PORTFOLIO ANALYSIS AND CALCULATION OF VaR AND ES RISK MEASURES

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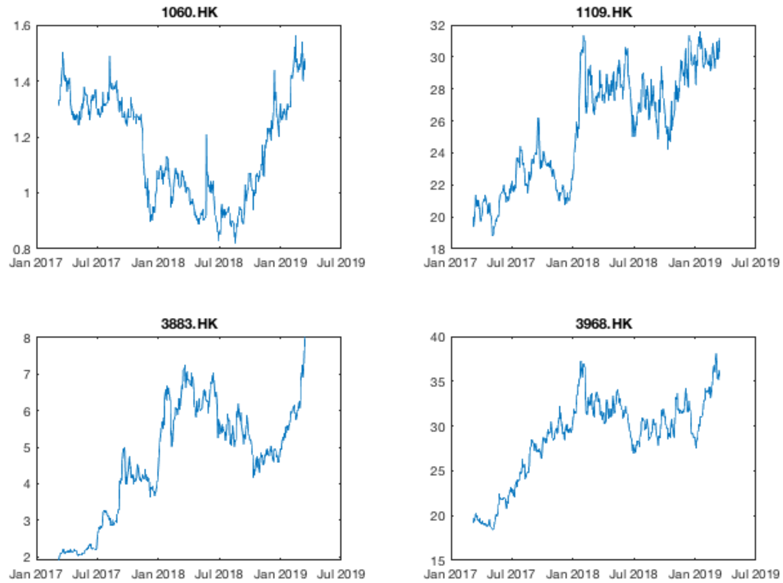
INTRODUCTION

The assignment given was to construct an optimal portfolio and assess the risk of the portfolio by calculating the expected loss. The loss distribution of the portfolio was calculated through the Variance-Covariance, Historical Simulation with Extreme Value Theory and Monte Carlo methods. The securities for this portfolio were chosen based on past performance of individual stocks; Alibaba Pictures Group Limited [1060.HK], China Resources Land Limited [1109.HK], China Aoyuan Group Limited [3883.HK], and China Merchants Bank Co., Limited [3968.HK], all of which are listed on the Hong Kong Stock Exchange. A risk-free asset was forgone to maximize returns. All of the data for the portfolio construction, VaR and ES was extracted from Yahoo Finance and ranges from 2017/03/08 to 2019/03/19. From the Monte Carlo method, the max-VaR was calculated as: ¥ 1042425.35936 and the max-ES as: ¥ 1048861.38826.

CONSTRUCTION OF OPTIMAL PORTFOLIO

The methodology for the construction of the portfolio was fairly simple. It consists of one small-cap, one mid-cap and two large cap stocks. Two high-growth, small to mid-cap stocks were chosen; Alibaba Pictures and China Aoyuan. These stocks were chosen for their high return and high-growth characteristics without being as volatile as stocks in other industries such as technology. Alibaba Pictures and China Aoyuan demonstrate strong returns over their lives without being too volatile. The other two stock selections consisted of large-cap stocks; China Merchant's Bank as a blue-chip stock and China Resources Land Limited as a stable large-cap stock. Both of these stocks minimize the risk of the portfolio due to their stability and provide steady returns for the portfolio. All of the assets' price history, logarithmic returns, and EWMA variances from 2017/03/08 to 2019/03/19 are shown in the figures below:


