

# Boundary Effects in Stochastic Cyclic Competition Models on a Two-Dimensional Lattice

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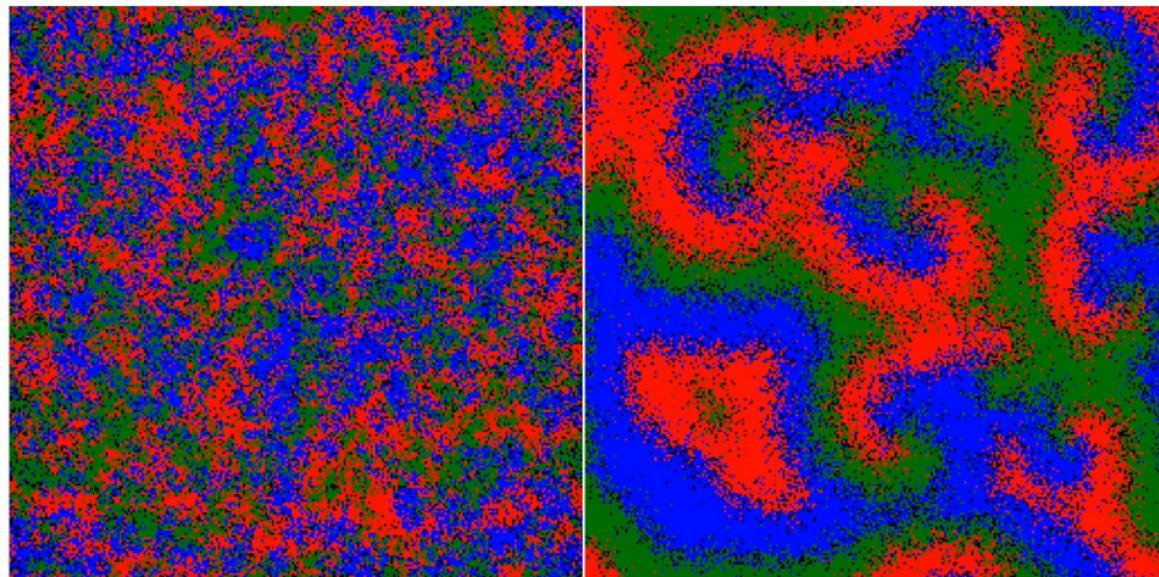
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# Motivation

Three species cyclic competition schemes motivated by examples in biology, population dynamics, and chemistry.

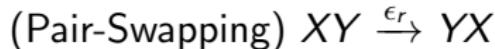
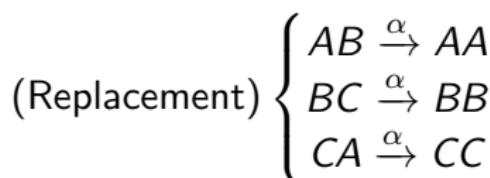


(a) Rock Paper Scissors

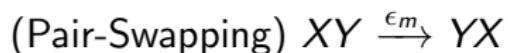
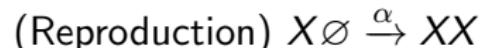
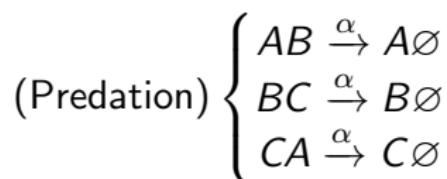
(b) May-Leonard

