Finite Geometries 2025
Seventh Irsee Conference

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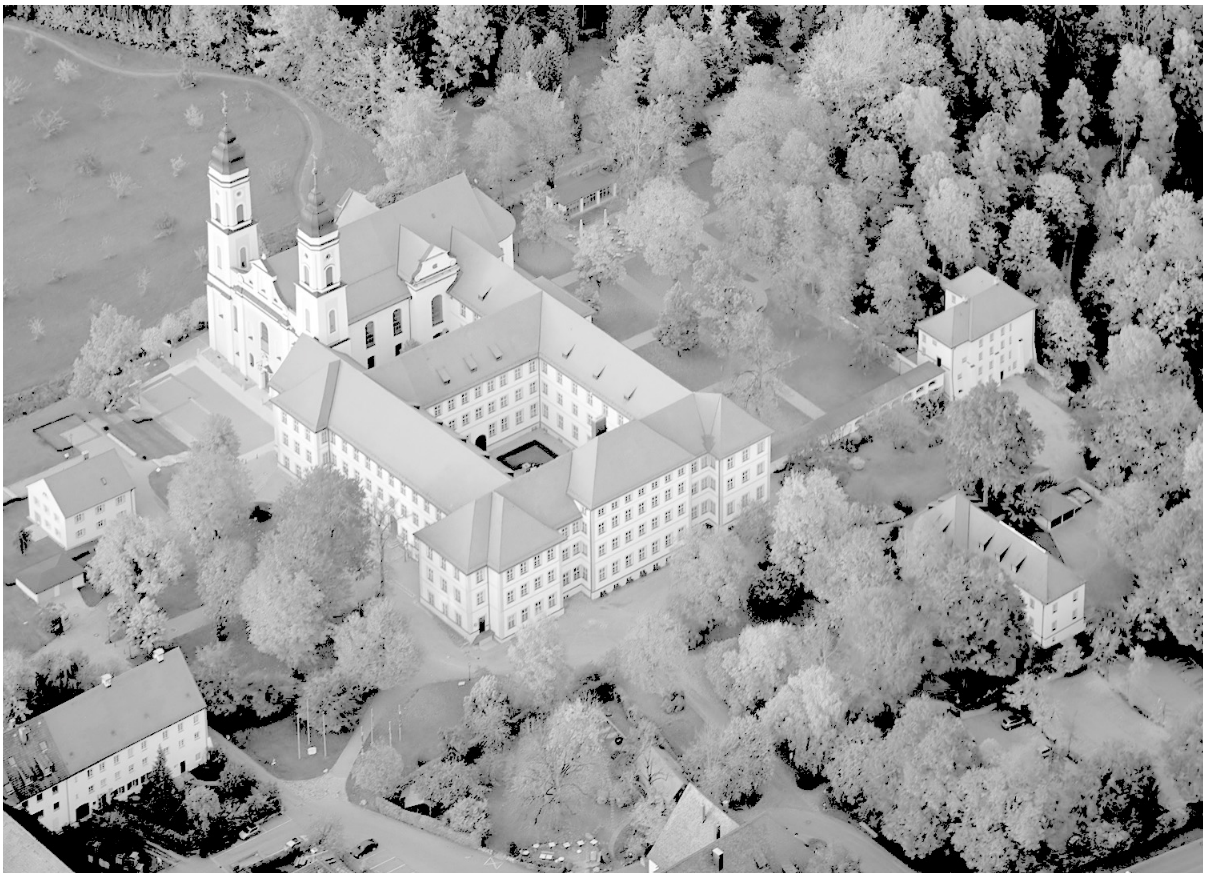
Organisers: Ilaria Cardinali - Michel Lavrauw - Klaus Metsch - Alexander Pott



General Information

The Seventh Irsee Conference, Finite Geometries 2025, continues a series of previous meetings which took place at the Isle of Thorns (2000), in Oberwolfach (2001), and in Irsee (2003, 2006, 2011, 2014, 2017, and 2022).

Conference topics

Combinatorial structures in Galois geometries; Finite Incidence Geometry; Algebraic curves and varieties over finite fields; Geometric and algebraic coding theory; Finite groups and geometries; Algebraic design theory.

Invited speakers

Angela Aguglia (Politecnico di Bari, Italy)
Eimear Byrne (University College Dublin, Ireland)
Alice Devillers (University of Western Australia, Australia)
Daniel Katz (California State University Northridge, USA)
Maria Montanucci (Technical University of Denmark)
Enes Pasalic (University of Primorska, Slovenia)
Jeroen Schillewaert (University of Auckland, New Zealand)
Tamás Szőnyi (Eötvös Loránd University, Hungary)
Jacques Verstraete (University of California San Diego, USA)

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Venue

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1 Invited abstracts

Exploring quasi-Hermitian varieties: properties and applications

Angela Aguglia

Politecnico di Bari

Quasi-polar spaces are combinatorial generalizations of classical polar spaces embedded in finite Desarguesian projective spaces. In this talk, we focus on the Hermitian case.

A quasi-Hermitian variety is any point set in $PG(n,q^2)$, $n \geq 2$, that has the same intersection numbers with hyperplanes as the non-singular Hermitian variety $H(n,q^2)$. These intersection numbers are either

$$(q^n + (-1)^{n-1})(q^{n-1} - (-1)^{n-1})/(q^2-1)$$

or

$$\frac{(q^n + (-1)^{n-1})(q^{n-1} - (-1)^{n-1})}{q^2-1} + (-1)^{n-1}q^{n-1}.$$

By definition, a non-singular Hermitian variety is a quasi-Hermitian variety, referred to as the classical quasi-Hermitian variety.

In fact, any point set $S \subset PG(n,q^2)$ with the same hyperplane intersection numbers as $H(n,q^2)$ also has the same number of points as $H(n,q^2)$ for $n > 2$. In the smallest case, $n = 2$, the point set S has size either q^3+1 or q^2+q+1, corresponding respectively to a unital or a Baer subplane, two combinatorial structures playing a fundamental role in finite geometry.

The systematic study of quasi-Hermitian varieties in higher dimensions began in the early 2010s. We will provide an overview of recent constructions of non-classical quasi-Hermitian varieties, equivalence results, and methodologies that may help to characterize non-singular Hermitian varieties within the broader class of quasi-Hermitian varieties. These results make use of several mathematical tools, including algebraic geometry over finite fields, group theory, and combinatorics.

The study of these varieties is motivated not only by their intrinsic geometric interest but also by their significant applications in areas such as coding theory, graph theory, and cryptography.

This talk is based on several joint works with A. Cossidente, L. Giuzzi, G. Korchmáros, G. Longobardi, A. Montinaro and V. Siconolfi.

q-Systems of Matrix Codes and an Application to the Critical Problem

Eimear Byrne

University College Dublin

(Joint work with John Sheekey)

A matrix code may be represented as a slice space of a 3-tensor, called its generator tensor. We use the generator tensor of a matrix code to introduce the notion of a q-system associated with the code, as an extension of the notion of a q-system of an $\mathbb{F}_{q^m}$-linear rank metric code. This approach allows one to describe the parameters of a matrix code in terms of the other slice spaces of its generator tensor, such as the rank weight of a codeword. Furthermore, the critical exponent of a representable q-polymatroid can be described in terms of an associated q-system. We use this to derive a new upper bound on the critical exponent, which is sharp.

Block-transitive designs admitting multiple invariant partitions

Alice Devillers

University of Western Australia

(Joint work with Seyed Hassan Alavi, Carmen Amarra, Ashraf Daneshkhah, Cheryl Praeger)

Combinatorial designs are incidence structures with points and blocks, where every pair of points is in a fixed number of blocks. There has been a renewed interest in recent years on designs admitting a group of automorphisms that is both flag-transitive (or sometimes just block-transitive) and point-imprimitive. In this talk I will present recent research about such designs which admit multiple systems of imprimitivity. For instance, for 2 systems of imprimitivity $\mathcal{C}_1, \mathcal{C}_2$, they can be either in a chain (each class of $\mathcal{C}_1$ is a subset of a class of $\mathcal{C}_2$), or grid-like (each class of $\mathcal{C}_1$ intersects each class of $\mathcal{C}_2$ in a single point). We determined conditions for a design to admit a flag-transitive or block-transitive group preserving a chain of partitions, and found infinitely many examples for any chain length. We also determined conditions for a design to admit a block-transitive group preserving a multi-dimensional grid structure, and found infinitely many examples for 2- and 3-dimensional grids. We know a single example that is four-dimensional. We also studied more general multiple systems of imprimitivity (not chains nor grid-like).

Differential analysis through a double cover using the unit circle in a finite field

Daniel J. Katz

California State University, Northridge

(Joint work with Kathleen R. O'Connor, Kyle Pacheco, and Yakov Sapozhnikov)

Let F be a finite field, let f be a function from F to F, and let a be a nonzero element of F. The discrete derivative of f in direction a is $\Delta_a f \colon F \to F$ with $(\Delta_a f)(x) = f(x+a) - f(x)$. The differential spectrum of f is the multiset of cardinalities of all the fibers of all the derivatives $\Delta_a f$ as a runs through F^*. Functions whose derivatives have fibers of small size (for example, planar and almost perfect nonlinear functions) are of interest in finite geometry and cryptography. If d is a positive integer, then the power function over F with exponent d is the function $f \colon F \to F$ with $f(x) = x^d$ for every $x \in F$. There is a small number of known infinite families of almost perfect nonlinear power functions. In this talk, we re-express the exponents for one such family in a more convenient form. This enables us to give the differential spectra of the functions in the family and, even more, to give a very precise determination of individual fibers of the derivatives. The key to the analysis is a double cover using the unit circle in a quadratic extension of a finite field.

