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Comparing administered and market-based water allocation systems through a consistent agent-based modeling framework

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ABSTRACT

Water allocation can be undertaken through administered systems (AS), market-based systems (MS), or a combination of the two. The debate on the performance of the two systems has lasted for decades but still calls for attention in both research and practice. This paper compares water users' behavior under AS and MS through a consistent agent-based modeling framework for water allocation analysis that incorporates variables particular to both MS (e.g., water trade and trading prices) and AS (water use violations and penalties/subsidies). Analogous to the economic theory of water markets under MS, the theory of rational violation justifies the exchange of entitled water under AS through the use of cross-subsidies. Under water stress conditions, a unique water allocation equilibrium can be achieved by following a simple bargaining rule that does not depend upon initial market prices under MS, or initial economic incentives under AS. The modeling analysis shows that the behavior of water users (agents) depends on transaction, or administrative, costs, as well as their autonomy. Reducing transaction costs under MS or administrative costs under AS will mitigate the effect that equity constraints (originating with primary water allocation) have on the system's total net economic benefits. Moreover, hydrologic uncertainty is shown to increase market prices under MS and penalties/subsidies under AS and, in most cases, also increases transaction, or administrative, costs.

1. Introduction

Water allocation can be undertaken through administered systems (AS), market-based systems (MS), or a combination of the two. The debate on the performance of these two systems has lasted for decades and still receives extensive attention in both research and practice. Many economists have advocated MS as a tool for managing the allocation of water as well as other commodities and resources, due to numerous advantages it has over alternative water allocation mechanisms. In an ideal market system, commodities can move from low value users to high value users, and therefore economic efficiency is achieved from both an individual and social point of view (Vaux and Howitt, 1984; Rosegrant and Binswanger, 1994; Becker, 1995; Kaiser and McFarland, 1997). However, since water markets do not often function efficiently in practice (Matthews, 2004), administered (centralized) allocation systems have been implemented much more frequently.

In fact, numerous studies have produced various explanations of this paradox using classic economic theories. In particular, they have focused on the following issues: 1) transaction costs, 2) water as a public or market good, and 3) the structure and performance of water markets. Rosegrant and Binswanger (1994) argued that the basic disadvantage of using market-based water allocation was its transactions costs which often exceed its social benefits. These transaction costs include the cost of infrastructure used to convey the traded water and the cost of establishing an institutional framework that assures both buyers and sellers the desired quantities transferred through the markets and regulates the externalities imposed on third parties through trading. Kanazawa (2003) attributed the inefficiency of water market to the fact that many water right systems were developed to resolve problems a century ago or earlier without considering the possible formation of a water market. Matthews (2004) corroborates this observation and points out that water markets did not always function efficiently because the features of some water rights were not designed for market transactions and, in particular, the impact of hydrological uncertainty was not adequately considered.

Meanwhile, the advantages and disadvantages of AS have also been widely discussed. Under an administered water allocation system, usually a governmental agency controls, regulates, and administers the allocation of water according to established laws

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(Karkkainen, 2001). A World Bank (1993) list of the advantages of AS includes: 1) Large capital requirements and economies of scale required for water infrastructure investments, 2) very long time horizons of investments, 3) the spatial interdependence of water uses within a river basin or aquifer, 4) an AS system is more appropriate for managing public goods, such as water, and 5) the strategic importance of water for national security and regional development. Additional reasons for adopting AS relate to ensuring allocation equity and sustainable system control since the primary responsibility for ensuring equitable and sustainable water resources management rests with governments or designated river basin or groundwater aquifer management authorities (Draper, 2008).

In practice, one disadvantage of AS is that the institution may develop its own internal goals, sometimes known as internalities (Young, 1986), that contrast the system-level water allocation goals for the water resources system at hand. Another disadvantage of AS is that the absence of marketplace incentives can lead to government inefficiencies (Draper, 2008). However, Rosegrant and Binswanger (1994) pointed out that, under AS, authorities might have the same incentive to improve the performance of the system when they are subject to political pressures.

Under AS, water users often violate the terms of their permits when there is inadequate enforcement and monitoring, which has led to both conflicts and environmental degradation (see Zhao et al., 2009 for an example of the Yellow River Basin in China). However, those violations might be treated as "rational crimes". Rational crime is defined as a violation of imposed regulations if the agent believes the potential benefits of the violation outweigh the potential consequences of being punished for committing the violation (Cooter and Ulen, 2000; Souza Filho et al., 2008).

This paper does not aim to resolve the paradox of the AS vs. MS debate, but rather present a systems approach to studying the problem that may yield some insights that could subsequently contribute to the establishment of effective water allocation systems in the real world. We study water allocation in the context of a complex system composed of various water users, or stakeholders, who share a common environment (e.g., watershed, basin or a region) (Dooley and Corman, 2002), are autonomous (i.e., they make decisions independently and have full control over their actions), and may have different and possibly conflicting goals. Specifically, we attempt to explain the general behavior of individual water users under AS and MS, respectively, and explain how individual agents' behavior influence the functionality of both government-controlled and market-based water allocation systems.

We conduct the comparative analysis of MS and AS through a consistent analytical agent-based modeling framework. Agent-based models (ABMs) are recognized for their capacity to detect system-level emergence (i.e., the performance and patterns of the system) based on agent-level behavior analysis. ABMs have been widely used in economics (e.g., Arthur, 1999; Martin and Sunley, 2007), ecology (e.g., Grimm et al., 2005), water resources (e.g. Souza Filho et al., 2008; Yang et al., 2009), and other complex systems analyses (e.g., Bithell and James, 2009). In this study, we apply the ABM approach to water allocation systems. The key components of the system include: 1) individual water users, which are defined as agents, 2) the natural environment of the geographical water planning unit (e.g., watershed, groundwater basin), including its climate, hydrology, and engineering infrastructure, 3) rules describing agent behaviors under the policies of local management institutions, and 4) system-level performance metrics such as system-wide net economic benefits, equity, and environmental performance. Moreover, interactions among agents as well as those between active human agents and the modeling environment are discussed in the context of either AS or MS.

The rest of this paper is organized as follows: First in Section 2, the properties of the analytical ABM used in this paper are introduced, and then in Section 3, these properties and model parameters are used to characterize agents' behaviors and detect system emergence. Next in Section 4, AS and MS are defined in the context of the ABM and the two systems are compared in terms of their management mechanisms, economic benefit and equity. In addition, the impact of hydrological uncertainty on water allocation is analyzed in Section 5. Finally, in section 6, conclusions are given regarding the effectiveness of the ABM and insights on the AS vs. MS paradox.

