Experimentation and Learning under Competitive Search

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We study competitive search equilibrium in markets where matches between workers and firms are an experience good and there is uncertainty about match productivity. Parties learn about the underlying match productivity through on-the-job experimentation. If the firms can offer fully flexible wages, the competitive search equilibrium is efficient. However, under fixed-wage contracts, experimentation may be sub-optimal, resulting in fewer vacancies compared to the efficient benchmark. Conditions under which inefficiency occurs depend on whether the firms can commit to contract duration. Without commitment, there is generically too little experimentation. Minimum wages and non-common priors lead to inefficiency even under flexible wages.

I. Introduction

Uncertainty about match productivity and costly search are ubiquitous in performance-driven professions, such as entrepreneurship, scientific research, and executive leadership. Parties involved in a match in these professions often continue to learn about the match productivity after a match is formed. If they believe the relationship is not productive enough, either party can end it. In this paper, we provide a framework to analyze endogenous experimentation duration and how it may be affected by contract and matching frictions in an environment with competitive search.

In our model, workers (or matches) can be of two types: productive or unproductive. The workers and firms share a common prior about match productivity. Workers typically produce ordinary output, but productive workers occasionally produce stellar output (or succeed). Output is publicly observable, hence if the worker produces stellar output, then all market participants learn that the worker is productive. Otherwise, they all update their beliefs, and the expected payoff from continuation declines. We are interested in the length of the initial learning period and who dissolves the match if no success arrives. We model the surplus-sharing arrangements after success in a stylized manner. We assume

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that a portion of the future match surplus accrues to the worker depending on their exogenous bargaining power. This modeling choice is not critical for our results, and we only require that the productive workers earn a higher wage after producing stellar output.

We account for the matching frictions through a matching function and search costs. Workers earn a safe outside option wage when not employed by the firm. Firms incur search costs when they are searching for a match. Firms post publicly observable contracts that specify the transfers to the workers, and workers direct their search effort to the most lucrative contracts. More lucrative contracts attract more workers. The queue length, which is the number of workers per vacancy, is contract-specific. For a given contract, the firms prefer a longer queue on account of lower search costs due to faster matching. We assume free entry by firms. That is, if firms make positive profit by posting a certain contract, more firms post this contract, leading to a shorter queue length until there is zero profit from posting another vacancy with this contract in equilibrium. The contract, the experimentation duration, and the measure of searching workers and firms are determined in equilibrium.

One of our goals is to compare equilibrium outcomes with the efficient benchmark which we define as one where a planner chooses the number of vacancies and the experimentation duration to maximize the total output net of dead-weight search cost. The efficient benchmark can be solved in two steps. The planner fist chooses the optimal experimentation duration beyond which continuing experimentation reduces the expected value of the match. Next, given the expected match value under optimal tenure, the planner chooses the number of vacancies that maximizes the expected flow output net of the search cost. The experimentation duration and the number of vacancies (or equivalently, the queue length) chosen by the planner constitute the efficient outcome.

Turning to equilibrium analysis, suppose first that firms can post contracts with a fully flexible wage schedule.[1] We show that such contracts can be fully summarized by the total payoff promised to the worker, and an experimentation duration. Since the firms are residual claimants, they choose experimentation duration efficiently. Each contract is associated with a queue of workers who apply to that contract. Because of free entry, firms must make zero profit. In equilibrium, in the payoff-queue length space, the workers'

[1] Such a compensation scheme eliminates concerns about the worker quitting because the firm can just adjust the compensation so that the worker earns just slightly more than her outside option payoff. Moreover, it subsumes compensation schemes involving startup and exit bonuses.

indifference curve is tangent to the isoprofit curve of the firms corresponding to zero profit. Using this construction, we show that a decentralized equilibrium with fully flexible wages exists and the equilibrium experimentation duration and the queue length coincide with the planner's solution. The intuition for this result is similar to that for the efficiency of competitive search in Moen (1997), which is closely related to the efficiency of competitive equilibrium.

However, firms may face institutional limitations that restrict the rate at which wages can grow during employment. For example, wages can only be raised annually. We account for such contractual frictions by analyzing the case of fixed-wage contracts. Since forced employment is rarely observed in reality, we assume that workers are free to opt out of employment. For fixed wage contracts whether the firms have the ability to commit to the experimentation duration becomes critical.

With commitment to experimentation duration, a contract specifies a constant wage and an experimentation duration. The worker quits before the contract term ends, if the continuation payoff from employment falls below her outside option. The worker's continuation payoff and hence the quitting time are increasing in the wage. For any contract duration, there is a wage threshold below which the worker quits before the contract expires. Firms take workers' quitting decisions into account and post contracts specifying a fixed wage for a set duration.

Competitive search equilibrium with fixed-wage contracts exists and fall into one of two categories. When search costs are below a threshold, when equilibrium experimentation duration expires, workers prefer further experimentation. We call this a *tenure equilibrium*, similar to the 'up or out' promotion system in many performance-centric professions. When search costs are above the threshold, workers quit while the firms would have preferred to retain the worker. We call this a *quitting equilibrium*.

A tenure equilibrium always results in optimal experimentation. If experimentation is shorter than the efficient one, then some firms can deviate and commit to a slightly longer contract duration and lower wage such that both the deviating firms and the workers are better off. With the optimal experimentation, the indifference curve of the worker, and the iso-profit curve of the firm in the wage-queue length space must be tangent to each other so the number of vacancies in a tenure equilibrium must also be efficient. This result provides a rationale for the tenure contracts commonly offered to academic researchers and professional athletes.[2] In contrast, a quitting equilibrium always results in an inefficient number of vacancies, and sub-optimal experimentation. For any contract duration, workers quit employment before the contract expires if the offered wage is below a threshold. Hence, there is a threshold wage corresponding to the optimal experimentation duration such that for any wage below this threshold, the worker's quitting time precedes the optimal experimentation duration. If the wage at the tangency between the worker's indifference curve and the firm's iso-profit curve is below the threshold wage, then the equilibrium features workers quitting employment before the optimal experimentation duration and is not efficient.

If firms cannot commit to the contract duration – e.g. due to weak enforcement of worker termination laws – then equilibrium is inefficient. To gain intuition for this result, we can draw an analogy with a two-player version of the two-arm bandit problem. The firm and the worker each pull either a risky or a safe arm. If they both pull the risky arm then they continue experimentation with the chance of a windfall gain. If either one pulls the safe arm, they separate. The worker takes the outside option, and the firm goes back to recruitment. If the fixed wage is below/above a threshold, the proportion of the expected surplus that accrues to the worker is too low/high, and the worker/firm pulls the safe arm sooner than optimal. Only at a unique threshold wage, they split the surplus in such a way that both parties prefer to dissolve the match simultaneously at the optimal experimentation duration.

The optimal experimentation in our model naturally lends itself to applications in innovation financing. The literature on entrepreneurial financing has shown that a common structure of such financing involves a period of unconditional financing despite low productivity, followed by a period of conditional financing, Ewens et al. (2018), Ewens et al. (2020). Our model where the firm continues the experimentation even as the worker remains unproductive seems to fit this stage of entrepreneurial financing where the monitoring and governance costs are high. There is empirical evidence provided by Ederer and Manso (2011) where extending this initial period of unconditional financing increases innovation, Acharya et al. (2014) who show that increasing dismissal costs increase match duration, and hence innovation, Tian and Wang (2014) show that innovation increases as the tolerance

[2] The rookie contracts in the NBA offer a fixed salary schedule based on the position of the player in the draft lottery. These contracts are continued without negotiation for a period of 4-5 years. Successful rookies generally turn free agents at the end of this period to negotiate a larger compensation for themselves.

of VC firms for this initial period of low productivity increases. Our model captures these settings fairly well and can speak to the efficiency of such experimentation. Naturally in our model, increasing the experimentation duration may lead to more instances of success, as documented in these empirical studies. However, we further highlight how contractual and search frictions play an important role in determining the efficiency of experimentation.