1060.HK
1109.HK
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Jan 2017
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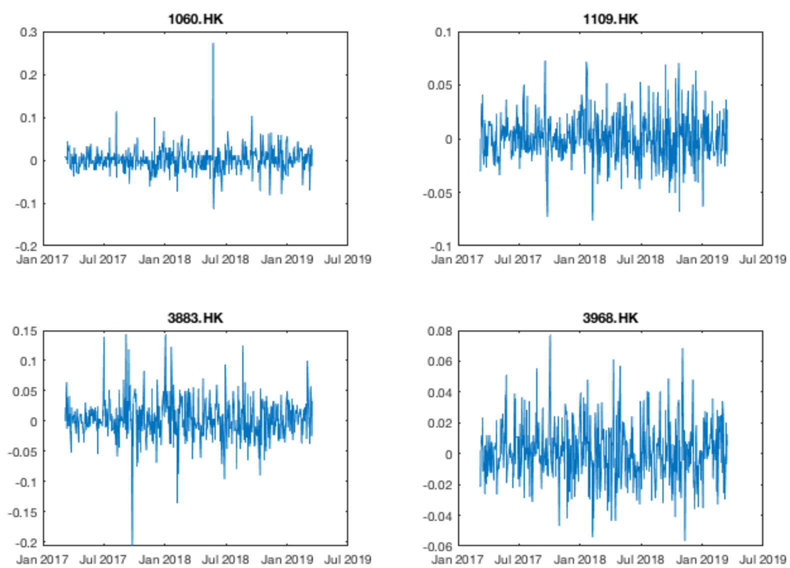
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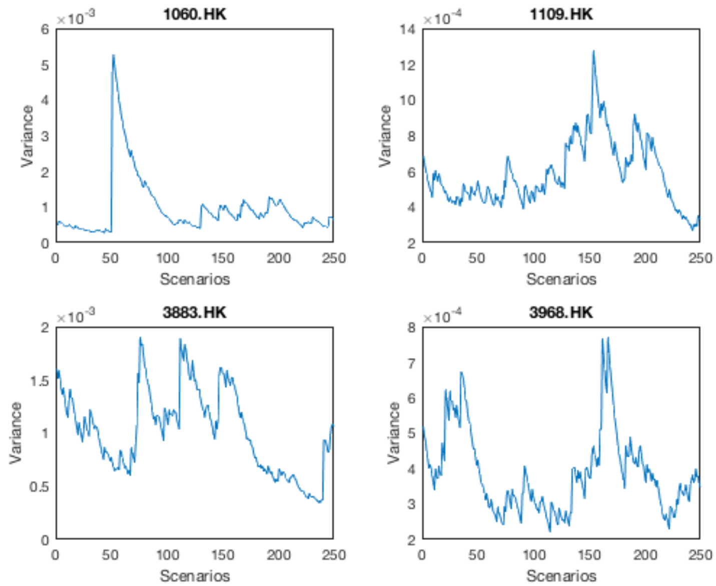
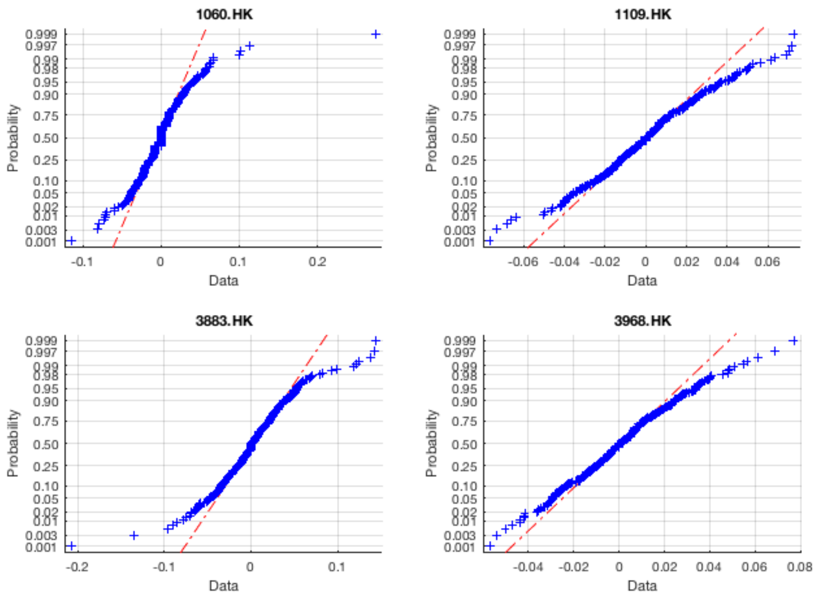