2. ABM formulation and properties

More than two centuries ago, Smith (1759) addressed the dynamics characterizing the interactions between individual social agents and society as a whole in his famous monograph "*Theory of Moral Sentiments*" (paragraph VI.II.42):

> "*The man of system, …, seems to imagine that he can arrange the different members of a great society with as much ease as the hand arranges the different pieces upon a chess-board. He does not consider that the pieces upon the chess-board have no other principle of motion besides that which the hand impresses upon them; but that, in the great chess-board of human society, every single piece has a principle of motion of its own, altogether different from that which the legislature might chuse [sic] to impress upon it. If those two principles coincide and act in the same direction, the game of human society will go on easily and harmoniously, and is very likely to be happy and successful. If they are opposite or different, the game will go on miserably, and the society must be at all times in the highest degree of disorder.*"

Water allocation problems are analogous to the "chess game" that Smith described. The success of a water allocation plan depends on the coincidence of the "two principles" – one representing the individual decisions of water users and the other one reflecting the goals of centralized water managers. Classic top-down "command and control" modeling approaches may not be suitable for analyzing such problems (Loucks and Beek, 2005) since they neglect the "principle" of individual decisions. On the other hand, bottom-up modeling approaches, such as ABM, emphasize root-level activities and information exchange between lower and higher levels. It is interesting that the concept of ABM exactly follows Smith's "chess game" ideas in exploring mechanisms that can synthesize the impacts of agent- (i.e., water user) and system-level (i.e., water manager) behaviors and their interactions. Furthermore, overall system-level behavior usually emerges from the responses and feedbacks resulting from agent interactions, which are often nonlinear. Thus water management systems can be defined as complex systems whose system-level behavior is not predictable as a function of its individual components alone. Moreover, water management systems are neither purely social nor purely physical systems, but instead coupled human-nature systems (Liu et al., 2007), given that their efficacy is based on the natural hydrologic variability as well as anthropogenic influences, such as engineering infrastructure and management institutions. ABM has been recognized as an effective approach for modeling complex, nonlinear systems through the use of nonlinear functions that derive "system emergence" (system behavior) from agents' behaviors (Bonabeau, 2002).

Although the application of ABM for simulating complex water resources systems has demonstrated some promise (e.g. Souza Filho et al., 2008; Yang et al., 2009), more successful demonstrations are still needed. In this section, we present a simple ABM-based mathematical formulation that explains the behavior of individual agents and illustrates how different combinations of agent behaviors can lead to certain system behaviors.

2.1. ABM formulation for water allocation

The agents' behavior is formulated following the penalty-based decentralized optimization framework provided by Inalhan et al. (2002). A complex system is decomposed into a number of sub-systems or agents, each of which is set to optimize its own unique objective:

$$\min F_i\left(x_i, \beta_i | \{x_j\}_i\right) = \min\left(\beta_i f_i(x_i) + P_i\left(x_i, \left|\{x_j\}_i\right)\right)\right) \quad (1)$$

where x_i is the decision variable of the ith sub-system, F_i is the penalized objective function, $\beta_i > 0$ is the local penalty parameter, $\{x_j\}_i \subseteq \{x_j | j \in N_i\}$ represents other sub-systems' decisions that interact with and constrain the ith sub-system, f_i is the primary utility function of the ith sub-system, and P_i is the penalty function that penalizes the violation of the interconnection constraints that affect the decisions of both each ith sub-system and the sub-systems (j) connected to each i.

Yang et al. (2009) extended the formulation (Eq. (1)) to solve a multiple agent water system problem, in which agents maximize their own utility subject to the penalty caused by the violation of constraints:

$$\max F_i\left(x_i, \beta_i | \{x_j\}_i\right) = \max\left(\beta_i f_i(x_i) - P_i\left(x_i, \left|\{x_j\}_i\right)\right)\right) \quad (2)$$

where $\beta_i > 0$is no longer a penalty parameter but instead acts as a local interest factor indicating the selfishness of an agent. Yang et al. (2009) analyzed the impacts of β_i on agents' behaviors through a hypothetical watershed management problem. Yang et al. (2012) further discussed the reciprocal relation between β_i and water price in a basin-scale water allocation case, and showed that β_i or water price) represented an interaction mechanism among agents. For simplicity, this study assumes an equal level of selfishness for all agents by setting $\beta_i = 1, \ \forall i$, and focuses on the penalty item. Then, we have:

$$\max F_i\left(x_i, \left|\{x_j\}_i\right.\right) = \max\left(f_i(x_i) - P_i\left(x_i, \left|\{x_j\}_i\right)\right)\right) \quad (3)$$

where F_i is net benefit of the ith agent, f_i is utility function of the ith agent, and x_i is water use of the ith agent. The penalty item P_i is replaced by a simple formula based on actual water use (x_i) and entitled water use (w_i).

$$P_i\left(x_i | \{x_j\}_i\right) = p(x_i - w_i)$$
$$p = \begin{cases} p_h & x_i \geq w_i \\ p_l & x_i \leq w_i \end{cases} \quad (4)$$

where w_i is the entitled water allocation for the ith agent, p_{h} is the penalty for $x_i \geq w_i$, and p_{l} is the subsidy for $x_i \leq w_i$.

Substituting Eq. (4) into Eq. (3), we can write the final agent formulation that reflects the rules governing the agent's behavior as:

$$\operatorname{argmax} F_i(x_i) = f_i(x_i) - p(x_i - w_i)$$
$$p = \begin{cases} p_h & x_i \geq w_i \\ p_l & x_i \leq w_i \end{cases} \quad (5)$$

When an agent uses water more than the entitled, the agent's benefit becomes penalized. In this case, Eq. (4) is consistent with Eq. (3). On the other hand, when an agent uses water less than the quantity to which they are entitled, the agent's benefit is subsidized.

2.2. Properties of the ABM

Applying the first-order condition to Eq. (5), we have:

$$\frac{\mathrm{d}F_i(x_i)}{\mathrm{d}x_i} = \frac{\mathrm{d}f_i(x_i)}{\mathrm{d}x_i} - p = f_i'(x_i) - p = 0 \quad (6)$$

and

$$\begin{aligned} f_i'(x_i) - p_{\mathrm{l}} &= 0 \quad \forall\ i \quad \text{if} \quad x_i \leq w_i \\ f_i'(x_i) - p_{\mathrm{h}} &= 0 \quad \forall\ i \quad \text{if} \quad x_i \geq w_i \end{aligned} \quad (7)$$

where $f_i'(x_i)$ represents the marginal profit of agent i. Eq. (7) reflects individual agents' behavior using N independent equations and $N+2$ variables (p_{h}, p_{l} and x_i).

The interactions between agents and those between agents and the environment can be represented by a mass balance equation and a cost recovery equation:

$$\sum_{i=1}^{N} (x_i - w_i) = 0 \quad (8)$$

$$p_{\mathrm{h}} = p_{\mathrm{l}} + C_{\mathrm{t}} \quad (9)$$

Eq. (8) means that the total water use $\sum_{i=1}^{N} x_i$ equals the total entitled water $\sum_{i=1}^{N} w_i$, which implies that, if some agents use water more than entitled, others use water less than entitled. With this assumption, agents are allowed to consume more water than to which they are entitled if and only if their total benefits will still increase after accepting a penalty, which follows the theory of "rational crime" or "rational violation" (Cooter and Ulen, 2000; Souza Filho et al., 2008), particularly with water use permits. Meanwhile, it also assumes that other agents are willing to use less water than their entitled allocation and transfer part of their allocation to the agents who exceed their entitlement when the payment per unit of the water transfer is greater than the loss caused by transferring one unit of water.

Eq. (9) implies a cost recovery constraint, which is necessary for the sustainability of public water systems because using public funds to subsidize service is often not sustainable (Bond, 1999). In this equation, C_{t} represents the transaction cost under MS or administrative cost under AS per unit water exchange, which will be discussed in greater detail later in this paper. Following Carl (1979), C_{t} can be explained as follows: "*these, then, represent the first approximation to a workable concept of transaction costs: search and information costs, bargaining and decision costs, policing and enforcement costs*" (Carl, 1979). Thus, $C_{\mathrm{t}} \geq 0$, $p_{\mathrm{h}} \geq p_{\mathrm{l}}$, and Eq. (9) are all based on an assumption that transaction costs or administrative costs are covered by the difference between the buying price and the selling price under MS and that between penalty and the subsidy under AS.