Finally, our model can rationalize the secular shortening of scientific careers and the increasing proportion of early-career researchers who never graduate to chief investigators.[3] If, in our model, the expected windfall gain from experimentation declines, the total match surplus is lower, and firms may post fewer vacancies. The expected windfall gain may decline because the return from stellar output is lower, i.e., lower worker productivity. Bloom et al. (2020) document the declining productivity of a researcher, with more researchers required to achieve the same research output. The optimal experimentation duration and the threshold wage decline because the windfall gain is lower. Our model provides a framework to analyze whether the shortening of careers is inefficient. If the workers quit before the contract term expires, then the equilibrium outcome is inefficient. Otherwise, if the researchers fail to renew their contracts on expiry, the observed outcomes can be attributed to diminishing returns and experimentation may not necessarily be inefficient.

In section VIII, we consider two extensions: mandated minimum wage and relaxing the common prior about workers' types. Mandated minimum wages may prevent the firm from offering very low initial wages. We show that the workers are weakly better off, as long as the minimum wage is below the threshold wage. If the minimum wage is above the threshold, the experimentation duration is shorter than optimal as firms, unable to commit to the tenure, prefer to fire workers sooner. As a result, the workers may be worse off as their post-match payoff declines because they get a shorter trial period to prove themselves. Next, we relax the common prior assumption and highlight that efficiency crucially depends on it. If either of the parties is excessively optimistic or pessimistic about match productivity, even under flexible wage schedules, experimentation is sub-optimal in equilibrium.

Related Literature: This paper builds upon the search and matching framework for wage determination as in Pissarides (1984). Rogerson et al. (2005) provide an excellent

[3] As of 2010, it takes 5 years for half of the cohort of young scientists to drop out, as opposed to 35 years in the 1960s (Milojević et al. (2018)).

survey of the search and matching literature. We employ a competitive search model as in Moen (1997), however, the match productivity in our case is not deterministic. Efficiency of equilibrium with flexible wage schedules in a dynamic environment with experimentation extends Moen (1997) and echoes others in the competitive search literature. Wright et al. (2021) provide a comprehensive survey of similar results in the literature. We show that even with contractual frictions, competitive search equilibrium may be efficient if firms commit to contract duration. However, if search frictions are sufficiently high, commitment is insufficient for efficiency due to sub-optimal experimentation and learning. This result complements few others on inefficiency of competitive search equilibrium with restricted contracts.[4]

Our model is related to the literature on strategic experimentation with multiple agents (e.g. Keller et al. (2005) and Keller and Rady (2010)). In those papers, the focus is on free-riding in experimentation with multiple agents, when experimentation is costly and agents observe the results of others' actions. This paper is also related to Halac et al. (2016). We show that contractual frictions may not lead to inefficiency if firms have full commitment power, however, only when search cost is sufficiently low. This inefficiency results without adverse selection or dynamic moral hazard, unlike Halac et al. (2016) where adverse selection, moral hazard, and private learning are necessary for inefficient experimentation. Gieczewski and Kosterina (2023) also study experimentation with multiple agents. Their model features a learning process similar to ours. If agents do not observe success, they become pessimistic over time and can exit. There are several differences between their model and ours. They restrict attention to voting as a decision rule for experimentation and do not allow transfers between agents. In our model, experimentation continues only if both agents choose to do so and they can enter into contracts stipulating how to split the surplus. In addition, their model does not have search frictions which as we highlight, may lead to inefficient experimentation. Hoppe-Wewetzer et al. (2023) also looks at learning and experimentation with multiple agents. Unlike us, their game has a winner-take-all structure and they focus on preemption motives in experimentation.

[4]For example, in Galenianos and Kircher (2009) workers can simultaneously apply to multiple markets and a worker who gets a high wage still enters the queue at low wages. The resulting externality leads to inefficiency. If, as in Kircher (2009), workers who get a low wage offer do not queue for a low wage, efficiency is restored. Another example is, Delacroix and Shi (2006) who show that if workers engage in costly on the job search, competitive search equilibrium can be inefficient. Menzio and Shi (2011) show that contracts that directly specify search activity, or specify transfers when workers quit, can restore efficiency.

II. Model

Time is continuous. There are two kinds of agents: workers and firms. The workers and the firms discount their future income at the rate ρ.

A. *Workers*

The measure of the continuum of workers is normalized to 1. The workers earn b_0 flow payoff per unit of time when they are not working for a firm. The workers may either be productive (p) or unproductive (l). Productive workers working for a firm produce a stellar output $\overline{y}$ at a Poisson rate λ and produce regular output $\underline{y}$ ($< \overline{y}$) otherwise.[5] Unproductive workers working for a firm always produce $\underline{y}$. [6] We assume that it is socially efficient for an unproductive worker to accept the outside option and the productive worker to be employed by a firm:

$$\underline{y} < b_0 < \underline{y} + \lambda\overline{y}. \tag{1}$$

Workers are born and die at a rate δ. If the workers quit employment or are fired, they continue to earn b_0 flow payoff. Hence a worker's continuation value after leaving employment is $v_0 = b_0/(\rho + \delta)$.

B. *Information and Learning*

The probability that a worker is productive is $\mu \in (0, 1)$ and this belief is common to workers and firms. When a worker is matched with a firm, her output is observable. A worker who produces $\overline{y}$ learns that she is productive and so does the firm. Moreover, the common belief that the worker is productive after working at the firm for time t having never produced $\overline{y}$ is given by

$$\gamma(\mu, t) = \frac{\mu e^{-\lambda t}}{(1-\mu) + \mu e^{-\lambda t}} = \frac{\mu}{\mu + (1-\mu)e^{\lambda t}}$$

Since $\gamma(\mu, t)$ is decreasing in t, over time, both the worker and the firm employing the worker, become less confident that the worker is of type p if the worker does not produce

[5] In the text, we sometimes refer to stellar and regular outputs as high and low.

[6] We can also interpret the types as match quality instead of inherent worker types. In that case, a successful match would produce a high output at the rate λ and an unsuccessful one always produces a low output.

$\overline{y}$.

C. Contracts

There is a continuum of ex-ante homogeneous firms. A subset of these firms enter the recruitment market and incur flow cost κ while searching. Firms post vacancies with contracts $\boldsymbol{\omega}$. In all the cases that we consider, contracts specify transfers to the worker, denoted by $w(t)$. The firm commits to making these payments, as long as the match between the firm and the worker lasts and the worker does not produce stellar output. If the firm fires the worker or the worker quits, the firm is no longer obligated to make these payments. If the firm has the ability to commit to the length of the contract, the contract also specifies the date T at which the contract ends if stellar output is not produced. The expected payoff of a firm from matching with a worker who is productive with probability μ under contract $\boldsymbol{\omega}$ is $V(\mu, \boldsymbol{\omega})$ and that of the worker is $v(\mu, \boldsymbol{\omega})$.

D. Bargaining

Under any contract, if at any time t, the worker produces $\overline{y}$, then both the firms and the worker update their beliefs about the worker and learn that the worker is productive. At this point, the worker's wage is determined through Nash bargaining with the worker's bargaining power β. We assume that if the negotiation breaks down without an agreement, the productive worker's disagreement payoff is v_p ($\geq v_0$). We take v_p as exogenously specified in the main paper. In Appendix B.B2 we provide micro-foundations for endogenizing v_p where we assume that in case of disagreement, workers' continuation value is determined through competitive search in a market where all workers are productive. Throughout the paper we assume free-entry, therefore the firms' disagreement payoff is 0. The total surplus from a productive match is $(\lambda\overline{y} + \underline{y})/(\rho + \delta)$. Hence, the worker's continuation payoff after the stellar output is,

$$v_h = v_p + \beta \left(\frac{\lambda\overline{y} + \underline{y}}{\rho + \delta} - v_p \right) \tag{2}$$

and the firm's continuation payoff is,

$$V_h = (1 - \beta) \left(\frac{\lambda\overline{y} + \underline{y}}{\rho + \delta} - v_p \right) \tag{3}$$

When a firm enters the recruitment market, it posts a contract $\boldsymbol{\omega}$ and commits to it. All workers can see the contracts posted by all the firms. We assume that workers direct their search to one of the posted contracts.

