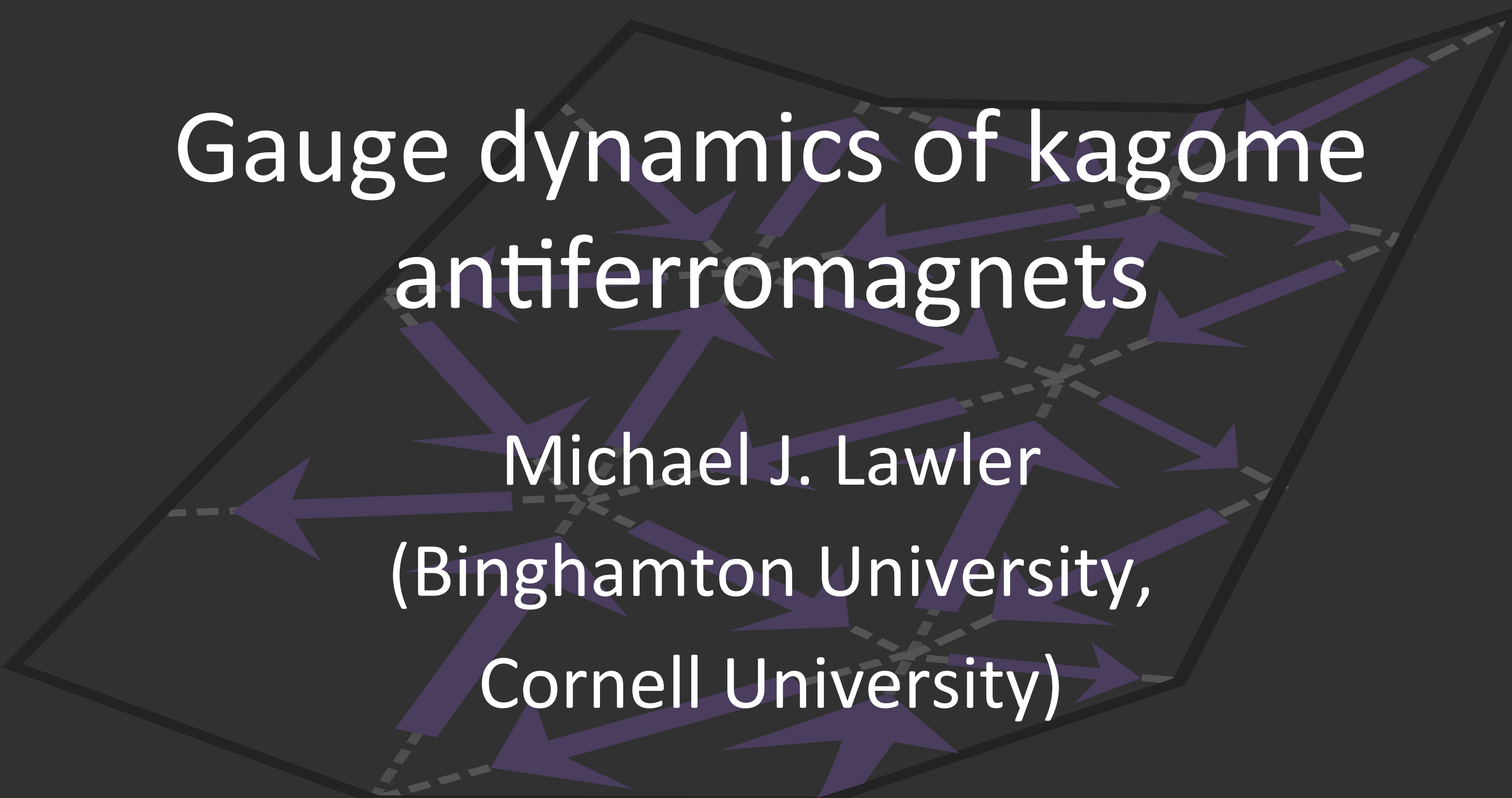
Gauge dynamics of kagome antiferromagnets



Michael J. Lawler
(Binghamton University,
Cornell University)

Outline

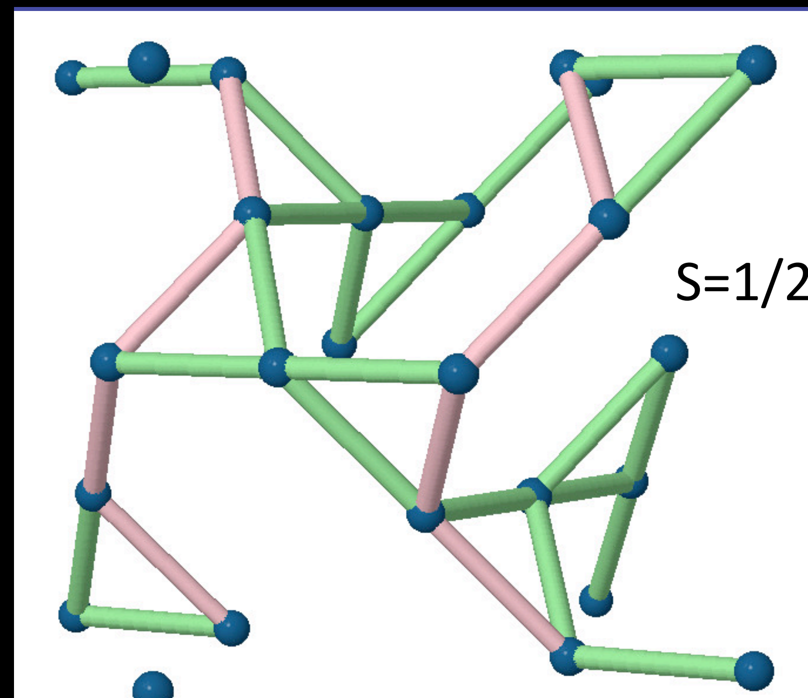
- Introduction to highly frustrated magnets
- Constrained spin models
  - Dirac's generalized Hamiltonian mechanics
  - Degrees of freedom counting
  - Edge states?
- Simulations of spin waves in kagome AFM
- Conclusions

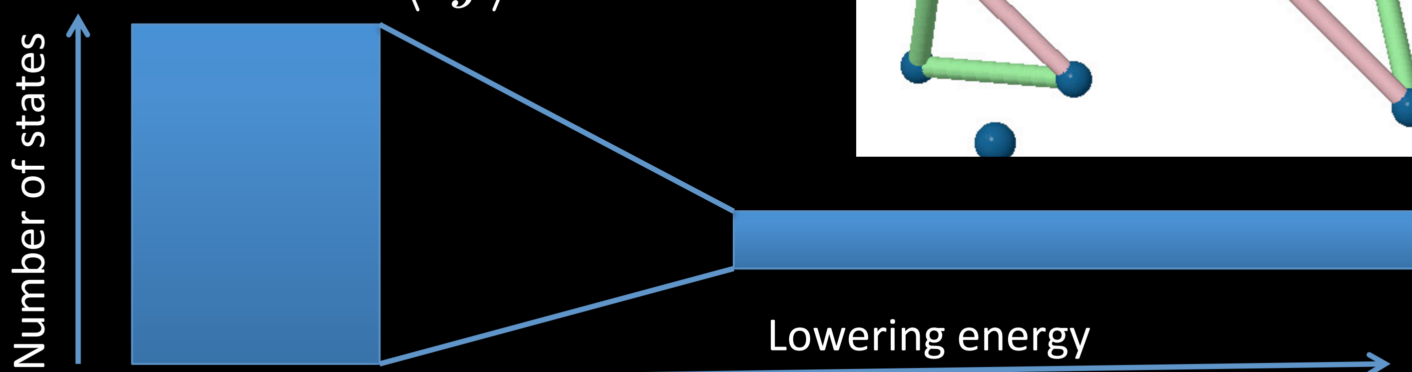
The problem of highly frustrated magnetism

- Nearest neighbor model just selects a low energy subspace!