After selecting the assets for the portfolio, it was a matter of minimizing the risk of the portfolio while maximizing the return. The methods chosen to carry out these objectives was through a slight modification of Markowitz's Portfolio Theory. To ensure a solid assessment of possible weighting schemes for the portfolio, a Monte Carlo simulation was used to create a distribution of portfolio means. Various portfolio means were generated through sampling different possible means of return for the individual assets. For each asset, the mean and standard deviation was calculated and then 10,000 possible means of return for each asset was generated through a Monte Carlo simulation. Then, 10,000 portfolio means of equally-weighted assets were generated from the return distributions of the individual assets. The next step in the optimization process was to minimize the objective function, which was the portfolio's variance subject to each asset's weights, through iterating through all 10,000 possible portfolio means. After creating 10,000 possible portfolios with different weighting schemes, the chosen optimal portfolio was the following: with a mean log return of 0.109016 and standard deviation of 0.017447.

1060.HK	**1109.HK**	**3883.HK**	**3968.HK**	**σP**	**μP**
0.153339	0.637413	0.089923	0.119325	0.017447	0.109016

It was decided to assess the normality of the asset's returns. The following figure demonstrates that all of the asset's returns were short-tailed, so it could be concluded that the probability distributions of loss will also not be long-tailed and the Monte Carlo method or EVT method will best assess the loss distributions of the portfolio. Also, since the returns are non-normal, the most accurate calculation of VaR and ES would probably come from the EVT method, as it does not assume the returns to be normally distributed. The next section will analyze, contrast and compare the three different methods used to calculate the VaR and ES.



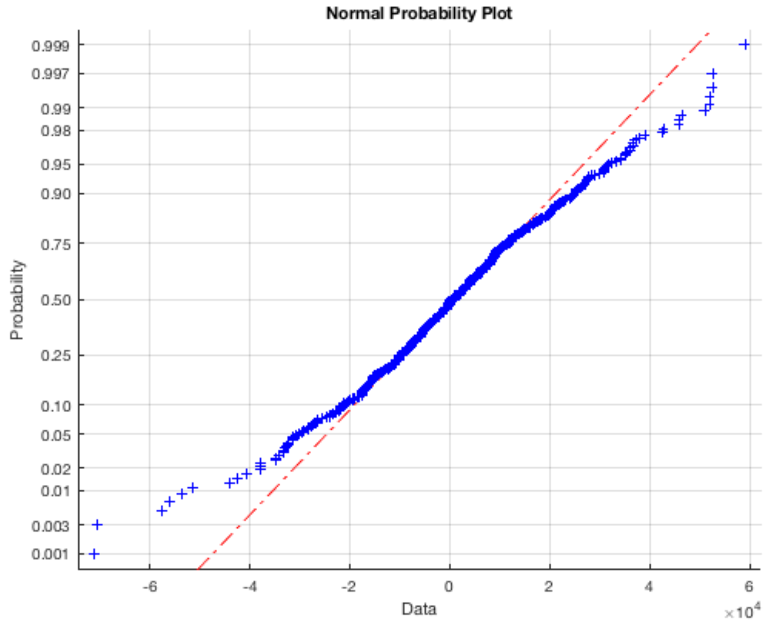
VARIANCE-COVARIANCE METHOD

Firstly, each of the lognormal returns were calculated and were given their respective portfolio weights in dollars. Then, the daily portfolio value was calculated using 501 days of returns. The variance of the portfolio was calculated through an EWMA variance by creating the returns' covariance matrix. A lambda of $\lambda = 0.94$ was used to calculate this. After calculating the standard deviation of the portfolio, the VaR and ES were calculated with a 99% confidence interval as per usual using the following equations:

$$VaR = N^{-1}(X)\sigma_P\sqrt{T}$$

$$ES = \sigma_P\sqrt{T}\frac{e^{-Y^2/2}}{\sqrt{2\pi}(1-X)}$$

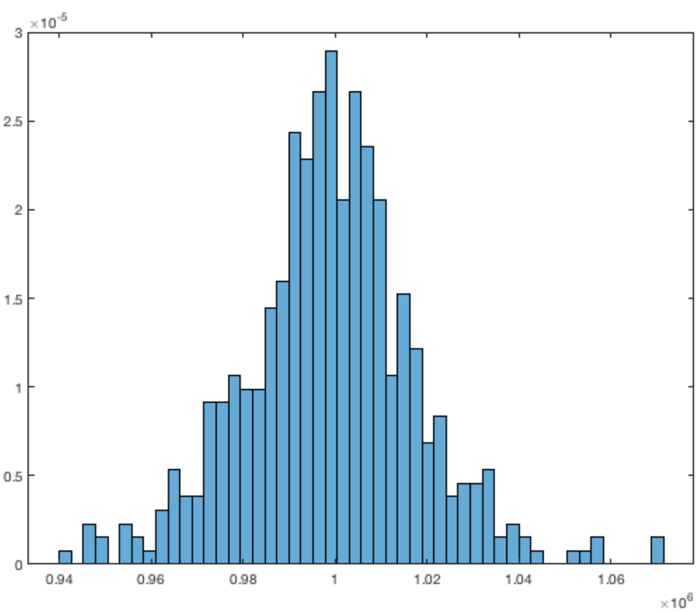
The VaR and ES of the portfolio calculated using the variance-covariance approach were respectively: ¥ 907449.88694 and ¥ 1039633.1391 – notably lower than the values calculated through the Monte Carlo method. It can be seen through the following chart that the returns of the portfolio were not normally distributed:



This assessment of the distribution of returns proved to be limitations towards the calculations of VaR and ES using the variance-covariance approach. The VaR and ES calculations were slightly understated due to the non-normality of the return distribution.

MONTE CARLO METHOD

The probability distribution of loss for the portfolio used in the Monte Carlo method is the following:



The portfolio value each day was calculated using t = 501 days of data, creating 500 scenarios for the portfolio. The loss distribution was created by subtracting the investment for the portfolio of ¥ 1000000 from the daily changes in the portfolio. The mean and standard deviation of the loss distribution were used to generate a probability distribution of values through Monte Carlo simulation. The amount of trials used in generating the loss distribution for the Monte Carlo simulation was n = 1000000. The losses were ranked from greatest to least and the VaR and ES of the portfolio calculated through this method were: ¥ 1042425.35936 and ¥ 1048861.38826 respectively. These values are notably higher than the VaR and ES calculated in the variance-covariance method. The amounts at risk seem to be higher using the Monte Carlo approach because of the large amount of n-trials used to generate an n = 1000000 loss distribution, simulating larger tails for the distribution and – in return – a higher expected loss. It should still be noted that the returns of the portfolio are not perfectly normally distributed, and this impacts the accuracy of the results from the Monte Carlo approach, though the Monte Carlo approach proves to be quite useful in scenario analysis and has definitely generated the worst-case scenarios for the portfolio.

HISTORICAL SIMULATION AND EVT

The portfolio's loss distribution was calculated in the same way as in the Monte Carlo simulation approach, however instead of simulating values of the loss distribution, the tail of the loss distribution was extrapolated using Extreme Value Theory to calculate the VaR and ES. The VaR and ES calculations using this approach were ¥ 1030454.12291 and ¥ 1030514.12252 respectively. Firstly, to carry out EVT the losses greater than the u = 0.95 confidence interval were used to extrapolate the tail of the loss distribution. Secondly, the xi and beta of the distribution were calculated through optimizing the values of xi and beta set to the constraints of the maximizing function and xi and beta being constrained to be positive:

$$\sum_{i=1}^{n_u} \ln\left[\frac{1}{\beta}\left(1+\frac{\xi(v_i - v_u)}{\beta}\right)^{-1/\xi-1}\right]$$

