

# Neutral fermions in kagome spin liquids and spin liquid crystals

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# Acknowledgements



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Theorique, UPS)



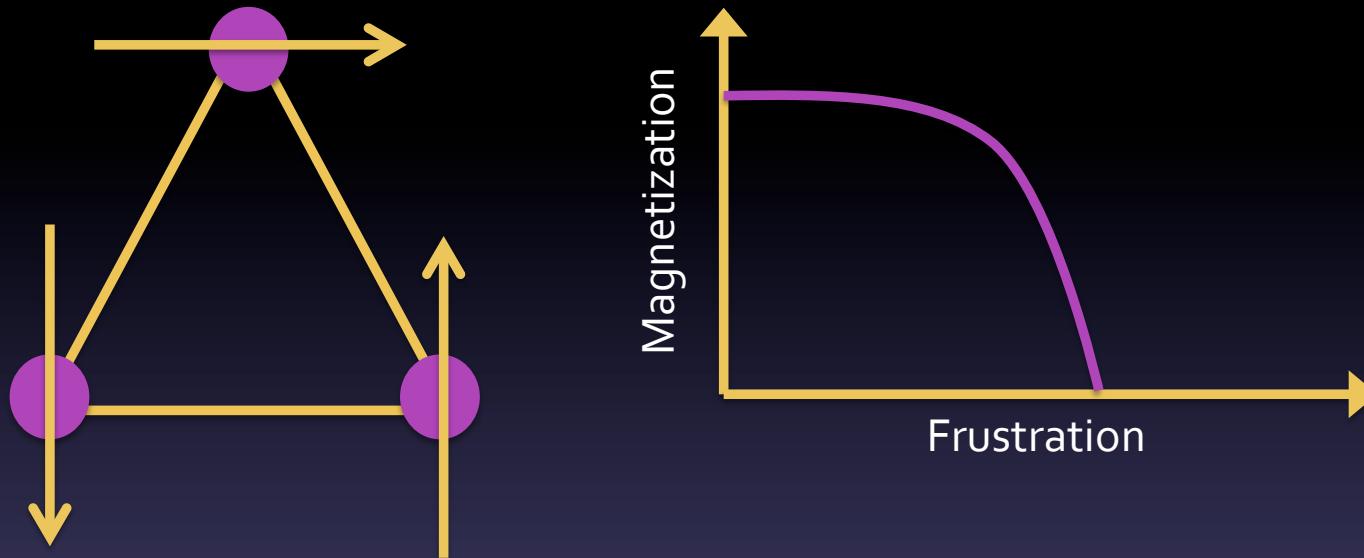
**Jesse Kinder**  
(CASE Western)



**Bryan Clark**  
(Microsoft Station Q,  
Kvali Institute of  
Theoretical Physics)

Eric Neunescamann (UC Berkeley)  
Garnet Chan (Princeton)

# FUNDAMENTAL PROBLEMS IN FRUSTRATED MAGNETISM

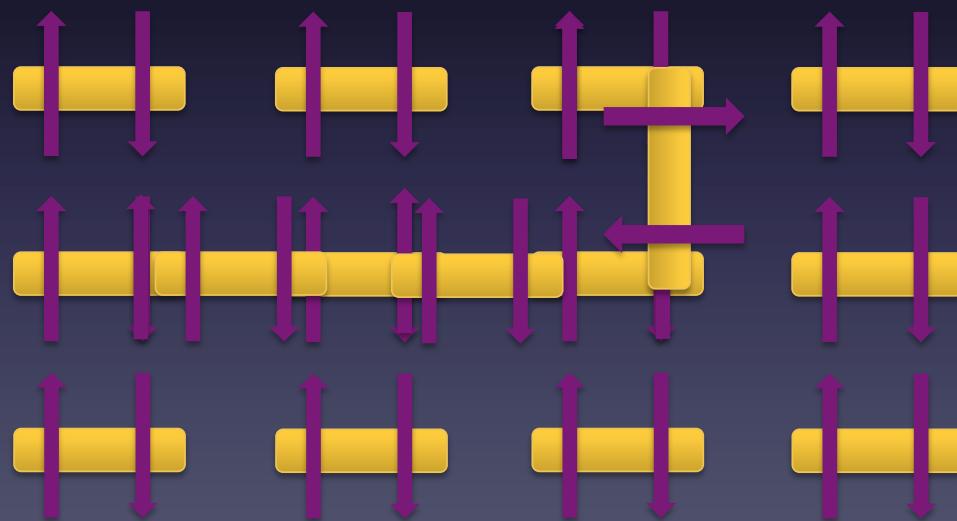


- CAN FRUSTRATION PRODUCE A SPIN LIQUID PHASE?
- IF SO, WOULD IT BE USEFUL?

# Cartoon view of spin liquids

Original idea from Anderson, 1973

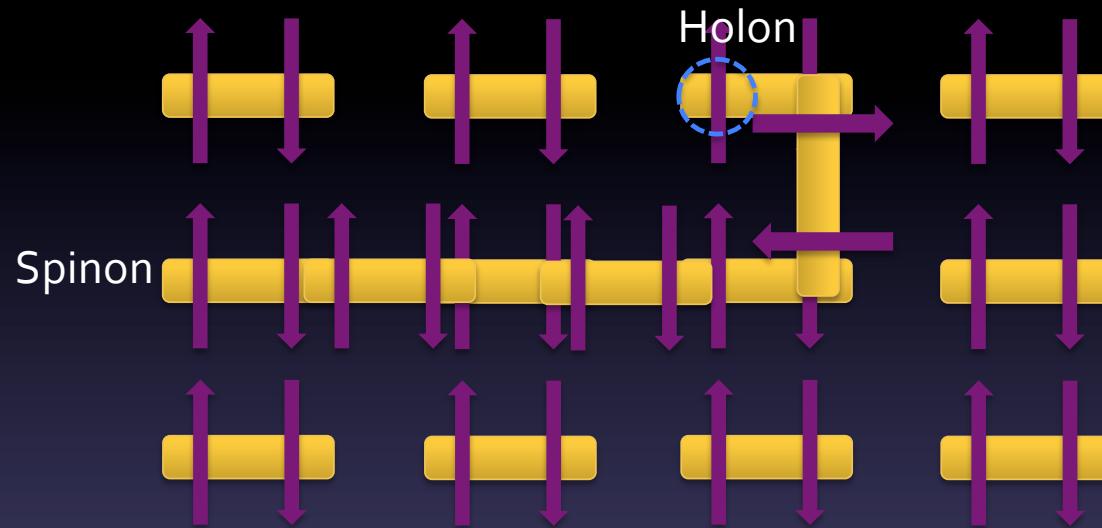
- Ground state: a superposition of singlet states
- Has spin  $\frac{1}{2}$  excitations called spinons