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

$Na_4Ir_3O_8$, Okamoto et. al. 2007

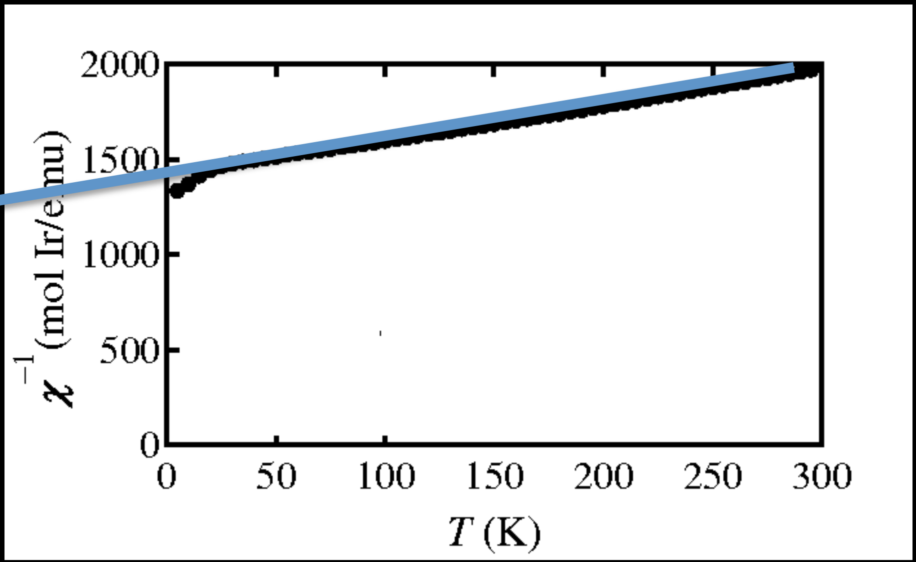




→ defines the "cooperative paramagnet" (Villain, 1979)

Cooperative paramagnets

$Na_4Ir_3O_8$, Okamoto et. al. 2007



Θ_{CW} = -650 K

$$\chi(T) = \frac{C}{T - \Theta_{CW}}$$

Frustration parameter:

$$f = \Theta_{CW}/T_c \approx 65$$

→ temperature range of the cooperative paramagnetic

At low energies, by what classical law do these spins move?

If we new the answer to this question, we could:

- Understand equilibration mechanisms
- Quantize and connect classical notions of frustration to the stability of quantum spin liquids
- Carry out a controlled spin wave expansion

Constrained spin models

Lawler, 2013

On the kagome lattice, we can write

$$H_{nn} = \frac{J}{2} \sum_{\langle ijk \rangle} \left(\vec{S}_i + \vec{S}_j + \vec{S}_k \right)^2 + const$$

So the low energy subspace of states obeys

$$\vec{\phi}_{ijk} \equiv \vec{S}_i + \vec{S}_j + \vec{S}_k = 0$$

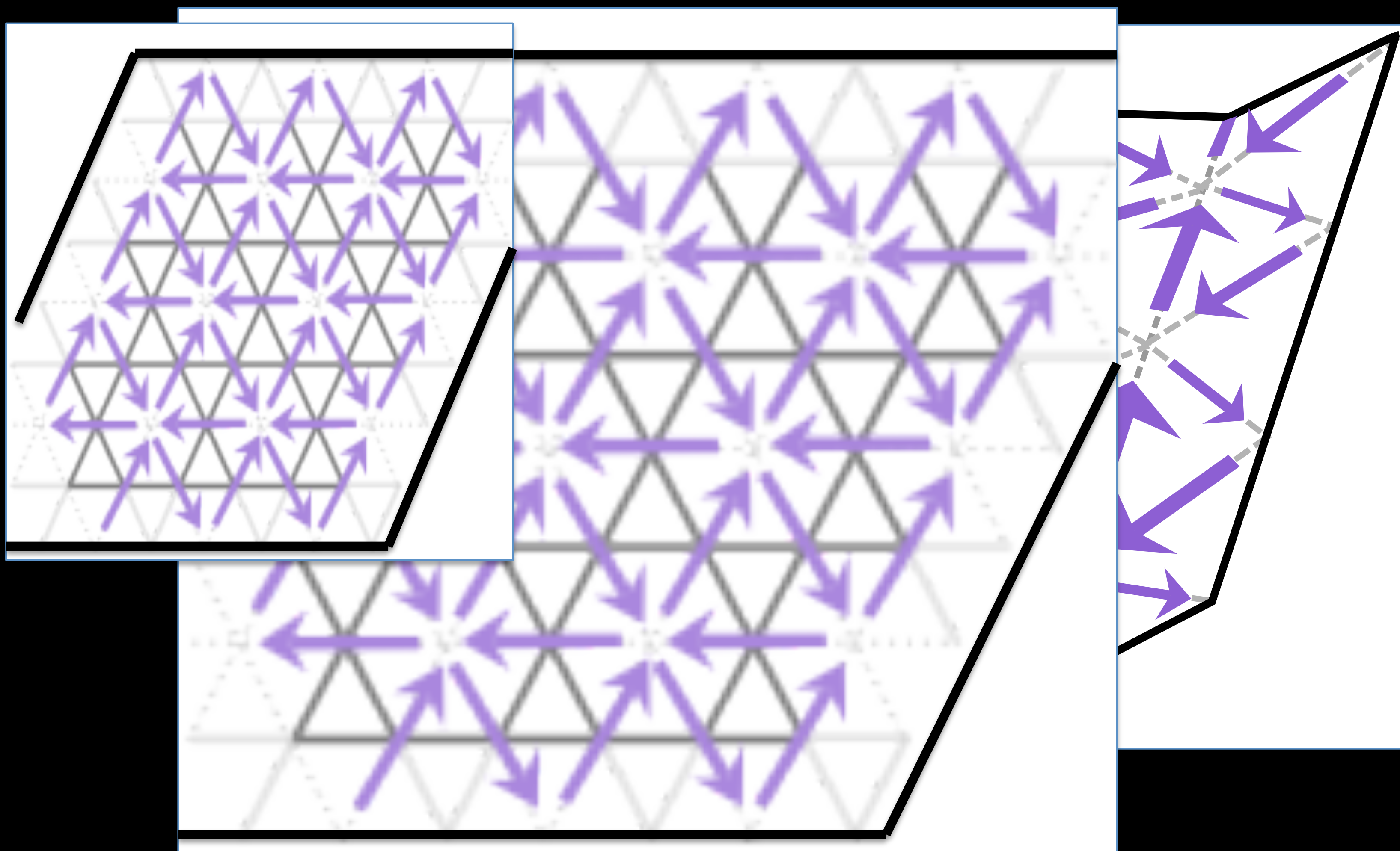
Lets then focus on the simpler model

$$H = H_{eff} - \sum_{\langle ijk \rangle} \vec{h}_{ijk} \cdot \vec{\phi}_{ijk}$$

Lagrange multipliers

Spin origami

Shender et. al, 1993



Constrained spin model is that of a fluctuating membrane!