Combining Eqs. (7)–(9), we have the following governing equation set for a water allocation system:

$$\begin{cases} f_i'(x_i) - p_{\mathrm{l}} = 0 \quad \forall\ i \quad \text{if} \quad x_i \leq w_i \\ f_i'(x_i) - p_{\mathrm{h}} = 0 \quad \forall\ i \quad \text{if} \quad x_i > w_i \\ \sum\limits_{i=1}^{N} (x_i - w_i) = 0 \\ p_{\mathrm{h}} = p_{\mathrm{l}} + C_{\mathrm{t}} \end{cases} \quad (10)$$

which has $N+2$ variables and $N+2$ independent equations and should result in a unique solution. However, this equation set is based on the assumption that *total water use* $\sum_{i=1}^{N} x_i$ equals the total available water $\sum_{i=1}^{N} w_i$, which is implicit in Eq. (8). In the real world, this assumption is viable only if the total *water demand* (i.e., desired water use) among all agents is greater than the total water available, as stated in Theorem 1 and proved in Appendix A.

Theorem 1. *Assuming* $(\hat{p}_l, \hat{p}_h, \hat{x}_1, \hat{x}_2, \ldots \hat{x}_N)$ *is the solution of the system governing equation set* (10), *and* $(x_1^*, x_2^*, \ldots x_N^*)$ *is a vector which satisfies* $f_i'(x_i^*) = 0, \forall\ i$, *the necessary condition for* $\hat{p}_l, \hat{p}_h > 0$ *is* $\sum_{i=1}^{N} w_i < \sum_{i=1}^{N} x_i^*$.where $(x_1^*, x_2^*, \ldots x_N^*)$ represents the agents' desired water use (or water demand) . This theorem states the necessary condition for developing a water allocation scheme, i.e., the total available water must be less than the maximum total water demand. Meanwhile, this theorem validates the assumption of "scarcity" in economics, i.e., regulation is needed only when a system has limited resources and therefore cannot satisfy the demands of all users in the system. Thus some market-based mechanisms or administrative measures (i.e. $\hat{p}_l, \hat{p}_h > 0$) are required to allocate water when it is a scarce resource as illustrated later in this paper in the discussion of the equilibrium state of a water allocation system. An ideal case is when a sufficient supply of water exists to meet the demand of all agents. In this case $\hat{p}_l = \hat{p}_h = 0$, which means no management or market is needed. In other words, a water allocation management system is necessary only if if a region is considered to be water stressed, i.e., the supply is less than the desired demand.

While the model represented by equation set (10) can be used to describe both market and administered systems based on the concept of an ABM, equation set (10) can also be understood as a partial equilibrium economic model of market system, which concentrates on the supply-demand equilibrium in an individual market, holding other things equal (Samuelson and Nordhaus, 2004). The first two equations in equation set (10) (i.e., Eq. (7)) reflect individual agents' behaviors, and the last two (i.e., Eqs. (8) and (9)) depict the environment in which individual agents interact to each other, with Eq. (8) representing the mass balance between supplies and demands and Eq. (9) describing the economic and policy regulations of the water allocation system.

Equation set (10) describes an analytical framework that represents the governing laws of the equilibrium state of a water allocation system. The existence of the equilibrium state of a general natural resource allocation management system has been proven through both experimental study (Smith, 1991) and numerical modeling (Dawid, 1999; Bosch-Domenech and Sunder, 2000). Simple bargaining rules (such as double auction) can be sufficient for an economic system consisting of multiple interrelated agents to attain a competitive equilibrium, even if agents have only limited information. In other words, the equilibrium state of a system is primarily a function of the rules for managing the system (Sunder, 2004), which will be discussed in details in following section.

3. Agent behaviors and system emergence

The formulation of the ABM describes a water allocation system from the perspective of complex system composed of autonomous agents, a shared environment, interactions among agents and between agents and the environment, as well as emergence at the system-level (i.e., the performance and patterns at the system-level). Eq. (7) shows the rules that characterize the behavior of individual agents, including agent autonomy and their response to environmental changes and behaviors of other agents. Eqs. (8) and (9) depict the environment that all agents share, with Eq. (8) characterizing its physical component (water balance) and Eq. (9) characterizing its economic component (cost recovery). Eq. (10), together with Theorem 1, comprises a set of governing equations that characterizes the system emergence – the equilibrium state resulting from agents' autonomy and interactions among the agents (the choice of water use, x_i) and between the agent and the environment (the determination of p_l, $p_h > 0$).

The equation set (10) can be further illustrated by a curve describing the behavior of water users with changing p_l and p_h, as shown in Fig. 1(a) (also see details in Appendix B). When $p_l = 0$, agents will use water up to their maximum water demand x_i^* if water is available. When p_l increases, an agent will decrease its water use according to the curve AB (labeled as "high water use curve," $x_i = g(p_l + C_t)$) curve AB in Fig. 1. This behavior will change when the water use of an agent is equal to their entitled water (w_i) at $p_l = f'(w_i) - C_t$. Then, the agent will keep its water use at a level of w_i until p_l increases to $f'(w_i)$ (curve BC). After that point, the low water use curve CD ($x_i = g(p_l)$) characterizes the agent's behavior. Finally, beyond $p_l = f'(0)$, the agent will not use any water. Specially, if the utility function $f_i(x_i)$ is exponential, x_i^* and $f'(0)$ will be positively infinite while if the utility function $f_i(x_i)$ is quadratic, the curves will be straight lines with linear functions of $x_i = g(\cdot)$. Fig. 1(b) has the same implications as Fig. 1(a), but the x-axis and y-axis are switched so that it has a comparable form with a conventional demand curve in microeconomics. Furthermore, if $C_t = 0$, segment BC will converge to one single point, and ABCD will become a continuous curve. In this case, the agent's behavior curve will have the same form as a typical continuous demand curve in economics. In hydro-economic models (Cai, 2008a; Harou et al., 2009), a continuous demand curve is used to derive a gross

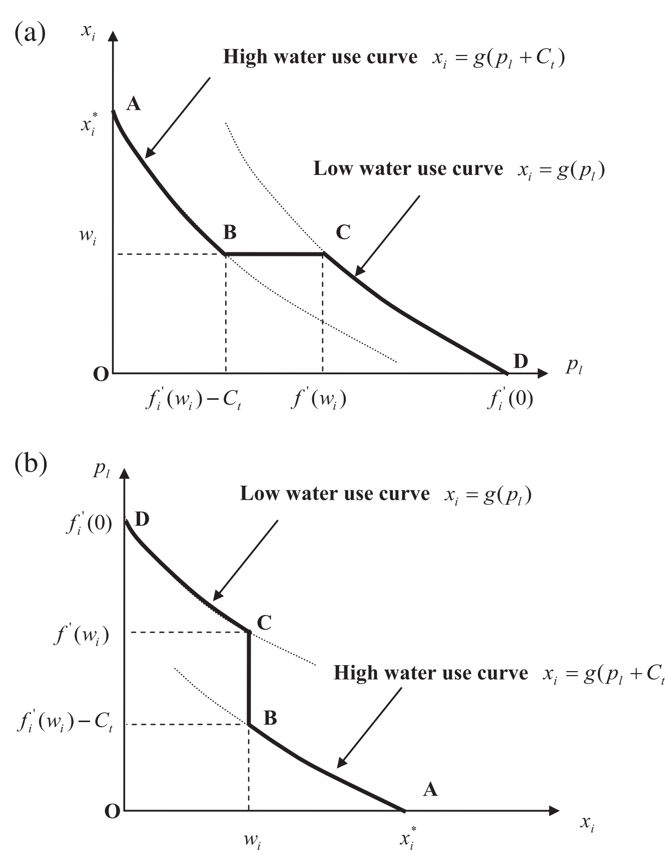

Fig. 1. The relationship between agent water use behaviors (x_i) and p_l.

benefit function of all agents to evaluate the benefit of the entire water use system.