- Has singlet excitations called photons or visions

# Useful for superconductivity?

Anderson, 1987



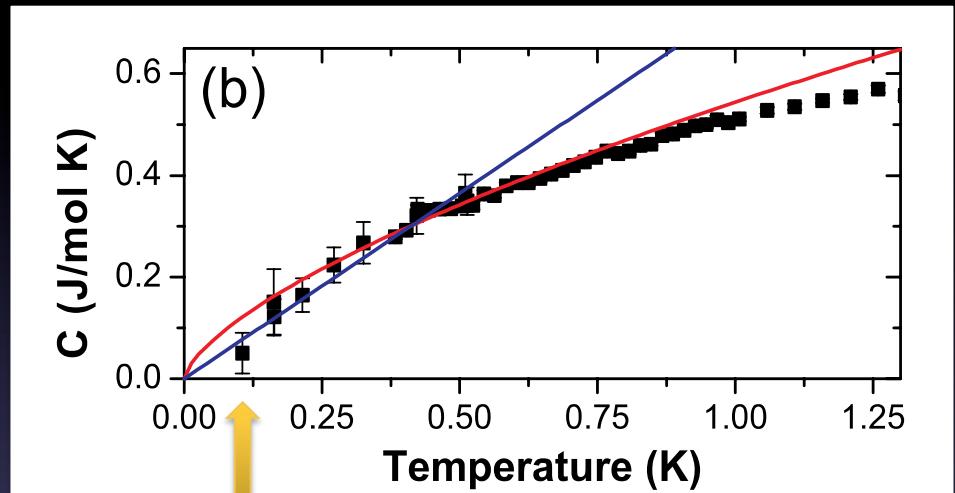
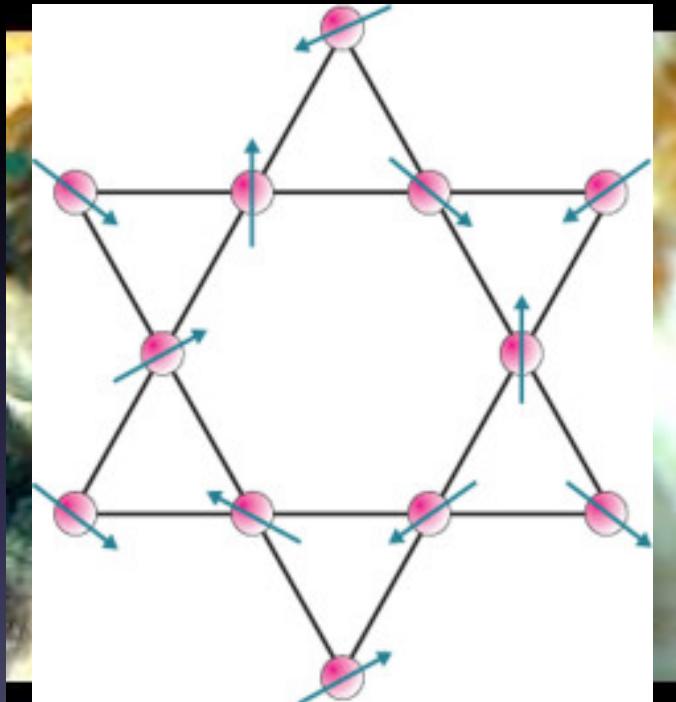
- Mobile singlets become Cooper pairs!
- Slave boson picture:  $c_{i\sigma} = b_i f_{i\sigma}^\dagger$   
coherence obtained when bosonic holons condense

Baskaran, Zou, Anderson, 1987

**SO, DO SPIN LIQUIDS EXIST ON  
FRUSTRATED LATTICES?**

# Evidence from experiment: Herbertsmithite

Helton et. al. PRL 2007



100 mK!

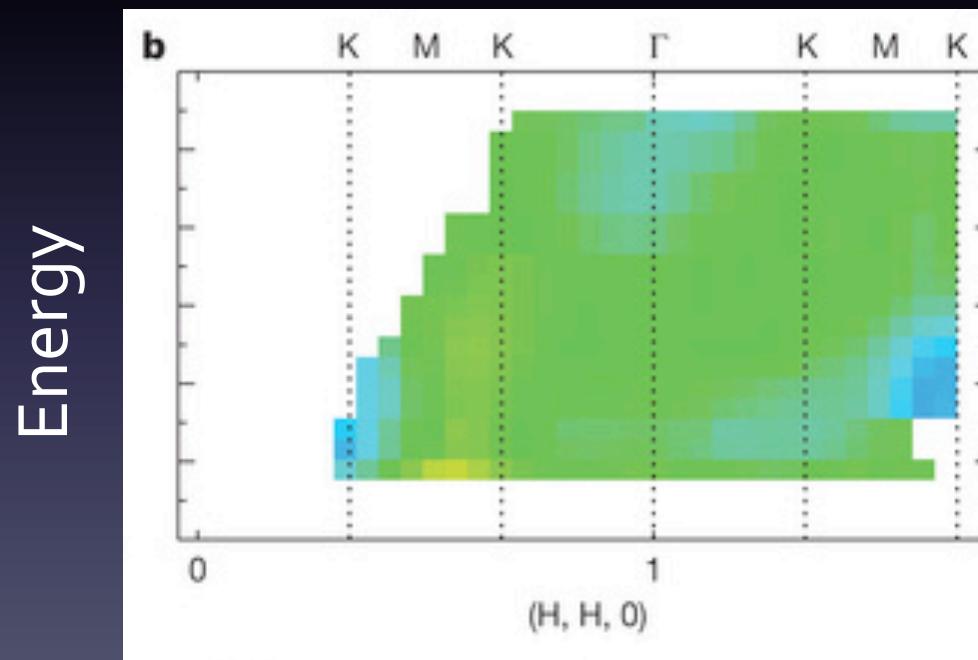
But  $J\vec{S}_i \cdot \vec{S}_j$  has  $J \approx 200K$ !

What is this new phase of matter?

# Certainly not magnetic order!

A recent neutron scattering study found

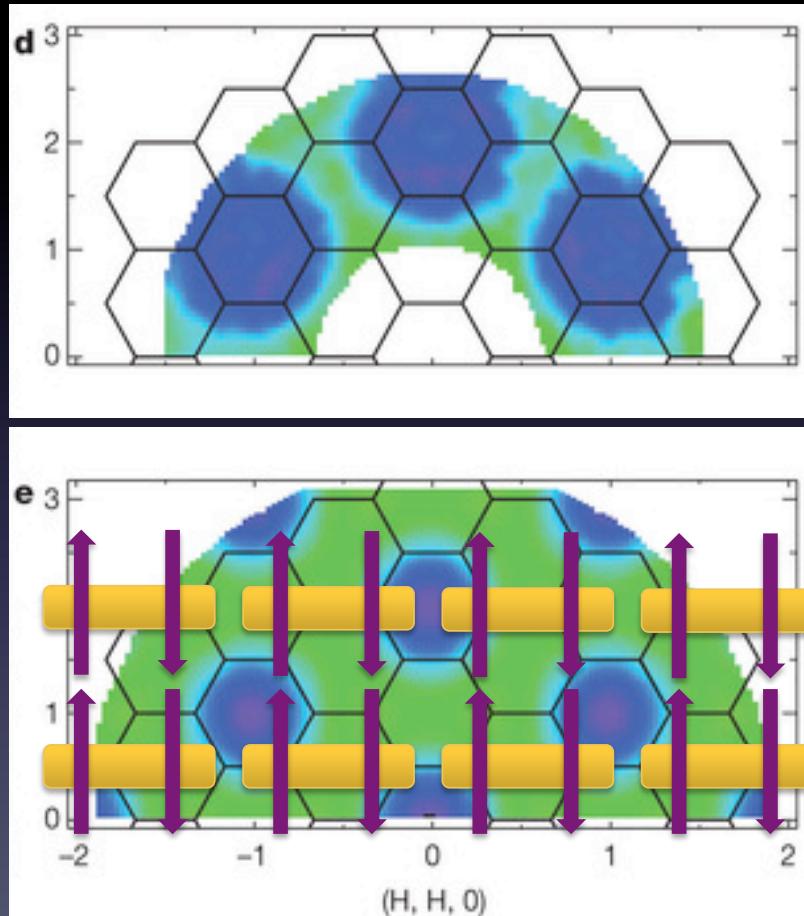
Han et. al. Nature 2012



No sign of spin waves!