Automorphism groups of algebraic curves in positive characteristic

Maria Montanucci

Technical University of Denmark, Department of Applied Mathematics and Computer Science

Algebraic curves in positive characteristic and their function fields have been a source of great interest ever since the seminal work of Hasse and Weil in the 1930s and 1940s. Many important and fruitful ideas have arisen out of this area, where number theory and algebraic geometry meet, including the famous application to error-correcting codes given by Goppa's AG codes.

Let $\mathcal{X}$ be a projective, geometrically irreducible, non-singular algebraic curve defined over an algebraically closed field $\mathbb{K}$ of positive characteristic p. Let $\mathbb{K}(\mathcal{X})$ be the field of rational functions on $\mathcal{X}$ (i.e. the function field of $\mathcal{X}$ over $\mathbb{K}$). The $\mathbb{K}$-automorphism group $Aut(\mathcal{X})$ of $\mathcal{X}$ is defined as the automorphism group of $\mathbb{K}(\mathcal{X})$ fixing $\mathbb{K}$ element-wise. The group $Aut(\mathcal{X})$ has a faithful action on the set of points of $\mathcal{X}$.

By a classical result by Schmid (1938), $Aut(\mathcal{X})$ is finite whenever the genus g of $\mathcal{X}$ is at least two. Furthermore it is known that every finite group occurs in this way, since, for any ground field $\mathbb{K}$ and any finite group G, there exists an algebraic curve $\mathcal{X}$ defined over $\mathbb{K}$ such that $Aut(\mathcal{X}) \cong G$ (see for example the work of Valentini-Madden, 1982).

This result raised a general problem for groups and curves, namely, that of determining the finite groups that can be realized as the $\mathbb{K}$-automorphism group of some curve with a given invariant. The most important such invariant is the *genus* g of the curve. In positive characteristic, another important invariant is the so-called *p-rank* of the curve, which is the integer $0 \leq \gamma \leq g$ such that the Jacobian of $\mathcal{X}$ has p^γ p-torsion points.

Several results on the interaction between the automorphism group, the genus and the p-rank of a curve can be found in the literature. A remarkable example is the work of Nakajima (1987) who showed that the value of the p-rank deeply influences the order of a p-Sylow subgroup of $Aut(\mathcal{X})$. Extremal examples with respect to Nakajima's bound are known from the work of Korchmáros-Giulietti (2017) and Stichtenoth (1973). The following open problem arose naturally:

Open Problem 1: How large can a d-group of automorphisms G of an algebraic curve $\mathcal{X}$ of genus $g \geq 2$ be when $d \neq p$ is a prime number? Is there a method to construct extremal examples as for the case $d = p$?

In his work Nakajima also analyzed the case of curves for which the p-rank is the largest possible (the so-called *ordinary curves*), namely $\gamma = g$, proving that they can have at most $84(g^2 - g)$ automorphisms. Since no extremal examples for this bound were found by Nakajima, also the following open problem arose naturally:

Open Problem 2: Is Nakajima's bound $|Aut(\mathcal{X})| \leq 84(g^2 - g)$, sharp for an ordinary curve $\mathcal{X}$ of genus $g \geq 2$?

Hurwitz (1893) showed that if $\mathcal{X}$ is defined over $\mathbb{C}$ then $|Aut(\mathcal{X})| \leq 84(g-1)$, which is known as the *Hurwitz bound*. This bound is sharp, i.e., there exist algebraic curves over $\mathbb{C}$ of arbitrarily high genus g whose automorphism group has order exactly $84(g-1)$. Well-known examples are the Klein quartic and the Fricke-Macbeath curve.

Roquette (1970) showed that Hurwitz bound also holds in positive characteristic p, if p does not divide $|Aut(\mathcal{X})|$. A general bound in positive characteristic is $|Aut(\mathcal{X})| \leq 16g^4$ with one exception: the so-called Hermitian curve. This result is due to Stichtenoth (1973). The quartic bound $|Aut(\mathcal{X})| \leq 16g^4$ was improved by Henn (1978). Henn's result shows that if $|Aut(\mathcal{X})| > 8g^3$ then $\mathcal{X}$ is $\mathbb{K}$-isomorphic to one of 4 explicit exceptional curves, all having p-rank equal to zero. A third natural open problem arose as a consequence of this result:

Open Problem 3: Is it possible to find a (optimal) function $f(g)$ such that the existence of an automorphism group G of $\mathcal{X}$ with $|G| > f(g)$ implies that $\mathcal{X}$ has p-rank zero?

Henn's result clearly implies that $f(g) \leq 8g^3$, but it is pleausible to believe that a quadratic bound with respect to g could also be found.

In this talk, we will describe our main contributions to the three problems mentioned above and more generally in understanding the relation between automorphism groups of algebraic curves in positive characteristic and the other invariants mentioned above. If time allows, applications of these results in determining isomorphism classes of algebraic curves over finite fields will also be discussed.

Bent functions - five decades later

Enes Pasalic

University of Primorska

(Joint work with S. Kudin, S. Polujan, F. Zhang)

Bent functions form a special class of Boolean functions in an even number of variables, notable for various combinatorial properties and applications in cryptography. For instance, their Hamming distance to the set of affine functions is maximal, and the bent property is equivalent to the fact that its support forms a Hadamard difference set. Moreover, the Cayley graphs constructed from bent functions are strongly regular.

The notion of bent functions was introduced by Rothaus in the mid sixties, whereas two primary constructions are due to Maiorana-McFarland [1] and Dillon [2]. In the last decade, a series of articles considered the design of bent functions that are provably outside the Maiorana-McFarland ($\mathcal{M}$) class. In this talk, using the notion of $\mathcal{M}$-subspaces, we will survey the most important achievements [12, 8, 5, 4, 10, 6, 3, 9, 7] regarding the design of bent functions that do not belong to the primary classes. A complete characterization of these objects mainly depends on the properties of so-called *bent sets* whose exact specification, especially in terms of induced $\mathcal{M}$-subspaces, remains unanswered.

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Constructing highly regular expanders from hyperbolic Coxeter groups

Jeroen Schillewaert

University of Auckland

(Joint work Marston Conder, Alexander Lubotzky and François Thilmany)

Expander graphs are sparse graphs with strong connectivity properties. Chapman, Linial and Peled asked whether there exist families of expander graphs with high levels of regularity, that is not only the number of edges containing a given vertex needs to be constant but also the number of triangles containing a given edge etcetera. We answer this question positively constructing families of expander graphs as quotient graphs of 1-skeleta of infinite polytopes (1-skeleton means only retain the vertex-edge information of the polytope). The latter are Wythoffian polytopes, which are obtained from Coxeter groups by decorating the associated Coxeter diagram. The specific higher regularity properties depend on this diagram. Expansion stems from superapproximation of the Cayley graphs associated to the Coxeter group, which is a number-theoretic way to study the rate of convergence of random walks on these graphs. The Cayley graphs and the 1-skeleta are quasi-isometric (that is equal on a large scale) which implies that one forms an expanding family if and only if the other does.

On a colouring problem for projective planes

Tamás Szőnyi

HUN-REN Rényi Institute, Eötvös University and University of Primorska

(Joint work with Aart Blokhuis, Ádám Markó, Zsuzsa Weiner)

Determining the chromatic number of a projective plane of order n is an easy exercise. It is 3 for the Fano plane and 2 for $n \geq 3$. In spite of this, there are several interesting problems for colourings of projective planes, already about 2-colourings. We will mostly be interested in 2-colourings and the colours we call red and blue. The classical result by Spencer [4] about the discrepancy of projective planes gives a probabilistic proof of the existence of a 2-colouring so that for every line ℓ the difference of the number of red and blue points on ℓ is at most $K\sqrt{n}$ (where K is an absolute constant and n is the order of the plane). Using the standard equations for the set R of red points it is easy to see that $\sum_{i=1}^{n^2+n+1}(r_i - b_i)^2$ only depends on the number of red points, where $\ell_1, \ldots, \ell_{n^2+n+1}$ is the list of lines and r_i denotes the number of red, b_i denotes the number of blue points on ℓ_i. In particular, the above sum is at least $(n+1)(n^2+1)$, which implies that $K \geq 1$. It also implies that there must be lines with $r_i \neq b_i$.

Probably motivated by the above result by Spencer, Erdős asked in the eighties the following problem: *What is the maximum number of lines that have the same number of red and blue points?* Let us call such a line *balanced*. It is again an easy exercise to show that there are always at least $n+1$ unbalanced lines, so the answer to Erdős' question is n^2. In case of equality, the unbalanced lines go through a point, half of them are entirely red, half of them are entirely blue (except the point itself). This colouring is called *trivial*.

It is a natural question to improve on the bound $n+1$ if the colouring is not trivial. This question was considered by Ádám Markó [3], who proved that when the number of unbalanced lines is at most $\frac{13}{8}n$, then the colouring must be trivial. It seems a natural conjecture that the next possible number of unbalanced lines is $2n+1$, and the corresponding colouring comes from the trivial one by changing the colour of one point.

In this talk we focus on Galois planes $\mathrm{PG}(2, q)$, so from now on the order of the plane will be denoted by q. In this case we could prove the bound $2q+1$ but could not describe the corresponding colouring.

A natural generalization of the problem is to prescribe the profile of the line (r, b) $(r + b = q + 1)$ and ask for the maximum number of lines with r red and b blue points. Such lines are called (r, b)-coloured. Many of our arguments work also in this more general case, if there are lines which have r' red points for some $r' \not\equiv r$ modulo p, where p is the characteristic of the ground field. Note that if we colour the lines of a dual hyperoval (q even) entirely blue, then every other line will have $b = (q+2)/2$ and $r = q/2$, so it is a colouring with $q+2$ not (r, b)-coloured lines.