Moreover, the length of curve BC represents the condition under which agents stay with their preexisting water allocation and do not trade water under MS or do not exchange water use permits under AS. As can be seen from Fig. 1, the length of BC, which is equal to the transaction, or administrative, costs C_t, will influence the agents' willingness to exchange water, i.e., a longer BC (a larger C_t) will induce a larger price or incentive interval without water exchange. An extreme case occurs when BC is as long as OD, in which case agents will not exchange water and will just use water as much as entitled. This is in agreement with the conclusion from previous studies (e.g., Rosegrant and Binswanger, 1994) that trading will not occur in water markets when transaction costs are too high, which is one of the most important reasons of why water markets are sometimes ineffective in practice. In particular, if $C_t = 0$, agents will always interact with others.

Based on the properties of the agent behavior curve, we prove that the equilibrium state of a water allocation system can be achieved following a simple bargaining rule regardless of the initial values of p_l and p_h, i.e., increasing p_l when the total water demand $\sum_{i=1}^{N} x_i$ is greater than the total water available $\sum_{i=1}^{N} w_i$ and decreasing p_l otherwise. This occurs automatically under MS and through administration under AS. The process is illustrated in Fig. 2 with an initial value of $p_l = 0$. The proof is given in Appendix C.

4. Comparison of AS and MS within a consistent modeling framework

In this section, we show that the same system behavior, in terms of the system's economic benefits and equity, can occur in either AS or MS allocation systems, even though the agents' (water use) behavior is driven by different mechanisms and responses to different system rules, i.e., market mechanisms and trading rules in MS, and rational violation mechanisms and cross- subsidy rules in AS.

4.1. Formal definition of AS and MS

The agent-based model used in this study can accommodate the mechanisms of water transfer used in both MS (water trading) and AS (cross-subsidies).

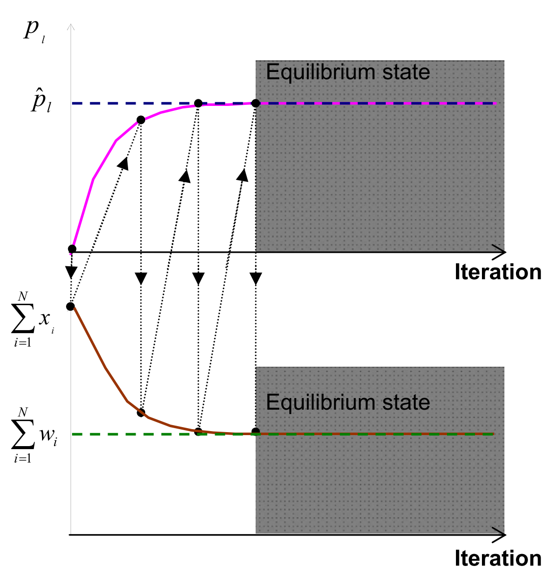

Fig. 2. Understanding the emergence of a system's equilibrium state.

Under MS, water users are supposed to be autonomous and behave rationally, i.e., they make decisions in a water market to maximize their utility subject to physical and social constraints. Water users or stakeholders in a market behave as agents with water trading as the core interaction mechanism among them.

To define AS in the context of the ABM, we need to introduce the so-called cross-subsidy mechanism (CSM), which means the use of gain from one activity to cover losses from another. CSM for water allocation is used around the world, including in Austria, Denmark, Finland, and the United Kingdom (OECD, 1999), in Spain, South Africa, Australia, Chile, and Israel (Saleth and Dinar, 2000), and in China (Chen et al., 2005). Increasing block tariff (IBT) system and penalty-incentive-based permit system are probably the two most popular forms to implement the CSM in AS. Usually, in an increasing block tariff (IBT) system, users will be charged at below-cost prices (i.e., subsidized prices) for water uses lower than a certain amount and at prices that incorporate a penalty for water use greater than a threshold value. Under IBT, rich-to-poor subsidies and industrial-to-agricultural subsidies are implicitly performed to reallocate water from users with lower marginal value to users with a higher marginal value (Boland and Whittington, 2000; Rogersa et al., 2002; Gomez-Limon and Riesgo, 2004). In a penalty-incentive-based permit system, water is allocated by permit, with penalties charged to users who exceed the entitled allocation and with subsidies provided to users whose use is less than the entitled.

Administrative cost recovery (Eq. (9)) is usually a concern with an IBT system (Saleth and Dinar, 2000; World Bank, 2002). Programs for cost recovery in the real world have been tested around the world. For example, Hooper (2003) and Koenig (2008) describe an innovative policy in Durban, South Africa in which a long-term attempt of cost recovery and equity was achieved through dynamically adjusting the IBTs of a so-called "free basic water policy" to reflect total administrative costs (Hooper, 2003). Furthermore, variable unit pricing (VUP) was developed as a tool for cost recover and economic efficiency (Loehman, 2008; Griffin, 2009), and can be understood as a continuous form of IBT.

These pricing/incentive mechanisms are used in some municipal water supply management cases. For instance, the city of Beijing issued a new IBT policy for urban water management in April 2010. The policy simultaneously penalized households who use water more than their quota and subsidized the households who use water less than their quota (Beijing Water Authority, 2011). When a subsidy for not using one's full entitlement does not exist ($p_l = 0$) two consequences may come up: 1) water users will use all of their entitled water with a certain amount of water wasting just because they have no motivation to save water. The proposed framework still holds if the penalty on overuse exists ($p_h > 0$ and $p_l = 0$); 2) if some water users (i.e., downstream users) cannot obtain what is entitled to them due to the overuse of others (i.e., upstream users), they will request for subsidies or compensations from the administrative system, which will act as a driving mechanism of the equilibrium for AS, as discussed later in this paper.

Utilizing the theoretical framework of rational violation, this paper essentially employs the concept of CSM to define the penalty/subsidy under AS, and discusses economic efficiency and cost recovery in a context similar to previous studies. However, the focus of this study is on the behavior of both individual water users (i.e., water saving, water overuse, water transfer, etc.) and the overall system performance (i.e., economic benefit, equity). In particular, AS is defined by employing CSM as the interaction mechanism between individual agents and the government or management agency. By specifying the parameters in Eq. (5), AS and MS are defined in Fig. 3.