# Evidence for valence bonds

Han et. al. Nature 2012



Energy Integrated  $S(q, w)$

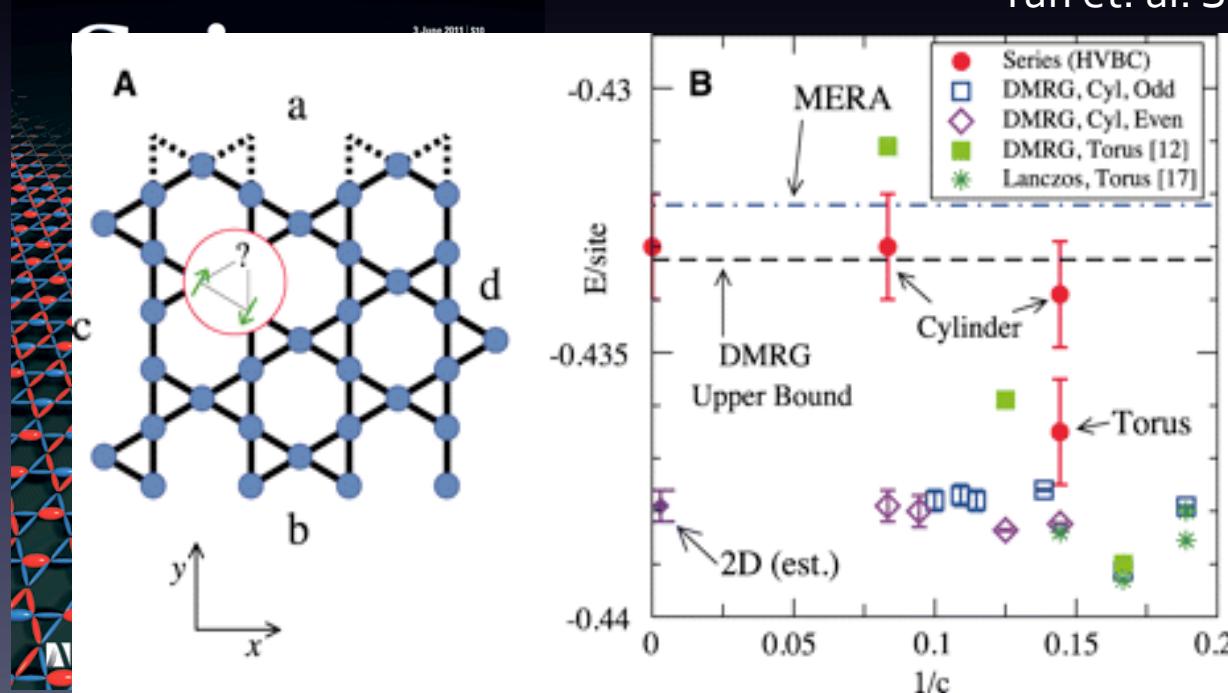
Prediction from an average  
over many valence bond  
crystal states

However, it is hard to see how the spinons behave!

# Evidence from numerics

A spin liquid was recently found in DMRG calculations!

Yan et. al. Science 2011

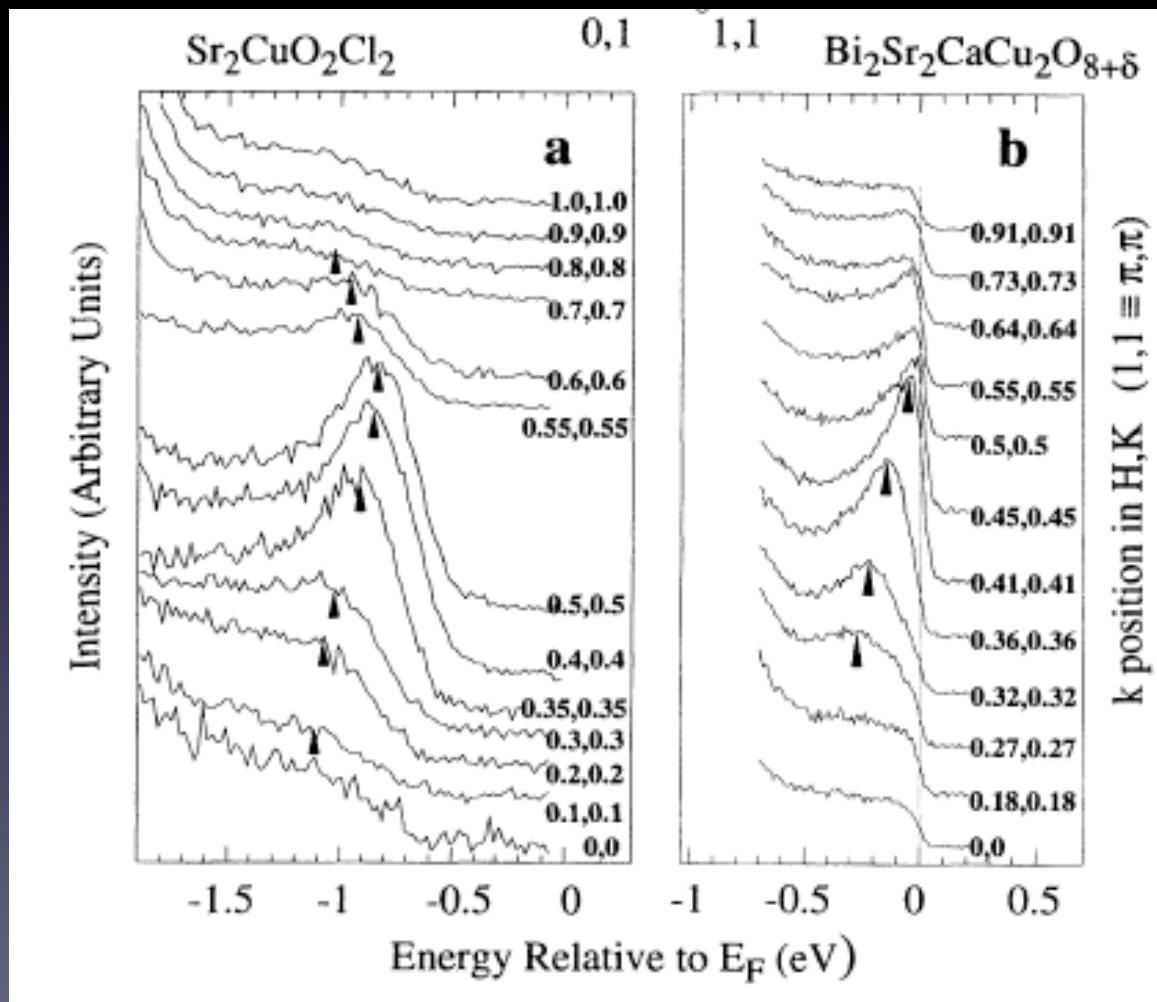


# Future directions for the field

- Do spinon's exist? Are they fermions?
- How can we find more spin liquid phases?
- Are there other exotic magnetic phases?
- Which spin liquid is it?
- Do they produce superconductivity?
- Are there broader implications for this research in other areas of physics?

# Do spinons exist?

ARPES on undoped cuprates: spinons and holons?

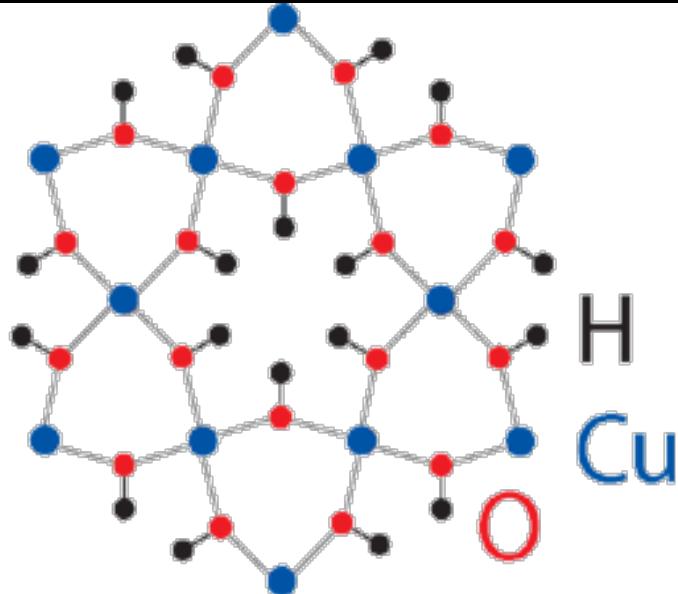
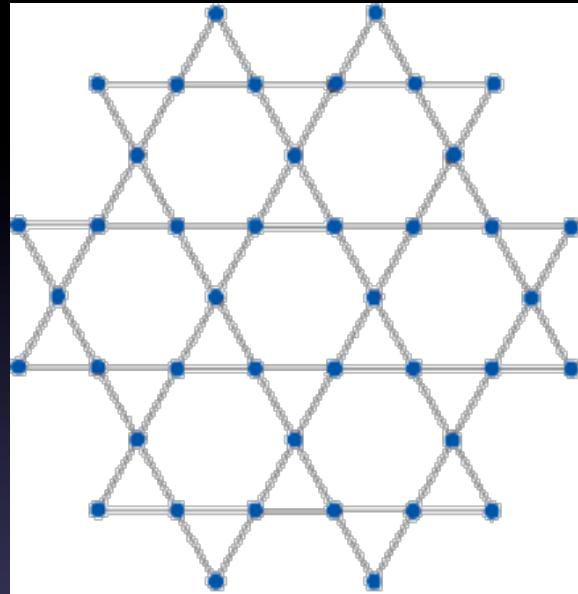