Constrained Hamiltonian Mechanics

Dirac, 1950,1958

Follow Dirac, and fix the Lagrange multipliers h_n by

$$\frac{d}{dt}\phi_m = \{\phi_m, H_{eff}\} - \sum_n \{\phi_m, \phi_n\} h_n = 0$$

This is a linear algebra problem! If

$$\det C_{mn} \equiv \det\{\phi_m, \phi_n\} \neq 0$$

We can invert and solve for h_n.

Otherwise, some combinations of h_n remain arbitrary!

Gauge dynamics

- A “gauge theory” in mechanics is one with multiple solutions to its equations of motion.
- Example: Maxwell electrodynamics
  - There are many solutions to the scalar and vector potential
  - The electric and magnetic fields evolve the same way for each solution

The single triangle model

This model has constraints

$$\vec{\phi} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3 = 0$$

and Hamiltonian

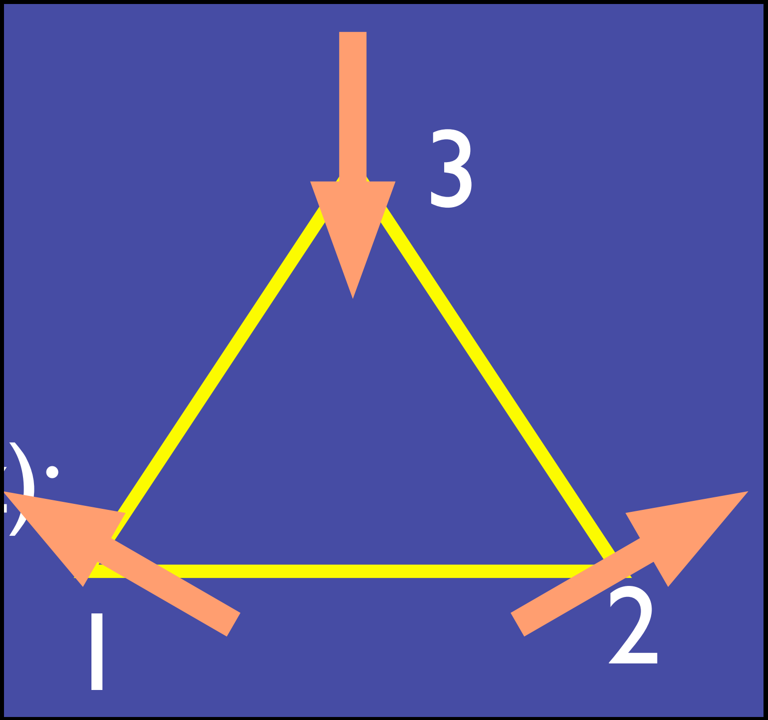
$$H = H_{eff} - \vec{h} \cdot \vec{\phi}$$

The constraints obey

$$\{\phi_x, \phi_y\} = \phi_z = 0$$

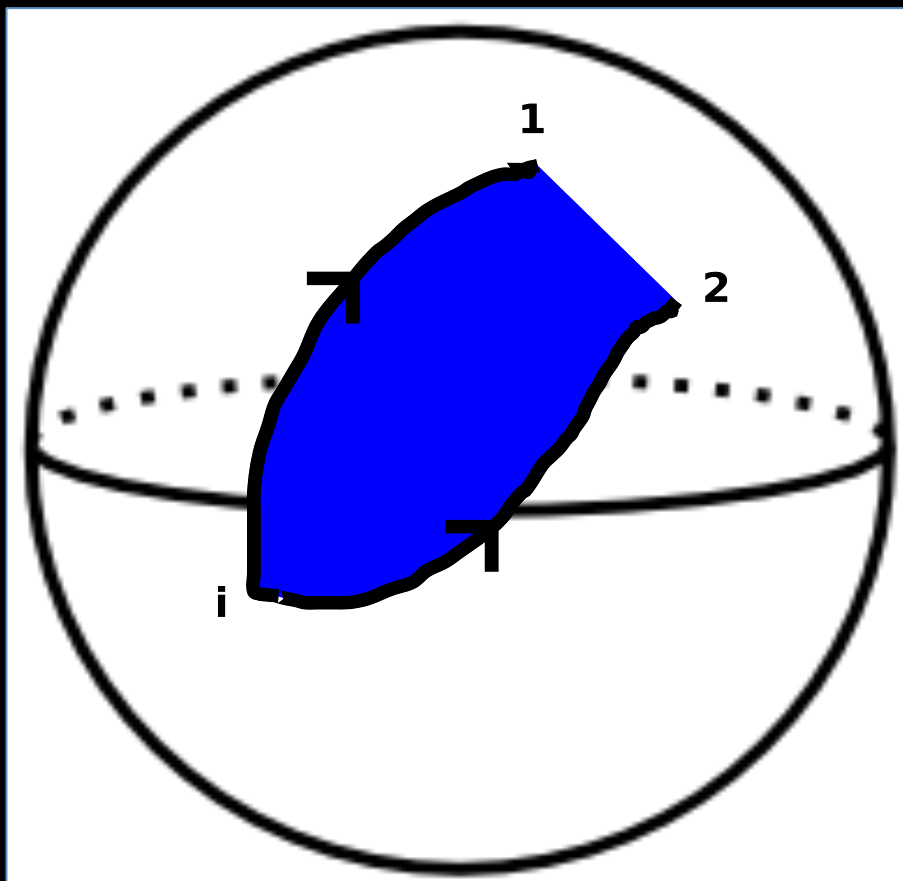
So

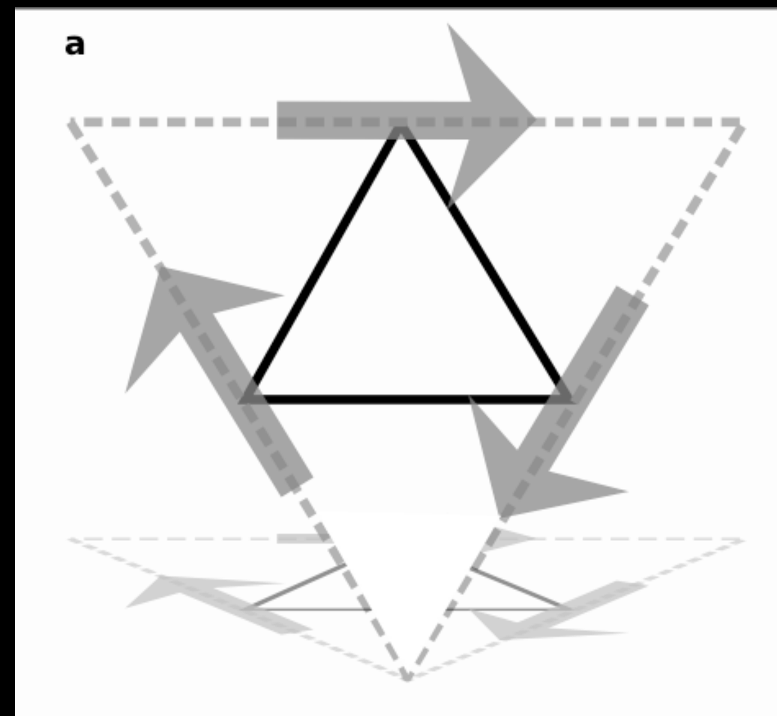
$$\frac{d\phi_a}{dt} = \{\phi_a, H_{eff}\} - h_b\{\phi_a, \phi_b\} = \{\phi_a, H_{eff}\} = 0$$



h_x, h_y and h_z are arbitrary!