Of course, for planes of prime order the above modulo p condition is almost automatical, so we have the strongest results in this case. Also, in the prime case we can associate a small weight codeword in the code generated by the lines of the dual plane to a colouring with many (r, b)-coloured lines. We know a codeword of weight $3p-3$ discovered by Bagchi (and independently by De Boeck and Vandendriessche), so one might suspect that there should be a colouring, too. Kiss and Somlai [2] proved that $R = \{(x, y) : y < x, 0 \leq x, y < p\} \cup \ell_\infty$ is a set in the Erdős case ($r = (p+1)/2$) such that the unbalanced lines are the affine lines with slope $0, 1, \infty$ (with one-one exception for each slope) and the line at infinity. So, the coordinates are considered as integers in the example. To have a theorem in this abstract, let us state our result for the prime case (see [1]).

Theorem 1 *Consider a colouring of* $\mathrm{PG}(2, p)$, $p \geq 19$ *prime, in red and blue. A line is called balanced if it contains exactly* $0 < r < p+1$ *red points. Suppose that there are at most* $\max\{3p+1, 4p-22\}$ *unbalanced lines. Then either the set of red or the set of blue points has one of the following structures:*

1. *r concurrent lines through a point P, the colour of P can be switched.*

2. *The structure described in case (1), with switching the colour of at most two points different from P.*

3. *One line together with at most three points outside of it.*

4. *A set, equivalent, up to affine transformation, with the affine part of the Kiss-Somlai example or its complement (the points of the line at infinity have the same colour; it can be both red or blue).*

Moreover, all these structures give a colouring with at most $\max\{3p+1, 4p-22\}$ *unbalanced lines.*

Finally, if r is small (a small constant times $\sqrt{p}$), then we can allow considerably more unbalanced lines and describe the colouring completely.

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The probabilistic method in finite geometry

Jacques Verstraete

University of California, San Diego

The probabilistic method is a non-constructive existence argument that was pioneered by Paul Erdős in the middle of the twentieth century. While the impact of the method is felt across all of mathematics, I will focus on applications in finite geometry in this talk. This includes the topics of blocking sets, maximal arcs, partial ovoids and spreads, and strong representative systems to mention a few. We will outline the central tools in this method, and give some simple geometric consequences as well as open problems.

2 Contributed abstracts

Extended field presentations of arcs and ovoids

Kanat Abdukhalikov

UAE University

Arcs and ovoids in finite projective spaces have canonical presentations in homogeneous coordinates. We revisit these presentations and provide their presentations in terms of extensions of basic fields, or in terms of polar coordinates. We provide constructions of hyperovals, Segre arcs, maximal arcs, and ovoids in $PG(3, q)$.

Eigenvalue bounds for the independence number of graph powers and an application to coding theory

Aida Abiad

Eindhoven University of Technology, Vrije Universiteit Brussel

(Joint work with A. Ravagnani and A. Khramova)

In this talk, eigenvalue bounds on the independence number of graph powers will be presented. We will then use such eigenvalue bounds to estimate the maximum size of a code in the sum-rank metric, illustrating how the spectral method can often improve the state of the art coding bounds.

Linear complete symmetric rank-distance codes

Nour Alnajjarine

University of Rijeka

(Joint work with Michel Lavrauw)

An $\mathbb{F}_q$-linear code of minimum distance d is said to be *complete* if it is not contained in any larger $\mathbb{F}_q$-linear code with the same minimum distance d. In this talk, we demonstrate the existence of 3 (resp. 6) $\mathbb{F}_q$-linear complete symmetric rank-distance (CSRD) codes in $M_{3\times 3}(\mathbb{F}_q)$ with $d = 2$, up to equivalence, for q odd (resp. even). Our approach is mainly geometric. We will also present some contributions of our results toward the classification of nets of conics in $\mathrm{PG}(2, q)$.

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Stabiliser codes over finite fields, associated geometries and entanglement in quantum states

Simeon Ball

Universitat Politècnica Catalunya

In this talk I will give a detailed construction of stabiliser codes over finite fields of prime order using sets of lines in projective spaces. One-dimensional stabiliser codes are known as graph states, since one can associate to such a stabiliser code a graph whose edges are weighted with elements of $\mathbb{F}_p$. In this talk I will prove that there is in fact a weighted graph associated to any stabiliser code. I will then endeavour to broaden the definition of stabiliser codes which will allow us to construct stabiliser code for mixed dimensional Hilbert spaces. I will also mention applications to entanglement measures and absolutely maximally entangled states.

The Kovalevski Configuration of a Quartic Curve over a Finite Field

Anton Betten

Kuwait University

Quartic curves with 28 bitangents may have points outside the curve where 4 bitangents meet. We call such a point a Kovalevski point. The number of such points is at most 63, but most curves over finite fields have far fewer such points. What are the possible configurations of Kovalevski points? To investigate this question, we proceed to classify these curves over small finite fields. This is facilitated by the relation to cubic surfaces with 27 lines. In particular, we will study the relation between Kovalevski points on quartic curves and Eckardt points on cubic surfaces. Recall that an Eckardt point of a cubic surface is a point lying on three lines of the surface.

Asymptotically optimal cyclic subspace codes

Chiara Castello

University of Campania "L. Vanvitelli"

(Joint work with Paolo Santonastaso)

Subspace codes, and in particular cyclic subspace codes, have gained significant attention in recent years due to their applications in error correction for random network coding. The Grassmannian $\mathcal{G}_q(n,k)$ is the set of all k-dimensional $\mathbb{F}_q$-subspaces of an n-dimensional vector space over $\mathbb{F}_q$. A constant dimension subspace code is a subset of $\mathcal{G}_q(n,k)$ endowed with the subspace distance. We may consider as n-dimensional vector space over $\mathbb{F}_q$ the extension field $\mathbb{F}_{q^n}$ of $\mathbb{F}_q$, so that a cyclic subspace code in $\mathcal{G}_q(n,k)$ is union of orbits of subspaces of $\mathbb{F}_{q^n}$ under the action of the multiplicative group of $\mathbb{F}_{q^n}$. In this talk, we introduce a new technique for constructing cyclic subspace codes with large cardinality and prescribed minimum distance. Using this new method, we provide new constructions of cyclic subspace codes in the Grassmannian $\mathcal{G}_q(n,k)$, where $k \mid n$ and n/k is a composite number, with minimum distance $2k-2$ and large size. Precisely, we prove that the resulting codes have sizes larger than those obtained from previously known constructions with the same parameters. Finally, we discuss the asymptotic behavior of the sizes of our constructions, showing that they achieve the Johnson type bound II for constant dimension subspace codes within certain parameters regime.

The even and odd sets of PG(2,8)

Kris Coolsaet

Ghent University (Belgium)

(Joint work with Arne Botteldoorn and Silvia Pagani)

An *even* (resp. *odd*) *set* in a projective plane is a set of points such that every line of the plane intersects that plane in an even (resp. odd) number of points.

We obtained, by computer, a full classification up to equivalence of the even and odd sets of the projective plane of order 8. The generation algorithm is non-standard in that it does not incrementally add points to previously generated sets but instead starts from a family of 'reduced' sets and takes subsequent symmetric differences with lines.

We give geometric descriptions of some of the resulting sets with the largest automorphism groups.

Even sets can be interpreted as code words in the dual of the binary projective code of the plane. Our results therefore provide an explicit weight enumerator for that code.

Graphs from hyperbolic quadrics

Jan De Beule

Vrije Universiteit Brussel

The graph $\mathrm{NO}^+(8,2)$ is a strongly regular graph with parameters $v = 120$, $k = 63$, $\lambda = 30$, and $\mu = 36$. Its vertex set is the set of points of $\mathrm{PG}(7,2)$ not on the quadric $\mathrm{Q}^+(7,2)$. Two vertices are adjacent if and only if the line they span is tangent to the hyperbolic quadric $\mathrm{Q}^+(7,2)$. In [1], it is mentioned that a strongly regular graph with the same parameters arises from a rank 7 action of the symmetric group $\mathrm{Sym}(7)$. We provide a description of this graph on the same vertex set as the graph $\mathrm{NO}^+(8,2)$, and we explain how the adjacency relation of $\mathrm{NO}^+(8,2)$ can be modified to obtain this graph. It turns out that the unique ovoid (and spread) of the quadric $\mathrm{Q}^+(7,2)$ plays a central role. Secondly, we consider a strongly regular graph, again with the same parameters, that is non-isomorphic to $\mathrm{NO}^+(8,2)$. Its vertices are the points of $\mathrm{Q}^+(7,2) \setminus \Pi$, Π a generator of $\mathrm{Q}^+(7,2)$. We discuss a geometrical argument why this graph is non-isomorphic with the graph $\mathrm{NO}^+(8,2)$.

The results obtained are joint work with Sam Adriaensen, Robert Bailey, Morgan Rodgers, and Antonio Cossidente, Giuseppe Marino, Francesco Pavese, and Valentino Smaldore.

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Binary code generated by the hyperbolic quadrics of $W(2n-1,q)$, q even

Bart De Bruyn

Ghent University

(Joint work with Devjyoti Das, Binod Kumar Sahoo and N. S. Narasimha Sastry)

Consider a vector space of dimension $2n$, $n \geq 2$, defined over the finite field of order q, that is equipped with a nondegenerate alternating bilinear form f. Denote by $\mathrm{PG}(2n-1,q)$ and $W(2n-1,q)$ the associated projective space and the symplectic polar space, respectively. For q even, let $\mathcal{H}$ denote the binary linear code spanned by those hyperbolic quadrics of $\mathrm{PG}(2n-1,q)$ with quadratic forms κ for which the associated symmetric bilinear form f_κ equals f, up to a nonzero factor. We characterize the codewords of minimum and maximum weights in $\mathcal{H}$ and its dual code $\mathcal{H}^\perp$. For all q, we also determine the minimum size blocking sets in $\mathrm{PG}(2n-1,q)$ with respect to the hyperbolic lines of $W(2n-1,q)$.