Matching is bilateral and therefore an agent contacts at most a single firm. We capture the search frictions in forming bilateral matches using a matching function. For u unemployed workers looking for a job and v firms searching for workers, the matching rate is given by a matching function $m(u, v)$ which is concave and homogeneous of degree 1. The queue length is $q = u/v$ and the market tightness is $1/q$. Under directed search, the queue length, $q(\boldsymbol{\omega})$ is endogenous and depends on all posted contracts. Define the transition rate from unemployed to employed by

$$\alpha_u(q) = \frac{m(u,v)}{u} = m\left(1, \frac{v}{u}\right) = m(1, q^{-1})$$

$\alpha_u(q)$ is decreasing in q, i.e. if the queue is longer, workers transition less rapidly to employment. Similarly define the arrival rate of workers to a vacancy by

$$\alpha_v(q) = \frac{m(u,v)}{v} = m\left(\frac{u}{v}, 1\right) = m(q, 1)$$

$\alpha_v(q)$ is increasing in q, i.e. if the queue is longer, the firm fills its vacancy faster. Note that, $\alpha_v(q) = q\alpha_u(q)$. The continuation payoff of a firm, $V^u(\mu, \boldsymbol{\omega}, q)$, from posting a contract $\boldsymbol{\omega}$ and facing a queue length q is

$$\rho V^u(\mu, \boldsymbol{\omega}, q) = -\kappa + \alpha_v(q)(V(\mu, \boldsymbol{\omega}) - V^u(\mu, \boldsymbol{\omega}, q)) \tag{4}$$

where $V(\mu, \boldsymbol{\omega})$ is the firm's payoff at the instant it is matched with a worker. Similarly, the continuation payoff of a worker, $v^u(\mu, \boldsymbol{\omega}, q)$ from applying to a contract $\boldsymbol{\omega}$ and facing a queue length q is,

$$(\rho + \delta)v^u(\mu, \boldsymbol{\omega}, q) = v_0(\rho + \delta) + \alpha_u(q)(v(\mu, \boldsymbol{\omega}) - v^u(\mu, \boldsymbol{\omega}, q)) \tag{5}$$

where $v(\mu, \boldsymbol{\omega})$ is the worker's payoff at the instant she is matched with a firm.

F. Equilibrium

Suppose measure v_m of firms post contracts $\boldsymbol{\omega}_m \in \boldsymbol{\Omega}$ with $m \in \{1, 2, \ldots, M\}$. Each worker directs her search based on the posted contracts and the search behavior of other workers. The measure v_m firms with contract $\boldsymbol{\omega}_m$ and the measure u_m workers applying for this contract constitute the sub-market m with $q_m = u_m/v_m$. In equilibrium, the workers must receive the same payoff, $\overline{v}$, from applying to any of the posted contracts. Hence, if $\boldsymbol{\omega}_m$ is a posted contract then $v^u(\mu, \boldsymbol{\omega}_m, q_m) = \overline{v}$. Substituting this worker indifference condition in Equation (5) gives us a relationship that governs the queue length $q_m = q(\mu, \boldsymbol{\omega}_m; \overline{v})$ in sub-market m. A contract that gives a higher expected payoff to the workers, attracts more workers increasing the queue length until the expected payoff of the workers from any posted contract is equalized across sub-markets.

Three conditions pin down a competitive search equilibrium. First condition requires the firms to post profit-maximizing contracts taking into account the workers' search behavior, i.e. the firms believe that the queue length is $q(\mu, \boldsymbol{\omega}; \overline{v})$. Since firms are homogeneous, in equilibrium they all post the same contract $\boldsymbol{\omega}^*$, and workers apply to this contract, resulting in a single sub-market. There is free-entry, so the second condition requires that the firms enter the recruitment market until the expected value of the vacancy is driven to 0. The third condition gives us the steady state number of unemployed workers. At each instance fraction $\alpha_u(q(\mu, \boldsymbol{\omega}; \overline{v}))$ of the unemployed workers transition to employment and fraction δ of the unemployed workers die. At the same time, mass δ of new unemployed workers are born. Equating the outflow to inflow gives us the third condition.

We summarize the equilibrium conditions in the next definition.

DEFINITION 1: *A competitive search equilibrium is the vector $(\boldsymbol{\omega}^*, q^*, \overline{v}, u)$ that satisfies,*

1) Firms' Profit Maximization:

$$(\boldsymbol{\omega}^*, q^*) = \underset{\{\boldsymbol{\omega}, q\}}{\operatorname{argmax}} \left\{ \frac{-\kappa + \alpha_v(q) V(\mu, \boldsymbol{\omega})}{\rho + \alpha_v(q)} \right\}$$

subject to the worker's search behavior,

$$v^u(\mu, \boldsymbol{\omega}, q) = \overline{v}$$

2) Free Entry:

$$V^*(\mu;\overline{v}) = \frac{-\kappa + \alpha_v(q^*)V(\mu,\boldsymbol{\omega^*})}{\rho + \alpha_v(q^*)} = 0$$

3) Worker population:

$$u = \frac{\delta}{\alpha_u(q^*) + \delta}$$

III. Flexible Wage

First, we consider the benchmark of fully-flexible wages. When the wage schedule can be chosen flexibly, the firm retains control over the duration of experimentation and can effectively choose the time, T, at which the worker's employment at the firm is terminated if the worker has not produced the high output until that time. The firm can choose the termination time because it can prevent the worker from quitting by offering a high enough wage and can induce the dissolution of the match by offering a low enough wage. Hence, without loss of generality, we can think of a flexible contract as a choice of duration T and a flow wage transfer to the worker at each instant over the duration of the contract, $\boldsymbol{\omega} := \{w(t)\}_{t=0}^{T}$. In what follows, to save from notation we continue to use $\boldsymbol{\omega}$ to refer to the contract, but note that $\boldsymbol{\omega}$, and therefore all the functions that depend on it, all depend on the contract duration T.

Let us examine the worker's and the firm's post-match expected payoffs under the variable wage contract. We denote the worker's post-match continuation payoff after an unproductive spell of length t by $v(\mu,\boldsymbol{\omega},t)$. The differential equation governing $v(\mu,\boldsymbol{\omega},t)$ is given by

$$-\frac{dv(\mu,\boldsymbol{\omega},t)}{dt} = w(t) + \lambda\gamma(\mu,t)(v_h - v(\mu,\boldsymbol{\omega},t)) - (\rho+\delta)v(\mu,\boldsymbol{\omega},t) \tag{6}$$

The above differential equation captures how the continuation payoff declines as the worker remains unproductive. The first two terms on the right-hand side capture the flow wage earned and the expected gain had the worker produced stellar output. The third term captures the flow continuation payoff which takes into account the effective discount rate, the sum of the death rate and the discount rate. Refer to Appendix B.B1 for a detailed derivation of (6). Solving the differential equation, and imposing the boundary condition that the value to the worker for all $t \geq T$ is v_0 we obtain:

$$v(\mu, \boldsymbol{\omega}) = \int_0^T (w(z) + \lambda\gamma(\mu, z)v_h)(\mu + (1-\mu)e^{\lambda z})e^{-(\rho+\delta+\lambda)z}dz \tag{7}$$
$$+ v_0 e^{-(\rho+\delta+\lambda)T}\left(\mu + (1-\mu)e^{\lambda T}\right)$$

We denote the firm's post-match continuation payoff after an unproductive spell of length t by $V(\mu, \boldsymbol{\omega}, t)$. The differential equation governing t by $V(\mu, \boldsymbol{\omega}, t)$ is given by

$$-\frac{dV(\mu, \boldsymbol{\omega}, t)}{dt} = \underline{y} + \lambda\gamma(\mu, t)(V_h + \overline{y} - V(\mu, \boldsymbol{\omega}, t)) - w(t) - (\rho + \delta)V(\mu, \boldsymbol{\omega}, t) \tag{8}$$

