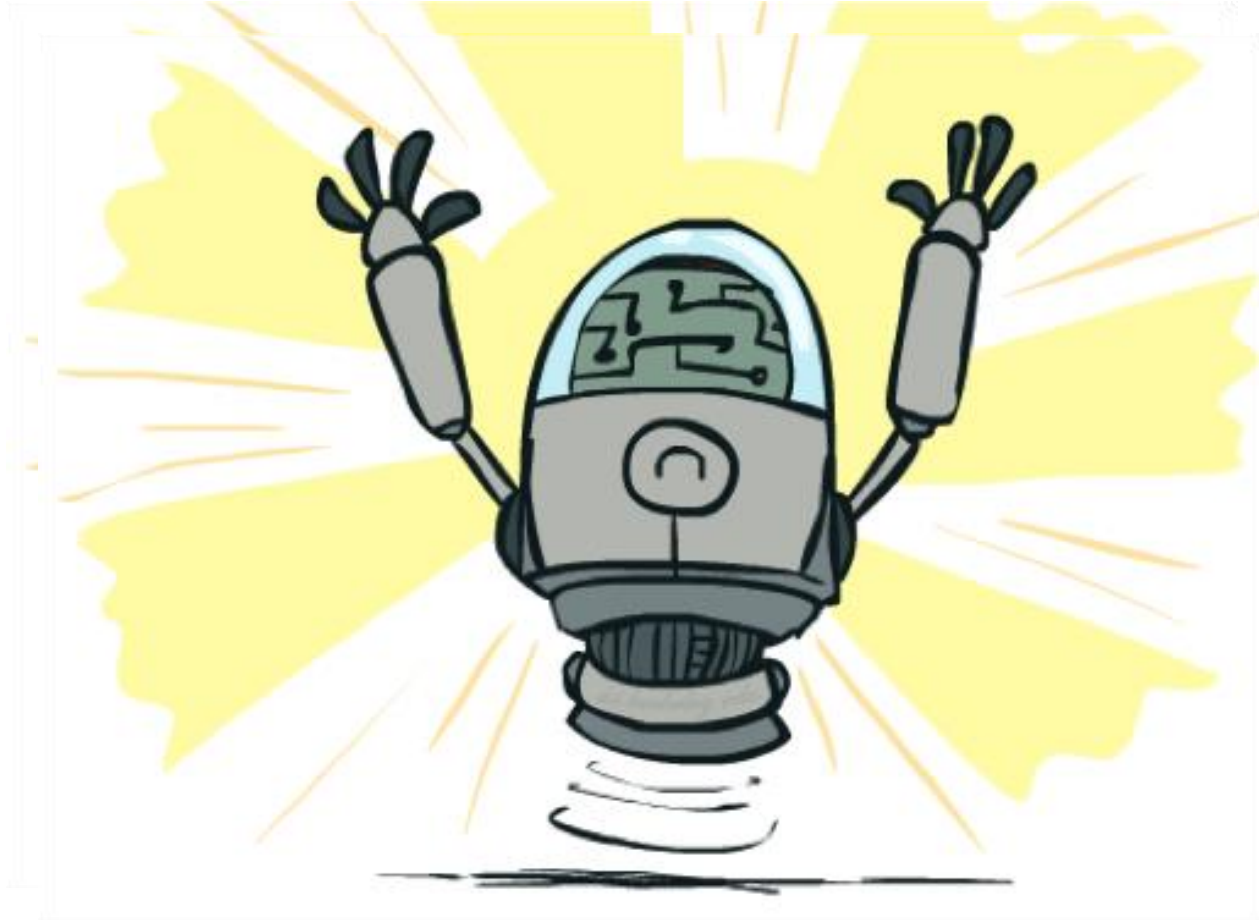


- Main idea: estimated heuristic costs \leq actual costs
 - Admissibility: heuristic cost \leq actual cost to goal
$$h(A) \leq \text{actual cost from A to G}$$
 - Consistency: heuristic “arc” cost \leq actual cost for each arc
$$h(A) - h(C) \leq \text{cost(A to C)}$$
- Consequences of consistency:
 - The f value along a path never decreases
$$h(A) \leq \text{cost(A to C)} + h(C)$$
 - A* graph search is optimal

