Boundary Effects in Stochastic Cyclic Competition Models on a Two-Dimensional Lattice

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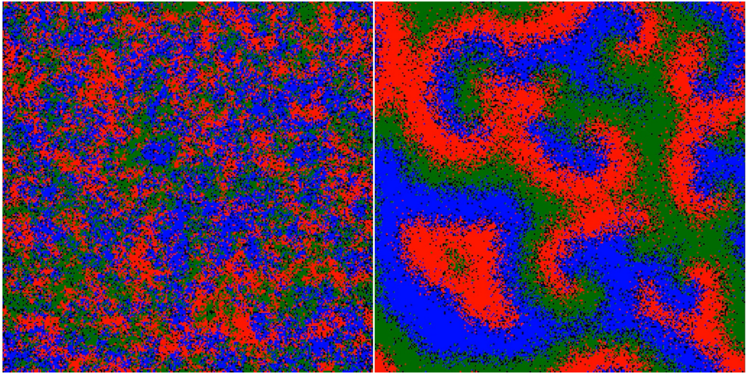
Motivation

Three species cyclic competition schemes motivated by examples in biology, population dynamics, and chemistry.



(a) Rock Paper Scissors (b) May-Leonard

Models

Rock-Paper-Scissors/Cyclic Lotka-Volterra (RPS) Model

$$\text{(Replacement)} \begin{cases} AB \xrightarrow{\alpha} AA \\ BC \xrightarrow{\alpha} BB \\ CA \xrightarrow{\alpha} CC \end{cases}$$

$$\text{(Pair-Swapping)}\ XY \xrightarrow{\epsilon_r} YX$$

May-Leonard (ML) Model

$$\text{(Predation)} \begin{cases} AB \xrightarrow{\alpha} A\varnothing \\ BC \xrightarrow{\alpha} B\varnothing \\ CA \xrightarrow{\alpha} C\varnothing \end{cases}$$

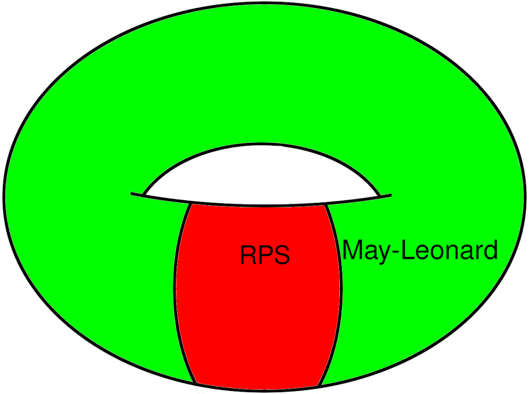
$$\text{(Reproduction)}\ X\varnothing \xrightarrow{\alpha} XX$$

$$\text{(Pair-Swapping)}\ XY \xrightarrow{\epsilon_m} YX$$

$$X \in \{A, B, C\},\ Y \in \{A, B, C, \varnothing\}$$

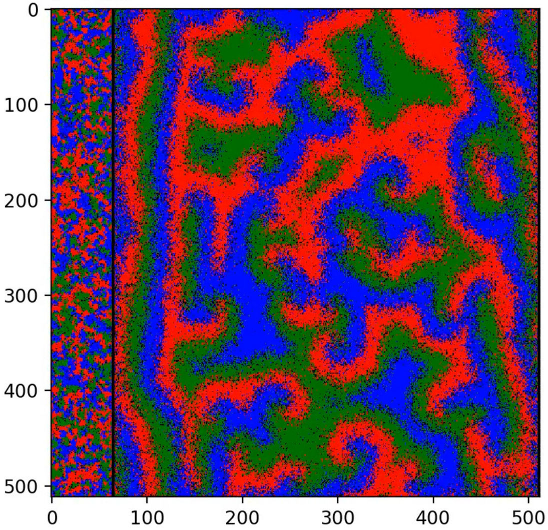
Combined System

- Periodic boundary conditions (i.e. toroidal topology)
- Maximum occupancy of one particle per lattice site.
- May-Leonard except for thin (width $= 64$) annular patch governed by RPS
- Random initial conditions.
- 512×512 lattice sites.
- $\alpha = 1.0$, $\epsilon_m = 5.0$. ϵ_r varied between simulations



System topology and layout

Combined System

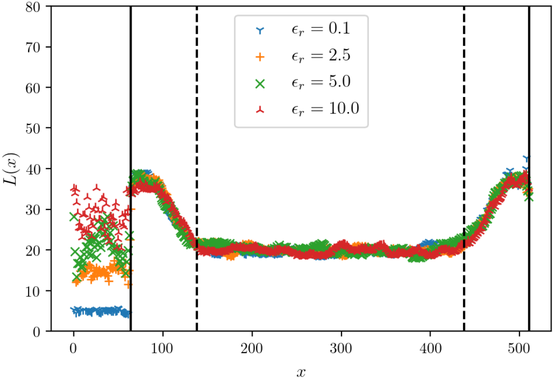


Correlation Lengths and Permeation Distance

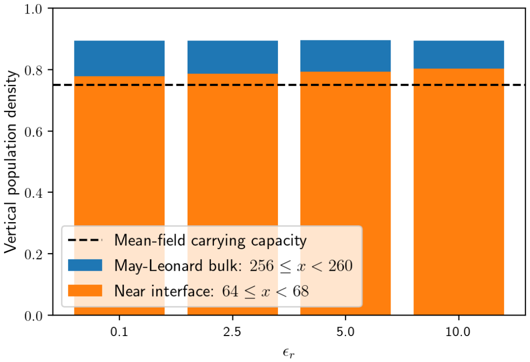
- Vertical correlation function $C(x,t,r)$ measured parallel to interface.
- $C(x,t,r)$ averaged over multiple runs and used to calculate the vertical correlation length $L(x)$.
- Plane-wave size and permeation distance are approximately equal for all for all ϵ_r tested.

$$C_{AA}(x,t,r) = \langle n_a(x,y,t) n_a(x,y+r,t)\rangle - (a(x,t))^2$$

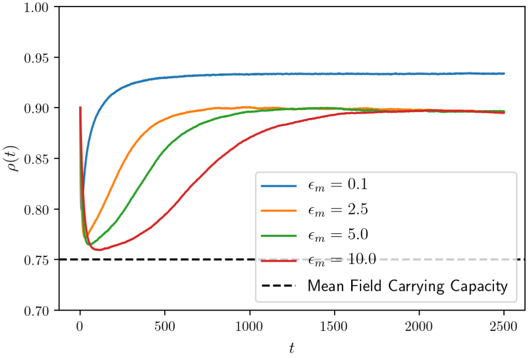
$$C(x,t,r) = \frac{C_{AA}(x,t,r) + C_{BB}(x,t,r) + C_{CC}(x,t,r)}{3}$$



Well Mixing Effects



Average net population densities near and far from the RPS-ML interface for various RPS mobility rates.



Net population density vs. time of pure ML lattice starting from from random initial condition

Conclusions

- Changing the microscopic rules from ML to RPS on a subsection of the lattice produces plane-waves emanating away from the interface.
- Size and permeation distance of plane-waves independent of RPS mobility
- Drop in net population density near the interface due to the system being more "well-mixed".
- Suggests periodic, localized mixing as a potential method for disruption of steady-state pattern formation in cyclic competition models.

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