Under AS, a governmental agency establishes and enforces a scheme that subsidizes and penalizes different agents based upon

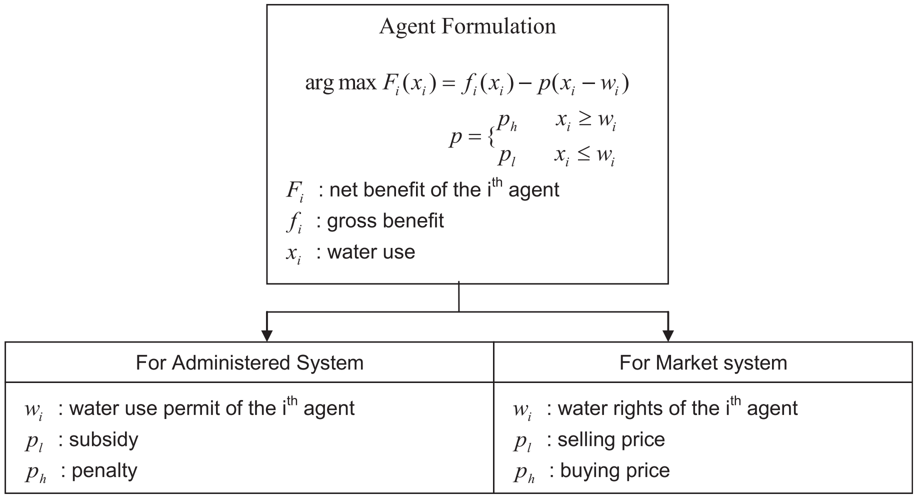

Fig. 3. Definition of AS and MS in the context of the ABM.

their water use. Thus, Eq. (5) can be interpreted using the theory of "*rational violation*", i.e., agents will violate their initial permit if they believe it can improve their net benefit while doing so. Following the theory of "*rational violation*", $(x_i - w_i)$ represents the quantified rational water use violation against the prescribed water use permits, including water overuse $(x_i > w_i)$ and water saving $(x_i < w_i)$. Therefore, p_h and p_l are terms that describe the degree of penalty and incentive on "rationalized" violations of water permits. The difference of the two items $(C_t = p_h - p_l)$ represents the administrative costs per unit water use violation in the context of cost recovery. Note that p_h and p_l are special realizations of IBT, as p_h reflects the tariff on water use above the permitted volume while p_l, which is usually lower than the costs of water delivery, implies a subsidy on water use below the permitted amount.

Under MS, a market mechanism replaces the management coordination under AS. Water prices p_h and p_l result from bargaining between agents through a market, and $(x_i - w_i)$ represents the water trade amount of the *i*th agent. It is positive for water purchased and negative for water sold. The terms p_h and p_l are system-level variables that depend on all agents' behaviors, and C_t can be understood as the transaction costs per unit of water traded.

Two common assumptions used in the application of AS and MS models are made for the analysis presented in this paper: 1) agents in the system are heterogeneous in water use efficiency and entitled water allocation (w_i), but homogeneous in other aspects, including water supply infrastructure and information access; 2) physical conditions (e.g., spatial relationships) and engineering infrastructures facilitate (storage and delivery channels) any water trade or exchange.

4.2. Driving mechanisms of the equilibrium state under AS and MS

The behavior of agents is affected by markets and governance under AS and MS, respectively. Even though agents perform in a similar manner in terms of their autonomy, they are operating in two different institutional environments in which system emergence arises from different driving mechanisms.

Under MS, the optimal state occurs when buyers and sellers achieve identical marginal benefits. In general, the market can be in one of two situations at any given instant. First, when p_l and p_h are lower than their corresponding values at equilibrium, the total water demand is greater than the total available water, i.e., $\sum_{i=1}^{N} x_i > \sum_{i=1}^{N} w_i$, leaving the demand of some water buyers unsatisfied. Thus, buyers whose water use is higher than their water rights $(x_i > w_i)$, must buy water at a higher price (p_h) so that they can gain additional benefit while sellers will sell water with a higher p_l to increase their net benefit. In this way, the total demand $\sum_{i=1}^{N} x_i$ decreases (because both the sellers and the buyers reduce their demand according to the agents' water use behavior curve shown in Fig. 1) and approaches $\sum_{i=1}^{N} w_i$ while the marginal water value of the sellers and buyers approaches an identical value as the system approaches its equilibrium. The other situation occurs when p_l and p_h are greater than their corresponding values under the system's equilibrium state and the total water demand is less than the total water available, $\sum_{i=1}^{N} x_i < \sum_{i=1}^{N} w_i$. Under this situation with declining buying and selling prices, buyers will increase water use by buying more water while sellers will also increase water use by keeping more water for their own use. This will cause the total demand to increase and approach $\sum_{i=1}^{N} w_i$.

Under AS, the social goal of fairness is the driving mechanism that brings the system toward an equilibrium state. Under the theory of rational violation, agents decide how much water they demand, as they can choose to accept a penalty for violating their water use permit or accept a subsidy for using less water than permitted. There exist two situations similar to the two situations under MS described in the previous paragraph. When p_l and p_h are lower than those under the equilibrium state, $\sum_{i=1}^{N} x_i > \sum_{i=1}^{N} w_i$, some users (e.g., upstream users that have water access advantages) will violate their permits. As a result, other agents, with relative water access disadvantages (e.g., downstream agents and ecological systems), will not obtain the volume of water entitled to them, causing conflicts over the fairness of the allocation scheme. When this issue catches the attention of the governmental agency, p_l and p_h will typically be increased through a series of political, legal and administrative procedures. In the real world, if the governmental agency responds late or ignores the inequity (e.g., even without any penalty or subsidy), social instability and ecosystem damage will eventually emerge and may even cause some irreversible environmental and economic losses. On the other hand, if p_l and p_h are larger than the ones under the equilibrium state, the total water demand will be less than the total water available, $\sum_{i=1}^{N} x_i < \sum_{i=1}^{N} w_i$. This is also possible, although it may not occur as often as the other situation discussed above. Under this situation, few users will use water in excess of the amount that they are permitted to use due to the high penalty while, on the other hand, some users may use water less than the permitted to obtain a high subsidy. As a result, some amount of water

$(\sum_{i=1}^{N} w_i - \sum_{i=1}^{N} x_i)$ will not be used, potentially compromising economic and domestic activities in the watershed. Moreover, this situation can cause financial problems for the governmental agency since the total subsidies awarded can be higher than the total penalties received (which can be zero), requiring a budget increase to maintain the incentive program. Consequently, this situation will finally force the governmental agency to reduce p_l and p_h.

For both situations under AS discussed above, the change toward the equilibrium state depends on effective monitoring of the system state, a channel for bottom-up information flow, and timely administrative responses through policies and regulations. All of these are collectively referred to the efficiency of governance, i.e., the so-called "super hand", while, under MS, it is argued that much of these are managed by the "invisible hand" that controls market dynamics.

4.3. System economic benefits

Based on Eq. (10) and Theorem 1, the total benefit of the whole system is derived as shown in Eq. (11) (Appendix D):

$$\sum_{i=1}^{N} F_i(\widehat{x}_i) = \sum_{i=1}^{N} f_i(\widehat{x}_i) - \frac{C_t}{2}\sum_{i=1}^{N}\left|\widehat{x}_i - w_i\right| \tag{11}$$

As defined in Theorem 1, $(\widehat{p}_l, \widehat{p}_h, \widehat{x}_1, \widehat{x}_2, \ldots \widehat{x}_N)$ is the solution corresponding to the equilibrium state of the system (Eq. (10)), $\widehat{p}_l$ and $\widehat{p}_h$ are the market prices (with MS) or penalty/subsidy (with AS), and $\widehat{x}_i$ is the water demand under the equilibrium state. Eq. (11) indicates that the system benefit can be improved in three ways: (1) improving individual agents' utility, $f_i(\widehat{x}_i)$, (2) reducing transaction costs with MS and administration costs with AS (C_t), and (3) adjusting the entitled water allocation (w_i) to one that is closer to the water allocation under the equilibrium state ($\widehat{x}_i$).