The interpretation of the differential equation is analogous to that of the worker's. The rate of decline in the firms' continuation payoff is sum of the low output just produced, $\underline{y}$, and the potential expected payoff gain had the worker produced a high output less the flow wage and the flow continuation payoff. We can solve the differential equation to derive the firm's payoff,

$$V(\mu, \boldsymbol{\omega}) = \int_0^T (\underline{y} - w(z) + \lambda\gamma(\mu, z)(V_h + \overline{y})) \cdot (\mu + (1-\mu)e^{\lambda z}) \cdot e^{-(\rho+\delta+\lambda)z}dz \tag{9}$$

We define the expected surplus from a match that dissolves at T if a stellar output does not occur as

(10)

$$V^p(\mu, T) = \mu\left(\frac{\lambda\overline{y} + \underline{y}}{\rho + \delta} - v_0\right)\left(1 - e^{-(\rho+\delta+\lambda)T}\right) + (1-\mu)\left(\frac{\underline{y}}{\rho + \delta} - v_0\right)\left(1 - e^{-(\rho+\delta)T}\right) + v_0$$

Substituting $V_h = (\lambda\overline{y} + \underline{y})/(\rho + \delta) - v_h$ in (9), integrating the first expression using (7), and rearranging the terms we obtain:

$$V(\mu, \boldsymbol{\omega}) = V^p(\mu, T) - v(\mu, \boldsymbol{\omega}) \tag{11}$$

The expected post-match payoff for the firm is the total expected surplus generated through the match, less the compensation to the worker. Next, we construct a wage path $\boldsymbol{\omega}$ such that the worker does not quit employment before time T. Consider the following wage,

$$w(t) = b + v_0(\rho + \delta) - \lambda\gamma(\mu, t)(v_h - v_0) \tag{12}$$

where $b \geq 0$ is some constant. Substituting the wage path into the equation (7) and imposing the boundary conditions, the value to the worker at any time t is,

$$
\begin{aligned}
(13) \quad v(\mu, \boldsymbol{\omega}, t) = v_0 + \gamma(\mu, t)\frac{b}{\rho+\delta+\lambda}\left(1 - e^{-(\rho+\delta+\lambda))(T-t)}\right) \\
+ (1-\gamma(\mu, t))\frac{b}{\rho+\delta}\left(1 - e^{-(\rho+\delta)(T-t)}\right)
\end{aligned}
$$

The second part of the above expression is increasing in b and $v(\mu, \boldsymbol{\omega}, t, T) \geq v_0$ for all $b \geq 0$ and $t \in [0, T]$. The expected value of the match for the worker and the firm at $t = 0$ is,

$$
(14) \qquad v(\mu, \boldsymbol{\omega}) = v_0 + \mu\frac{b}{\rho+\delta+\lambda}\left(1 - e^{-(\rho+\delta+\lambda))T}\right) + (1-\mu)\frac{b}{\rho+\delta}\left(1 - e^{-(\rho+\delta)T}\right)
$$

$$
(15) \qquad V(\mu, \boldsymbol{\omega}) = V^p(\mu, T) - v(\mu, \boldsymbol{\omega})
$$

Note that there could be multiple wage paths $\{w(t)\}_{t=0}^{T}$ for a given T that give the same expected payoff to the worker. We remain agnostic about the wage path offered, as long as the expected continuation payoff of the worker during employment $v(\mu, \boldsymbol{\omega}, t) \geq v_0$ for all $t \in [0, T]$. Therefore, the contract can be fully characterized in terms of the promised payoff, and the contract duration, i.e. $\boldsymbol{\omega} = (\hat{v}, T)$. The firms' payoff is,

$$
(16) \qquad V(\mu, \hat{v}, T) = V^p(\mu, T) - \hat{v}
$$

The contract duration that maximizes firms' post-match value for a given $\hat{v}$,

$$
\overline{T} = \arg\max_{T} V(\mu, \hat{v}, T) = \arg\max_{T} V^p(\mu, T)
$$

Note that the optimal contract duration does not depend on the offered expected value to the worker $\hat{v}$. Examining the expression for $V^p(\mu, T)$ in Equation (11), we can see that it is hump-shaped in T. The first expression is increasing and the second expression is decreasing in T. As the duration of the contract (or experimentation) is increased incrementally, the expected marginal benefit comes from the worker producing the stellar output, and the marginal loss due to the worker failing to do so while forgoing the higher outside option wage. For lower values of T the marginal benefit is greater than the loss and the difference between them declines as the firm becomes more pessimistic. The firm will choose the contract duration so that the marginal benefit equals the marginal loss.

(17)

$$\frac{\partial V^p(\mu,T)}{\partial T} = e^{-(\rho+\delta+\lambda)T}\left[\underbrace{\mu(\rho+\delta+\lambda)\left(\frac{\lambda\overline{y}+\underline{y}}{\rho+\delta}-v_0\right)}_{\text{Marginal Benefit}} - \underbrace{(1-\mu)e^{\lambda T}\left(v_0(\rho+\delta)-\underline{y}\right)}_{\text{Marginal Loss}}\right]$$

Therefore, the optimal $\overline{T}$ has the following form.

$$\overline{T} = \frac{1}{\lambda}\left[\ln\left(\frac{\mu}{1-\mu}\right) - \ln\left(\frac{\underline{\mu}_p}{1-\underline{\mu}_p}\right)\right] \tag{18}$$

where the threshold belief $\underline{\mu}_p$ is given by

$$\lambda\underline{\mu}_p\left(\overline{y}+\frac{\lambda\overline{y}+\underline{y}}{\rho+\delta}-v_0\right) = v_0(\rho+\delta)-\underline{y}.$$

In words, $\underline{\mu}_p$ is the belief at which the marginal benefit from increasing the experimentation duration is equal to the marginal loss when $T=0$ (or immediate match dissolution). That is if the initial belief about the productivity was $\underline{\mu}_p$, the match would be immediately dissolved. The firm chooses a longer contract duration if the difference between the log-odds ratio of the belief that the worker is productive and the threshold belief is higher. Observe that for a given difference between the log-odds ratio, the experimentation duration is shorter if the speed of learning, λ is higher. The firm's value-maximizing experimentation duration is increasing in the worker's type μ, the output values $(\overline{y},\underline{y})$, and decreasing in v_0. The following proposition specifies the variable wage contract offered by the firm $\boldsymbol{\omega}$ which is characterized by the offered contract duration $\overline{T}$ and the offered expected income $\hat{v}$.

PROPOSITION 1: *The firm offers a variable wage contract $\boldsymbol{\omega}$ such that the worker's expected income upon matching is $\hat{v}\geq v_0$ and the contract duration is $\overline{T}$ given by* (18)*. The continuation value $v(\mu,\boldsymbol{\omega},t)\geq v_0$ for all $t\in[0,\overline{T}]$.*

A. Competitive Search

With competitive search and flexible wages, the firms offer contracts, $\boldsymbol{\omega} \equiv (\hat{v}, \overline{T})$ to the workers. The value to the worker when starting the job ($t = 0$) is,

$$v(\mu, \boldsymbol{\omega}) = \hat{v}$$

The value to the firm is

$$V(\mu, \boldsymbol{\omega}) = V^p(\mu, \overline{T}) - \hat{v}$$

Hereafter, we use the contract $\boldsymbol{\omega}$ and the summary statistics $(\hat{v}, \overline{T})$ that fully characterize it, interchangeably. The firms' expected income from posting a vacancy with contract $(\hat{v}, \overline{T})$ is

$$\rho V^u(\mu, \boldsymbol{\omega}, q) = -\kappa + \alpha_v(q)(V(\mu, \boldsymbol{\omega}) - V^u(\mu, \boldsymbol{\omega}, q))$$

The workers' expected income in the same sub-market is

$$(\rho + \delta)v^u(\mu, \boldsymbol{\omega}, q) = v_0 + \alpha_u(q)(\hat{v} - v^u(\mu, \boldsymbol{\omega}))$$