Optimality of A* Search

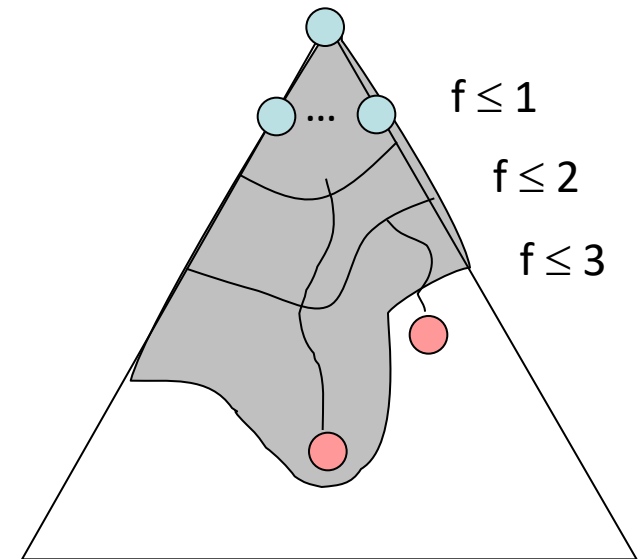
- With a admissible heuristic, Tree A* is optimal.
- With a consistent heuristic, Graph A* is optimal.
 - See slides for details.
- With $h=0$, the same proof shows that UCS is optimal.