Laughlin, PRL, 1997:  
electron decays into  
a spinon and a holon

$$c_{i\sigma} = b_i f_{i\sigma}^\dagger$$

Wells et. al., 1995

# ARPES on Herberthsmithite?



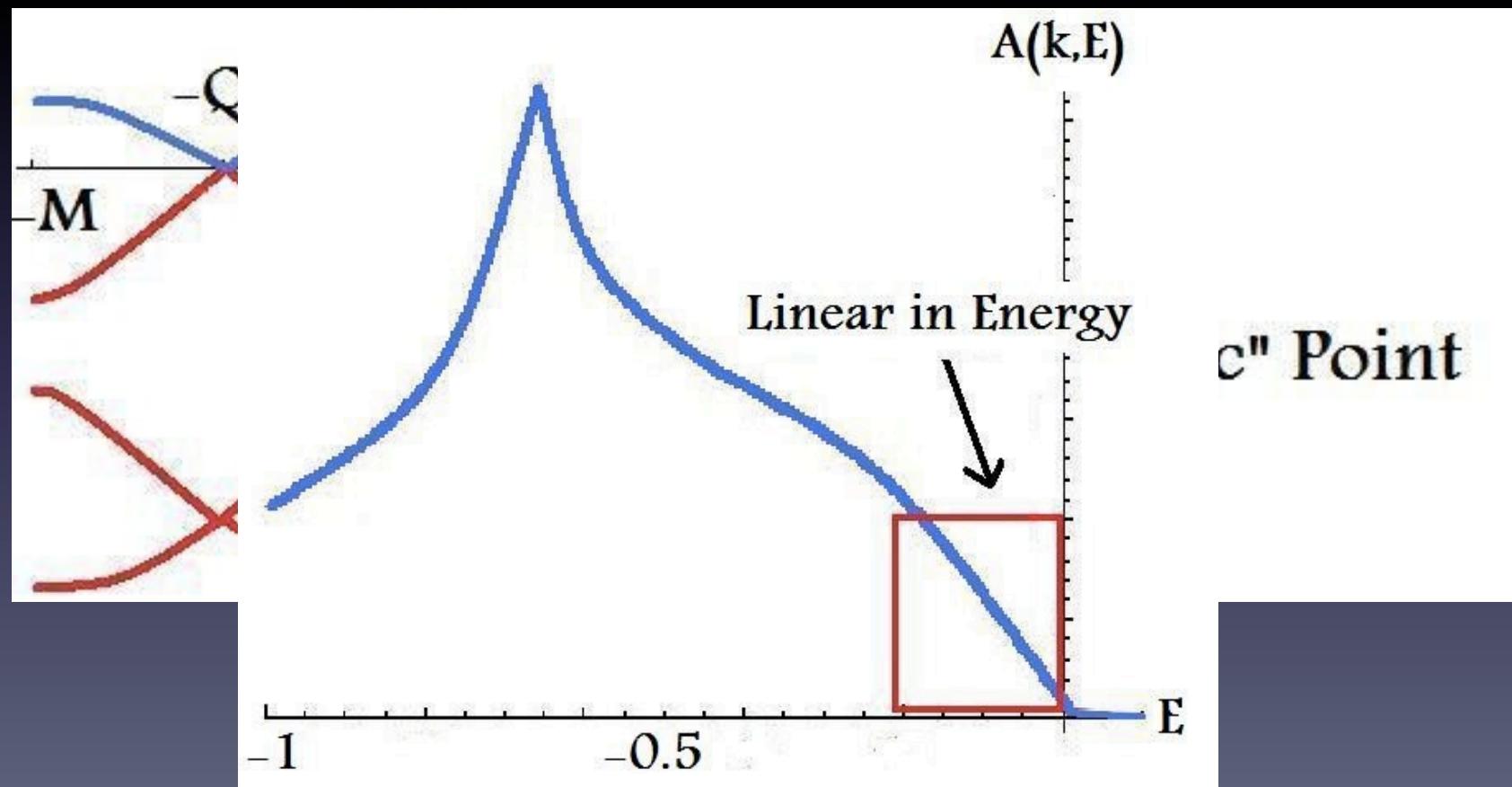
Leading theory of spinons in herbertsmithite proposes they are Dirac fermions interacting with a maxwell-like photon

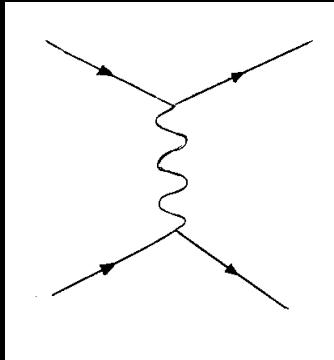
Ran et. al. PRL. 2007

# Predictions of Free Dirac spinons

Pujari and Lawler, PRB, 2013

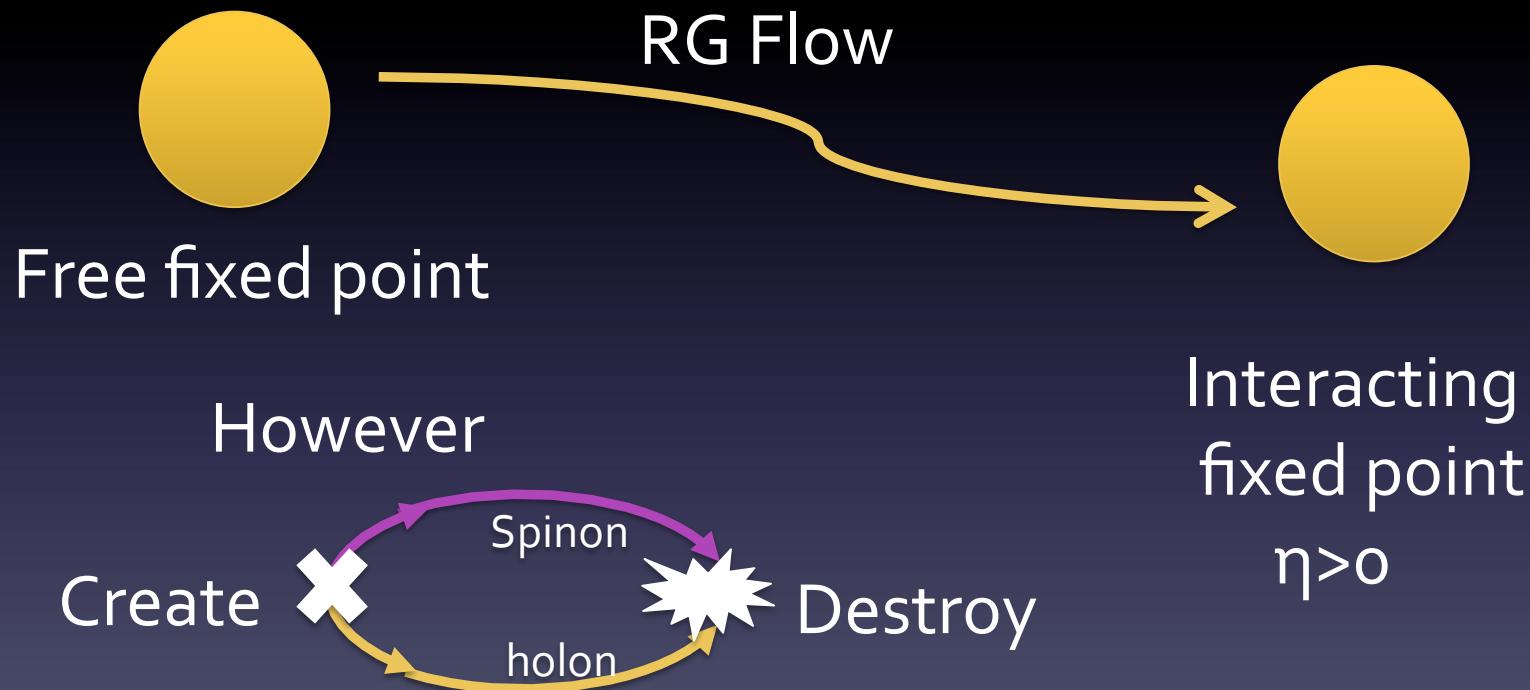
Apikor Odishaion of the Dirac node in k-space:





# Interacting Dirac Spinons

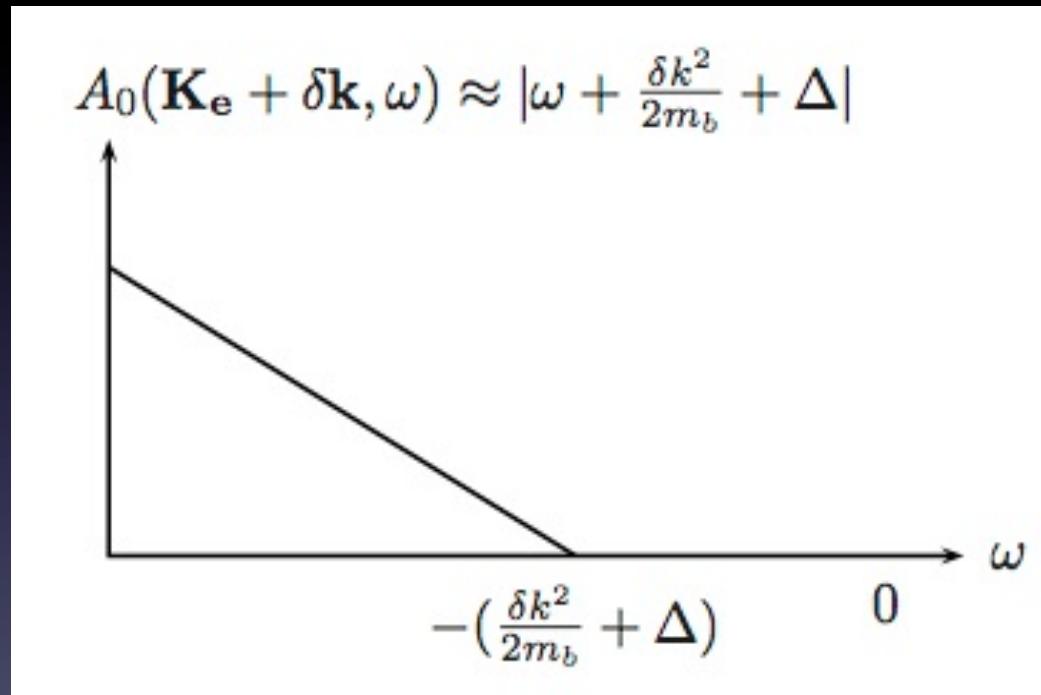
Pujari and Lawler, PRB, 2013



Electron correlations unrenormalized!

# Contact interaction

Tang, Fisher, Lee, PRB 2013



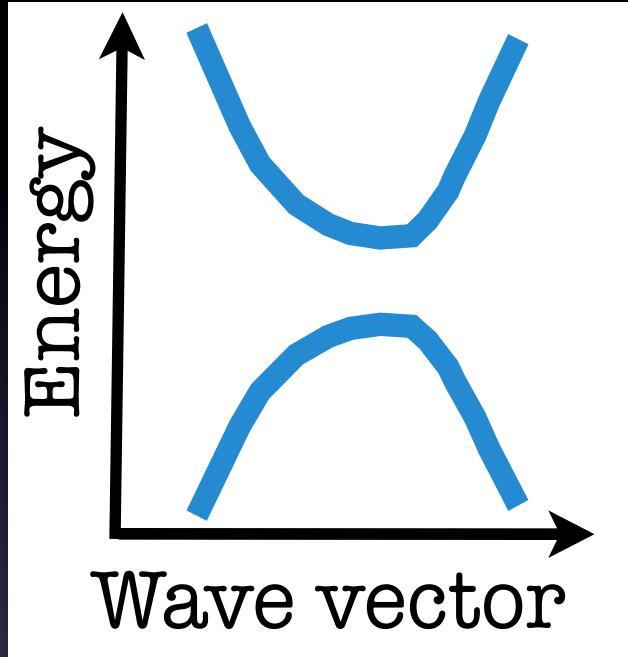
Justified by crude contact interaction model  
with no understanding of gauge invariance.

# Future directions for the field

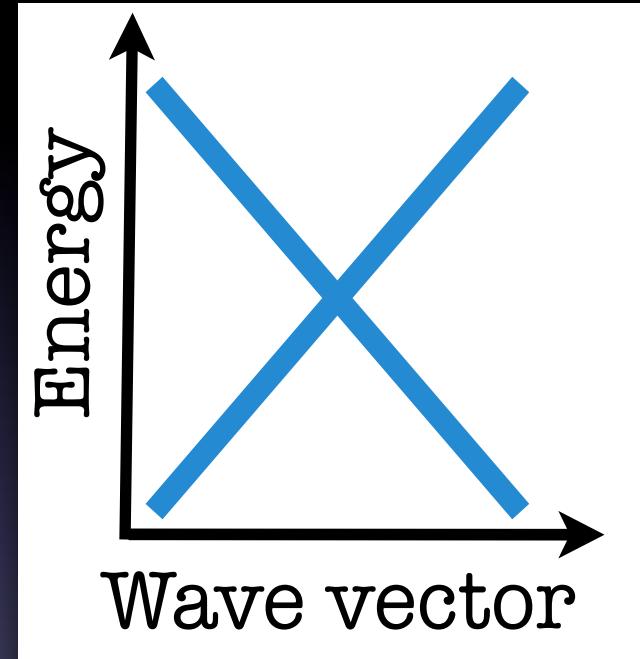
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- How can we find more spin liquids?
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# Instability of Dirac spinons