As discussed earlier, the following equation determines the queue length in the sub-market $q(\mu, \boldsymbol{\omega}; \overline{v})$ so that

$$v^u(\mu, \boldsymbol{\omega}, q(\mu, \boldsymbol{\omega}; \overline{v})) = \overline{v}$$

PROPOSITION 2: *The competitive search equilibrium under the variable wage contract is the tuple $(\hat{v}^*, \overline{T}, q^*, \overline{v}^*, u^*)$ and the competitive search equilibrium with variable wage contracts exists.*

PROOF:
The firms maximize,

$$V^*(\mu; \overline{v}) = \max_{(\hat{v}, q, T)} \left\{ \frac{-\kappa + \alpha_v(q)(V^p(\mu, T) - \hat{v})}{\rho + \alpha_v(q)} \right\}$$

subject to

$$\frac{v_0(\rho + \delta) + \alpha_u(q)\hat{v}}{\rho + \delta + \alpha_u(q)} = \overline{v}$$

Note that for a given $\hat{v}$, the firm maximization with respect to T is equivalent to maximizing $V^p(\mu, T)$. Therefore, $T = \overline{T}$. Next, we show that $V^*(\mu; \overline{v})$ is a well-defined function. The expected payoff to the workers that keeps them indifferent across sub-markets is $\overline{v} \in [v_0, V^p(\mu, \overline{T})]$. The contract is $\boldsymbol{\omega} = (\hat{v}, \overline{T})$ and the queue length corresponding to the workers' indifference, $q(\mu, \boldsymbol{\omega}; \overline{v})$. If the firm offers $\hat{v} = \overline{v}$, the workers must get matched without waiting, so that $q(\mu, \boldsymbol{\omega}; \overline{v}) = 0$, and the expected income from a vacancy is $V^u(\mu, \boldsymbol{\omega}, q(\mu, \boldsymbol{\omega}; \overline{v})) = -\kappa/\rho < 0$. If the firm offers all of the post-match surplus to the workers, $\hat{v} = V^p(\mu, \overline{T})$ the expected income from a vacancy is $V^u(\mu, \boldsymbol{\omega}, q(\mu, \boldsymbol{\omega}; \overline{v})) = -\kappa/(\rho + \alpha_v(q(\mu, \boldsymbol{\omega}; \overline{v})) < 0$. The firms expected income $V^u(\mu, \boldsymbol{\omega}, q(\mu, \boldsymbol{\omega}; \overline{v}))$ is continuous in $\hat{v} \in [\overline{v}, V^p(\mu, \overline{T})]$. A continuous function on a compact domain attains a maximum. Therefore, $V^*(\mu; \overline{v})$ is well-defined. Because the firms choose $\hat{v}$, subject to the constraint on $(\hat{v}, q)$ that keeps the workers indifferent, the firms' value maximizing $\hat{v}$ is the point of tangency between the indifference curve of the workers corresponding to their expected payoff $\overline{v}$ and the iso-profit curve of the firm in the $\hat{v}$-q space.

By the application of the theorem of maximum, $V^*(\mu; \overline{v})$ is continuous in $\overline{v}$, and applying the envelope theorem implies that $V^*(\mu; \overline{v})$ is decreasing in $\overline{v}$. Finally, the free-entry condition pins down the equilibrium $\overline{v}^*$. When $\overline{v} \to v_0$, $V^*(\mu; \overline{v}) = V^p(\mu, \overline{T}) - v_0 > 0$ and when $\overline{v} \to V^p(\mu, \overline{T})$, $V^*(\mu; \overline{v}) = -\kappa/\rho < 0$.[7] Therefore, by the intermediate value theorem, there must exist a $\overline{v}^* \in [v_0, V^p(\mu, \overline{T})]$ such that the free-entry condition $V^*(\mu; \overline{v}^*) = 0$ holds. Note that the equilibrium $(\hat{v}^*, q^*)$ is the point of tangency between the iso-profit curve of the firm corresponding to 0 profit and the workers' indifference curve corresponding to the expected payoff $\overline{v}^*$. The equilibrium contract is $\boldsymbol{\omega}^* = (\hat{v}^*, \overline{T})$, and $q^* = q(\mu, \boldsymbol{\omega}^*; \overline{v}^*)$. The workers' entry condition pins down the equilibrium unemployment u^*.

IV. Fixed Wage Contracts with Commitment to Contract Duration

Firms often face contractual frictions that prevent them from offering a fully flexible wage schedule. Firms may face limitations on how fast the wage can grow, i.e., the wages may be revised only at an annual frequency or at the end of the contract term. To account for such contractual limitations, we assume that firms can only use contracts where the wage is constant throughout the contract duration. Such contracts are also observed in the real world, as documented by Ewens et al. (2020), who show that venture capitalist firms commonly award fixed wage contracts to entrepreneurs until a verifiable signal about the quality of the venture is produced within a specific period. Note that, unlike flexible wage contracts, with fixed wage contracts firm's ability to commit to the duration of the contract is important. In this section, we assume that firms have this commitment power and relax

[7] This is because $\overline{v} \to V^p(\mu, \overline{T})$ implies $\hat{v} \to V^p(\mu, \overline{T})$ and $q(\boldsymbol{\omega}; \overline{v}) \to 0$.

this assumption in Section VI. Throughout the paper, we assume that workers can quit at any point during the contract duration if they prefer their outside option. Therefore, workers may quit before the contract term ends. The contract term is irrelevant if the worker quits before the contract term expires. Both, the wage and the contract duration are determined in equilibrium.

A. *Worker's Quitting Decision*

We denote a fixed wage contract by $\boldsymbol{\omega} = (w, T)$ where w is the wage and T is the contract duration. To solve for the quitting time, in this sub-section we temporarily assume that the firms offer lifetime employment ($T \to \infty$). We then solve for profit maximizing contracts taking the worker's quitting time as a constraint on the contract duration.

Denote the worker's quitting time by t_q. We substitute a fixed wage in Equation (6) to derive the differential equation governing the worker's post-match continuation payoff $v(\mu, w, t, t_q)$ when the worker is employed is,

$$-\frac{dv(\mu, w, t, t_q)}{dt} = w + \lambda\gamma(\mu, t)v_h - (\rho + \delta + \lambda\gamma(\mu, t))v(\mu, w, t, t_q) \tag{19}$$

Solving the differential equation, and imposing the boundary condition that the value to the worker for all $t \geq t_q$ is v_0 we obtain:

$$\begin{aligned} v(\mu, w, t, t_q) - v_0 = \gamma(\mu, t) &\left(\frac{w + \lambda v_h}{\rho + \delta + \lambda} - v_0\right)\left(1 - e^{-(\rho+\delta+\lambda)(t_q - t)}\right) \\ &+ (1 - \gamma(\mu, t))\left(\frac{w}{\rho + \delta} - v_0\right)\left(1 - e^{-(\rho+\delta)(t_q - t)}\right) \end{aligned} \tag{20}$$

By setting $t = 0$ in the previous expression, we get the expected post-match payoff as a function of the quitting time,

$$\begin{aligned} v(\mu, w, t_q) - v_0 = \mu &\left(\frac{w + \lambda v_h}{\rho + \delta + \lambda} - v_0\right)\left(1 - e^{-(\rho+\delta+\lambda)t_q}\right) \\ &+ (1 - \mu)\left(\frac{w}{\rho + \delta} - v_0\right)\left(1 - e^{-(\rho+\delta)t_q}\right) \end{aligned} \tag{21}$$

The expected payoff comprises two terms. The first term captures the payoff if the worker is productive and the second term captures the payoff if the worker is unproductive. The worker chooses t_q to maximize $v(\mu, w, t_q)$. We denote,

$$\overline{T}_q(\mu, w) = \arg\max_{t_q} v(\mu, w, t_q)$$

Observe from (21) that, if the offered wage is above $v_0(\rho+\delta)$, then $\overline{T}_q(\mu, w) \to \infty$. This is because for all $w \geq v_0(\rho+\delta)$, regardless of the belief μ, the worker is better off working for the firm, and therefore never quits. On the other hand, if the offered wage is below $v_0(\rho+\delta+\lambda) - \lambda v_h$ then $\overline{T}_q(w, \mu) = 0$. This is because for all $w < v_0(\rho+\delta+\lambda) - \lambda v_h$, even if the match is productive for sure, the worker is not getting compensated adequately and she is better off quitting right away. Combining these observations, without loss of generality, we restrict attention to $w \in (v_0(\rho+\delta+\lambda) - \lambda v_h, v_0(\rho+\delta))$. If the offered wage w is in this range, increasing the quitting time provides a marginal benefit if the worker is productive, however, if the worker is unproductive, then the worker incurs a marginal loss. As the quitting time increases, the marginal benefit declines faster than the marginal loss as the worker puts less weight on being productive. So the value function is hump-shaped in the quitting time. For the worker to accept employment, the marginal benefit $\lambda\mu(v_h - v_0)$ must exceed the marginal loss $(v_0(\rho+\delta) - w)$ at $t_q = 0$ leading to the following lemma.

