

Boundary Effects in Stochastic Cyclic Competition Models on a Two-Dimensional Lattice

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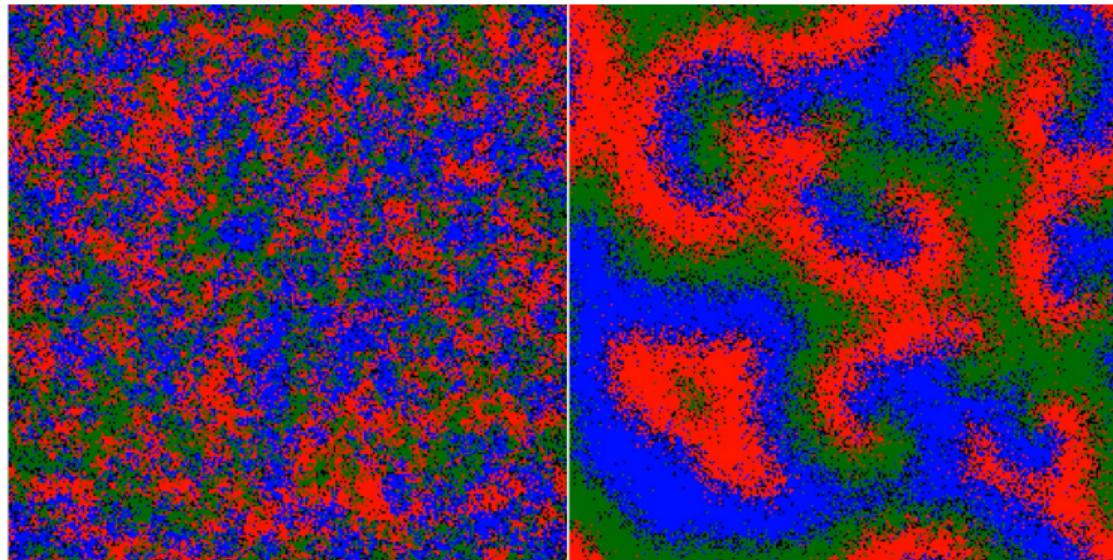
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Motivation

Three species cyclic competition schemes motivated by examples in biology, population dynamics, and chemistry.



(a) Rock Paper Scissors

(b) May-Leonard

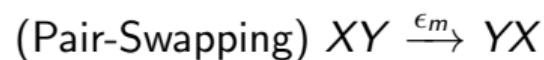
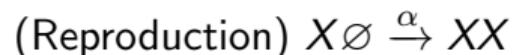
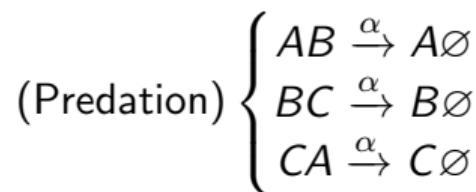
Models

Rock-Paper-Scissors/Cyclic Lotka-Volterra
(RPS) Model



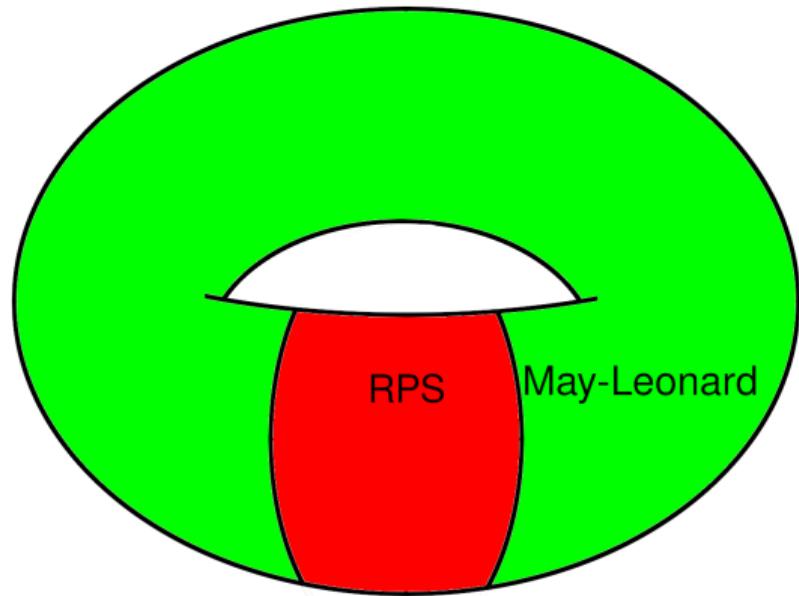
$$X \in \{A, B, C\}, \quad Y \in \{A, B, C, \emptyset\}$$