Lu, Ran, Lee, PRB 2012



“Cooper”  
Pairing



Spin liquid in DMRG

$U(1)$  Dirac State

DMRG → Dirac state unstable at low temperatures

# But how?

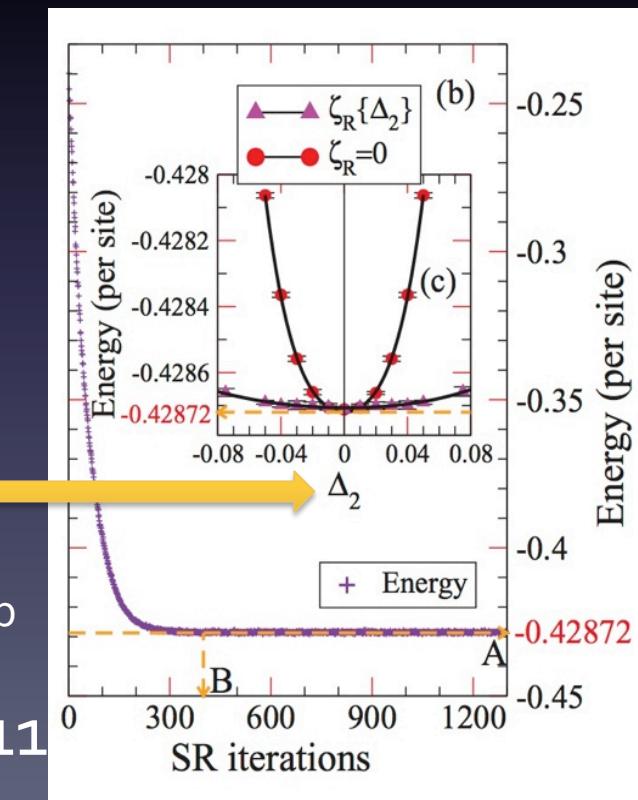
State	$8 \times 8 \times 3$ lattice	$12 \times 12 \times 3$ lattice
U(1)-Dirac spin liquid	-0.42866(2)	-0.42863(2)
VBS state ( $ \chi_1/\chi_2  = 1.05$ )	-0.42848(2)	-0.42844(2)
VBS state ( $ \chi_1/\chi_2  = 0.95$ )	-0.42846(2)	-0.42846(2)
Chiral spin liquid ( $\theta = 0.05$ )	-0.42857(2)	-0.42853(2)

Spin liquid	$8 \times 8 \times 3$ lattice	$12 \times 12 \times 3$ lattice
SL-[ $\frac{\pi}{2}, 0$ ]	-0.4010(1)	-0.4010(1)
SL-[ $\pm \frac{\pi}{2}, 0$ ]	-0.3907(1)	-0.3910(1)
SL-[ $\frac{\pi}{2}, \pi$ ]	-0.3814(1)	-0.3822(1)
SL-[ $0, 0$ ]	-0.4115(1)	-0.4121(1)
SL-[ $0, \pi$ ]	-0.42866(2)	-0.42863(2)

Ran et. al., PRL 2007

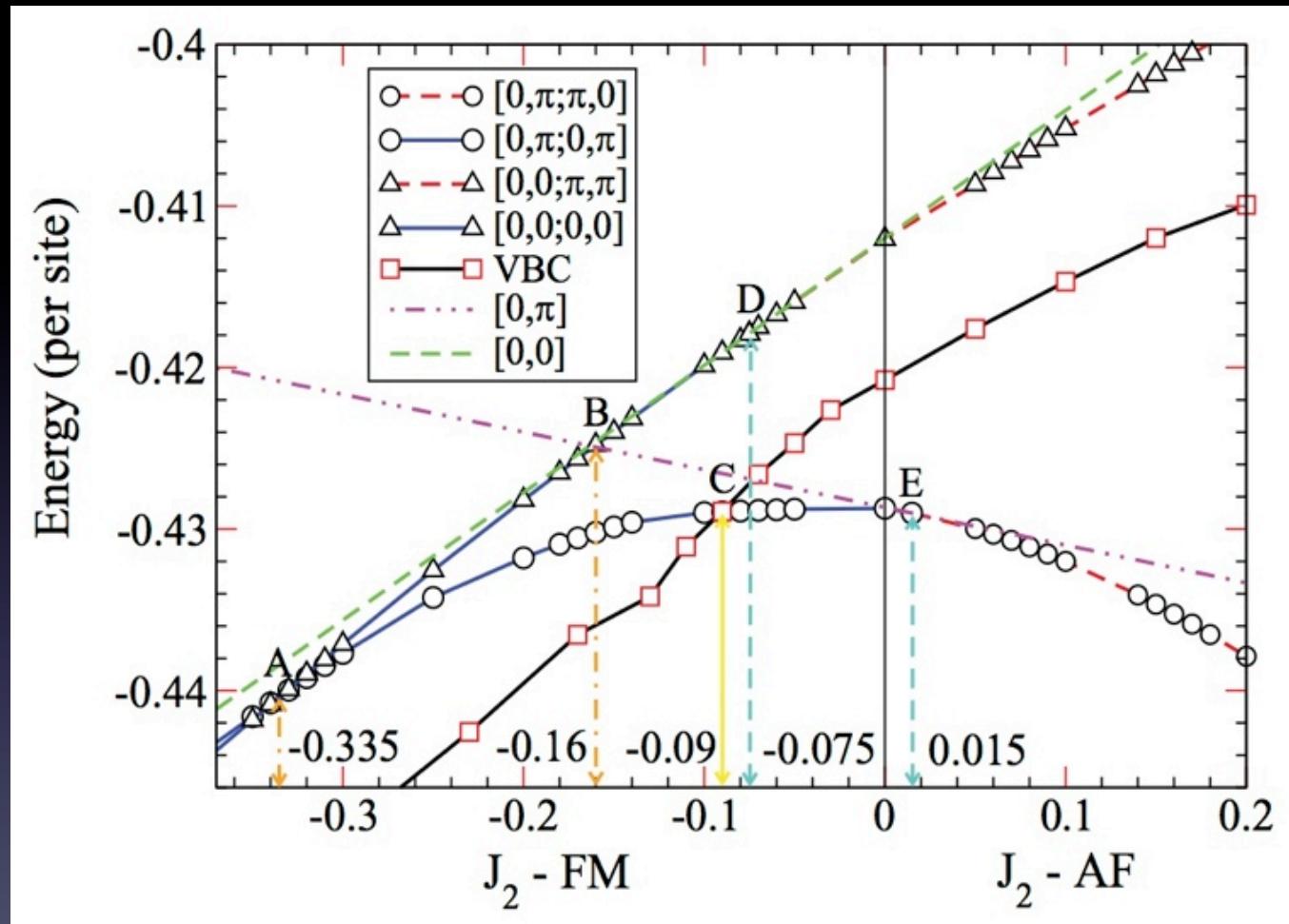
BCS pairing gap

Yqbal, Becca, Poilblanc, PRB, July 2011



# But how?

Yqbal, Becca, Poilblanc, PRB, March 2011



Dirac spin liquid wins again at  $J_2 = 0$

# Look at all modified BCS wave functions!

Clark et. al.,  
under review at PRL

Lu, Ran, Lee PRB 2010

No.	$\eta_{12}$	$\Delta_s$	$u_\alpha$	$u_\beta$	$u_\gamma$	$\tilde{u}_\gamma$	Label	Gapped?
1	+1	$\tau^2, \tau^3$	$Z_2[0,0]A$	Yes				
2	-1	$\tau^2, \tau^3$	$\tau^2, \tau^3$	$\tau^2, \tau^3$	$\tau^2, \tau^3$	0	$Z_2[0,\pi]\beta$	Yes
3	+1	0	$\tau^2, \tau^3$	0	0	0	$Z_2[\pi,\pi]A$	No
4	-1	0	$\tau^2, \tau^3$	0	0	$\tau^2, \tau^3$	$Z_2[\pi,0]A$	No
5	+1	$\tau^3$	$\tau^2, \tau^3$	$\tau^3$	$\tau^3$	$\tau^3$	$Z_2[0,0]B$	Yes
6	-1	$\tau^3$	$\tau^2, \tau^3$	$\tau^3$	$\tau^3$	$\tau^2$	$Z_2[0,\pi]\alpha$	No
7	+1	0	0	$\tau^2, \tau^3$	0	0	—	—
8	-1	0	0	$\tau^2, \tau^3$	0	0	—	—
9	+1	0	0	0	$\tau^2, \tau^3$	0	—	—
10	-1	0	0	0	$\tau^2, \tau^3$	0	—	—
11	+1	0	0	$\tau^2$	$\tau^2$	0	—	—
12	-1	0	0	$\tau^2$	$\tau^2$	0	—	—
13	+1	$\tau^3$	$\tau^3$	$\tau^2, \tau^3$	$\tau^3$	$\tau^3$	$Z_2[0,0]D$	Yes
14	-1	$\tau^3$	$\tau^3$	$\tau^2, \tau^3$	$\tau^3$	0	$Z_2[0,\pi]\gamma$	No
15	+1	$\tau^3$	$\tau^3$	$\tau^3$	$\tau^2, \tau^3$	$\tau^3$	$Z_2[0,0]C$	Yes
16	-1	$\tau^3$	$\tau^3$	$\tau^3$	$\tau^2, \tau^3$	0	$Z_2[0,\pi]\delta$	No
17	+1	0	$\tau^2$	$\tau^3$	0	0	$Z_2[\pi,\pi]B$	No
18	-1	0	$\tau^2$	$\tau^3$	0	$\tau^3$	$Z_2[\pi,0]B$	No
19	+1	0	$\tau^2$	0	$\tau^2$	0	$Z_2[\pi,\pi]C$	No
20	-1	0	$\tau^2$	0	$\tau^2$	$\tau^3$	$Z_2[\pi,0]C$	No