LEMMA 1: *Let $\underline{\mu}(w)$ be defined as:*

$$\underline{\mu}(w) := \frac{1}{\lambda}\left(\frac{v_0(\rho+\delta) - w}{v_h - v_0}\right).$$

A worker with prior belief μ accepts employment, i.e. $\overline{T}_q(\mu, w) > 0$, if and only if $\mu > \underline{\mu}(w)$.

Intuitively, the threshold belief is the ratio of the flow marginal loss and the marginal benefit. Workers are more likely to join if the post-success payoff is much higher relative to the loss in earnings due to the low flow wage during the experimentation phase. The following proposition characterizes the worker's quitting time.

PROPOSITION 3: *If the firm does not fire the worker, the workers endogenously quit employment at time $\overline{T}_q(\mu, w)$,*

$$\overline{T}_q(\mu, w) = \begin{cases} 0 & w \leq v_0(\rho+\delta) - \lambda\mu(v_h - v_0) \\ \frac{1}{\lambda} \cdot \left[\ln\left(\frac{\mu}{1-\mu}\right) - \ln\left(\frac{\underline{\mu}(w)}{1-\underline{\mu}(w)}\right)\right] & w \in (v_0(\rho+\delta) - \lambda\mu(v_h - v_0), v_0(\rho+\delta)) \\ \infty & w \geq v_0(\rho+\delta) \end{cases}$$

The quitting time is increasing in the belief of the worker, μ and w so that workers who have stronger beliefs about being productive quit later for a given wage and workers with a given belief quit later when offered a higher wage.

The proof of Proposition 3 is in Appendix B.B1. The workers' quitting time is the difference between the log odds ratio of the worker being productive $\left(\frac{\mu}{1-\mu}\right)$ and the threshold belief $\left(\frac{\underline{\mu}(w)}{1-\underline{\mu}(w)}\right)$ at a given wage, weighted by the inverse of the speed of learning λ. The higher the difference between the worker's type and the threshold belief, the worker works longer before quitting. Moreover, for a given difference between the log odds ratios, a higher speed of learning would reduce the quitting time because the worker learns about her type, or the match productivity faster.

B. *Threshold Wage*

Recall from Section III that the contract duration, $\overline{T}$, chosen by a planner who maximizes the expected payoff from a match is

$$\overline{T} = \frac{1}{\lambda}\left[\log\left(\frac{\mu}{1-\mu}\right) - \log\left(\frac{\underline{\mu}_p}{1-\underline{\mu}_p}\right)\right]$$

where $\underline{\mu}_p$ solves,

$$\underline{\mu}_p\left(\frac{\lambda}{\rho+\delta}\right)\left(\overline{y} + \frac{\lambda\overline{y}+\underline{y}}{\rho+\delta} - v_0\right) = v_0 - \frac{\underline{y}}{\rho+\delta}$$

Whether the worker quits earlier or later than the optimal experimentation duration depends on her wage, since the quitting time $\overline{T}_q(\mu, w)$ increases with w. Let $\tilde{w}$ be the threshold wage such that the quitting time is exactly the optimal experimentation duration, i.e. $\overline{T}_q(\mu, \tilde{w}) = \overline{T}$. At $\tilde{w}$ the threshold beliefs must be equalized, $\underline{\mu}_p = \underline{\mu}(\tilde{w})$, hence

$$\tilde{w} = v_0(\rho+\delta)\left(1 - \frac{\underline{\mu}_p}{\underline{\mu}(0)}\right)$$

In the above expression, $\underline{\mu}(0)$ is the threshold belief at which the worker immediately quits if the offered wage is 0 and it is the outside option flow payoff relative to the post-success payoff gain. This implies that for a given value of the total post-success match surplus, if the post-success payoff to the worker is lower, then the threshold belief corresponding to 0 wage is higher. Hence, the worker must be paid a higher wage to experiment optimally before quitting. If the equilibrium wage is below this threshold, then the worker quits sooner than the optimal experimentation duration. Note that this threshold wage is independent of the initial belief μ because the initial belief increases the optimal experimentation time and the quitting time by the same amount.

C. Competitive Search Equilibrium

The competitive search equilibrium is the tuple $(w^*, T^*, q^*, \overline{v}^*, u^*)$ as in Definition 1, where the firms must maximize their value by choosing the contract features (w, T) taking into account the search behavior of the workers, $q(\mu, w, T; \overline{v})$. The free-entry condition pins down $\overline{v}^*$, and the workers' entry condition u^*, the equilibrium level of unemployment.

PROPOSITION 4: *The competitive search equilibrium,* $(w^*, T^*, q^*, \overline{v}^*, u^*)$ *when the firms offer fixed-wage contracts* $\boldsymbol{\omega} = (w, T)$ *exists.*

The proof of existence follows similar arguments as in Proposition 2 with the modification that the contracts are fixed-wage contracts.[8]

Note that without loss of generality, we can restrict attention to contracts (w, T) such that $T \leq \overline{T}_q(\mu, w)$. When $T = \overline{T}_q(\mu, w^*)$, the firm prefers (weakly) a longer contract duration but the worker quits at T. When $T < \overline{T}_q(\mu, w^*)$, the worker prefers a longer contract duration but the firm fires the worker at T. Hence, there are two types of equilibria.

1) **Quitting Equilibrium** where $T^* = \overline{T}_q(\mu, w^*)$.

2) **Tenure Equilibrium** where $T^* < \overline{T}_q(\mu, w^*)$.

Lemma 2 shows that the type of equilibrium that arises depends on whether the equilibrium wage is above or below the threshold wage.

LEMMA 2: *If the equilibrium contract wage is such that,* $w^* \leq \tilde{w}$*, then the equilibrium is a quitting equilibrium, i.e.* $T^* = \overline{T}_q(\mu, w^*)$*. Otherwise, the equilibrium is a tenure equilibrium.*

PROOF:

For $w^* \leq \tilde{w}$, suppose that $T^* < \overline{T}_q(\mu, w^*)$. At the equilibrium wage, $\partial V(\mu, w^*, T^*)/\partial T > 0$ and $\partial v(\mu, w^*, T^*)/\partial T > 0$. Therefore, a subset of firms could always deviate to $T =$

[8] A detailed proof is given in Appendix A.

$\overline{T}_q(\mu, w^*)$. The deviating firms and the workers will be better off with this new contract. Hence, for $w^* \leq \tilde{w}$, $T < \overline{T}_q(\mu, w^*)$ cannot be an equilibrium. We have already ruled out $T > \overline{T}_q(\mu, w^*)$ because the worker quits at $\overline{T}_q(\mu, w^*)$ and the offered contract term is irrelevant. Therefore, if $w^* \leq \tilde{w}$, then $T^* = \overline{T}_q(\mu, w^*)$.