Under both MS and AS, agents are motivated to behave in a way that maximizes their individual benefits due to their autonomy, which, according to Eq. (11) will also improve the system's overall benefits.. However, the other two components of Eq. (11) indicate that different mechanisms drive MS and AS toward their equilibrium states. As mentioned earlier, the buying price, p_h, should usually be greater than selling price, p_l, due to transaction costs. For AS, C_t reflects administration costs, including information acquisition, bargaining and decision, and policing and enforcement costs. Usually, the penalty is higher than the subsidy and the surplus $(\widehat{p}_h - \widehat{p}_l)$ comprises all of \\the administrative costs. In an AS, the governmental agency determines $\widehat{p}_l$ and $\widehat{p}_h$ while, with MS, $\widehat{p}_l$ and $\widehat{p}_h$ are the outcomes of the market. This indicates that the financial feasibility of these two systems is maintained in distinct ways. In particular, the infrastructure maintenance costs that are critical to the stability of the systems constitute part of the transaction costs for MS and administrative costs for AS. Usually, individual agents are responsible for maintaining their own water infrastructure when they are the sole owners of the sources (e.g., small reservoirs, pumping wells, etc.) while allocations are typically managed at the system-level if the system is dependent upon large-scale infrastructure (e.g., large multi-reservoir systems with long-distance diversion canals), regardless of the tradability of water rights (AS vs. MS). In this latter case, the quality of infrastructure maintenance depends upon government effectiveness.

With respect to the third option, improving the overall system benefits by adjusting the water allocations to which each agent is entitled is obviously more feasible and flexible with AS than MS. The adjustment of water rights is usually a very costly and time consuming legal issue and can often not be achieved while a governmental agency may have a much easier time in making reasonable adjustments to previous allocations.

In summary, between AS or MS, it is not necessary for one to be more advantageous than the other in terms of the overall economic benefit, due to their advantages and disadvantages that influence system benefits through the three ways as discussed above.

4.4. Equity versus economic benefit

Economic efficiency and social equity are among the most important objectives for water management and, in many instances, there is a tradeoff between them. Equity in water allocation can be reflected from the various perspectives (Cai, 2008b). In this paper, we refer the equity to the initial water allocation, or the entitled water allocation (w_i), i.e., a water right with MS and a permit with AS. In Eq. (11), if it can be assumed that $C_t = 0$, we have:

$$\sum_{i=1}^{N} F_i(\widehat{x}_i) = \sum_{i=1}^{N} f_i(\widehat{x}_i) \tag{12}$$

which means that, if $C_t = 0$, the overall economic benefit of the system will depend on individual agent benefits and will have nothing to do with their initial water rights or water permits. This follows the Coase Theorem in Economics, which states: "*If there are zero transaction costs, the efficient outcome will occur regardless of legal entitlement*" (Coase, 1960). Thus, the system's economic benefit would ultimately not be affected by initial water allocations if transaction costs are zero. In other words, the equity in initial water rights allocation will not influence the overall economic benefit under MS if there are no transaction costs. Of course, this is impossible in the real world. Thus, under MS, the tradeoff between equity in initial allocation and overall economic benefit depends on transaction costs. For AS, if $C_t = 0$, the initial water allocations will have no impact on the system's economic benefit and thus the equity objective can be completely fulfilled without compromising economic efficiency. However, this is not realistic or sustainable from a long-term point of view due to administrative costs. Thus, an important policy implication of Eq. (12) is that reducing either transaction costs under MS or administrative costs under AS will mitigate the effects of an equity constraint on the system's economic benefits.

5. The impacts of the uncertainty of water availability

The uncertainty of water availability affects water allocation substantially. Hydrologic variability is a natural property of water systems, which means the total water availability of system is uncertain in the real world. Defining a random variable, W, for the total available water, $\overline{W}$ for the expected value of W (which can be estimated by agents based on forecasts), and ε for random bias, we have:

$$W = \overline{W} + \varepsilon \tag{13}$$

For simplicity, assuming ε is symmetrically distributed (e.g., with a normal distribution), then

$$\begin{aligned} E[\varepsilon] &= 0 \\ E\left[\varepsilon^2\right] &= \sigma^2 \end{aligned} \tag{14}$$

where $E[\cdot]$ means expected value, σ is standard deviation, and σ^2 is variance of ε.

The key challenge with employing uncertainty analysis through this model framework is associated with risk management mechanisms, i.e., the relationship between system uncertainty and agent uncertainty. In practice, the uncertainty of an agent's water availability is based on the definition of risk particular to their allocation scheme i.e., water rights with MS and water permit with AS. We consider two cases: one in which risk is shared among agents based on the ratio of their water allocation to total available water, which is defined as the equal-risk-share mechanism (ERSM); the other is to assign risk by priority of water uses, i.e., low risk for water uses with high priority, which is defined as the priority-risk-assignment mechanism (PRAM). These two mechanisms correspond to some risk management mechanisms commonly adopted in MS and AS. Under MS, ERSM is related to fractional flow rights, under which all users have the equal priority of water use while PRAM is related to appropriative water rights or prioritized steady use rights (Eheart and Lyon, 1983), under which senior users have priority over junior users. In AS, ERSM and PRAM are also related to two risk management mechanisms, respectively, the lentic system for large lakes or reservoirs with the identical priority for all water users, and the lotic system for rivers with different water use priorities determined by locations and timing of land settlement (Eheart and Lyon, 1983).

The uncertainty impact is different under ERSM and PRAM. In this study, we present a preliminary analysis of ERSM and summarize some previous studies that offer a similar type of analysis of PRAM.

For agents, water availability uncertainty under ERSM can be described as:

$$\begin{aligned} w_i &= \overline{w}_i + \varepsilon_i \\ \varepsilon_i &= \beta_i \varepsilon \\ \beta_i &= \frac{\overline{w}_i}{\overline{W}} \end{aligned} \tag{15}$$

where $\overline{w}_i$ is the expected value, ε_i is the random noise of w_i, the entitled water allocation of the ith agent, β_i is the fraction of random noise with the ith agent (ε_i) relative to the random noise of the whole system. With the random variables, the agent model can be formulated as follows:

$$\begin{aligned} &\operatorname{argmax} E[F_i(x_i)] = E[f_i(x_i) - p(x_i - w_i)] \\ &p = \begin{cases} p_{\mathrm{h}} & x_i \geq w_i \\ p_{\mathrm{l}} & x_i \leq w_i \end{cases} \end{aligned} \tag{16}$$

Let

$$\overline{x}_i = x_i - \varepsilon_i \tag{17}$$

$\overline{x}_i$ can be considered as the water use corresponding to $\overline{w}_i$. According to the derivation in Appendix E, we update the deterministic system in equation set (10):

$$\begin{cases} f_i'(\overline{x}_i) + 0.5 f_i''(\overline{x}_i) \cdot \beta_i^2 \sigma^2 - p_{\mathrm{l}} = 0 \quad \forall\, i \quad \text{if} \quad \overline{x}_i \leq \overline{w}_i \\ f_i'(\overline{x}_i) + 0.5 f_i''(\overline{x}_i) \cdot \beta_i^2 \sigma^2 - p_{\mathrm{h}} = 0 \quad \forall\, i \quad \text{if} \quad \overline{x}_i > \overline{w}_i \\ \sum\limits_{i=1}^{N} (\overline{x}_i - \overline{w}_i) = 0 \\ p_{\mathrm{h}} = p_{\mathrm{l}} + C_{\mathrm{t}} \end{cases} \tag{18}$$

