Boundary Effects in Stochastic Cyclic Competition Models on a Two-Dimensional Lattice

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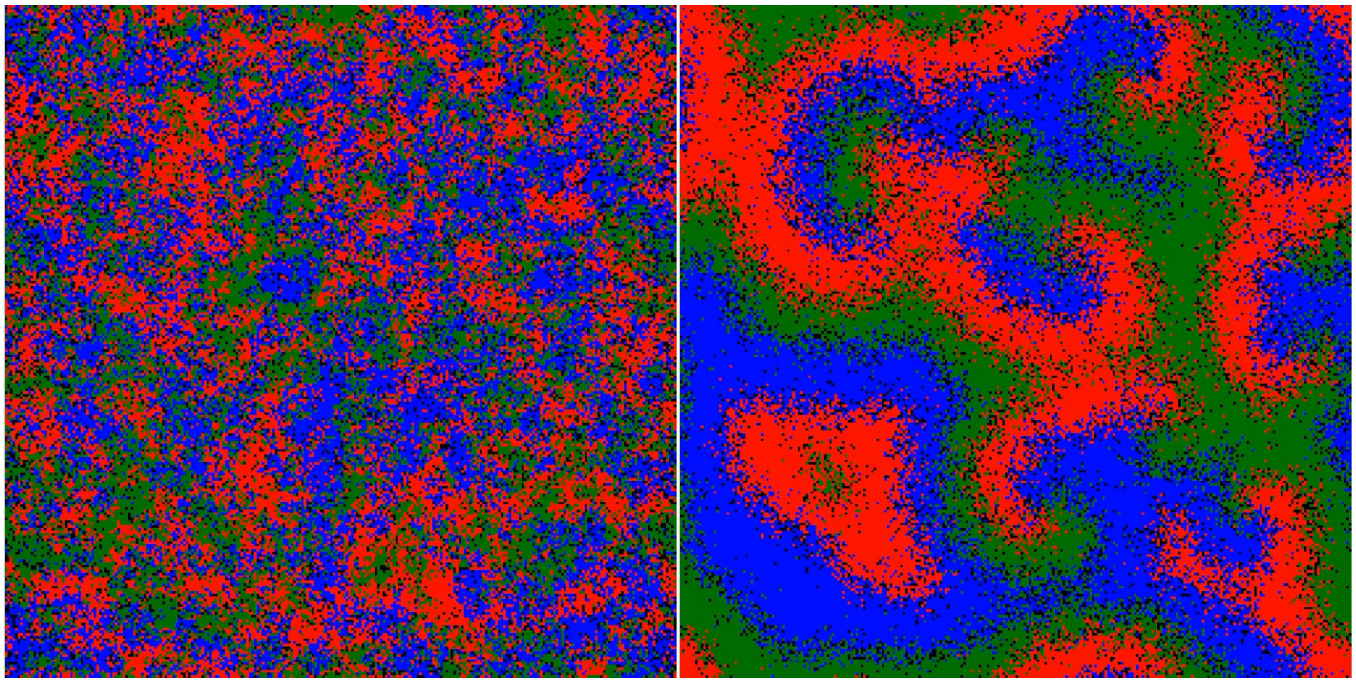
Typical Behavior



(a) $\epsilon_r = 1.25$ (b) $\epsilon_m = 5.0$

Figure: Typical steady state snapshots of Rock-Paper-Scissors (a) and May-Leonard (b) systems

Models

Three species cyclic competition schemes motivated by examples in biology, population dynamics, and chemistry.