Cameron-Liebler sets of generators in the Klein quadric $Q^+(5,q)$

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(Joint work with Jonathan Mannaert and Leo Storme)

In 1982, Cameron and Liebler introduced specific line classes in $\mathrm{PG}(3,q)$ when investigating the orbits of the subgroups of the collineation group of $\mathrm{PG}(3,q)$. They found that these Cameron-Liebler sets can be defined in many equivalent ways; some combinatorial, geometrical or algebraic in nature.

A Cameron-Liebler line set $\mathcal{L}$ in $\mathrm{PG}(3,q)$ is a set of lines, such that every line spread in $\mathrm{PG}(3,q)$ has the same number of lines in common with $\mathcal{L}$.

The examination of these Cameron-Liebler line sets in $\mathrm{PG}(3,q)$ started the motivation for defining and investigating Cameron-Liebler sets in other contexts, including the context of finite classical polar spaces [1, 2].

In this talk I will focus on Cameron-Liebler sets in these finite classical polar spaces, in particular in the hyperbolic quadric $Q^+(5,q)$. I will present some non-trivial examples of Cameron-Liebler sets of generators in $Q^+(5,q)$, which were recently found by using the Klein correspondence. This project is joint work with J. Mannaert and L. Storme [3].

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Codes from the point-hyperplane geometry of PG(V)

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(Joint work with Ilaria Cardinali)

Let V be a vector space of dimension $n+1$ over the finite field $\mathbb{F}_q$ and Γ be the point-hyperplane geometry of PG(V), i.e. the geometry whose points are pairs $(x, \xi) \in \mathrm{PG}(V) \otimes \mathrm{PG}(V^*)$ with $x \in \xi$. For any automorphism σ of $\mathbb{F}_q$, the map $\varepsilon_\sigma : \Gamma \to \mathrm{PG}(V \otimes V^*)$ sending $(x, \xi) \to (x^\sigma, \xi)$ is a projective embedding. Denote by Λ_σ its image, and by $\mathcal{C}(\Lambda_\sigma)$ the projective codes defined by Λ_σ. By studying the interplay between the codes and the geometry, we prove the following.

Theorem 1

- *If $\sigma = 1$, then the code $\mathcal{C}(\lambda_1)$ has parameters $[N_1, k_1, d_1]$ given by*

$$N_1 = \frac{(q^{n+1}-1)(q^n-1)}{(q-1)^2}, \qquad k_1 = n^2 + 2n, \qquad d_1 = q^{2n-1} - q^{n-1};$$

- *If $\sigma \neq 1$, then the code $\mathcal{C}(\lambda_\sigma)$ has parameters $[N_\sigma, k_\sigma, d_\sigma]$ given by*

$$N_1 = \frac{(q^{n+1}-1)(q^n-1)}{(q-1)^2}, \qquad k_1 = n^2 + 2n + 1.$$

$$d_\sigma = \begin{cases} q^3 - \sqrt{q}^3 & \textit{if } \sigma^2 = 1 \textit{ and } n = 2, \\ q^{2n-1} - q^{n-1} & \textit{if } \sigma^2 \neq 1 \textit{ or } n > 2. \end{cases}$$

- *For all $\sigma \in \mathrm{Aut}(\mathbb{F}_q)$, the codes $\mathcal{C}(\Lambda_\sigma)$ are minimal and admit an automorphism group isomorphic to the central product $\mathrm{PSL}(n+1, q) \cdot \mathbb{F}_q^*$.*

We also provide the complete weight list for the code $\mathcal{C}(\Lambda_1)$ and characterize the word of lowest and second lowest weight for the general codes $\mathcal{C}(\Lambda_\sigma)$.

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[3] I. Cardinali, L. Giuzzi, On minimal codes arising from projective embeddings of point-line geometries., Tech. rep. (in preparation).

Classification of low-degree ovoids

Giovanni Giuseppe Grimaldi

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(Joint work with Daniele Bartoli, Nicola Durante, Marco Timpanella)

An ovoid of a finite classical polar space $\mathcal{P}$ is a subset of points meeting any generator of $\mathcal{P}$ in exactly one point. Any such pointset can be parametrized by multivariate polynomials. In this talk, we provide classification results regarding low-degree ovoids of $Q^+(5,q), Q(6,q)$ and $Q^+(7,q)$, i.e., the polynomial degrees are low compared to the size of the field. The main tool is the investigation of absolutely irreducible components of a suitable hypersurface attached to the ovoid.

The Geometry of Codes for Random Access in DNA Storage

Anina Gruica

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(Joint work with Maria Montanucci and Ferdinando Zullo)

In this talk, we will explore the random access problem in the context of DNA data storage. The focus is on understanding how many DNA strands need to be read to reliably decode a specific piece of information requested by the user from a large pool of encoded strands. The information strands are encoded using an error-correcting code, and the encoded strands are read uniformly at random during sequencing. One of the key questions is: how many reads are needed, on average, to recover a particular information strand? We refer to this average as the *Random Access Expectation*. Since sequencing is expensive, a major challenge is to design codes for which this expectation is small, relative to the dimension k of the code.

We introduce new techniques to study this problem for arbitrary codes, highlighting its combinatorial and geometric nature. These results provide insight into which structural properties help reduce the random access expectation. Inspired by this, we introduce geometric objects that allow us to construct codes with improved performance in reducing the random access expectation. In particular, we present a novel construction for $k = 3$ that outperforms all previous constructions aiming to reduce the random access expectation.

Erdős-Ko-Rado problems and Uniqueness

Philipp Heering

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(Joint work with Jan De Beule, Jesse Lansdown, Sam Mattheus and Klaus Metsch)

The Erdős-Ko-Rado problem is a cornerstone of extremal combinatorics, given a suitable notion of "intersection", it asks the following questions: What is the maximum size of a set of intersecting objects? What is their structure? We will focus on the latter question. Algebraic methods have been highly effective in addressing the size question. We discuss an algebraic tool that allows us to determine the structure in certain cases, even if the underlying association scheme is not commutative. Our objects will be chambers in finite spherical buildings, in particular finite projective spaces and finite classical polar spaces and our notion of intersection will be non-oppositeness. We also discuss cases in which the algebraic approach fails.

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New Distance-Biregular Graphs

Ferdinand Ihringer

SUSTech

(Joint work with Blas Fernández, Sabrina Lato, Akihiro Munemasa)

Distance-biregular graphs are the bipartite generalization of distance-regular graphs. They include various well-known objects such as distance-regular graphs, generalized polygons, or quasi-symmetric designs. Excluding these objects objects which are studied in their own right, no new distance-biregular graph has been found for over 30 years. Here we present two new constructions using finite geometry. One based on Mathon's perp system, one utilizing hyperovals. Furthermore, we will give a new non-existence condition and some interesting small parameters.

About Reed-Muller codes $RM_q(2,2)$.

Trygve Johnsen

UiT-The Arctic university of Norway

(Joint work with Sudhir Ghorpade, Rati Ludhani, Rakhi Pratihar)

We demonstrate a technique, using homological algebra and matroid theory, for finding both the usual weight spectra and higher weight spectra of linear codes. We apply this technique to find those spectra for the Reed-Muller codes $RM_q(2,2)$ for all prime powers q.

The incidence matrix of a q-ary graph

Relinde Jurrius

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(Joint work with Michela Ceria)

Ever since the re-discovery of q-matroids from rank-metric codes, the question has been open what the q-analogue of a graph is and how this object relates to a q-matroid. In this talk we will propose an answer to this question by discussing a q-ary graph and its incidence matrix, which then can be viewed as the representation of a q-matroid.

We propose the following definition of a q-ary matroid. Let $V = \mathbb{F}_q^v$ and let E be a set of 2-dimensional subspaces of V, the **edges**. Then (V, E) is a **q-ary graph** if for all $c_1, c_2 \in \mathbb{F}_q$ the **q-graph property** holds: If $\langle \mathbf{x}, \mathbf{y_1} \rangle$ and $\langle \mathbf{x}, \mathbf{y_2} \rangle$ are (adjecent) edges, then $\langle \mathbf{x}, c_1\mathbf{y_1} + c_2\mathbf{y_2} \rangle$ is also an edge.

This means that a **vertex** (i.e., a 1-dimensional subspace of V) together with all its adjacent vertices precisely gives all 1-dimensional subspaces in some subspace of V. This is a generalisation of k-regular q-ary graphs [1].

The incidence matrix of a q-ary graph is a matrix over the extension field $\mathbb{F}_{q^v}$ with primitive element α, such that every column is the **representation** of an edge. This means that the rank support of the vector is the edge, hence it has rank weight 2; and the vector is orthogonal to $\left[1, \alpha, \alpha^2, \ldots, \alpha^{v-1}\right]^{\mathsf{T}}$. To reflect the q-graph property, we furthermore ask that if $\langle \mathbf{x}, \mathbf{y_1} \rangle$ and $\langle \mathbf{x}, \mathbf{y_2} \rangle$ are edges represented by $\mathbf{v}_1$ and $\mathbf{v}_2$, then the edge $\langle \mathbf{x}, c_1\mathbf{y_1} + c_2\mathbf{y_2} \rangle$ is represented by $c_1\mathbf{v}_1 + c_2\mathbf{v}_2$.

By use of examples we will motivate this definition and show some of its nice properties. For example, it exists, and once for one edge a representation is chosen, the rest of the representations are fixed up to multiplication by an element of $\mathbb{F}_q$. Furthermore we discuss several open questions and possible research directions, one of them being: what is the geometric interpretation of all this?