Largest known catalog  
of kagome spin liquids

$$\hat{\mathcal{P}} \prod_{ij} \left( 1 + \phi_{ij} f_{i\uparrow}^\dagger f_{j\downarrow}^\dagger \right) |0\rangle$$


Initial values

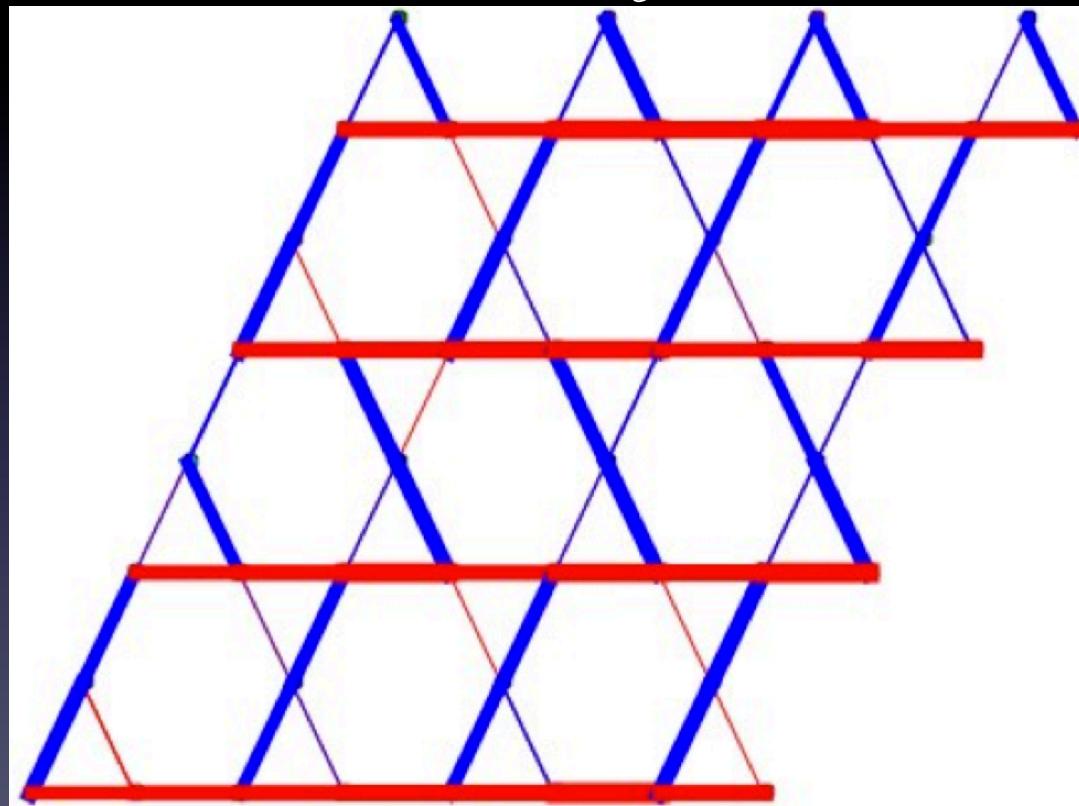


Minimize  $N^2$   
Variational  
Parameters

New Ground State?

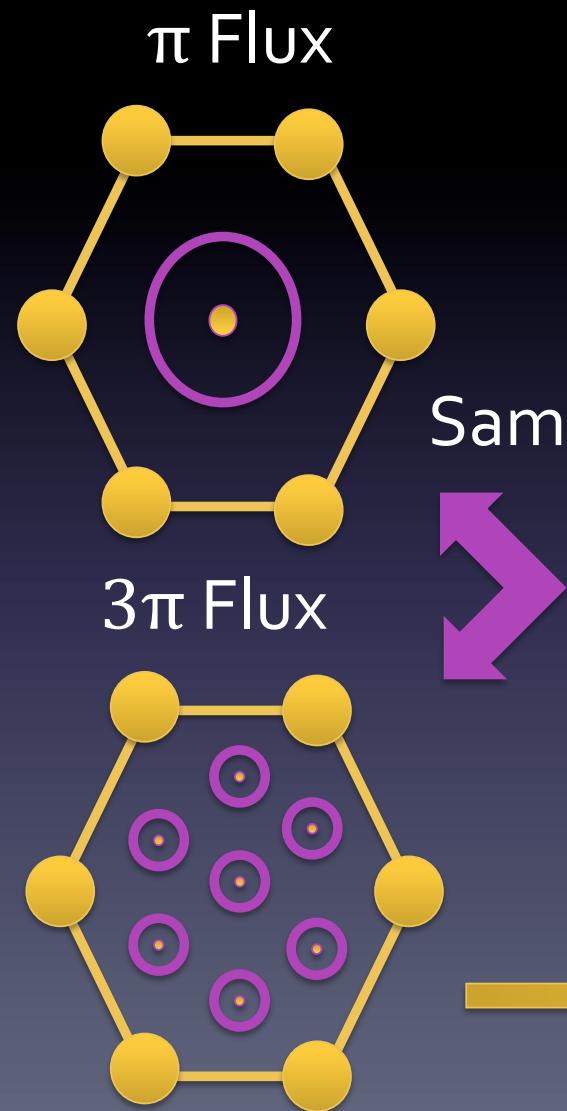
# Smectic spin liquid!