Optimality of A* Graph Search



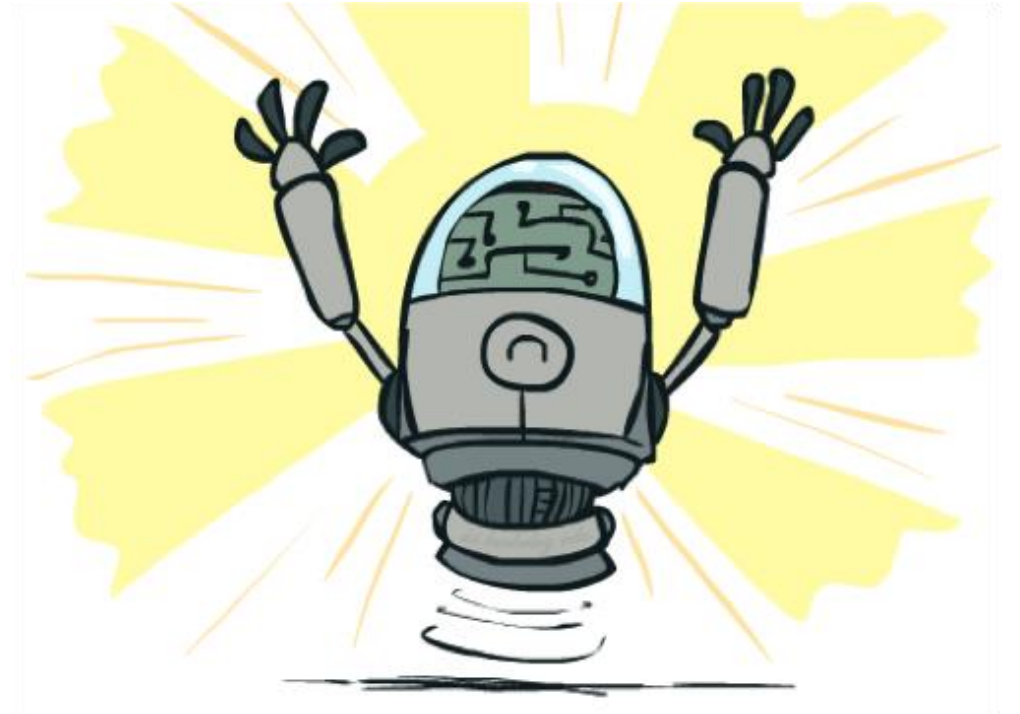
Optimality of A* Graph Search

- Sketch: consider what A* does with a consistent heuristic:
 - Fact 1: In tree search, A* expands nodes in increasing total f value (f-contours)
 - Fact 2: For every state s, nodes that reach s optimally are expanded before nodes that reach s suboptimally
 - Result: A* graph search is optimal