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The paired construction for Boolean functions in the Johnson scheme

Michael Kiermaier

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(Joint work with Jonathan Mannaert and Alfred Wassermann)

Let V be a finite set of size n. We consider real functions on the *slice* $\binom{V}{k}$, which are also known as functions in the Johnson scheme. For $I \subseteq J \subseteq V$, the characteristic function of the set of all $K \in \binom{V}{k}$ with $I \subseteq K \subseteq J$ is called *basic*. In this talk, we investigate a construction arising as the sum of two "opposite" basic functions. In essentially all cases, these *paired* functions are Boolean.

We will determine the exact degree – regarding a representation by an n-variable polynomial – of all paired functions. First, we settle the middle layer case $n = 2k$ by identifying and combining various relations among the degrees involved. Then the general case is reduced to the middle layer situation by means of derived, reduced, and dual functions, see [1] for the terminology.

Remarkably, in certain situations, the degree is strictly smaller than what is guaranteed by the elementary upper bound for the sum of functions. This makes paired functions good candidates for fixed-degree Boolean functions of small support size. As it turns out, for $n = 2k$ and even degree $t \notin \{0, k\}$, paired functions provide the smallest known non-zero Boolean functions, surpassing the t-pencils, which is the smallest known construction in all other cases.

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Designs of perfect matchings

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(Joint work with John Bamberg)

It is well-known that the complete graph K_{2n} on $2n$ vertices can always be decomposed into perfect matchings, called a 1-factorisation. In such a decomposition, every edge of K_{2n} appears in exactly 1 perfect matching. This was generalised by Jungnickel and Vanstone to *hyperfactorisations*. These are sets of perfect matchings such that every pair of disjoint edges of K_{2n} appears in a constant number of perfect matchings. Hyperfactorisations are examples of Cameron's *partition systems* and were rediscovered by Stinson who called them *hyperresolutions*. We generalise all these ideas to λ-factorisations of K_{2n} and characterise them algebraically as Delsarte designs in an association scheme using the theory of Gelfand pairs. We use this characterisation to derive divisibility conditions and non-existence results. Furthermore, we explore a connection to finite geometry, giving rise to explicit constructions of λ-factorisations.

Additive codes attaining the Griesmer bound

Sascha Kurz

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Additive codes may have better parameters than linear codes. However, still very few cases are known and the explicit construction of such codes is a challenging problem. Here we show that a Griesmer type bound for the length of additive codes can always be attained with equality if the minimum distance is sufficiently large. This solves the problem for the optimal parameters of additive codes when the minimum distance is large and yields many infinite series of additive codes that outperform linear codes.

Graphs of simplex codes

Mariusz Kwiatkowski

University of Warmia and Mazury in Olsztyn

This talk presents an overview of recent results concerning the *graphs of simplex codes*, with particular emphasis on their realization as *induced subgraphs of the Grassmann graph.* The vertex set of these graphs consists of all linear $[n, k]_q$ simplex codes, viewed as k-dimensional subspaces of $\mathbb{F}_q^n$, where 2 subspaces are adjacent if their intersection is $k-1$- dimensional, as in the classical Grassmann graph.

We focus on the *structure and classification of maximal cliques* in these graphs. Two natural families of cliques are*stars* and *tops*, inherited from the Grassmannian graph. A *star* is the set of all codes (subspaces) containing a fixed $(k-1)$-dimensional subspace, while a *top* consists of all k-dimensional codes contained in a fixed $(k+1)$-dimensional subspace. The stars of the graphs of simplex codes are uniform but the classification of tops is a hard problem.

On Intersection Families in Projective Hjelmslev Geometries

Ivan Landjev

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(Joint work with Emiliyan Rogachev and Assia Rousseva)

Let R be a finite chain ring with $|R| = q^m$, $R/\mathrm{rad}R \cong \mathbb{F}_q$, and consider a fixed left R-module ${}_RM$. A family $\mathcal{F}$ of submodules of ${}_RM$ of the same shape κ is called τ-intersecting if the intersection of every two submodules from $\mathcal{F}$ contains a submodule of shape τ. We consider two classical questions for τ-intersecting families:

(1) What is the maximal size of a τ-intersecting family of κ-subspaces of ${}_RM$?

(2) What is the structure of a τ-intersecting family of maximal size?

We present results for the special cases where ${}_RM = {}_RR^n$, $\kappa = m^k$, $\tau = m^t$.

Boolean degree t functions in the q-Johnson scheme

Jonathan Mannaert

Vrije Universiteit Brussel

(Joint work with Michael Kiermaier and Alfred Wassermann)

In 1982, Cameron and Liebler investigated certain sets of special line classes in $\mathrm{PG}(3, q)$, and gave several equivalent characterizations. Due to their interesting geometric and algebraic properties, these Cameron-Liebler line classes got a lot of attention. This resulted in several generalizations of this concept. An important tool is the interpretation of the generalizations as Boolean functions in the *Johnson*, denoted by $J(n, k)$, and *q-Johnson schemes*, denoted by $J_q(n, k)$. This interpretation connects Cameron-Liebler sets to Boolean functions of degree 1 in the proper scheme. Here the degree of the function is defined by the lowest natural number t for which the function can be written as a linear combination of the rows of the (t-space)-(k-space) incidence matrix of $J_q(n, k)$ or $J(n, k)$ respectively.

While, due to its connection with the standard Cameron-Liebler problem, many results are known for Boolean degree 1 functions, this is not the case for general degree. A natural next step is to investigate how the well known properties from degree 1 translate to higher degree. This will be the main focus of the talk. The results that will be discussed can be found in [1].

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Derangement graphs and the intersection density of permutation groups

Karen Meagher

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(Joint work with A. Sarobidy Razafimahatratra, Raghu Pantangi, Cody Solie, Pablo Spiga and Pham Huu Tiep)

Two permutations are **intersecting** if they both map some i to the same point, equivalently, permutations σ and π are intersecting if and only if $\pi^{-1}\sigma$ has a fixed point. A set of permutations is called intersecting if any two permutations in the set are intersecting. For any transitive group the stabilizer of a point is an intersecting set. The **intersection density** of a permutation group is the ratio of the size of the largest intersecting set in the group, to the size of the stabilizer of a point. If the intersection density of a group is 1, then the stabilizer of a point is an intersecting set of maximum size. Such groups are said to have the **Erdős-Ko-Rado property**.

One effective way to determine the intersection density of a group is to build a graph whose vertices are the elements of the group and the edges are defined so that the cocliques (or the independent sets) in the graph are exactly the intersecting sets in the group. This graph is called the **derangement graph** for the group. The focus of this talk is to demonstrate several ways we can use the derangement graph to find the intersection density of a group.

On the flag-transitive automorphism groups of 2-designs with λ prime

Alessandro Montinaro

University of Salento

A 2-(v, k, λ) *design* $\mathcal{D}$ is a pair $(\mathcal{P}, \mathcal{B})$ with a set $\mathcal{P}$ of v points and a set $\mathcal{B}$ of blocks such that each block is a k-subset of $\mathcal{P}$ and each two distinct points are contained in exactly λ blocks. An automorphism of $\mathcal{D}$ is a permutation of the point set which preserves the block set. The set of all automorphisms of $\mathcal{D}$ with the composition of permutations forms a group, denoted by $\mathrm{Aut}(\mathcal{D})$. For a subgroup G of $\mathrm{Aut}(\mathcal{D})$, G is said to be *point-primitive* if G acts primitively on $\mathcal{P}$, and said to be *point-imprimitive* otherwise. A *flag* of $\mathcal{D}$ is a pair (x, B) where x is a point and B is a block containing x. If $G \leq \mathrm{Aut}(\mathcal{D})$ acts transitively on the set of flags of $\mathcal{D}$, then we say that G acts *flag-transitively* on $\mathcal{D}$.

If $\lambda = 1$, then any flag-transitive automorphism group G of $\mathcal{D}$ is also point-primitive by a famous result of Higman and McLaughlin [5] dating back to 1961, and a classification of the pair $(\mathcal{D}, G)$ was achieved in 1990 by Buekenhout et al. [3], except when v is a power of a prime and $G \leq A\Gamma L_1(v)$. If $\lambda > 1$, there are 2-designs admitting a flag-transitive point-imprimitive automorphism group as shown, for instance, in [4].

In my talk, based on the recent results contained in [1, 2, 6], I will give an overview on the flag-transitive automorphism groups of 2-designs with λ prime, both in the primitive and imprimitive case, present some constructions, and provide some classification results.

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On the Q-polynomial property of bipartite graphs with a uniform structure

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(Joint work with B. Fernández, R. Maleki, Š. Miklavič)

Let Γ denote a finite, connected graph with vertex set X. Fix $x \in X$ and let $\varepsilon \geq 3$ denote the eccentricity of x. For mutually distinct scalars $\{\theta_i^*\}_{i=0}^{\varepsilon}$ define a diagonal matrix $A^* = A^*(\theta_0^*, \theta_1^*, \ldots, \theta_\varepsilon^*) \in \mathrm{Mat}_{\mathrm{X}}(\mathbb{R})$ as follows: for $y \in X$ set $(A^*)_{yy} = \theta^*_{\partial(x,y)}$, where ∂ denotes the shortest path-length distance function of Γ. We say that A^* is a *dual adjacency matrix candidate of* Γ *with respect to* x if the adjacency matrix $A \in \mathrm{Mat}_{\mathrm{X}}(\mathbb{R})$ of Γ and A^* satisfy

$$A^3A^* - A^*A^3 + (\beta+1)(AA^*A^2 - A^2A^*A) = \gamma(A^2A^* - A^*A^2) + \rho(AA^* - A^*A)$$

for some scalars $\beta, \gamma, \rho \in \mathbb{R}$.

Assume now that Γ is *uniform with respect to* x in the sense of Terwilliger [1]. In this talk, we give sufficient conditions on the uniform structure of Γ such that Γ admits a dual adjacency matrix candidate with respect to x. As an application of our results, we show that the full bipartite graphs of both Hamming and dual polar graphs are Q-polynomial in the sense of Terwilliger [2].