# Models

Rock-Paper-Scissors/Cyclic  
Lotka-Volterra (RPS) Model



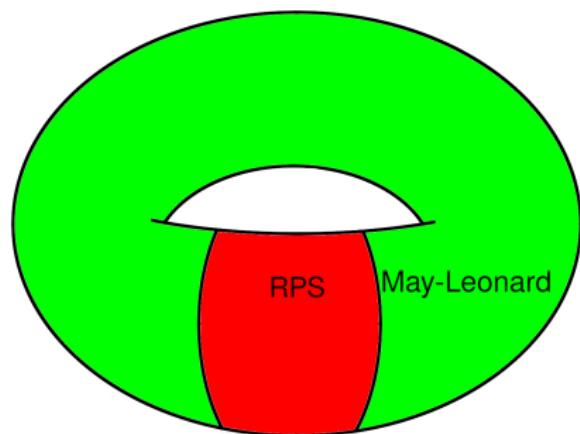
May-Leonard (ML) Model



$$X \in \{A, B, C\}, \quad Y \in \{A, B, C, \emptyset\}$$

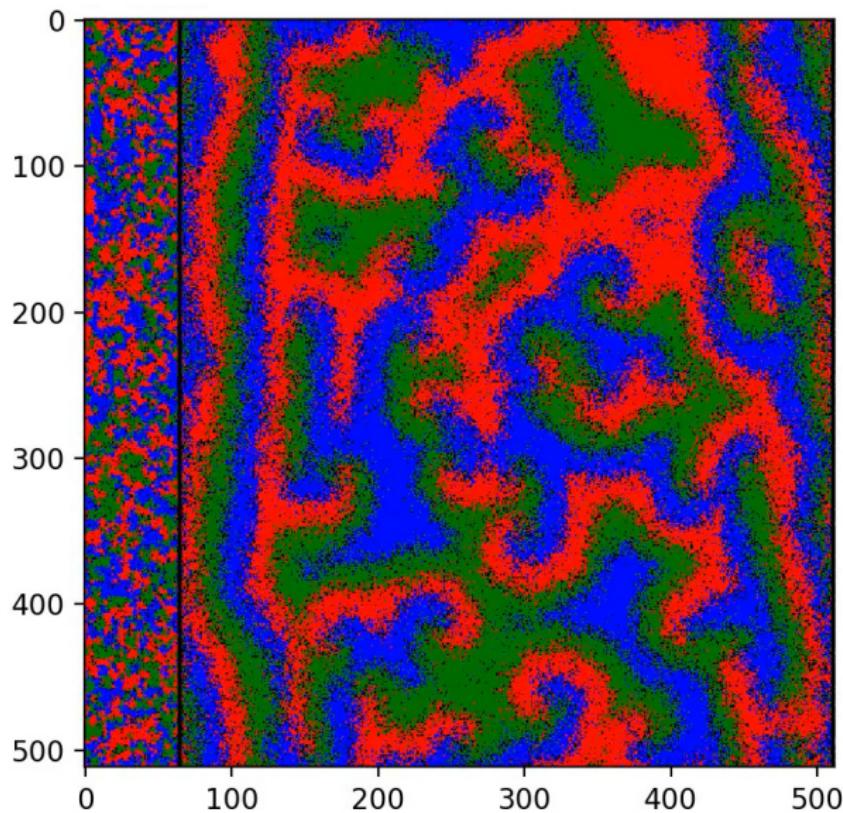
# Combined System

- Periodic boundary conditions (i.e. toroidal topology)
- Maximum occupancy of one particle per lattice site.
- May-Leonard except for thin (width = 64) annular patch governed by RPS
- Random initial conditions.
- $512 \times 512$  lattice sites.
- $\alpha = 1.0$ ,  $\epsilon_m = 5.0$ .  $\epsilon_r$  varied between simulations