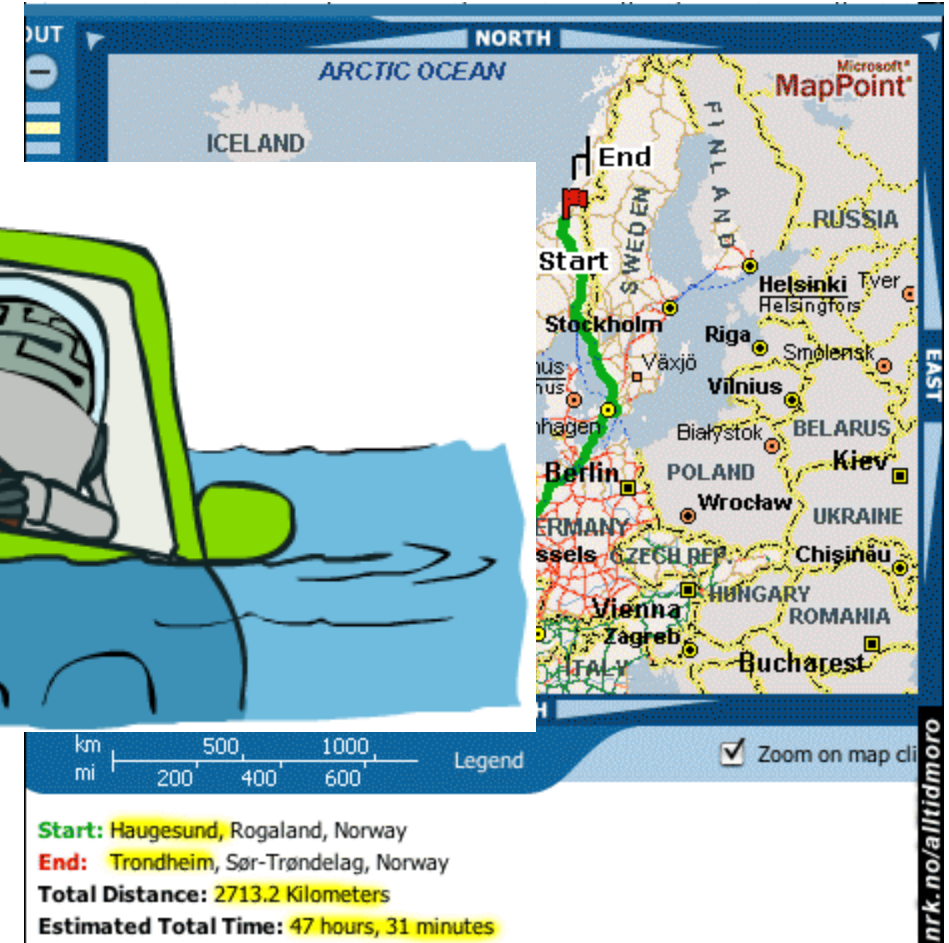
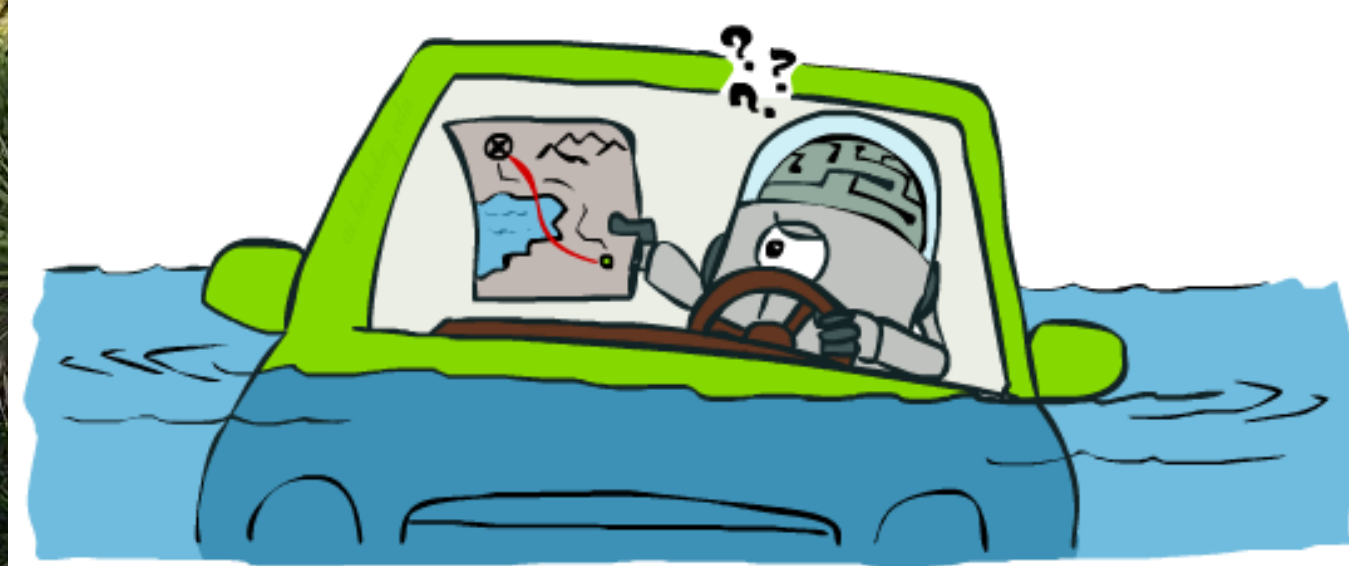
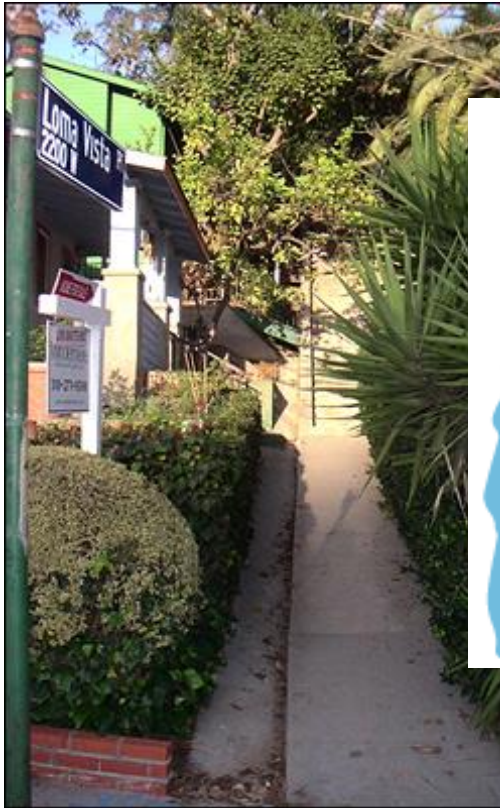


Optimality

- Tree search:
 - A^* is optimal if heuristic is admissible
 - UCS is a special case ($h = 0$)
- Graph search:
 - A^* optimal if heuristic is consistent
 - UCS optimal ($h = 0$ is consistent)
- Consistency implies admissibility
- In general, most natural admissible heuristics tend to be consistent, especially if from relaxed problems



Search Gone Wrong?