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Strongly regular graphs with 2-transitive two-graphs

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(Joint work with Robert F. Bailey and Valentino Smaldore)

A *two-graph* is a pair (V, T), where T is a set of unordered triples of a vertex set V, such that every (unordered) quadruple from V contains an even number of triples from T. Given a simple graph $\Gamma = (V, E)$, the set of triples T of the vertex set V, whose induced subgraph has an odd number of edges, forms the *associated two-graph* of Γ. Finite two-graphs with 2-transitive automorphism groups G have been classified by Taylor [1]:

1) **Linear type:** $G \cong P\Sigma L(2, q)$, for $q \equiv 1 \pmod 4$;

2) **Unitary type:** $G \cong P\Gamma U(3, q)$, for $q \geq 5$ odd;

3) **Ree type:** $G \cong Ree(q) \rtimes \mathrm{Aut}(\mathbb{F}_q)$, for $q = 3^{2e+1}$, $e \geq 1$;

4) **Symplectic type:** $G \cong Sp(2m, 2)$, $m \geq 3$;

5) **Sporadic type:** $G \cong HS$ or $G \cong Co_3$.

6) **Affine polar type:** $G \cong \mathbb{F}_2^{2m} \rtimes Sp(2m, 2)$.

Theorem 1 *Let $\Gamma = (V, E)$ be a strongly regular graph with associated two-graph $\mathcal{T} = (V, T)$. Write $H = \mathrm{Aut}(\Gamma)$ and $G = \mathrm{Aut}(\mathcal{T})$. If G is 2-transitive and H is a transitive maximal subgroup of G, then one of the following holds:*

(i) $\mathcal{T}$ is of affine or symplectic type.

(ii) $\mathcal{T}$ is of linear type, $q = p^{2e}$, $p \equiv 3 \pmod 4$, $H \cong C_{\frac{q+1}{2}} \rtimes C_{4e}$.

(iii) $\mathcal{T}$ is of unitary type, $q = 5$, $H \cong A_7$.

(iv) $\mathcal{T}$ is of sporadic type, $G \cong HS$, $H \cong M_{22}$.

In case (ii), the graph Γ is a srg $\left(q+1, \frac{q \pm \sqrt{q}}{2}, \frac{(\sqrt{q} \pm 1)^2}{4} - 1, \frac{(\sqrt{q} \pm 1)^2}{4}\right)$. It is known that orthogonal arrays $OA\left(\sqrt{q}, \frac{\sqrt{q}+1}{2}\right)$ yield graphs with these parameters. We conjecture that Γ cannot be obtained in this way.

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On sets of points of PG(n, q) with few intersection numbers

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Having few intersection sizes with all the members of one (or more than one) prescribed family of subspaces of PG(n, q) is a property common to most of classical objects of finite projective geometry (such as e.g. algebraic varieties, sets of points of the union of a family of skew lines in PG$(3, q)$ and subgeometries), so it is natural to ask if it is possible to reconstruct their structure starting from assumptions on their intersections sizes with all the members of one (or more than one) family of subspaces and possibly assuming some extra combinatorial or geometric condition.

Moreover, every set of points of PG(n, q) with few intersection sizes with respect to the family of the hyperplanes of PG(n, q) is the geometric counterpart of a class of linear codes, whose weight distribution is associated with the distribution of the intersection sizes of the given set.

Thus, there is a wide literature devoted to sets of points of PG(n, q) with few intersection "numbers" with all the subspaces of PG(n, q) of one (or more than one) prescribed dimension, mainly consisting of characterization results as well as of classification and existence results.

In this talk, I will survey some recent results on these sets and I will present a result on sets of points of PG(n, q) with exactly two intersection sizes with all the hyperplanes of PG(n, q) and a (combinatorial) characterization of the complement of the set of points of a hyperbolic quadric in PG$(3, q)$.

The Second Minimum Size of a Finite Subspace Partition

Esmeralda Năstase

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(Joint work with Papa Sissokho)

Let $V = V(d,q)$ denote the vector space of dimension d over $\mathbb{F}_q$. A *subspace partition* $\mathcal{P}$ of V, also known as a *vector space partition*, is a collection of nonempty subspaces of V such that each nonzero vector of V is in exactly one subspace of $\mathcal{P}$. Motivated by applications of *minimum blocking sets* and *maximal partial t-spreads*, Beutelspacher [3] proved a simple (yet useful) lemma which states that the minimum possible size over all (nontrivial) subspace partition of V is $\delta_q(d) = q^{\lceil d/2 \rceil} + 1$ if $d \geq 2$. An interesting extension of this lemma consists of determining the *(first) minimum size* $\sigma_q(d,t)$ of any subspace partition of V in which the largest subspace has dimension t, with $1 \leq t < d$. The exact value of $\sigma_q(d,t)$ has been determined in [1],[2],[4],[5].

In the quest for additional and more refined structural information, we extend Beutelspacher's lemma and determine the *second minimum size* $\sigma'_q(d,t)$ of any subspace partition of V, when $r = 0$, $t + r$ is even, or $d < 2t$, where $r \equiv d \pmod t$ and $0 \leq r < t$.

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Combinatorics of Ferrers diagrams in the Etzion-Silberstein conjecture

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(Joint work with Hugo Sauerbier Couvée)

In 2009 Etzion and Silberstein [1] provided a combinatorial upper bound on the largest dimension of a space of matrices over a finite field whose nonzero matrices are supported on a given Ferrers diagram and all have rank lower bounded by a fixed positive integer r. In the same paper, they also conjectured that such an upper bound is always tight. Since then, their conjecture has been verified in a number of cases, but as of today it still remains widely open. In this work, we investigate the notion of reducibility of Ferrers diagrams: a diagram $\mathcal{D}$ reduces to $\mathcal{D}'$ if an optimal matrix space supported on $\mathcal{D}$ can be obtained by shortening and/or inclusion of an optimal matrix space supported on $\mathcal{D}'$. This gives a natural notion of irreducibility of Ferrers diagrams, and the validity of the conjecture for irreducible diagrams implies the validity of the full conjecture. Moreover, following this notion, we can provide the Hasse diagram of Young's lattice with an orientation, producing a directed graph in which sources correspond to irreducible diagrams. Using combinatorial arguments, we show that these irreducible diagrams can be fully characterized as the integer points of a convex polytope.

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Intersection of irreducible curves and the Hermitian curve

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(Joint work with Peter Beelen, Mrinmoy Datta and Maria Montanucci)

Sørensens conjecture, as proven in [1], gives an upper bound on the number of $\mathbb{F}_{q^2}$-rational intersection points of a Hermitian surface in $\mathbb{P}^3$ and a surface of degree d. Moreover, it states that the upper bound is reached only when the surface is a union of d planes, i.e., when the surface is (highly) reducible. Based on this, one might think that something similar holds for the Hermitian curve $\mathcal{H}_q$ in $\mathbb{P}^2$. Bézout's theorem gives an upper bound on the number of $\mathbb{F}_{q^2}$-rational intersection points of $\mathcal{H}_q$ and a plane curve $\mathcal{C}_d$ of degree d, and a natural guess could be that this bound is reached only when $\mathcal{C}_d$ is reducible. A computer search reveals that this is true for $(q, d) \in \{(2, 2), (3, 2), (2, 3)\}$, but it turns out to be false in general.

The case $d = 1$ is trivial, and it follows from the work in [2], that there exists an irreducible curve of degree d intersecting the Hermitian curve in $d(q + 1)$ distinct $\mathbb{F}_{q^2}$-rational points for $d = 2$ and $q \geq 4$. In this talk, we will show that such a curve also exists for $q \leq d \leq q^2 - q + 1$, $d = \lfloor (q + 1)/2 \rfloor$ and $d = 3$ with $q \geq 3$, as well as for other small values of d compared to q.

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On line-parallelisms of PG$(3, q)$

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(Joint work with Paolo Santonastaso)

Let PG$(3, q)$ denote the three-dimensional projective space over the finite field with q elements. A *line-spread* of PG$(3, q)$ is a collection $\mathcal{S}$ of mutually skew lines such that every point of PG$(3, q)$ lies on exactly one line of $\mathcal{S}$. A *parallelism* of PG$(3, q)$ is a set Π of mutually skew line-spreads of PG$(3, q)$ such that every line of PG$(3, q)$ is contained in precisely one line-spread of Π. A *regulus* of PG$(3, q)$ is the set of transversals to three pairwise skew lines, and consists of $q + 1$ pairwise skew lines. A spread $\mathcal{S}$ is *regular* or *Desarguesian* if each line not in $\mathcal{S}$ meets $\mathcal{S}$ in the lines of a regulus. A *Hall spread* is obtained from a Desarguesian spread by switching a regulus with its opposite regulus. Infinite families of parallelisms of PG$(3, q)$ consisting of regular spreads were constructed by Penttila and Williams in 1998 in the case when $q \equiv 2 \pmod 3$. Most of the other known infinite families lie in the class of parallelisms that have one Desarguesian spread $\mathcal{D}$ and admit an elementary abelian group E of order q^2 which stabilizes a line of $\mathcal{D}$. Other examples are obtained from parallelisms in this class by means of a derivation process. In the case when q is even, the parallelisms in the previous class comprise one Desarguesian spread $\mathcal{D}$ and $q^2 + q$ Hall spreads, which are constructed by switching the $q^2 + q$ reguli through the fixed line of $\mathcal{D}$. In this talk I will present a characterization of the parallelisms of PG$(3, q)$ admitting E as an automorphism group and having one Desarguesian spread $\mathcal{D}$ and $q^2 + q$ Hall spreads, which are obtained by switching the $q^2 + q$ reguli through a fixed line of $\mathcal{D}$.