$$\langle \Psi | \vec{S}_i \cdot \vec{S}_j | \Psi \rangle$$

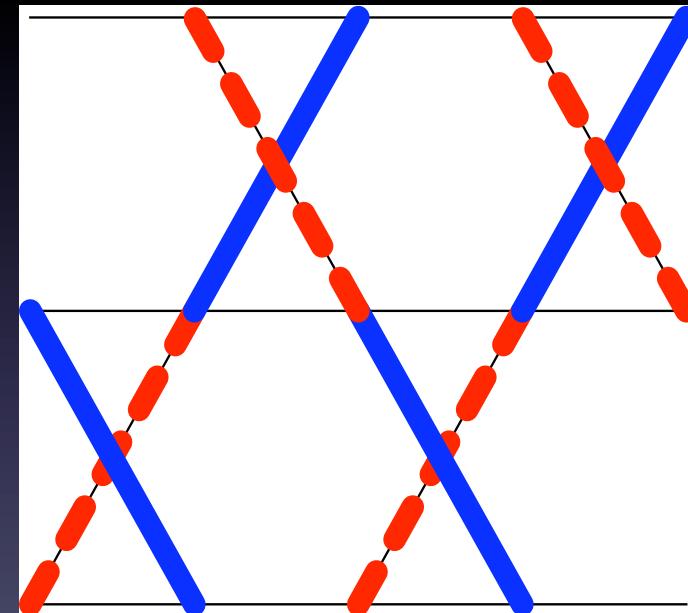


Also, we find evidence for Cooper pairing of the spinons

# Proliferation of Monopoles



Monopole operator found in 3D CFT  
Hermele, et. al. PRB 2008

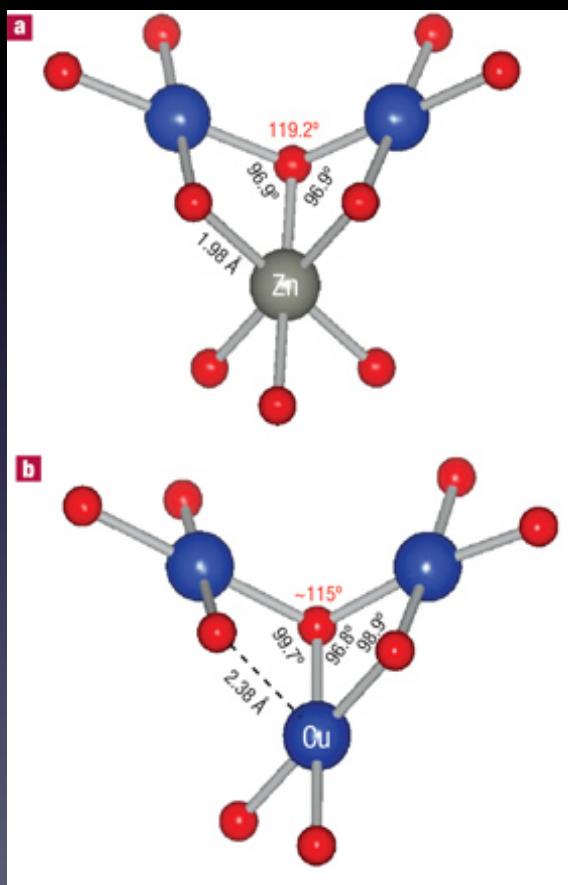


Monopole wave function

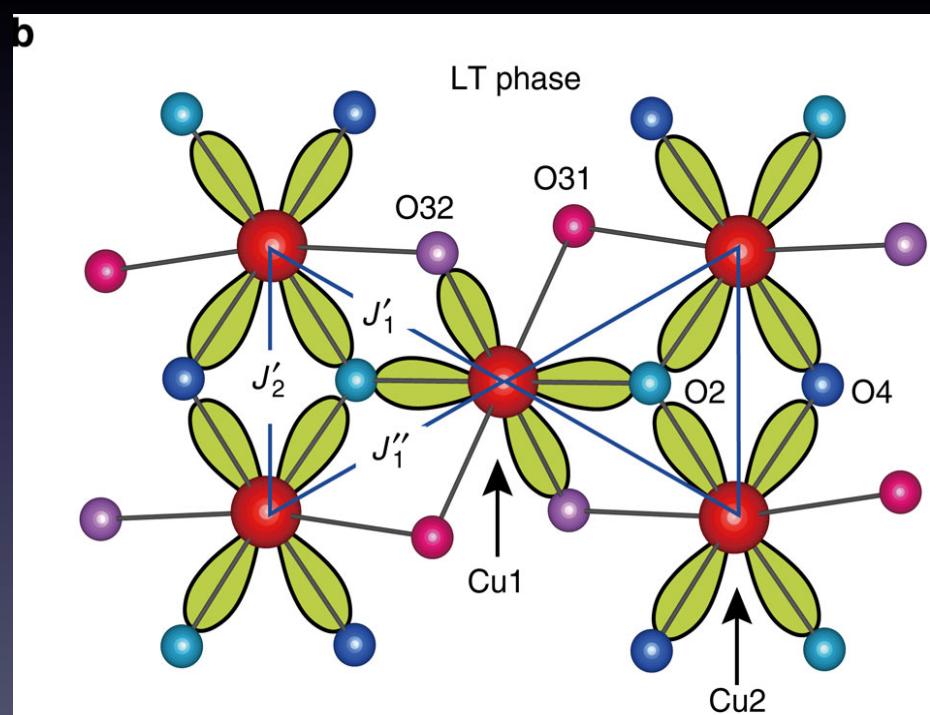
Clark, et. al., under review at PRL

# Distorted Kagome Systems

Zn free Herbertsmithite  
(clinoatacamite)



Single Crystals of  
Volborthite



Yoshida et. al. Nat. Comm. 2012

S.-H. Lee, Nat. Mat. 2007

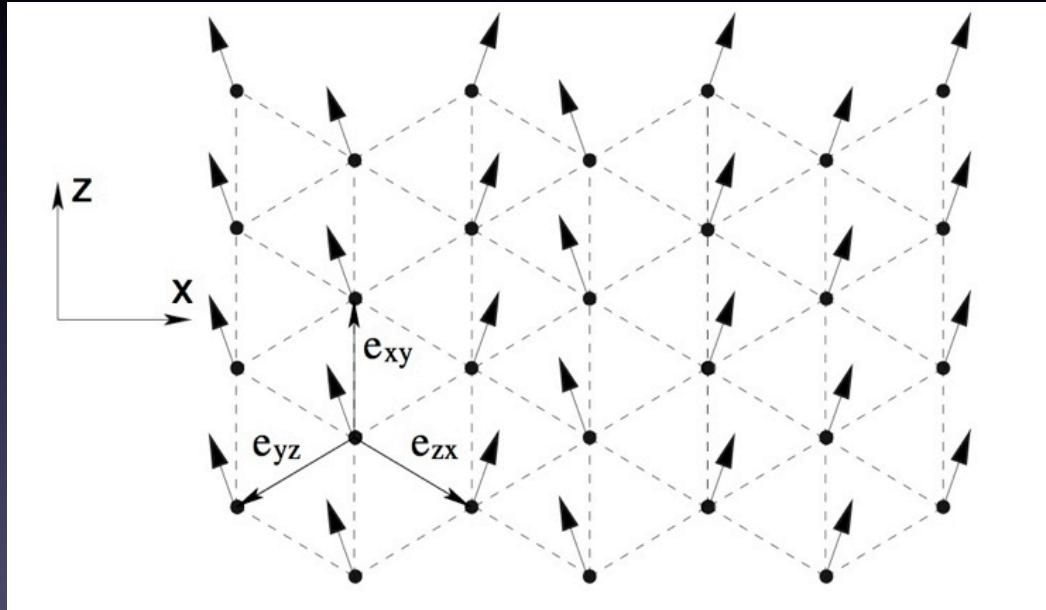
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# Orbital Frustration in LiNiO<sub>2</sub>

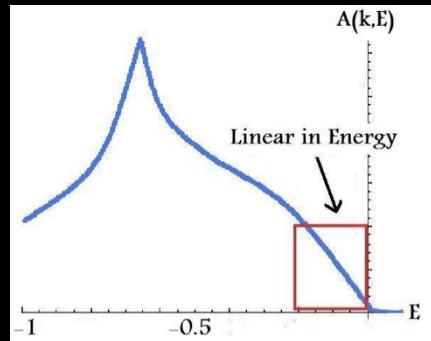
- Well known Li-ion Battery material  
(Ohzuku et. al. J. Electrochem. Soc., 1993)

Pseudospin eg  
Orbitals don't  
order

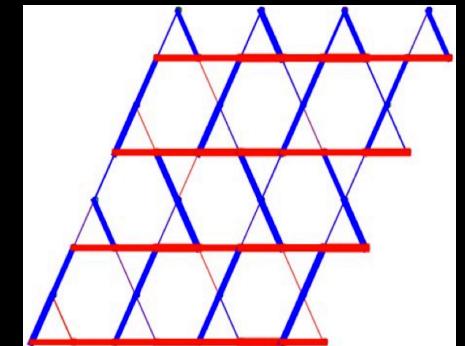


Mostovoy and Khomskii, J. of Phys. A, 2003

So does frustration help?



# Conclusions



- Substantial evidence for spin liquid phases but exotic properties still undetected
- Proposed a signature for fermionic spinons in ARPES
- Proposed the existence of spin liquid crystals that also have fermionic spinons