On the other hand, if $w^* > \tilde{w}$ and $T^* = \overline{T}_q(\mu, w^*) > \overline{T}$, then the firms can deviate to a new contract (w', T') where $w' > w^*$ and $\overline{T} \leq T' < \overline{T}_q(\mu, w^*)$ such that $v(\mu, w^*, \overline{T}_q(\mu, w^*)) = v(\mu, w', T')$. The workers' indifference along with $T' \in \left[\overline{T}, \overline{T}_q(\mu, w^*)\right)$ implies that $V(\mu, w', T') > V(\mu, w^*, \overline{T}_q(\mu, w^*))$. This follows because with the workers indifferent, reducing the contract duration improves the overall match surplus $V^p(\mu, T)$ $(\partial V^p(\mu, T)/\partial T < 0$ for all $T > \overline{T})$, and increases the firms' expected payoff. The deviating firms are better off and the workers are indifferent. Therefore, for $w^* > \tilde{w}$, $T^* = \overline{T}_q(\mu, w^*)$ cannot be an equilibrium.

The above lemma shows that if the equilibrium wage is below the threshold wage $\tilde{w}$, then the equilibrium features workers quitting. However, if the equilibrium wage strictly exceeds this threshold then the offered contract term binds and the workers are fired by the firm if they do not succeed before the term expires. But what is the equilibrium contract term offered by the firms and how does it compare with the optimal experimentation duration $\overline{T}$?

To answer this question, note that Lemma 2 implies that if $T^* < \overline{T}_q(\mu, w^*)$ it must be that $w^* > \tilde{w}$. From the definition of $\tilde{w}$, this implies that the optimal experimentation duration is shorter than the quitting time, $\overline{T} < \overline{T}_q(\mu, w^*)$. If $T^* > \overline{T}$, then the firms can deviate to a contract offering $\boldsymbol{\omega}' = (w', T')$ where $\tilde{w} < w^* < w'$ and $\overline{T} < T' < T^*$ such that $v(\mu, w', T') = v(\mu, w^*, T^*)$, so that the workers are indifferent. The workers' indifference along with $T' \in \left(\overline{T}, T^*\right)$ implies that $V(\mu, w', T') > V(\mu, w^*, T^*)$. This follows because with the workers indifferent, reducing the contract duration improves the overall match surplus $V^p(\mu, T)$ $(\partial V^p(\mu, T)/\partial T < 0$ for all $T > \overline{T})$, and increases the firms' expected payoff. The workers are indifferent, and the deviating firms are better off. Hence, $T^* > \overline{T}$ cannot be an equilibrium contract duration. An exactly similar argument applies for $T^* < \overline{T}$ where the firm deviates to offer $\tilde{w} < w' < w^*$ and $\overline{T} > T' > T^*$ which keeps the workers indifferent and the deviating firms are better off. This implies that $T^* < \overline{T}$ cannot arise in equilibrium either. Therefore, it must be that $T^* = \overline{T}$ for all $w^* > \tilde{w}$, giving us the following theorem.

THEOREM 1: *The tenure equilibrium results in optimal experimentation, i.e.* $T^* = \overline{T}$, *and the quitting equilibrium results in sub-optimal experimentation, i.e.* $T^* < \overline{T}$

The reader may refer to Appendix A for a more detailed proof. The tenure equilibrium contract is $\boldsymbol{\omega}^* = (w^*, \overline{T})$ with $w^* > \tilde{w}$, otherwise $\boldsymbol{\omega}^* = (w^*, \overline{T}_q(\mu, w^*))$. From Definition 1 the equilibrium contract, and the queue length are the tangency point between the indifference curve of the worker corresponding to the payoff $\overline{v}^*$, and the iso-profit curve of the firms corresponding to 0 profit (due to free-entry).[9] The following proposition highlights the equilibrium split of the surplus between the worker and the firm,

PROPOSITION 5: *The post-match expected payoff for the worker in equilibrium is,*

$$v(\mu, w^*, T^*) = \overline{v}^* + \frac{1 - \varepsilon(q^*)}{1 - \varepsilon(q^*)(1 - \Theta(\mu, w^*))} \cdot (V^P(\mu, T^*) - \overline{v}^*) \tag{22}$$

where $\varepsilon(q) = -q\alpha_u'(q)/\alpha_u(q)$ is the elasticity of the arrival rate of jobs and

$$1 - \Theta(\mu, w^*) = \frac{\frac{\partial V^p(\mu, T^*)}{\partial T} \cdot \frac{dT^*}{dw}}{\frac{dv(\mu, w^*, T^*)}{dw}} \in (0, 1]$$

is the marginal gain in total surplus from incremental wage increase relative to the marginal increase in worker's payoff.

Proposition 5 shows that the equilibrium post-match payoff of the worker can be interpreted as a solution to the Nash-Bargaining problem between the firm and the worker, in which $(1-\varepsilon(q^*))/(1-\varepsilon(q^*)(1-\Theta(\mu, w^*)))$ is analogous to the worker's bargaining power and $\overline{v}^*$ is the disagreement payoff of the worker. The firm's bargaining power is $(\varepsilon(q^*)\Theta(\mu, w^*))/(1-\varepsilon(q^*)(1 - \Theta(\mu, w^*)))$ and the disagreement payoff is 0. First, note that if $\varepsilon(q)$ is weakly increasing in q, then as q declines, the worker's bargaining power is higher.[10] Second, in addition to the bargaining power of workers due to the elasticity of matching rates, the endogenous experimentation also plays a role in determining the worker's share. In quitting equilibrium, an increase in wage leads to an increase in the contract duration and raises the total surplus. If this increase is large relative to the direct increase in the worker's payoff, then the worker's bargaining power is higher, and the worker parts with a larger proportion of the net surplus. This effect exists due to endogenous experimentation and learning. Note that in a tenure equilibrium, the split of the surplus only depends on matching rate elasticity as in the Hosios Efficiency Condition (Hosios (1990)) for dynamic

[9]If not, then at (w^*, q^*), the indifference curve of the worker intersects the 0 profit iso-profit curve either from below or above. If the former, then there is a wage, queue length pair, $(w', q') > (w^*, q^*)$ such that $v^u(\mu, w^*, q^*) = v^u(\mu, w', q')$ but $V^u(\mu, w', q') > 0$. Therefore, (w^*, q^*) cannot be an equilibrium. An exactly similar argument applies for the latter case when the indifference curve intersects the iso-profit from above. only that $(w', q') < (w^*, q^*)$.

[10]The elasticity of job arrival rate, $\varepsilon(q)$ is increasing in q for standard matching functions such as Cobb-Douglas and the urn-ball matching function

search. In the next section, we compare the equilibrium surplus allocation with that of a planner and check whether the competitive search equilibrium is efficient.

V. Efficiency

Moen (1997) shows that without learning and experimentation competitive search with wage posting is efficient. However, with workers and firms learning about the worker's type through experimentation, it is possible that in equilibrium, the match dissolves too soon or too late and the equilibrium is no longer efficient.

The planner's goal is to maximize the total surplus generated in the economy taking the search frictions (i.e. the matching function and search cost) as given. To achieve this goal, the planner chooses the number of vacancies and the experimentation duration subject to the evolution of unemployed workers in the economy. In other words, the planner solves the following dynamic optimization:

$$\max_{q(t),T} \int_0^\infty u(t) \left(v_0(\rho+\delta) + \alpha_u(q(t)) V^p(\mu,T) - \frac{\kappa}{q(t)} \right) e^{-\rho t} dt$$

subject to the law of motion,

$$\dot{u}(t) = \delta - (\delta + \alpha_u(q(t)))u(t).$$

The planner's problem is stationary and the only state variable is the number of unemployed workers, which evolves depending on the choice of (q,T). We denote the planner's value function by $\overline{V}(\mu,u)$. The planner's solution satisfies the following HJB equation,

$$\rho \overline{V}(\mu,u) = \max_{q,T} \left\{ u \left(v_0(\rho+\delta) + \alpha_u(q) \cdot V^p(\mu,T) - \frac{\kappa}{q} \right) + \frac{d\overline{V}(\mu,u)}{du} \dot{u} \right\} \tag{23}$$

where $\dot{u} = \delta - (\delta + \alpha_u(q))u$. The first expression on the RHS is the flow output generated in the economy net of search cost. The second part captures the change in continuation payoff, given the law of motion for the measure of unemployed workers.[11]

To solve for the value function of the planner, $\overline{V}(\mu,u)$, the solution to (23), we guess and verify

$$\overline{V}(u) = A + B \cdot u$$

[11] The mapping in (23) is a contraction, therefore, the value function is unique.