Characterising the natural embedding of the twisted triality hexagons

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(Joint work with Geertrui Van de Voorde)

Generalised polygons play an important role in incidence geometry, building theory and graph theory. A (weak) generalised n-gon can be defined as a point-line geometry such that the incidence graph has diameter n and girth $2n$. Here, we want to focus on generalised hexagons. Up to duality, only two classes of finite thick generalised hexagons are known: the *split Cayley hexagons* of order (q, q) and the *twisted triality hexagons* of order (q^3, q). In [1], Thas and Van Maldeghem characterised the natural embedding of the split Cayley hexagons in $\mathrm{PG}(6, q)$ using intersection numbers. Later, this was improved slightly by Ihringer in [2].

In this talk we investigate the twisted triality hexagon, learn more about its natural embedding and obtain similar results. In particular we prove the following:

A set of lines satisfies a list of properties (such as for example (Sd) *every solid is incident with either 0, 1,* $q+1$ *or* $2q+1$ *lines of the set*) if and only if it is the set of lines of a naturally embedded twisted triality hexagon.

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Additive Codes and Projective Geometries

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(Joint work with Simeon Ball and Michel Lavrauw)

In this talk, I will describe the relationship between additive codes and projective geometries, and discuss some new results we have obtained by viewing additive codes through this geometric perspective. The results of this talk can be found in [1] and [2].

Let $\mathbb{F}_q$ denote the finite field with q elements. An *additive code* of length n over $\mathbb{F}_{q^h}$ is a subset C of $(\mathbb{F}_{q^h})^n$ that is closed under addition. Such an additive code is linear over the subfield $\mathbb{F}_q$ and therefore has size q^r for some r. The notation $[n, r/h, d]_q^h$ is used to denote such an additive code with minimum distance d. We show that any $[n, r/h, d]_q^h$ additive code is equivalent to a projective h-$(n, r, d)_q$ system, which is defined as a multiset $\mathcal{S}$ of n subspaces of $PG(r-1, q)$ of dimension at most $h-1$, such that each hyperplane of $PG(r-1, q)$ contains at most $n-d$ elements of $\mathcal{S}$, and some hyperplane contains exactly $n-d$ elements.

Using this correspondence, we prove several new bounds for additive codes. In particular, we establish two analogs of the classical Griesmer bound for linear codes, extending these results to the additive case. Our Griesmer-type bound allows us to derive new restrictions on the length and dimension of additive maximum distance separable (MDS) codes. Notably, these bounds permit slightly longer codes than their linear counterparts.

Finally, we present several constructions of additive codes that leverage the relationship with projective geometries. These include families of MDS additive codes that meet our new bounds, as well as a new construction of an additive code with integral parameters that exceeds the parameters of the best-known linear codes.

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Cameron-Liebler sets of generators in polar spaces with rank $d > 3$

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(Joint work with Maarten De Boeck, Jozefien D'haesseleer)

Cameron-Liebler sets in projective and polar spaces are collections of subspaces having interesting regularity properties, which arise from their close relation to the eigenspaces of the corresponding association schemes, see [1, 2]. In this talk, we will look at the definitions of Cameron-Liebler sets of generators in the finite classical polar spaces. There are a limited number of non-trivial examples that have recently been constructed, see [3]. For some polar spaces $\mathcal{Q}$ with rank $d > 3$, examples can be constructed by considering certain regular sets of $(d-3)$-spaces in an embedded polar space $\mathcal{Q}'$ of rank $d-1$. Such methods have been used to construct Cameron-Liebler sets in $\mathcal{Q}(8,q)$ for all odd q; for $\mathcal{Q}^+(7,q)$ when q is odd, or when $q = 2^{2h+1} \geq 8$; and for $\mathcal{Q}^+(9,q)$ for all odd q. We will describe these construction methods, and outline some strategies to hopefully in the future construct examples in higher dimensional spaces.

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On the Reducibility of Minihypers and the Extension Problem for Arcs and Codes

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(Joint work with Ivan Landjev and Leo Storme)

An (n, w)-minihyper $\mathcal{F}$ in $\mathrm{PG}(r, q)$ is called reducible if it can be represented as a sum $\mathcal{F} = \mathcal{F}' + \chi_T$, where χ_T is the characteristic function of a j-dimensional subspace, and $\mathcal{F}'$ is a minihyper with parameters $(n - v_{j+1}, w - v_j)$. Here $v_k = (q^k - 1)/(q - 1)$. The results by Hill [1], Hill and Lizak [2], Maruta[3] on the extendabilty of linear codes and ars can be viewed as reducibility theorems for minihypers. In this talk, we present the following result which can be viewed as a generalization of some of the known extension theorems.

Theorem 1 *Let $\mathcal{F}$ be an (n, w)-minihyper in $PG(r, q)$, $q = p^h$, with $w \equiv n - q^j \pmod{q^{j+1}}, j \geq 0$. Assume $\mathcal{F}$ has the following properties:*

(1) $\mathcal{F}(H) \equiv n - q^j$ or $n \pmod{q^{j+1}}$ for every hyperplane H in $PG(r, q)$;

(2) for every hyperplane H with $\mathcal{F}(H) \equiv n - q^j \pmod{q^{j+1}}$, $\mathcal{F}|_H = \mathcal{F}_1 + \chi_S$ for a unique $(j-1)$-dimensional subspace S and a divisible minihyper $\mathcal{F}_1$ with divisor q^j;

(3) for every hyperplane H with $\mathcal{F}(H) \equiv n \pmod{q^{j+1}}$, $\mathcal{F}|_H$ is divisible with divisor q^j.

Then $\mathcal{F}$ is a reducible minihyper and the subspace of reduction T is uniquely determined.

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Intersecting codes in the rank metric

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(Joint work with Daniele Bartoli, Martino Borello, and Giuseppe Marino)

In this talk we introduce and investigate rank-metric intersecting codes, a new class of linear codes in the rank-metric context, inspired by the well-studied notion of intersecting codes in the Hamming metric [1, 2].

A rank-metric code is said to be intersecting if any two nonzero codewords have supports intersecting non trivially. We explore this class from both a coding-theoretic and geometric perspective, highlighting its relationship with minimal codes, MRD codes, and Hamming-metric intersecting codes.

We derive structural properties, sufficient conditions based on minimum distance, and geometric characterizations in terms of 2-spannable q-systems. We establish upper and lower bounds on code parameters and show some constructions, which leave a range of unexplored parameters.

Finally, we connect rank-intersecting codes to other combinatorial structures such as $(2, 1)$-separating systems and frameproof codes.

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Design switching on graphs

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(Joint work with Ferdinand Ihringer)

A switching method is a local transformation, used to obtain cospectral graphs (graphs with the same adjacency spectrum). It needs a switching set with some conditions. Abiad and Haemers [2] found a switching method that uses a switching set of size seven. I present a new combinatorial description of this switching method, based on the Fano plane, as described in [1].

The operation can in fact be generalized to a switching method based on *any* combinatorial design. This also generalizes other previously known switching methods such as the one in [3, Section 7.1], when applied to the point-hyperplane design of a projective space.

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Goppa codes from a Singer cycle

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(Joint work with Gabor Korchmáros and Federico Romaniello)

Let $\mathcal{C}$ be a non-singular plane curve defined over a finite field. A linear code arises from any two disjoint subsets of points of $\mathcal{C}$, say D and G, as follows: Take a divisor $\mathtt{G}$ with support G, where $\mathtt{G}$ is a formal sum a points of G with integer coefficients, and let $\mathcal{L}(\mathtt{G})$ be the Riemann-Roch space of $\mathcal{C}$ associated with $\mathtt{G}$, and fix an order $(P_1, \ldots, P_N)$ of the points in D, and assume $D \cup G$ be the set of all points of $\mathcal{C}$. Then evaluating the functions $f \in \mathcal{L}(\mathtt{G})$ on $D = (P_1, P_2, \ldots, P_N)$ produces a linear code of length N and dimension $\dim(\mathcal{L}(\mathtt{G}))$ which is called an AG (algebraic geometry) code. From previous work it has emerged that the best choice for $\mathcal{C}$ in order to obtain well performing AG-codes is the Hermitian curve of equation $Y^q + Y - X^{q+1} = 0$ over $\mathbb{F}_{q^2}$. Interesting cases occur when G is an orbit of a large subgroup Γ in the automorphism group $PGU(3,q)$ of $\mathcal{C}$. So far, the following cases have been worked out: the 1-point stabilizer [1], the chord stabilizer, $\Gamma \cong PGL(2,q)$ [5], and $\Gamma \cong PSU(3,q_0)$ for $q = q_0^3$, [4]. In this talk, we consider the case where G is an orbit of a Singer cycle of $PG(2,q^2)$ of length $q^2 - q + 1$.

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Erdős-Ko-Rado sets on the hyperbolic quadric $\mathcal{Q}^+(4n+1,q)$

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(Joint work with Laure Schelfhout)

The *hyperbolic quadric* $\mathcal{Q}^+(4n+1,q)$ is the non-singular quadric in $\mathrm{PG}(4n+1,q)$, with standard equation $X_0X_1 + X_2X_3 + \cdots + X_{4n}X_{4n+1} = 0$. This quadric contains points, lines, planes, ..., $2n$-spaces. The $2n$-spaces contained in the hyperbolic quadric $\mathcal{Q}^+(4n+1,q)$ are called the *generators*.

The set of generators of the quadric $\mathcal{Q}^+(4n+1,q)$ can be partitioned into two equivalence classes, called the class of the *Latin* and the class of the *Greek* generators. Two generators of $\mathcal{Q}^+(4n+1,q)$ belong to the same equivalence class if and only if they intersect in even dimension.

An *Erdős-Ko-Rado set* S of the quadric $\mathcal{Q}^+(4n+1,q)$ is a set of generators which pairwise intersect in at least one point.

The largest Erdős-Ko-Rado sets of the hyperbolic quadric $\mathcal{Q}^+(4n+1,q)$ are the class of the Latin and the class of the Greek generators.