System topology and layout

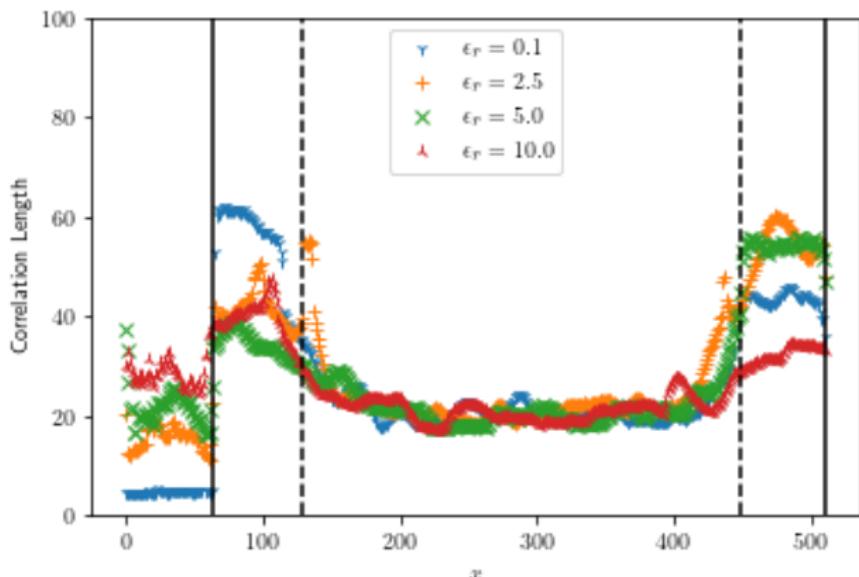
# Combined System



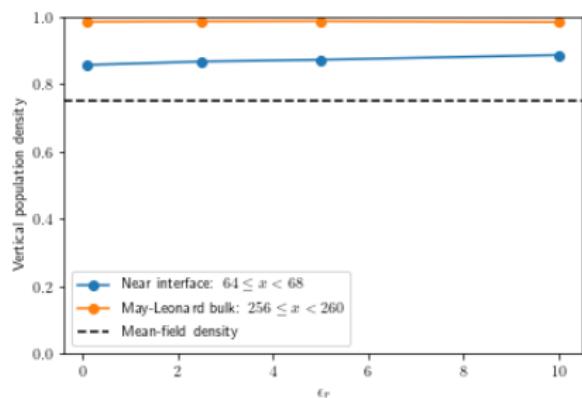
# Correlation Lengths and Permeation Distance

- Vertical correlation function  $C(x, t, r)$  measured parallel to interface.
- $C(x, t, r)$  averaged over multiple runs and used to calculate the vertical correlation length  $L(x)$ .
- Low  $\epsilon_r$  corresponds to higher  $L(x)$  near interface.
- Permeation distance of the plane-waves are approximately equal for all  $\epsilon_r$  tested

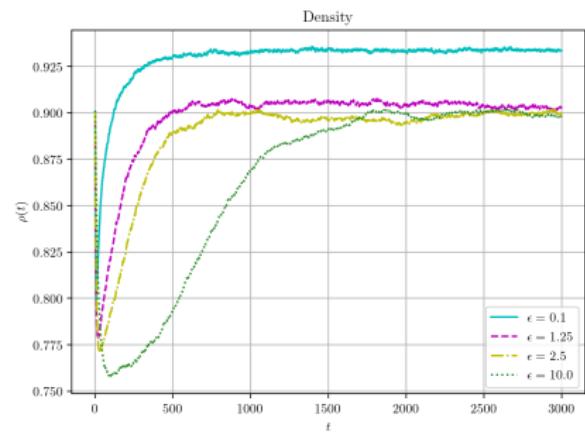
$$C_{AA}(x, t, r) = \langle n_a(x, y, t)n_a(x, y + r, t) \rangle - \langle n_a(x, t) \rangle^2$$
$$C(x, t, r) = \frac{C_{AA}(x, t, r) + C_{BB}(x, t, r) + C_{CC}(x, t, r)}{3}$$



# Well Mixing Effects



Average net population densities near and far from the RPS-ML interface for various RPS mobility rates.



Net population density vs. time of pure ML lattice starting from random initial condition

# Conclusions

- Changing the microscopic rules from ML to RPS on a subsection of the lattice produces plane-waves emanating away from the interface.
- Size of the plane waves is apparently dependant on RPS mobility
- Drop in net population density near the interface due to the system being more “well-mixed”.
- Suggests periodic, localized mixing as a potential method for disruption of steady-state pattern formation in cyclic competition models.

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