May-Leonard (ML) Model



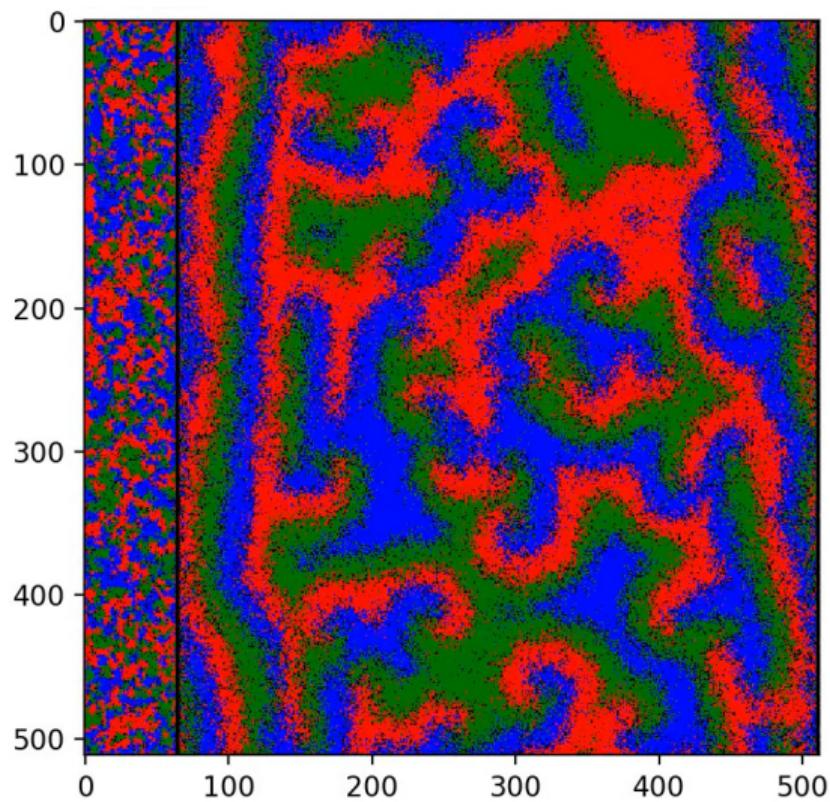
Combined System

- Periodic boundary conditions (i.e. toroidal topology)
- Maximum occupancy of one particle per lattice site.
- May-Leonard except for thin (width = 64) annular patch governed by RPS
- Random initial conditions.
- 512×512 lattice sites.
- $\alpha = 1.0$, $\epsilon_m = 5.0$. ϵ_r varied between simulations