Map all solutions





Spin origami construction

Physical observables evolve the same way
independent of the choice of the arbitrary functions

Degrees of freedom counting

- How many physical observables are there?
  - Dirac discovered

$$N_{canonical} = D - M - N_L$$

where

- D: the number of unconstrained coordinates
- M: the number of constraint functions ϕ_m
- N_L: the number of arbitrary Lagrange multipliers

Two polarizations of light

- Consider electricity and magnetism
  - D = 8

$$\phi, \quad \vec{A}, \quad \pi_0 = \frac{\delta L}{\delta \dot{\phi}}, \quad \pi_a = E_a = \frac{\delta L}{\delta \dot{A}_a}$$

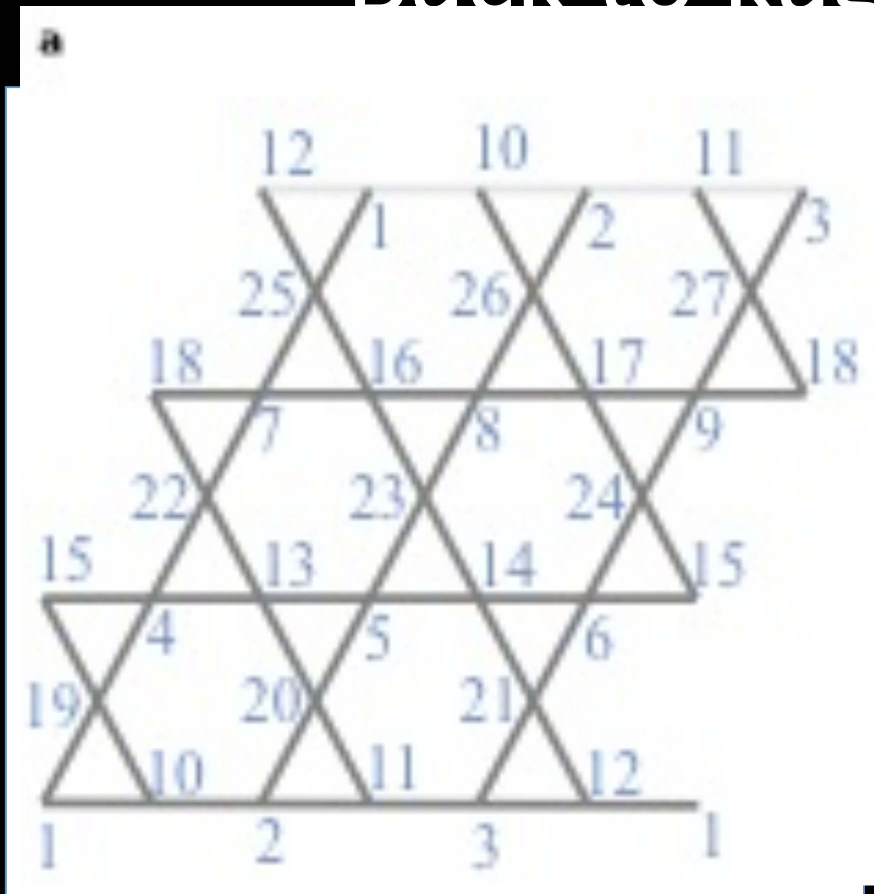
  - M = 2

$$\pi_0 = 0, \quad \nabla \cdot \vec{E} - \rho = 0$$

  - N_L = 2 (the above two constraints commute)

So $N_{canonical}$ = 8 – 2 – 2 = 4 → two polarizations of light!

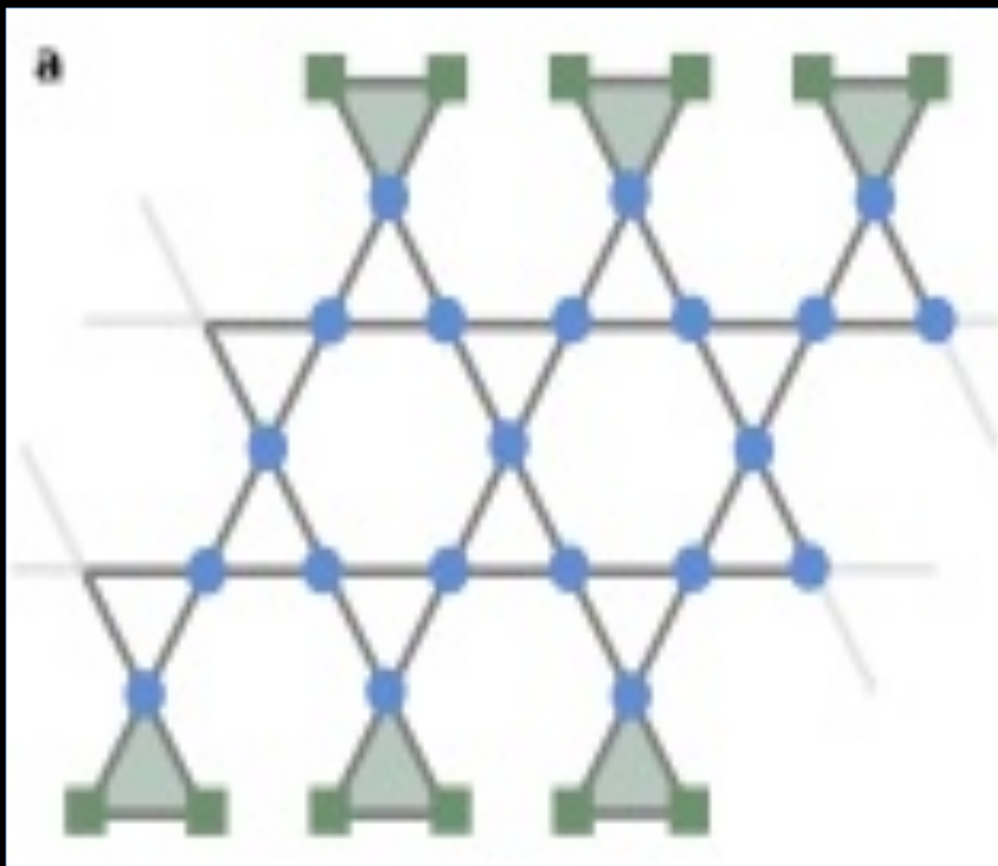
Back to kagome: pbc's



For every spin configuration that satisfies the constraints:

$N_{canonical} = 0!$

Edge states?



Open boundary conditions

$N_{canonical}$ = number of dangling triangles

But a local mechanical object requires a position *and* a momentum coordinate!