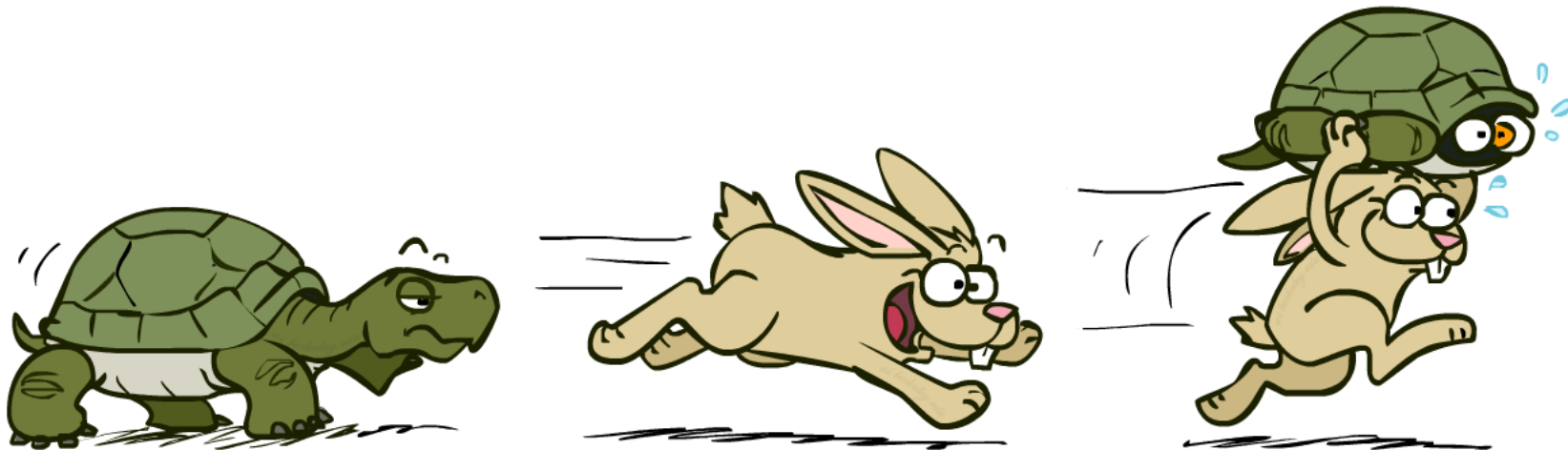


A*: Summary



A*: Summary

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible / consistent heuristics
- Heuristic design is key: often use relaxed problems