System topology and layout

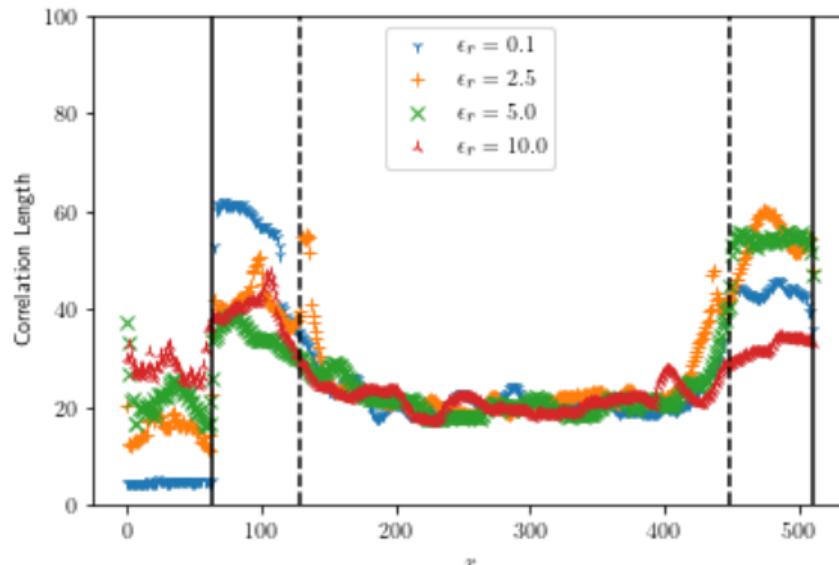
Combined System



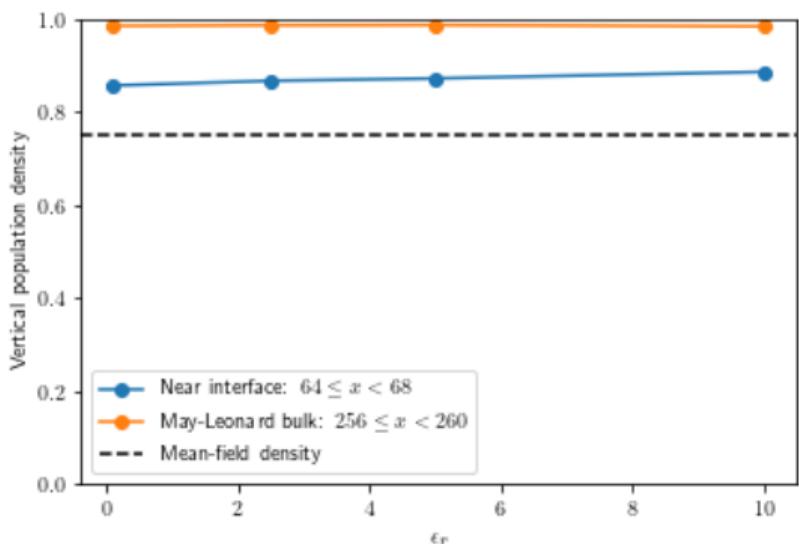
Correlation Lengths and Permeation Distance

- Vertical correlation function $C(x, t, r)$ measured parallel to interface.
- $C(x, t, r)$ averaged over multiple runs and used to calculate the vertical correlation length $L(x)$.
- Low ϵ_r corresponds to higher $L(x)$ near interface.
- Permeation distance of the plane-waves are approximately equal for all ϵ_r tested

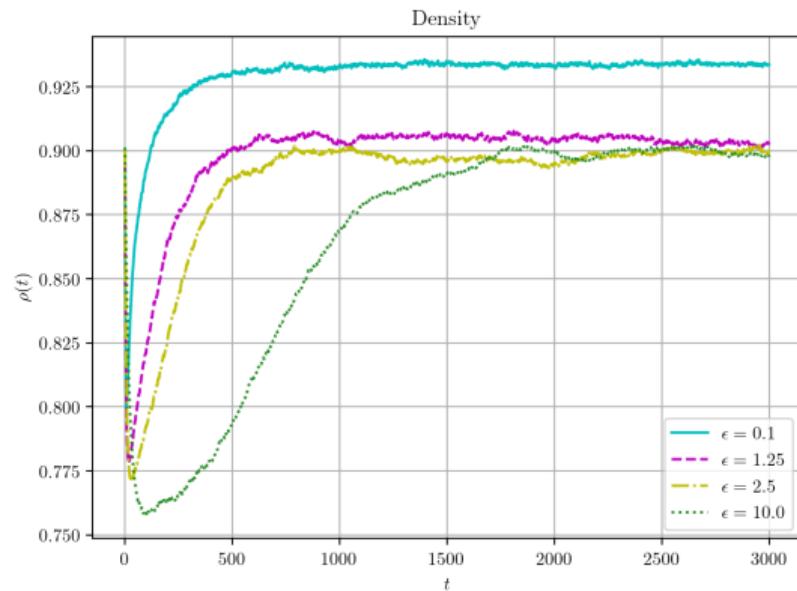
$$C_{AA}(x, t, r) = \langle n_a(x, y, t)n_a(x, y + r, t) \rangle - \langle n_a(x, t) \rangle^2$$
$$C(x, t, r) = \frac{C_{AA}(x, t, r) + C_{BB}(x, t, r) + C_{CC}(x, t, r)}{3}$$



Well Mixing Effects



Average net population densities near and far from the RPS-ML interface for various RPS mobility rates.



Net population density vs. time of pure ML lattice starting from a random initial condition

Conclusions

- Changing the microscopic rules from ML to RPS on a subsection of the lattice produces plane-waves emanating away from the interface.
- Size of the plane waves is apparently dependant on RPS mobility
- Drop in net population density near the interface due to the system being more “well-mixed”.
- Suggests periodic, localized mixing as a potential method for disruption of steady-state pattern formation in cyclic competition models.

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