Chern-Simon’s electrodynamics

- Similar to “doubled” Chern-Simon’s electrodynamics in 2 spatial dimensions

$$\vec{E} = 0, B = 0$$

- Changes only the statistics of particles
- Quantum model has long range entanglement
- Proposed to govern Z_2 spin liquids (Xu and Sachdev, 2009)

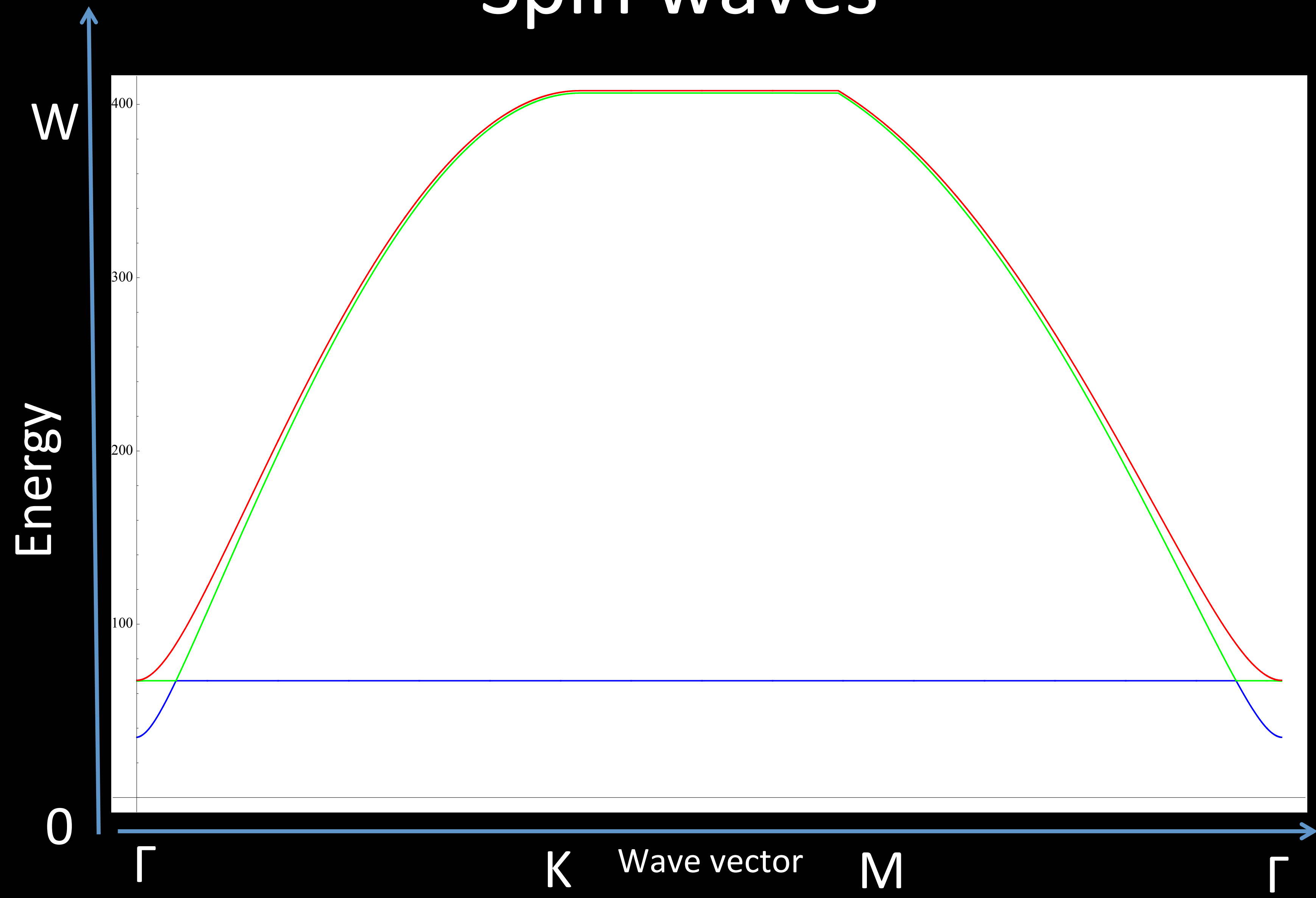
Ordinary Kagome antiferromagnets

Now consider an ordinary kagome antiferromagnet with Hamiltonian

$$H = \sum_{\langle ij \rangle} \left[J \vec{S}_i \cdot \vec{S}_j + \vec{D}_{ij} \cdot \vec{S}_i \times \vec{S}_j \right]$$

How is the discovered gauge dynamics important here?

Spin waves



What do the eigenmodes corresponding to gauge modes look like?

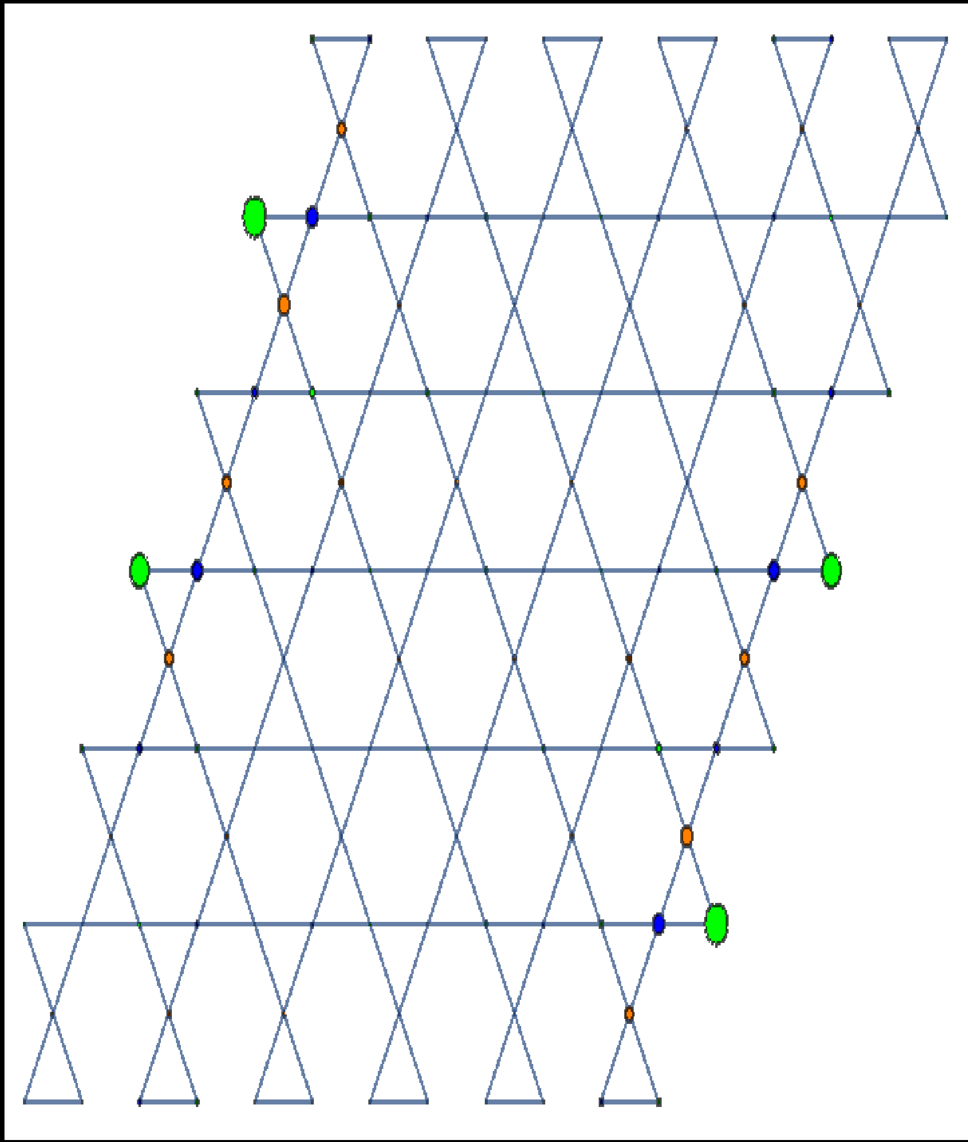
Motion of spins along the side edges

Blue Dots: Spin A

Green Dots: Spin B

Orange Dots : Spin C

Size of dots is proportional to motion of the spins



What do the eigenmodes corresponding to canonical modes look like?