- Rock-Paper-Scissors (RPS) Model

$$(\text{Replacement}) X_n X_{n+1} \xrightarrow{\zeta} X_n X_n$$
$$(\text{Hopping}) XY \xrightarrow{\epsilon_r} YX$$

- May-Leonard (ML) Model

$$(\text{Predation}) X_n X_{n+1} \xrightarrow{\sigma} X_n \varnothing$$
$$(\text{Reproduction}) X \varnothing \xrightarrow{\mu} XX$$
$$(\text{Hopping}) XY \xrightarrow{\epsilon_m} YX$$

n represents the species index where $X_4 = X_1$

Combined System

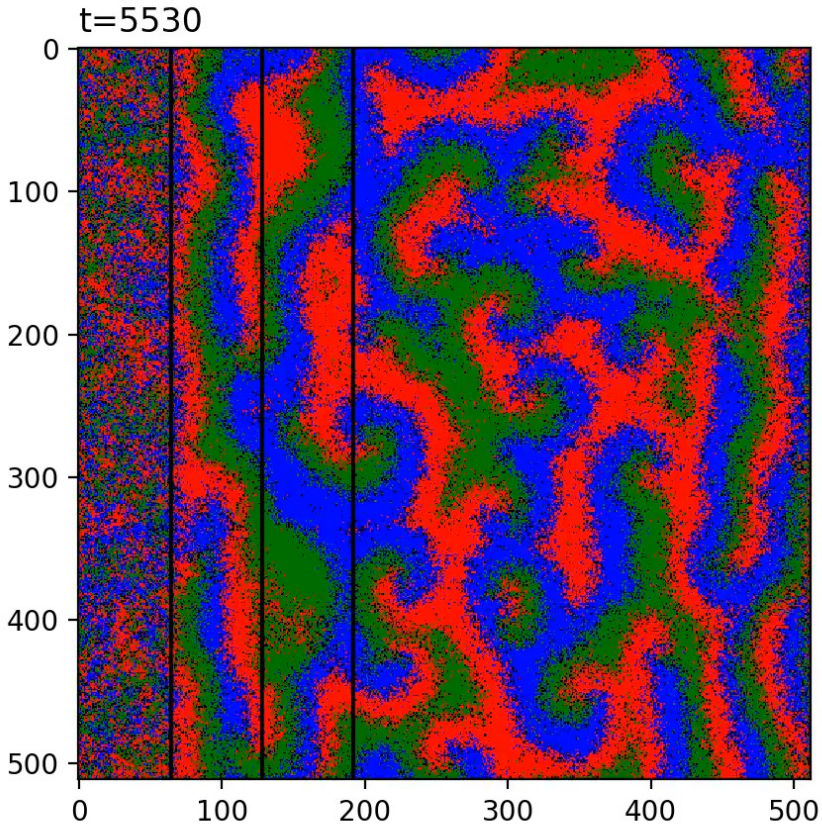


Figure: Plane wave formation

Correlation Lengths and Permeation Distance

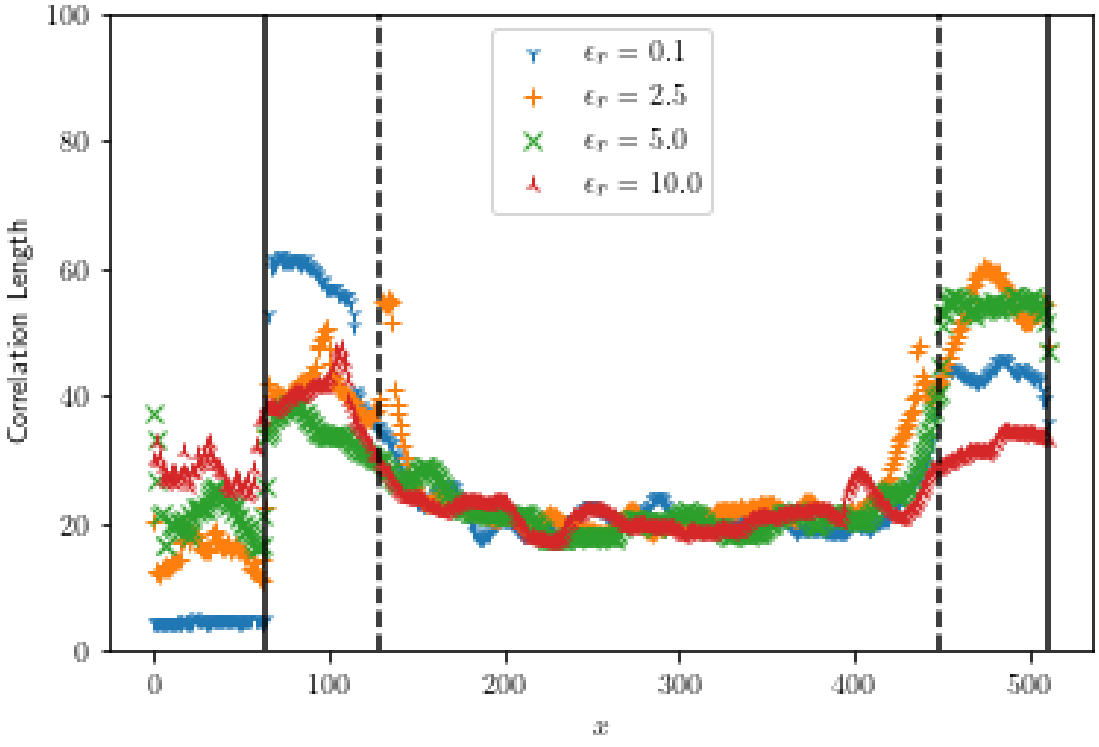


Figure: Correlation length

Well-Mixing Effects
Boundary Effects

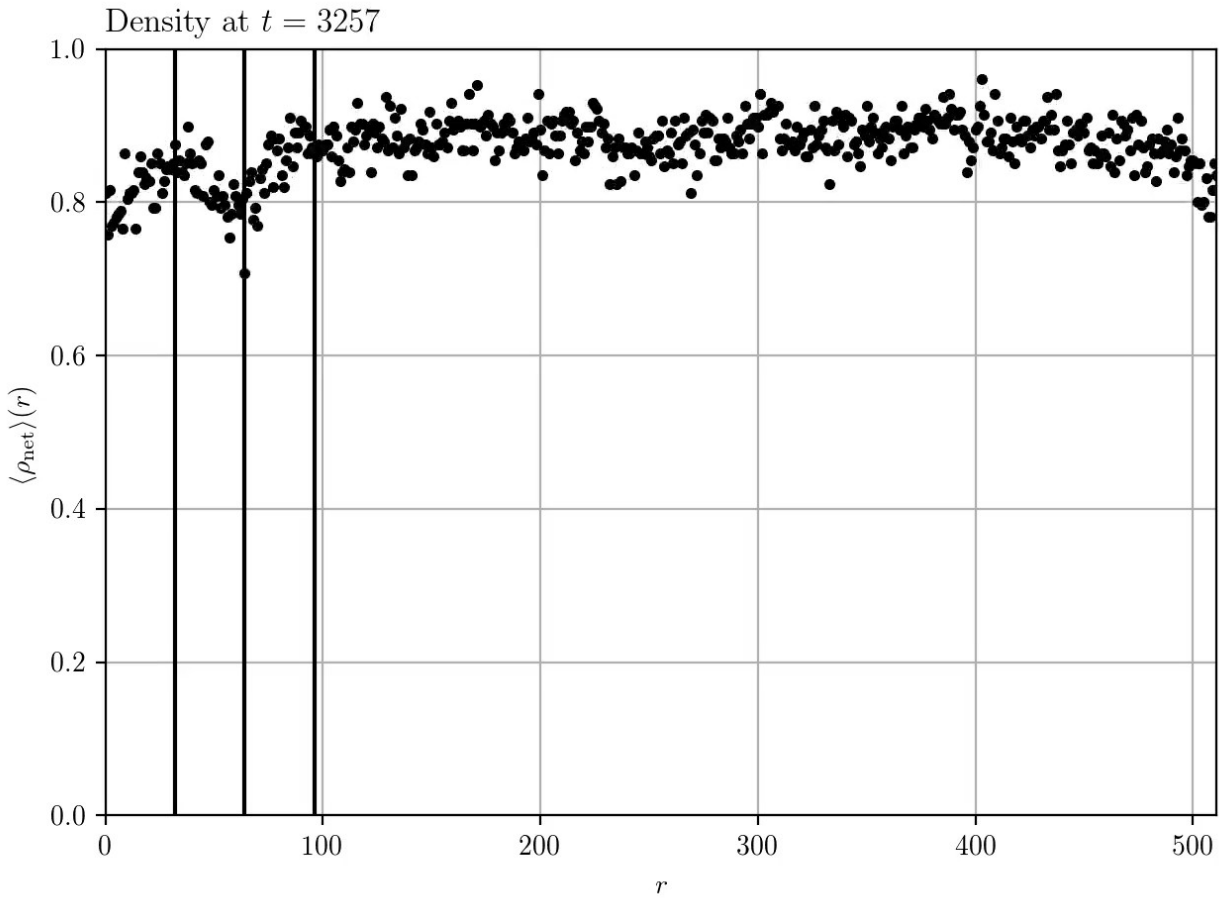


Figure: Prominent drop in net population density

Well-Mixing Effects

Boundary Effects cont.