Tree Search Pseudo-Code

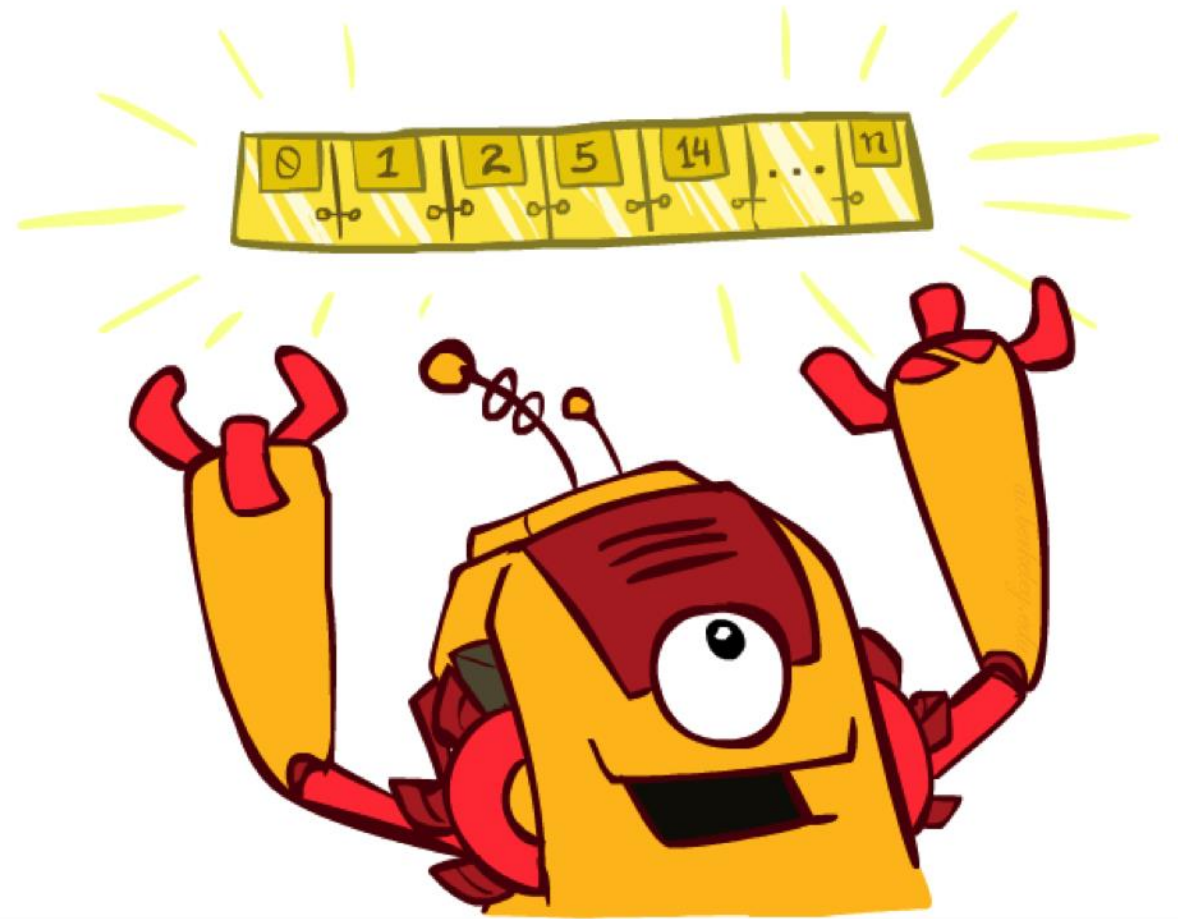
```
function TREE-SEARCH(problem, fringe) return a solution, or failure
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
  loop do
    if fringe is empty then return failure
    node ← REMOVE-FRONT(fringe)
    if GOAL-TEST(problem, STATE[node]) then return node
    for child-node in EXPAND(STATE[node], problem) do
      fringe ← INSERT(child-node, fringe)
    end
  end
```

Graph Search Pseudo-Code

```
function GRAPH-SEARCH(problem, fringe) return a solution, or failure
  closed  $\leftarrow$  an empty set
  fringe  $\leftarrow$  INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
  loop do
    if fringe is empty then return failure
    node  $\leftarrow$  REMOVE-FRONT(fringe)
    if GOAL-TEST(problem, STATE[node]) then return node
    if STATE[node] is not in closed then
      add STATE[node] to closed
      for child-node in EXPAND(STATE[node], problem) do
        fringe  $\leftarrow$  INSERT(child-node, fringe)
      end
    end
  end
```