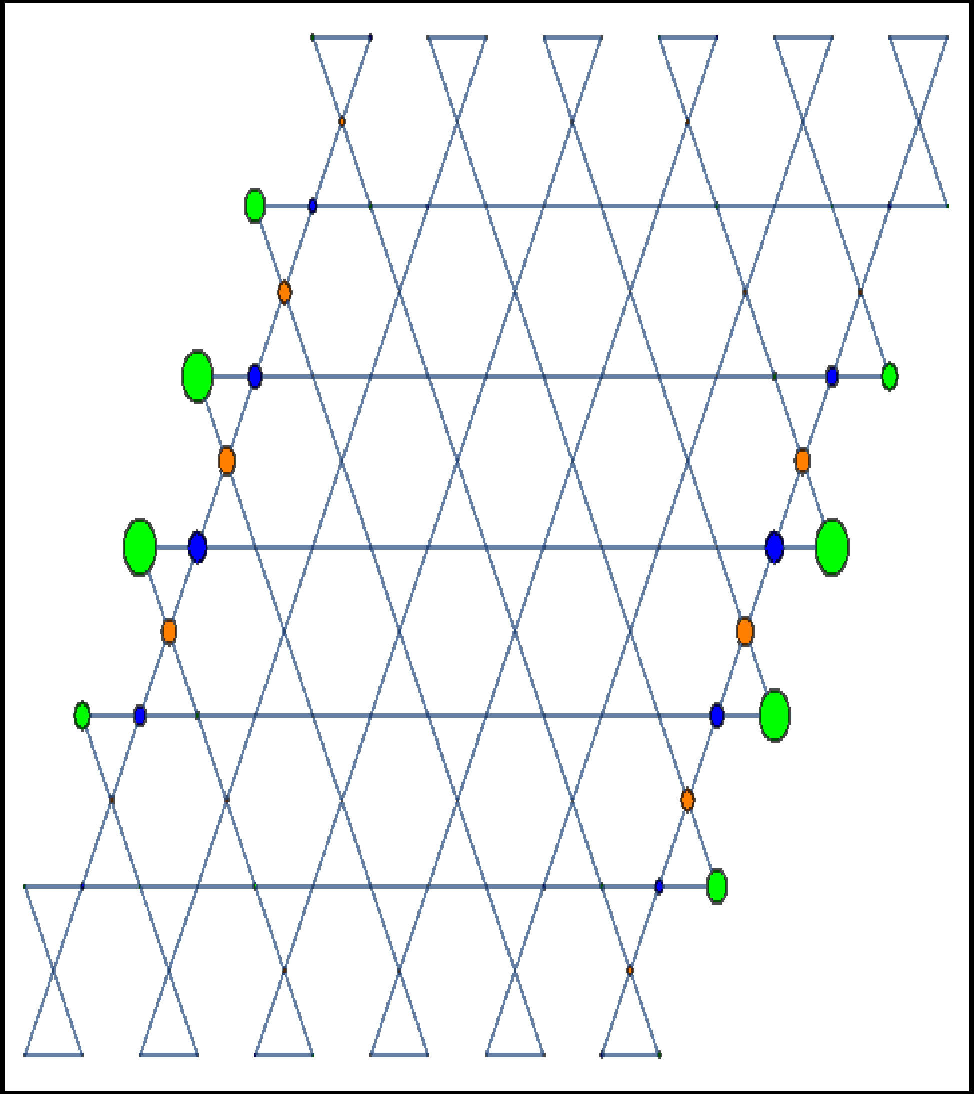
Motion along side edges – Proposed canonical edge states