We can compare equation set (18) under uncertain conditions with equation set (10), which is developed for a deterministic condition. Since $f''(\cdot) \geq 0$ (see the explanation by You and Cai, 2008 and the analysis of Kimball, 1990), the second term $(0.5 f_i''(\overline{x}_i) \cdot \beta_i^2 \sigma^2)$ in the first two equations of equation set (18) is positive. Thus the market prices or management penalty/subsidy (p_{h} and p_{l}) will be larger under uncertain conditions than a deterministic condition. This can be interpreted as the tendency of all agents to reserve more water to mitigate a future water supply shortage, because the extra loss of one unit water shortage is greater than the additional profit from one additional unit of water surplus according to the properties of the utility function. This increases both the equilibrium prices (both selling price and buying price) in MS and the value of violations (both penalty and subsidy) in AS. Moreover, the transaction and administrative costs are both expected to increase due to the management complexity that this uncertainty causes. The agent behavior curves under deterministic and uncertain conditions are compared in Fig. 4.

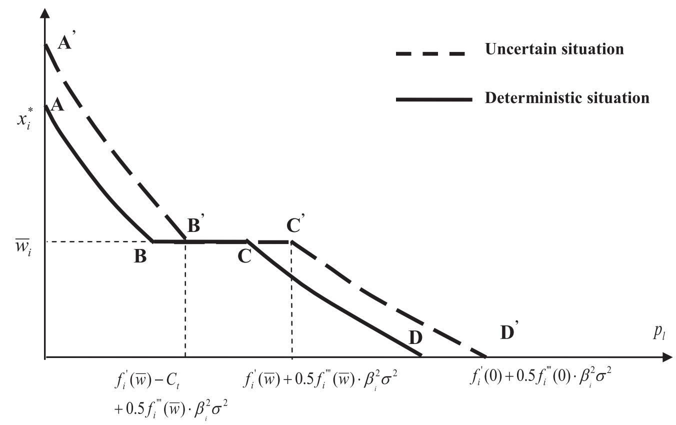

Fig. 4. The relationship between agent water use behaviors (x_i) and p_{l} under uncertain water availability.

On the other hand, PRAM is more complex than ERSM, since an agent's uncertainty is coupled with priorities, utility functions and initial allocated water or water rights in very complex manners. Burness and Quirk (1979) theoretically analyzed the efficiency of a typical PRAM known as the Appropriative Water Rights system. With some reasonable assumptions, they drew two important conclusions: (1) a user with senior rights claims a larger quantity of water than a user with junior rights does, and (2) an "equal sharing" system is more economically efficient than an appropriative system. Unfortunately, most MS use a priority-based risk assignment mechanism, which reflects Matthews' (2004) conclusion that uncertainty is a major barrier preventing the widespread use of MS. On the other hand, ERSM is easier to implement for AS and leads to higher system efficiency according to Burness and Quirk (1979).

6. Conclusions

AS and MS for water allocation can be defined through a consistent agent-based modeling framework in which the model parameters can be interpreted to represent different component of each type of system, i.e., water rights and trading prices with MS and water use permit and penalty/subsidy with AS. Considering cost recovery, the transaction costs with MS are related to the difference between buying prices and selling prices in the water market while the administrative costs with AS are related to the difference between penalties and subsidies. Comparable to the market mechanism under MS, the exchange of entitled water allocations under AS is justified by the theory of rational violation (i.e., using more or less water than permitted and being willing to accept penalty or subsidy) and implemented through cross-subsidies. While agents are characterized to be autonomous in both systems, MS and AS each adopt different institutional environments for agents – markets for water trade under MS and a system to manage rational violations under AS. An agent's behavior depends on both its own interests and its interactions with other agents and the environment under the constraints of market prices and economic incentives (penalty/subsidy) under MS and AS, respectively.

With non-zero interaction costs, i.e., transaction costs in MS or administrative costs in AS, interactions among agents will stop at a specific range of prices or penalties/subsidies, characterized by the magnitude of transaction cost or administrative cost, with the upper bound equal to the marginal value of water use at the level of entitled water allocation ($p_{\mathrm{l}} = f'(w_i)$) and the lower bound equal to the difference of the upper bound minus the interaction cost ($p_{\mathrm{l}} = f'(w_i) - C_{\mathrm{t}}$). Therefore, when the transaction cost is zero, the agent's behavior curve will be continuous and the trade and exchange will occur at any water price or penalty/subsidy. On the other hand, with a non-zero transaction cost, agents will stay at their entitled water allocations without trading or exchanging water during the range specified above (Fig. 1).

This paper also uncovers valuable insights on emergent system behavior, especially ones regarding the equilibrium price, overall economic benefits and equity. The system benefit can be improved by increasing individual agent benefits, reducing transaction costs in MS or administrative costs in AS, and adjusting entitled water allocations. Moreover, equity concerns related to initial water allocation will not influence the overall economic benefit of the system if there are not any transaction costs, i.e., with zero transaction costs, any initial water allocation will be changed for free to achieve the same overall economic benefit under which the marginal value of water use for all agents will be identical. Thus reducing transaction costs under MS or administrative costs under AS will mitigate the tradeoff between equity and system's overall economic benefit.

Finally, hydrologic uncertainty makes the analysis and operation of any water allocation management system even more complex. It is demonstrated that hydrologic uncertainty increases market prices under MS and penalties/subsidies under AS. As a result, this method illustrates that hydrologic forecasts are valuable for improving water allocation under both MS and AS.

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Appendix A

A utility function $f_i(x_i)$ is generally concave. Its first derivative represents the marginal benefit while its second derivative is negative due to diminishing marginal utility (You and Cai, 2008). To further explain the third derivative, we introduce the concept of prudence defined by Kimball (1990), which represents the strength of the precautionary saving motive. Generally, utility function has following properties:

$$f'(\cdot) > 0$$

$$f''(\cdot) \le 0$$

$$f'''(\cdot) \ge 0$$

The most commonly used utility functions matching these properties are the quadratic function and exponential function.

$$f_i(x_i) = a_i x_i^2 + b_i x_i + c_i \quad a_i < 0 \quad \text{and} \quad b_i > 0$$

$$f_i(x_i) = k_i x_i^{d_i} \quad k_i > 0 \quad \text{and} \quad 0 < d_i < 1$$

For Theorem 1, if $\widehat{p}_{\mathrm{l}}, \widehat{p}_{\mathrm{h}} > 0$, then according to Eq. (7) we have:

$$f_i'(\widehat{x}_i) > 0$$

Since $f_i'(x_i^*) = 0$ and $f_i(x_i)$ is concave, then we have:

$$\widehat{x}_i < x_i^*$$

According to Eq. (8), we have:

$$\sum_{i=1}^{N} w_i = \sum_{i=1}^{N} \widehat{x}_i < \sum_{i=1}^{N} x_i^*$$

Appendix B

(1) Curve AB:

According to Eqs. (7) and (9), we have:

$$f_i'(x_i) - p_{\mathrm{h}} = 0 \Rightarrow x_i = g_i(p_{\mathrm{h}}) = g_i(p_{\mathrm{l}} + C_{\mathrm{t}}) \quad \forall\ i \quad \text{if} \quad x_i \ge w_i$$

which is the function for the curve AB.