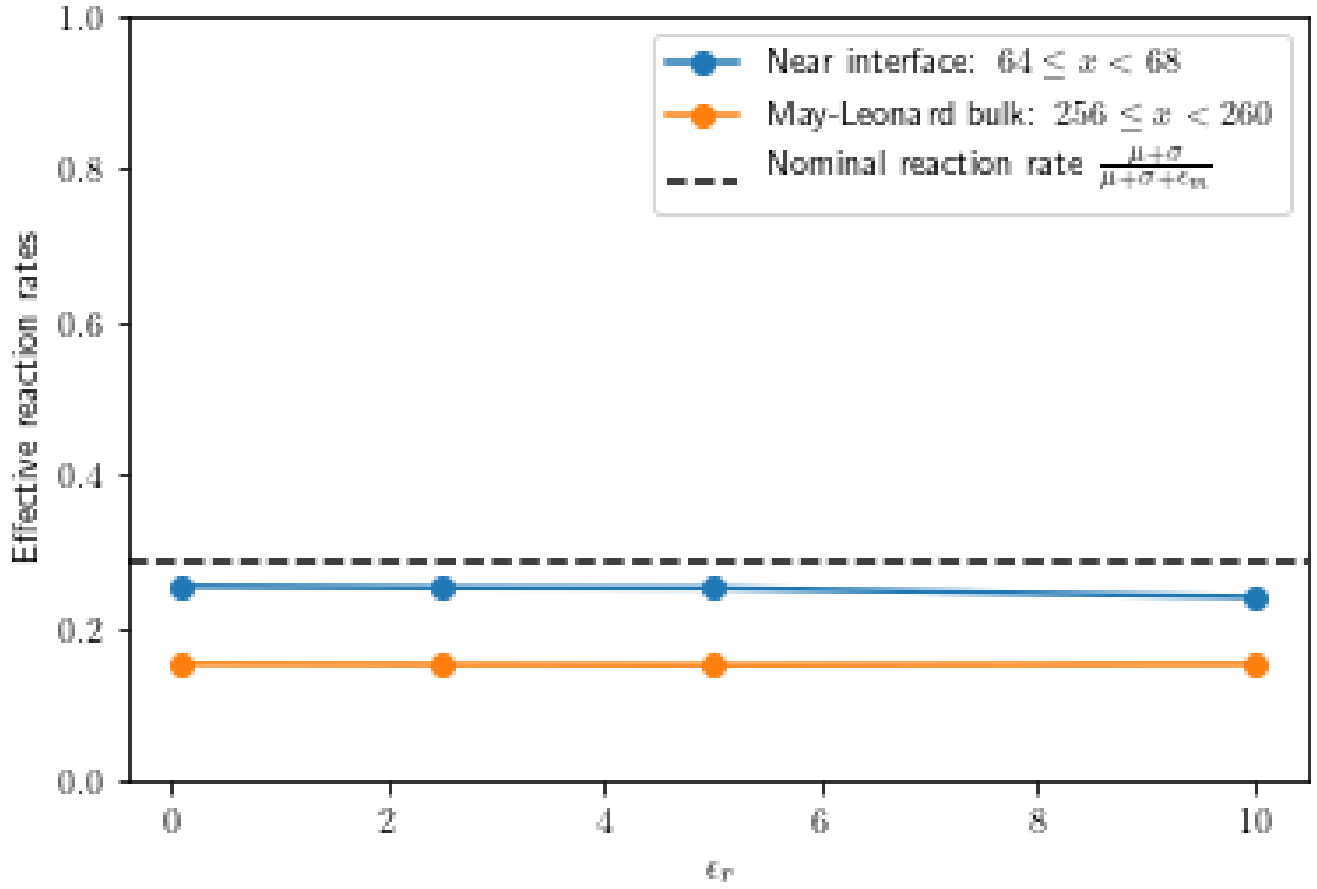


Figure: Relative reproduction + predation rates near the boundary vs. in May-Leonard bulk

Well-Mixing Effects
Transient Effects

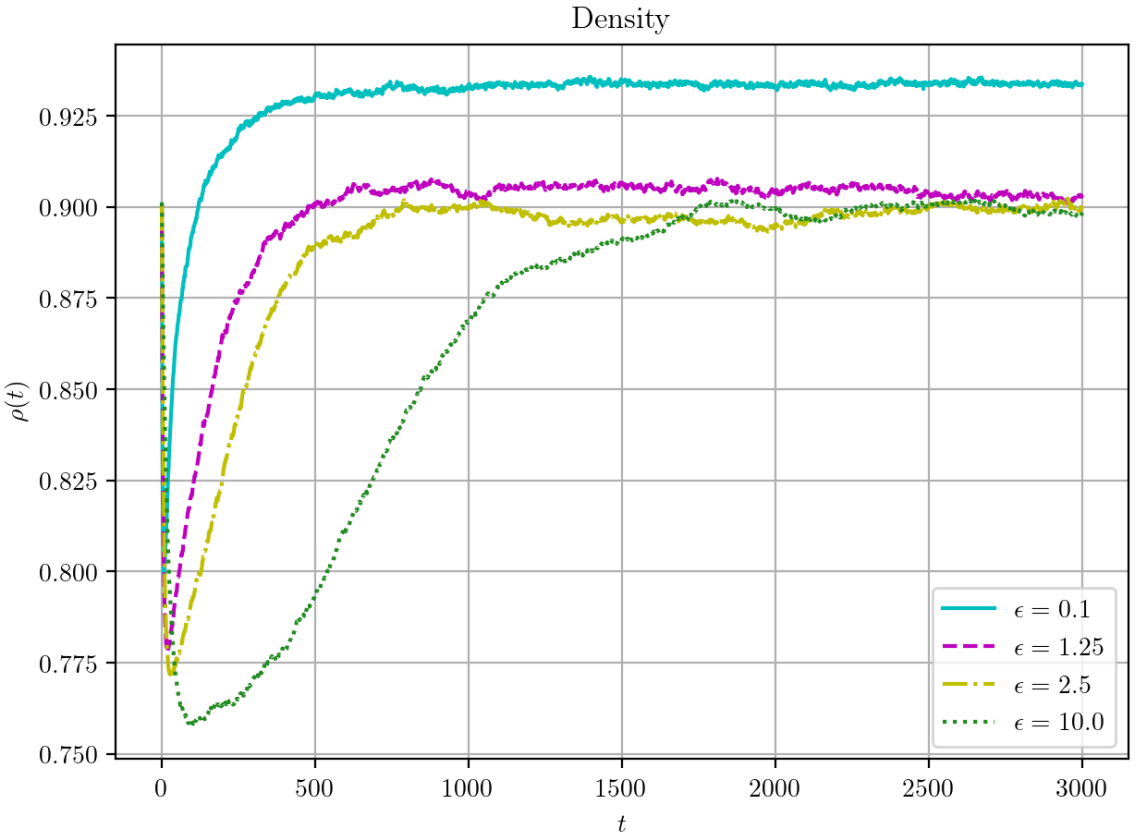


Figure: From random initial conditions ML model approaches mean-field density before relaxing to steady state.

Conclusions and questions

- Competion between different models can influence their long term behaviors.
- Disruptions in pattern formation caused by "mixing" of particles the boundary.