Blue Dots: Spin A

Green Dots: Spin B

Orange Dots : Spin C

Size of dots is proportional to motion of the spins



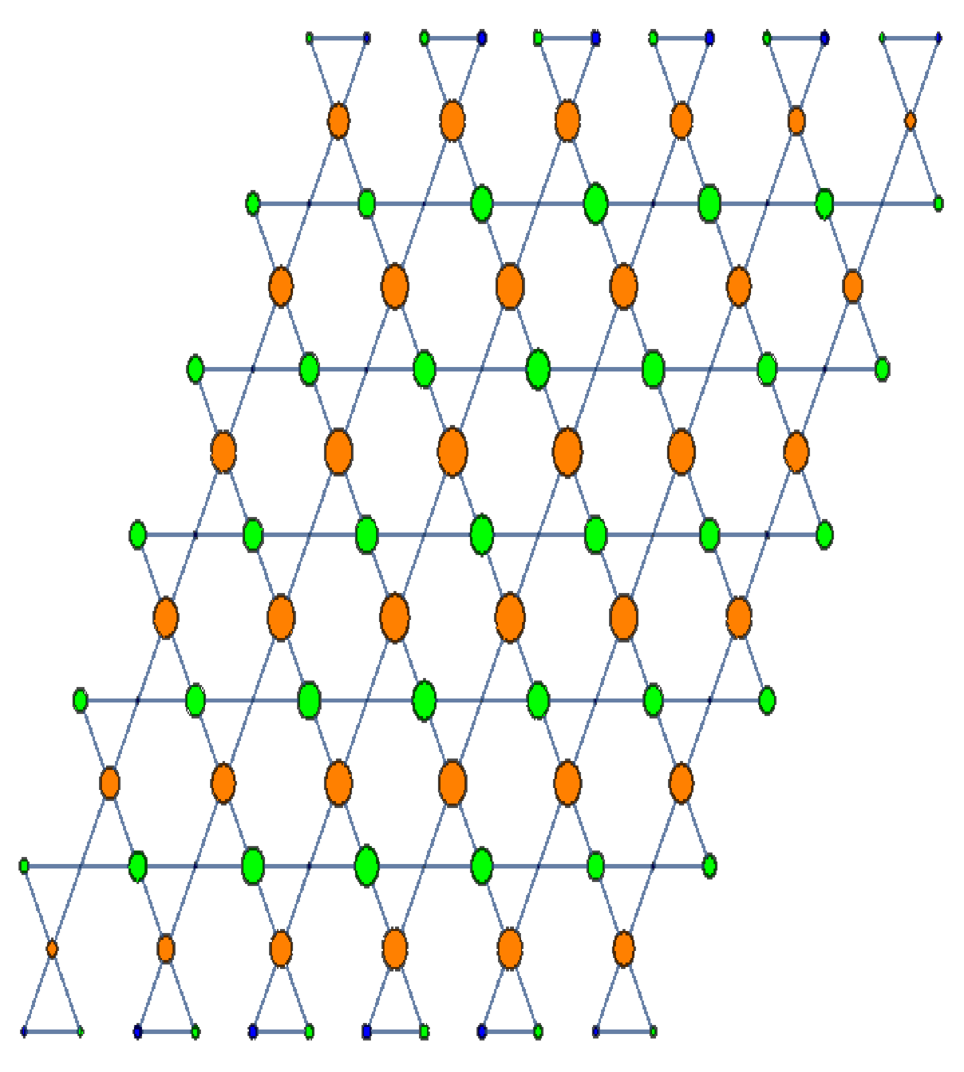
Motion of bulk - Folding

Blue Dots: Spin A

Green Dots: Spin B

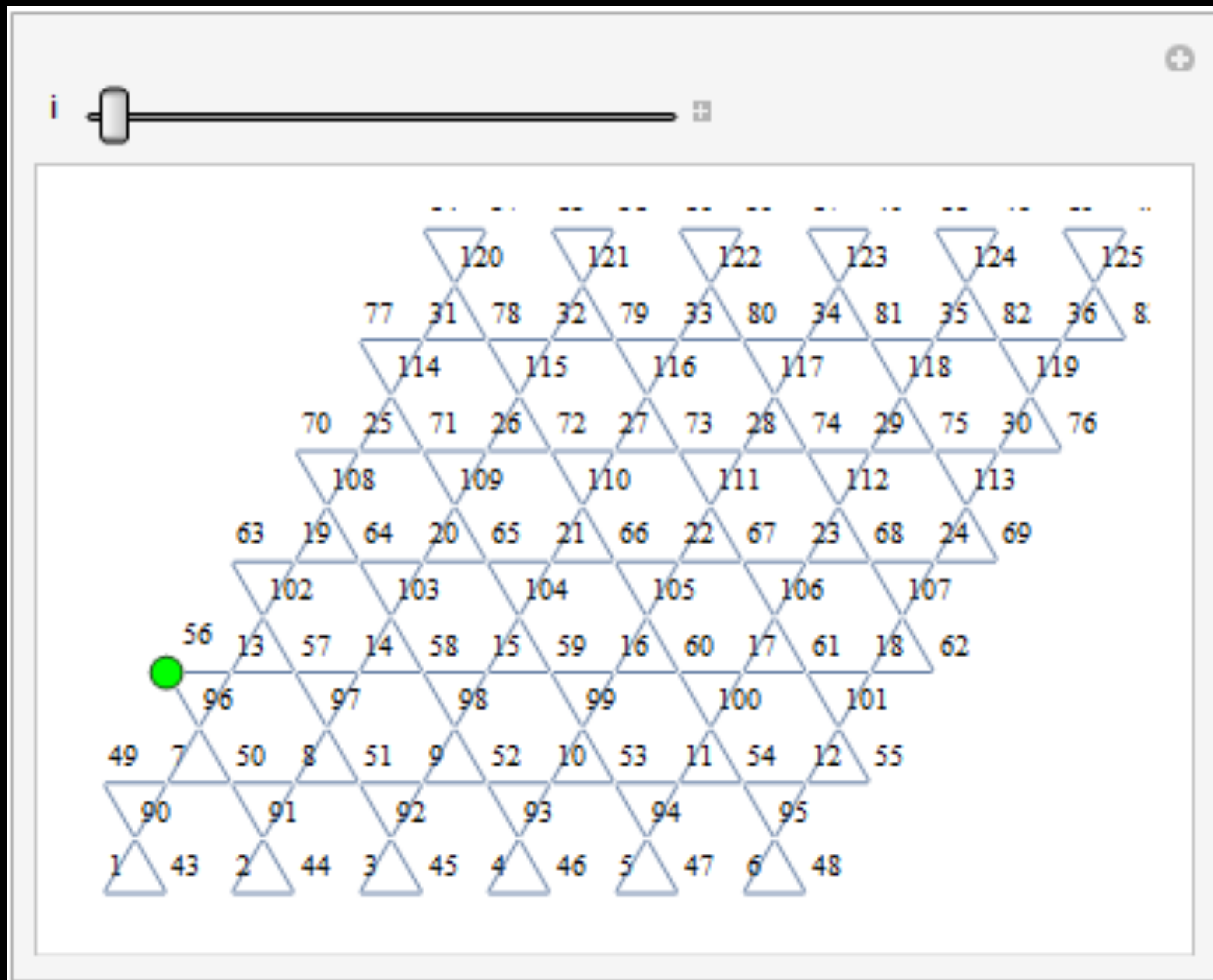
Orange Dots : Spin C

Size of dots is proportional to motion of the spins

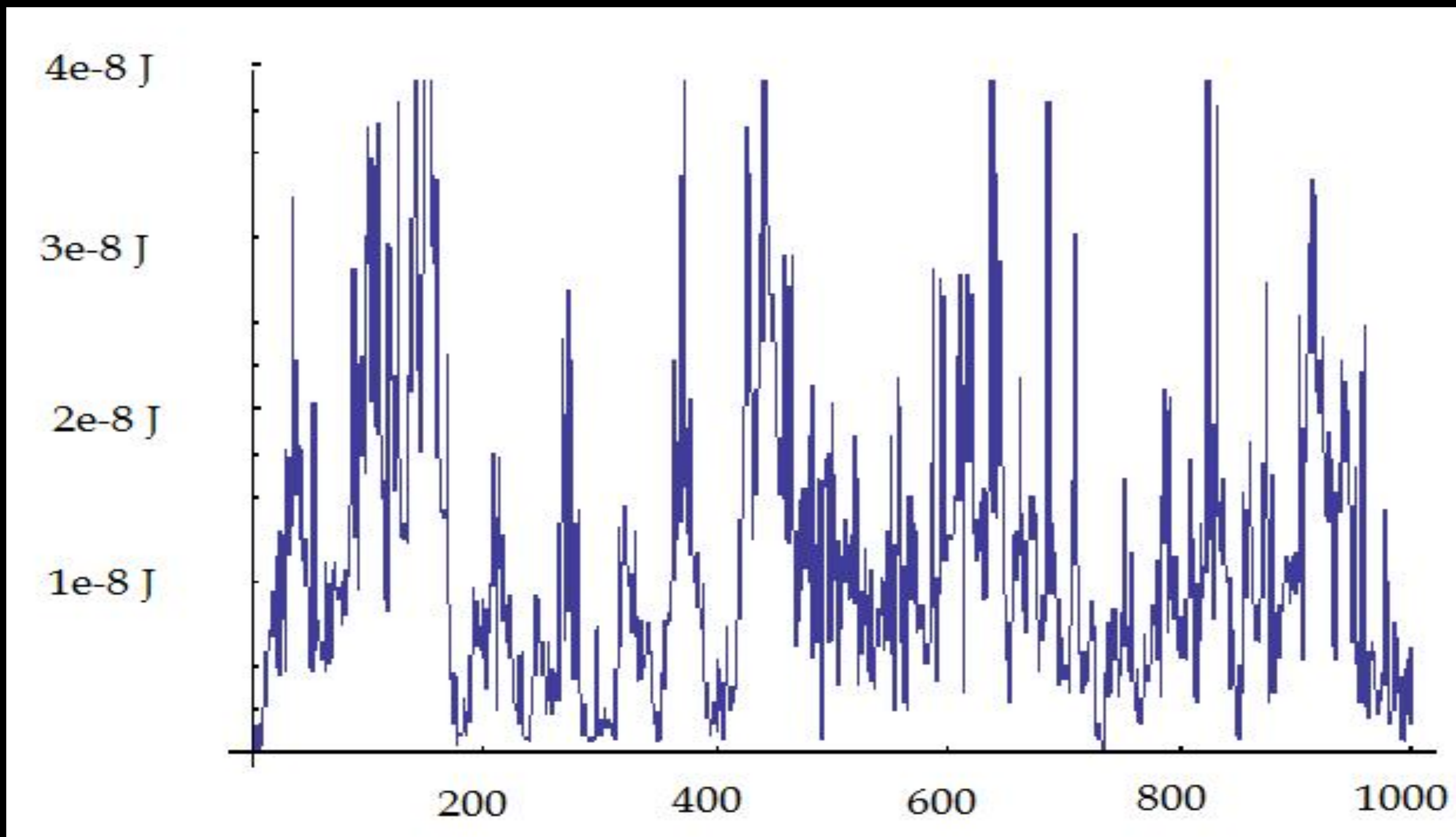


Just a global spin rotation mode!