The vi were 25 values greater than the u = 0.95 confidence interval, the vu was the 95th percentile of loss, and the beginning trial values of xi and beta were 0.3 and 40. MATLAB's fmincon optimization function was used. Subject to this maximization problem, xi was estimated as 4.515007383282020e-08 and the beta was estimated as 59.999598877127660. The VaR and ES were then calculated using: (with a q = 0.99, n = 500, nu = 25).

$$VaR = u + \frac{\beta}{\xi}\left\{\left[\frac{n}{n_u}(1-q)\right]^{-\xi} - 1\right\}$$

$$ES = \frac{VaR + \beta - \xi u}{1-\xi}$$

The VaR and ES results calculated using historical simulation and EVT are in-between the results from the latter two methods and should prove to be the most accurate since this approach does not have any limitations in terms of the loss distribution. The empirical loss distribution is non-normal and EVT extrapolates the right tail of the portfolio's loss distribution with a high confidence level. While – for this portfolio – the variance-covariance method understates the VaR and ES and the Monte Carlo method overstate, the Historical Simulation method with EVT gets it just right.

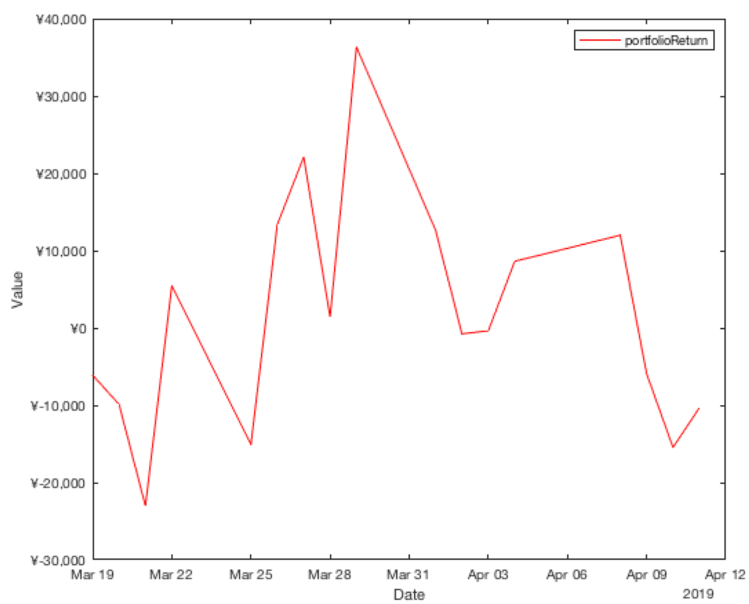
SHORTFALLS & EXPECTED PERFORMANCE

While the methods used should prove to be quite accurate, optimizations of the portfolio and calculations of the VaR and ES could be more robust and accurate. For the future, further ideas and recommendations to improve would be to try to optimize the portfolio subject to minimizing the ES or conditional Value-at-Risk while maximizing the expected return of the portfolio. A shortfall of Markowitz's Portfolio Theory is that standard deviation is not exactly an appropriate risk measure since it measures standard deviation in terms of losses as well as gains. To improve VaR and ES calculations, more scenario analysis could be used along with methods such as bootstrapping and ensuring bunching in data is not a problem, although all this would require a larger amount of data and confidence in the selection of criteria for calculations and optimizations.

It is expected that the portfolio should do fairly well, while not having a huge return due to the nature of the low-risk, and subtle returns of the portfolio. Based on past performances and historical data, the portfolio should provide a steady amount of return.

PERFORMANCE SUBJECT TO L FUNCTION AND CONCLUSIONS

The portfolio return over t = 15 days subject to the L function is shown in the figure below, demonstrating a negative return over 15 days.



The L function is defined as:

$$L = \left[\frac{1}{31}\sum_{t=15}^{15}\sum_{i=1}^{n} a_i \Delta x_{i,t} - \frac{1}{2}\left[0.7\sigma_{p,t}\right]^2\right] - \frac{1}{P}\left[0.1 * \max(VaR_{t=16}) + 0.2 * \max(ES_{t=16})\right]$$

Subject to this performance function, the L value over the period of 15 days is:

- ¥ 6656065.80318988