ML@B
SAP Algorithm Outline

Notation:

- D: Data matrix
  - D_i: Datapoint of format $\{\text{close price} - \text{open price}, \text{low}, \text{high}, \text{volume}, F_{i-1}\}$
- F: Trade value
  - $F_i = D_i \cdot \Theta = D_{i_0}\Theta_0 + D_{i_1}\Theta_1 + D_{i_2}\Theta_2 + D_{i_3}\Theta_3 + D_{i_4}\Theta_4$
  - The 0th - 3rd components are calculated at initialization, the fourth component is calculated later. D_{i_4} is F_{i-1} which is currently not fully calculated.
  - $F'_i = D_{i_0}\Theta_0 + D_{i_1}\Theta_1 + D_{i_2}\Theta_2 + D_{i_3}\Theta_3$
  - $F_i = F'_i + \Theta_4 F_{i-1} = \sum_{j=0}^{i} \Theta_4^{i-j} F'_j$ (This form is very similar to the Prefix Sum)
- R_i: The reward function at time step i
  - $R_i = F_{i-1}D_0 - \delta|F_i - F_{i-1}|$
- S': The Differential Sharpe ratio
  - S the Sharp ratio is obtained by considering $S_T = \frac{\text{Average}(R_t)}{\text{Standard Deviation}(R_t)}$
  - S' is obtained by considering exponential moving averages of the returns and standard deviation of returns above, and expanding to first order in the decay rate η:
  - $$\frac{\partial S_t}{\partial \eta} = \frac{B_{t-1}\Delta A_t - \frac{1}{2}A_{t-1}\Delta B_t}{(B_{t-1} - A_{t-1}^2)^{3/2}}$$
  - $A_t = A_{t-1} + \eta\Delta A_t = A_{t-1} + \eta(R_t - A_{t-1})$
  - $B_t = B_{t-1} + \eta\Delta B_t = B_{t-1} + \eta(R_t^2 - B_{t-1})$
- Performance function
  - Want to maximize performance functions instead of minimizing a cost function

- $U_T = U(R_1, R_2, \cdot, R_t)$ is the cumulative result of the differential sharpe ratio
- For on-line learning we'll only worry about $\frac{\partial U_t}{\partial R_t}$ which is the differential sharpe ratio for only the current timestep

- Gradient Equations Derived from Above
  - $$\frac{\partial R_t}{\partial F_t} = \begin{cases} -1 \text{ if } F_{t-1} < F_t \\ +1 \text{ else} \end{cases}$$
  - $$\frac{\partial R_t}{\partial F_{t-1}} = D_{t_0} - \begin{cases} -1 \text{ if } F_{t-1} < F_t \\ +1 \text{ else} \end{cases} = D_{t_0} - \frac{\partial R_t}{\partial F_t}$$
  - $$\frac{\partial F_t}{\partial \Theta} = D_t$$
  - $$\frac{\partial U_t}{\partial R_t} = \frac{B_{t-1} - A_{t-1} R_t}{(B_{t-1} - A_{t-1}^2)^{3/2}}$$
  - $$\frac{\partial U_t}{\partial \Theta} = \sum_{i=1}^{t} \frac{\partial U_t}{\partial R_t} \left(\frac{\partial R_t}{\partial F_t} \frac{\partial F_t}{\partial \Theta} + \frac{\partial R_t}{\partial F_{t-1}} \frac{\partial F_{t-1}}{\partial \Theta} \right)$$