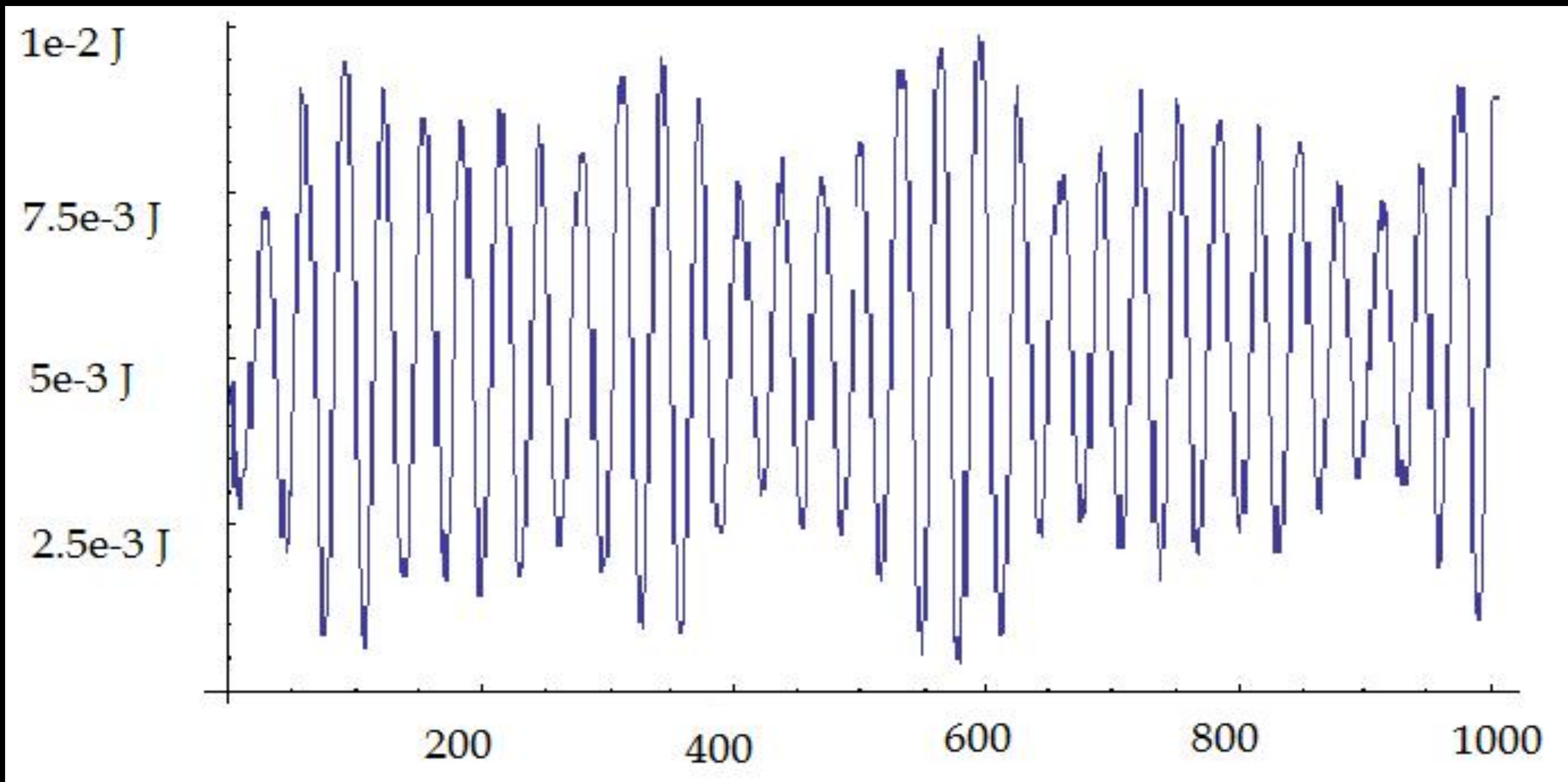
Simulation of edge excitations



Energy in the “gauge” modes

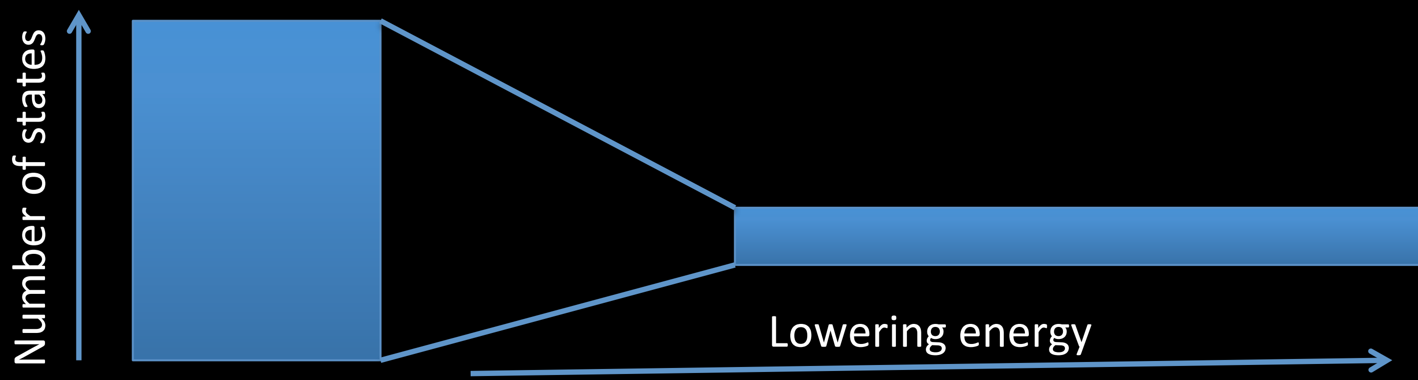


Energy in the “canonical” modes



Conclusions

- Spins constrained to classical ground states of HFMs obeys a kind of electrodynamics.



- Conjecture: frustration is important for the formation of a quantums spin liquid phase.

Strongly correlated metals

- Some strongly correlated metals are also gauge theories.
- Examples:
  - Double occupancy constraint implies

$$\hat{G}_i|phys\rangle = \hat{n}_{i\uparrow}\hat{n}_{i\downarrow}|phys\rangle = 0, [\hat{G}_i, \hat{G}_j] = 0$$

  - No nearest neighbor constraint of spinless fermions

$$\hat{G}_{ij}|phys\rangle = \hat{n}_i\hat{n}_j|phys\rangle = 0, [\hat{G}_{ij}, \hat{G}_{kl}] = 0$$