(2) Curve CD:

According to Eqs. (7) and (9), we have:

$$f_i'(x_i) - p_{\mathrm{l}} = 0 \Rightarrow x_i = g_i(p_{\mathrm{l}}) \quad \forall\ i \quad \text{if} \quad x_i \le w_i$$

This is just the curve CD.

(3) Curve BC:

Curve BC can be represented by $x_i = w_i$ if $f_i'(w_i) - C_{\mathrm{t}} < p_{\mathrm{l}} < f_i'(w_i)$.

Assuming $x_i < w_i$, then according to Eq. (7) and concave utility, we have:

$$f_i'(w_i) < f_i'(x_i) = p_{\mathrm{l}}$$

which conflicts with $p_{\mathrm{l}} < f_i'(w_i)$;

Further, if we assume $x_i > w_i$, then according to Eqs. (7) and (9) and concave utility we have:

$$f_i'(w_i) > f_i'(x_i) = p_{\mathrm{h}} = p_{\mathrm{l}} + C_{\mathrm{t}}$$

This also conflicts with $f_i'(w_i) - C_{\mathrm{t}} < p_{\mathrm{l}}$. So $x_i < w_i$ and $x_i > w_i$ can be proven to be incorrect.

Thus, we must have:

$$x_i = w_i \quad \forall\ i \quad \text{if} \quad f_i'(w_i) - C_{\mathrm{t}} < p_{\mathrm{l}} < f_i'(w_i)$$

which is the function for curve BC.

Appendix C

Assuming $(\widehat{p}_l, \widehat{p}_h, \widehat{x}_1, \widehat{x}_2, \ldots \widehat{x}_N)$ is solution of equation set (10), set $(p_l^1, p_h^1, x_1^1, x_2^1, \ldots x_N^1)$ and $(p_l^2, p_h^2, x_1^2, x_2^2, \ldots x_N^2)$ are vectors that come from the bargaining process that satisfy the agent behavior curves.

First, we assume $p_{\mathrm{l}}^1 < p_{\mathrm{l}}^2 < \widehat{p}_{\mathrm{l}}$, according to the agent behavior curve shown in Fig. 1, we have $x_i^1 \ge x_i^2 \ge \widehat{x}_i$, then:

$$\sum_{i=1}^{N} x_i^1 \ge \sum_{i=1}^{N} x_i^2 \ge \sum_{i=1}^{N} \widehat{x}_i = \sum_{i=1}^{N} w_i$$

This equation means we will get closer to satisfying the water balance constraint for the system by increasing p_{l}^1 to p_{l}^2, and also come closer to the equilibrium state that the unique solution of the system governing equation set suggests.

A similar proof can be done for the case where $p_l^1 > p_l^2 > \widehat{p}_l$. Hence, it is obvious that increasing p_l when $\sum_{i=1}^N x_i > \sum_{i=1}^N w_i$ and decreasing p_l when $\sum_{i=1}^N x_i < \sum_{i=1}^N w_i$, leads to the equilibrium state when a suitable iteration step length is used.

Appendix D

According to Eq. (5), for the whole system we have:

$$\sum_{i=1}^N F_i(\widehat{x}_i) = \sum_{i=1}^N f_i(\widehat{x}_i) - \widehat{p}_l \sum_i (\widehat{x}_i - w_i)\Big|_{\widehat{x}_i \le w_i} - \widehat{p}_h \sum_i (\widehat{x}_i - w_i)\Big|_{\widehat{x}_i > w_i}$$

where $\sum_i (\widehat{x}_i - w_i)|_{x_i \le w_i}$ stands for the total water savings in AS or the total amount of sold water in MS, and $\sum_i (\widehat{x}_i - w_i)|_{x_i \le w_i}$ represents total water overuse in AS or the total amount of purchased water in MS. Considering the water trade balance shown in Eq. (5), we have:

$$-\sum_i (\widehat{x}_i - w_i)\Big|_{x_i^* \le w_i} = \sum_i (\widehat{x}_i - w_i)\Big|_{x_i^* > w_i} = \frac{1}{2}\sum_{i=1}^N \left|\widehat{x}_i - w_i\right|$$

Then, we get:

$$\begin{aligned}\sum_{i=1}^N F_i(\widehat{x}_i) &= \sum_{i=1}^N f_i(\widehat{x}_i) - \frac{\widehat{p}_h - \widehat{p}_l}{2}\sum_{i=1}^N \left|\widehat{x}_i - w_i\right| \\ &= \sum_{i=1}^N f_i(\widehat{x}_i) - \frac{C_t}{2}\sum_{i=1}^N \left|\widehat{x}_i - w_i\right|\end{aligned}$$

Appendix E

The deterministic system governing equation set (10) should be updated to incorporate uncertain conditions. Equation set (10) states:

$$\begin{cases} E\left[f_i'(x_i)\right] - p_l = 0 & x_i \le w_i \\ E\left[f_i'(x_i)\right] - p_h = 0 & x_i > w_i \\ \sum_{i=1}^N (x_i - w_i) = 0 \\ p_h = p_l + C_t \end{cases}$$

According to Eq. (17), we have:

$$\sum_{i=1}^N (x_i - w_i) = \sum_{i=1}^N (x_i - \overline{w}_i - \varepsilon_i) = \sum_{i=1}^N (\overline{x}_i - \overline{w}_i)$$

And we also have:

$$\begin{aligned} E\left[f_i'(x_i)\right] &= E\left[f_i'(\overline{x}_i + \varepsilon_i)\right] \\ &= E\left[f_i'(\overline{x}_i) + f_i''(\overline{x}_i)\cdot\varepsilon_i + f_i''(\overline{x}_i)\cdot\frac{(\varepsilon_i)^2}{2!} + f_i''''(\overline{x}_i)\cdot\frac{(\varepsilon_i)^3}{3!} + \cdots\right]\end{aligned}$$

Neglecting the higher-order terms, that is:

$$E\left[f_i'(x_i)\right] = f_i'(\overline{x}_i) + f_i''(\overline{x}_i)\cdot E[\varepsilon_i] + 0.5 f_i''(\overline{x}_i)\cdot E\left[\varepsilon_i^2\right]$$

Considering Eqs. (15) and (16), we have:

$$E\left[f_i'(x_i)\right] = f_i'(\overline{x}_i) + 0.5 f_i''(\overline{x}_i)\cdot\beta_i^2\sigma^2$$

Thus we can derive Eq. (18):

$$\begin{cases} f_i'(\overline{x}_i) + 0.5 f_i''(\overline{x}_i)\cdot\beta_i^2\sigma^2 - p_l = 0 & \forall\ i \quad \text{if} \quad \overline{x}_i \le \overline{w}_i \\ f_i'(\overline{x}_i) + 0.5 f_i''(\overline{x}_i)\cdot\beta_i^2\sigma^2 - p_h = 0 & \forall\ i \quad \text{if} \quad \overline{x}_i > \overline{w}_i \\ \sum_{i=1}^N (\overline{x}_i - \overline{w}_i) = 0 \\ p_h = p_l + C_t \end{cases}$$

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