M. De Boeck classified the second-largest maximal Erdős-Ko-Rado sets of the hyperbolic quadric $\mathcal{Q}^+(4n+1,q)$. They are, up to equivalence, the set of Latin generators intersecting a fixed Greek generator in at least one point, with the fixed Greek generator included [1].

In this talk, we present the classification of the third-largest maximal Erdős-Ko-Rado sets of the hyperbolic quadric $\mathcal{Q}^+(4n+1,q)$ [2].

Theorem 1 *Select a particular $(2n-2)$-space Ω contained in the hyperbolic quadric $\mathcal{Q}^+(4n+1,q)$. Select the set S of all Greek generators through Ω and all Latin generators intersecting Ω in at least one point.*

All third-largest maximal Erdős-Ko-Rado sets on the hyperbolic quadric $\mathcal{Q}^+(4n+1,q)$ are, up to equivalence, equal to such a set S.

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On line parallelisms in PG$(n, 2)$

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(based on joint work with Philipp Heering)

In this talk, we show how certain APN functions can be used to construct line parallelisms in PG$(n, 2)$. These parallelisms can be seen as a generalization of the parallelisms given by Baker [1] and of Johnson and Montinaro [2].

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On the minimum weight of some geometric codes

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(Joint work with: B. Csajbók, G. Longobardi and G. Marino)

Assume p is a prime and m, h are two positive integers. Let $\Sigma = \mathrm{PG}(m, q)$ be the m-dimensional projective space over the Galois field $\mathbb{F}_q$ where $q = p^h$, and denote by the symbol $\mathcal{D}_\Sigma(m, q)$ the $2-(v, q+1, 1)$ design of points and lines of Σ; hence, with $v = \frac{q^{m+1}-1}{q-1}$. The p-ary code $\mathcal{C} = \mathcal{C}_\Sigma(m, q)$ associated with such a design is the $\mathbb{F}_p$-subspace generated by the incidence vectors of the blocks of the corresponding design. Also, the dual $\mathcal{C}^\perp$ of $\mathcal{C}$ is the $\mathbb{F}_p$-subspace of vectors of $\mathbb{F}_q^v$ which are orthogonal to all vectors of $\mathcal{C}$ (under the standard inner product). These are particular examples of so called *geometric codes*.

Unlike for codes derived from the designs of points and subspaces of Σ, the situation regarding the minimum weight of geometric codes is not as clear, and therefore its study is more challenging. In [3] the authors reduced this problem to the above mentioned case of points and lines of a projective space of suitable dimension. In [1] Bagchi and Inamdar proven that the minimum weight of $\mathcal{C}_\Sigma^\perp(m, q)$ is bounded from below by the value $2\left(\frac{q^m-1}{q-1}\left(1-\frac{1}{p}\right)+\frac{1}{p}\right)$.

This type of problem in coding theory can be quite naturally translated into one concerning with the cardinality of sets or *multi-sets* of points in projective or affine space with special intersection properties with respect to certain subspaces, as shown for instance in [2]. Using this geometrical approach and exploiting properties of certain kind of polynomial, in this talk, we will show a significant improvement of the bound stated in 2002 by Bagchi and Inamdar, in the case when $h > 1$, and $m, p > 2$.

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Anzahl theorems for formed spaces

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(Joint work with Maarten De Boeck)

Glasby, Niemeyer and Praeger [4, 5] (and later Glasby, Ihringer and Mattheus [3]) derived lower bounds for the probability of spanning a non-degenerate classical space by two non-degenerate subspaces. This problem is motivated by algorithms to recognise classical groups.

More precisely, given a vector space V and a quadratic, symplectic, or unitary form f on V, these authors determine lower bounds on the proportion of pairs (U, U') of non-degenerate subspaces U, U' with respect to f, such that U and U' are trivially intersecting and $\langle U, U' \rangle$ is a non-degenerate subspace of V among all such pairs of non- degenerate subspaces (U, U'). In recent work, together with Maarten De Boeck, we have improved on those results by deriving the exact formulae for this proportion for symplectic, hermitian [1] and odd characteristic quadratic forms [2]. In this talk, I'll present the main ideas behind the results along with some comments on the even characteristic case.

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Group testing via residuation and partial geometries

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(Joint work with Oliver W. Gnilke, Marcus Greferath, Cornelia Rößing)

We consider the problem of non-adaptive group testing and describe it in terms of residuated pairs on partially ordered sets. This formulation naturally leads to an efficient decoding algorithm. Furthermore, we demonstrate how non-linear codes can be constructed from these residuated pairs, enabling error correction within the group testing framework. Finally, we show how group testing schemes can be derived from the incidence matrices of partial linear spaces and inversive planes.

Relative Difference Sets from Almost Perfect Nonlinear Functions

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In this talk we will show that the image set of certain Almost Perfect Nonlinear (APN) functions is a relative difference set. Examples include relative difference sets arising from APN functions described in [1].

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Existence of t-designs in polar spaces for all t

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A finite classical polar space of rank n consists of the totally isotropic subspaces of a finite vector space over $\mathbb{F}_q$ equipped with a nondegenerate form such that n is the maximal dimension of such a subspace. A t-(n, k, λ) design in a finite classical polar space of rank n is a collection Y of totally isotropic k-spaces such that each totally isotropic t-space is contained in exactly λ members of Y. Nontrivial examples are currently only known for $t \leq 2$. We show that t-(n, k, λ) designs in polar spaces exist for all t and q provided that $k > \frac{21}{2}t$ and n is sufficiently large enough. The proof is based on a probabilistic method by Kuperberg, Lovett, and Peled, and it is thus nonconstructive.

Regular fat linearized polynomials

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(Joint work with Valentino Smaldore and Ferdinando Zullo)

Let U be an $\mathbb{F}_q$-subspace of the vector space $\mathbb{F}_{q^n}^m$. The $\mathbb{F}_q$-linear (or simply linear) set associated with U is the subset $L_U = \{\langle v\rangle_{\mathbb{F}_{q^n}} : v \in U,\, v \neq 0\}$ of $\mathrm{PG}(m-1, q^n)$. If $f \in \mathbb{F}_{q^n}[X]$ is an $\mathbb{F}_q$-linearized polynomial, then $L_f = L_{U_f} \subset \mathrm{PG}(1, q^n)$, where $U_f = \{(x, f(x))\colon x \in \mathbb{F}_{q^n}\}$. The weight of a point $P = \langle v\rangle_{\mathbb{F}_{q^n}}$ in a linear set L_U is defined as $\dim_{\mathbb{F}_q}(\langle v\rangle_{\mathbb{F}_{q^n}} \cap U)$. Particular interest has been shown in $\mathbb{F}_q$-linear sets in the projective line $\mathrm{PG}(1, q^n)$ in which the number r of points with weight greater than one is small. If $r = 0$, we call them scattered linear sets. In the case $r = 1$ and there is a point of weight $i > 1$, then the set is called an i-club. A regular (r, i)-fat linear set is one that has precisely r points with weight greater than one, and all of these points have weight i. Therefore, scattered linear sets and clubs are special types of regular fat linear sets. A regular (r, i)-fat polynomial is an $\mathbb{F}_q$-linearized polynomial f such that L_f is regular (r, i)-fat. For $r > 1$ and $i > 2$, few examples or results are found in the literature. In this talk I will describe some properties and provide examples of such regular fat linear sets. Some of these sets are also R-q^t-partially scattered, as defined in [1].

References

[1] G. Longobardi and C. Zanella: Partially scattered linearized polynomials and rank metric codes. Finite Fields and Their Applications 76 (2021) 101914.

New maximal additive d-codes on symmetric matrices over finite fields

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(Joint work with Wei Tang)

Let q be an odd prime power and let $X(n,q)$ denote the set of symmetric matrices over $\mathbb{F}_q$. A subset $\mathcal{C}$ of $X(n,q)$ is called a *d-code* if the rank of $A-B$ is at least d for any distinct A and B in $\mathcal{C}$. It has been proved by Schmidt [1, 2] that if $\mathcal{C}$ is additive, then

$$|\mathcal{C}| \leq \begin{cases} q^{n(n-d+2)/2}, & 2 \mid n-d; \\ q^{(n+1)(n-d+1)/2}, & 2 \nmid n-d. \end{cases}$$

Additive d-codes meeting the bounds above are called *maximal*. There are very few known constructions of maximal additive d-codes in $X(n,q)$. In this talk, we summarize the known constructions and present a new family of them.

References

[1] K. U. Schmidt. Symmetric bilinear forms over finite fields of even characteristic. *Journal of Combinatorial Theory, Series A*, 117(8):1011–1026, 2010.

[2] K. U. Schmidt. Symmetric bilinear forms over finite fields with applications to coding theory. *Journal of Combinatorial Theory, Series A*, 42:635–670, 2015.

Representability of uniform q-matroids

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(Joint work with Gianira Alfarano, Martino Borello, Relinde Jurrius, Alessandro Neri and Olga Polverino)

q-Matroids, the q-analogue of matroids, have been intensively investigated in recent years in coding theory due to their close connection with rank metric codes. Indeed, in 2018 it was shown by Jurrius and Pellikaan that a rank metric code induces a q-matroid that captures many of the code's invariants. In this talk we will deal with the direct sum of q-matroids, a concept recently introduced by Ceria and Jurrius, with a particular focus on the question of representability. We will show that representing the direct sum of t uniform q-matroids is equivalent to constructing special linear sets which are almost scattered with respect to the hyperplanes. In addition, we will give explicit constructions of such linear sets, implying as a byproduct that the direct sum of uniform q-matroids is always representable over a certain extension of $\mathbb{F}_q$. We conclude the talk by discussing about the smallest field extension of $\mathbb{F}_q$ over which such a representation exists.

3 Participants

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