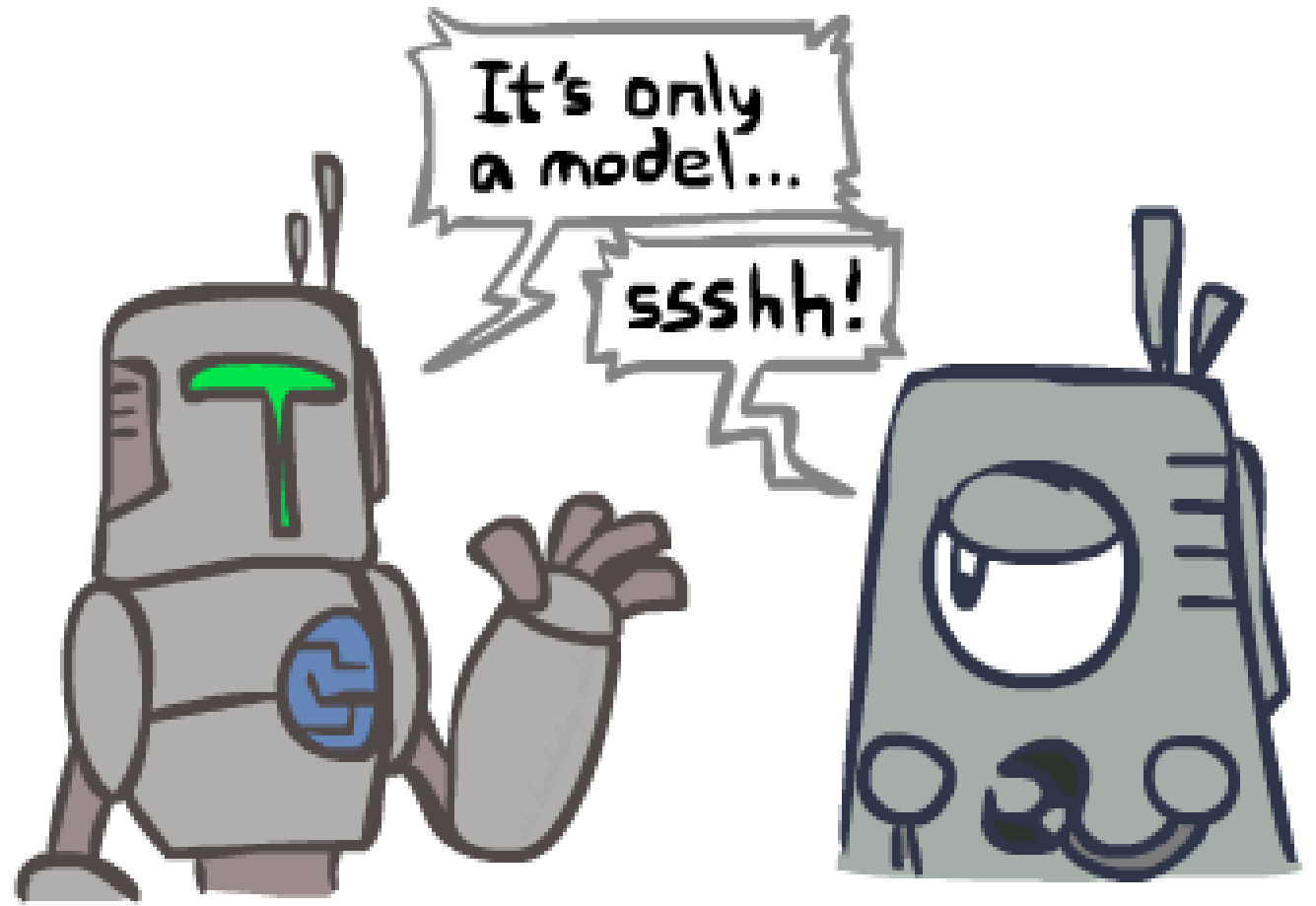

The One Queue

- All these search algorithms are the same except for fringe strategies
 - Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
 - Practically, for DFS and BFS, you can avoid the $\log(n)$ overhead from an actual priority queue, by using stacks and queues
 - Can even code one implementation that takes a variable queuing object



Search and Models

- Search operates over models of the world
 - The agent doesn't actually try all the plans out in the real world!
 - Planning is all “in simulation”
 - Your search is only as good as your models...



Search Gone Wrong?

