Boundary Effects in Stochastic Cyclic Competition Models on a Two-Dimensional Lattice

M. Lazarus Arnau, Shannon Serrao, Uwe C. Täuber

Department of Physics (MC 0435) and Center for Soft Matter and Biological Physics
Virginia Tech, Blacksburg, Virginia 24061

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We study noise-induced and -stabilized spatial patterns in two distinct stochastic population model variants for cyclic competition of three species, namely the Rock-Paper-Scissors (RPS) and the May-Leonard (ML) models. In two dimensions, it is well established that the ML model can display (quasi-)stable spiral structures, in contrast to simple species clustering in the RPS system. Our ultimate goal is to impose control over such competing structures in systems where both RPS and ML reactions are implemented. To this end, we have employed Monte Carlo computer simulations to investigate how changing the microscopic rules in a subsection of a two-dimensional lattice influences the macroscopic behavior in the rest of the lattice. Specifically, we implement the ML reaction scheme on a torus, except on a ring-shaped patch, which is set to follow the cyclic Lotka-Volterra predation rules of the RPS model. There, we observe a marked disruption of the usual spiral patterns in the form of plane waves emanating from the RPS region, up to a characteristic distance that is set by the diffusion rate in the RPS patch. Furthermore, the overall population density drops considerably in the vicinity of the interface between both regions.

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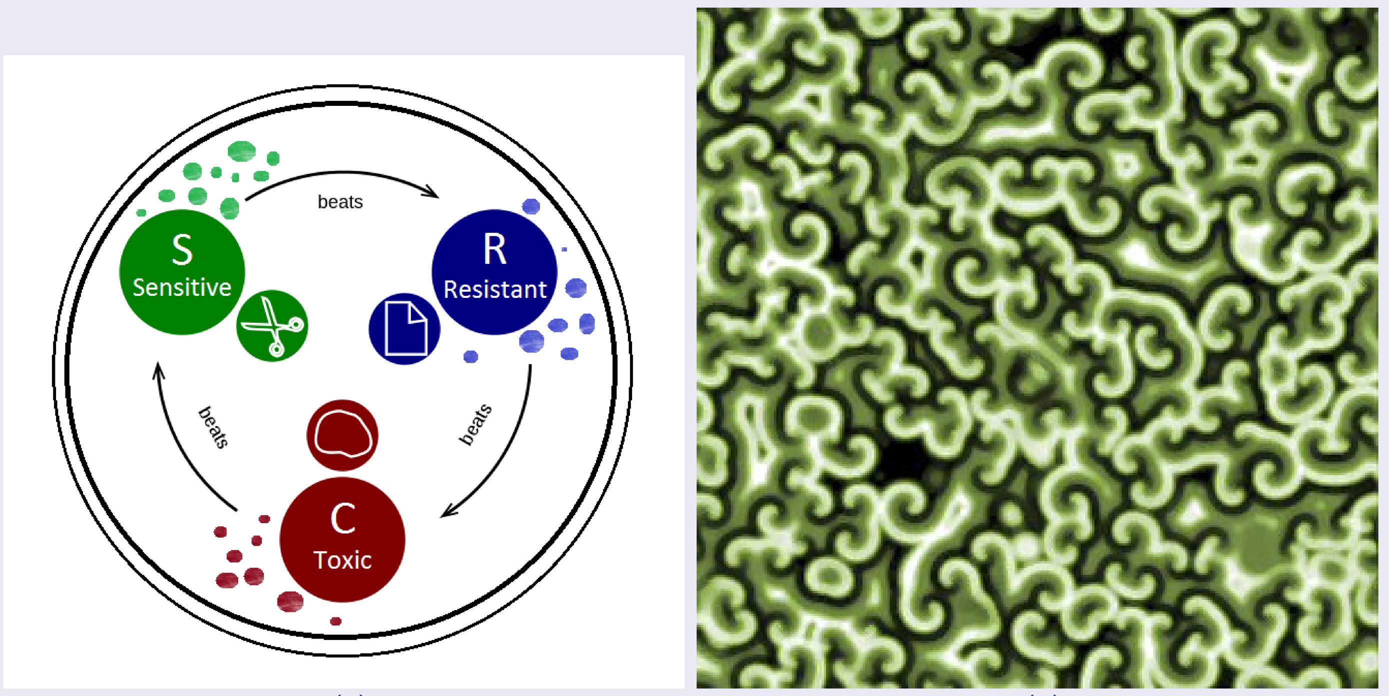
Introduction



(a) (b)

Figure 1: (a) Cyclic dominance in *E. -Coli* (b) Spiral pattern formation in the Belousov-Zhabotinsky reaction

A number of systems in the fields of ecology, epidemiology, and chemistry follow a paradigm of cyclic dominance (e.g. certain subspecies of Lizards in California, experiments on cyclically competing *E. -Coli* bacteria, and Belousov Zhabotinsky reactions). The formation of noise-induced and -stabilized spiral patterns in this class of system is captured by the spatially-extended May-Leonard (ML) model. The formation of these spirals is in stark contrast to the species clustering seen in the Rock-Paper-Scissors (RPS).

The RPS Model

The RPS Model is defined by the following binary reactions:

- Replacement Reaction:

$$\begin{gathered} AB \to AA \\ BC \to BB \\ CA \to CC \\ \text{with rate } \sigma_r \end{gathered}$$

- Pair Swap / Diffusion Reaction:

$$\begin{gathered} XY \to YX \text{ where } X, Y \in \{A, B, C, \varnothing\} \\ \text{with rate } D_r \end{gathered}$$

The May-Leonard Model

The ML Model is defined by the following binary reactions:

- Predation Reaction:

$$\begin{gathered} AB \to A\varnothing \\ BC \to B\varnothing \\ CA \to C\varnothing \\ \text{with rate } \sigma_m \end{gathered}$$

- Reproduction Reaction:

$$\begin{gathered} A\varnothing \to AA \\ B\varnothing \to BB \\ C\varnothing \to CC \\ \text{with rate } \mu \end{gathered}$$

- Pair Swap / Diffusion Reaction:

$$\begin{gathered} XY \to YX \text{ where } X, Y \in \{A, B, C, \varnothing\} \\ \text{with rate } D_m \end{gathered}$$

Previous research has shown that the the May-Leonard system has a (quasi-)stable reactive fixed point around $(a^*, b^*, c^*) = \frac{\mu}{3\mu+\sigma}(1,1,1)$.

Simulation

We initialize a lattice of size $L_x \times L_y$ (unless stated otherwise assume $(L_x, L_y) = (256, 512)$) with each cell having a probability $p(A) = p(B) = p(C) = \rho_0/3$ (where $/rho_0$ is the initial net particle density) of being initialized to any given species. The lattice is given a toroidal topology (i.e. $x = 0$ is equivalent to $x = L_x$ and likewise for y). The simulation then proceeds according to the following algorithm:

1. A random coordinate (x, y) is selected from a uniform distribution and time is advanced by $\delta t = P^{-1}$ (where $P = L_x \times L_y$).
2. If that lattice point is not empty, then a nearest neighbor is chosen at random. If the cell is empty, the simulation returns to step 1.
3. One of the possible reactions (according to whether the lattice point is governed by the ML or RPS model) is selected at random, and excecuted if possible.

The simulation is hence run for a predetermined amount of time.

In cases where there are both RPS and ML lattice points, all lattice points in the range $0 \leq y \leq d_i$ are governed by the RPS model and all remaining lattice points are governed by the ML model.

Normal Pattern formation