We can interpret A as the continuation value if there are no unemployed workers, and B, the flow payoff generated by a single unemployed worker. Note that we have suppressed μ but (A, B) are functions of μ along with the other parameters. We solve for the value function, using the first-order condition with respect to (q, T),

$$\varepsilon(q)\alpha_u(q)B = \varepsilon(q)\alpha_u(q)V^p(\mu, T) - \frac{\kappa}{q}\ ,\ T = \overline{T} \tag{24}$$

and the envelope theorem,

$$(\rho + \delta + \alpha_u(q))B = v_0(\rho + \delta) + \alpha_u(q)V^p(\mu, \overline{T}) - \frac{\kappa}{q} \tag{25}$$

Combining the two conditions, (24) and (25),

$$\frac{(\rho + \delta)(V^p(\mu, \overline{T}) - v_0)}{\kappa} = \frac{(\rho + \delta) + (1 - \varepsilon(q))\alpha_u(q)}{\alpha_v(q)\varepsilon(q)} \tag{26}$$

Equation (26) pins down the queue length chosen by the planner. If $\varepsilon(q)$ is weakly increasing in q, then the right-hand side is strictly decreasing in q. The left-hand side is the flow post-match surplus relative to the search cost and is independent of the queue length. This gives a unique solution, q. Notice that the efficient (q, T) are independent of u, which verifies our initial guess about the planner's value function.

To see how this condition compares to the equilibrium, note that Theorem 1 states that in the tenure equilibrium, $T^* = \overline{T}$, and the tangency condition (22) simplifies to,

$$v(\mu, w^*, \overline{T}) = \overline{v}^* + (1 - \varepsilon(q^*)) \cdot \left(V^p(\mu, \overline{T}) - \overline{v}^*\right)$$

This is because $\Theta(\mu, w^*) = 1$ when $T^* = \overline{T}$. To further simplify the tangency condition to eliminate w^*, we make the following substitutions,

$$(\rho+\delta+\alpha_u(q^*))(v(\mu, w^*, \overline{T}) - \overline{v}^*) = (\rho+\delta)(v(\mu, w^*, \overline{T}) - v_0) = (\rho+\delta)\left(V^p(\mu, \overline{T}) - v_0 - \frac{\kappa}{\alpha_v(q^*)}\right)$$

The first equality comes from the worker's indifference condition and the second from the free-entry. Rearranging the terms, results in

$$\frac{(\rho + \delta)(V^p(\mu, \overline{T}) - v_0)}{\kappa} = \frac{(\rho + \delta) + (1 - \varepsilon(q^*))\alpha_u(q^*)}{\alpha_v(q^*)\varepsilon(q^*)} \tag{27}$$

This is identical to Equation (26). A higher post-match value relative to the search cost induces more firms to enter, and the queue length declines. Therefore, we have the following theorem,

THEOREM 2: *The competitive search equilibrium is efficient*

1) *under fully flexible wage contracts.*
2) *under fixed-wage contracts if the equilibrium wage is high enough, $w^* \geq \tilde{w}$, or the offered contract duration binds.*

The proof follows from Proposition 2 and Theorem 1 because the equilibrium tenure is optimal, $T^* = \overline{T}$. As shown in arguments preceding the Theorem 2, if the equilibrium tenure is optimal, then the equilibrium tangency condition coincides with (26), and therefore the equilibrium is efficient.

To understand the intuition behind this result, note that under directed search, the firms internalize the workers' search behaviour and their preference for more lucrative contracts and shorter queue. With free-entry, firms out-compete each other by posting more lucrative contracts and doing so until the expected payoff from a vacancy is driven to 0. In equilibrium, the firms' and the workers' marginal rates of substitution of queue lengths for the contract payoff are equalized. As long as the experimentation under this tangency contract is $\overline{T}$, the planner cannot change the queue to make the firms (or workers) better off without making the workers (or firms) worse off. With fully flexible wages, the firms offer contracts that allow for optimal experimentation (Proposition 2), and hence the equilibrium queue length is the same as the efficient benchmark. With fixed-wage contracts, as shown in Theorem 1, under a tenure equilibrium the experimentation duration is optimal. Therefore, a tenure equilibrium with fixed wages is efficient.

Note that if the equilibrium fixed-wage is below the threshold wage $\tilde{w}$, or in other words the wage is so low that workers end up quitting, then the post-match surplus is lower and hence, the expected benefit from faster job arrival is lower. As a result, fewer firms post vacancies and the equilibrium queue length is longer than the efficient benchmark. The next result (Corollary 1) highlights that such an inefficient outcome arises in equilibrium if the search cost faced by firms is sufficiently high.

COROLLARY 1: *If κ is greater than $\overline{\kappa}$, then the fixed-wage competitive search equilibrium* *with contract duration commitment is not efficient. The threshold $\overline{\kappa}$ is such that the q that solves the equation* (26) *with $\kappa = \overline{\kappa}$, satisfies the free-entry condition at $w = \tilde{w}$.*

To better understand this result, note that for efficiency the queue length must solve (26). This queue length when substituted in the free-entry condition,

$$v(\mu, w, \overline{T}) = V^p(\mu, \overline{T}) - \frac{\kappa}{\alpha_v(q)}$$

gives us the wage at the point of tangency, conditional on $T^* = \overline{T}$ because the RHS is independent of w and the LHS is increasing in w. As long as this wage is higher than the threshold $\tilde{w}$, the experimentation duration $\overline{T}$, the queue length, and the wage are consistent with the equilibrium. We can substitute for q (say $\tilde{q}$ which is increasing in κ) that solves (26) into the free-entry condition.

$$v(\mu, w, \overline{T}) = V^p(\mu, \overline{T}) - \frac{\varepsilon(\tilde{q})(\rho + \delta)}{(\rho + \delta + (1 - \varepsilon(\tilde{q}))\alpha_u(\tilde{q}))}(V^p(\mu, \overline{T}) - v_0)$$

Notice that as κ increases, $\tilde{q}$ increases and the left-hand side of the above equation declines. It follows that the wage solving the free-entry declines. Therefore, there must exist a $\overline{\kappa}$ such that for all $\kappa > \overline{\kappa}$, the wage is below the threshold $\tilde{w}$ and therefore in equilibrium $T^* < \overline{T}$. For all $\kappa \geq \overline{\kappa}$, $w^* \geq \tilde{w}$ and $T^* = \overline{T}$. Put differently, under a tenure equilibrium as the search cost faced by the firms increases, fewer firms enter, and the equilibrium wage declines. There is a threshold search cost at which the equilibrium wage in the tenure equilibrium coincides with the threshold wage $\tilde{w}$. For all search cost values above this threshold, the equilibrium entails quitting, and sub-optimal experimentation.

To summarize, we find that the competitive search equilibrium is efficient if the firms can choose fully flexible wage contracts, or under fixed wages if the workers do not quit in equilibrium. It is important to point out that the equilibrium duration is optimal because the firms can commit to the contract duration. In the next section, we analyze the equilibrium when firms cannot commit to any arbitrary contract duration.

VI. Fixed Wages Without Commitment to Contract Duration

In this section, we analyze fixed-wage contracts when the firms cannot commit to a contract duration to understand how commitment affects the equilibrium. In practice, such a situation may arise when contract duration commitments cannot be enforced legally. For example, in developing countries with poor legal infrastructure or countries where worker termination laws are not strictly enforced. Without commitment, a firm may deviate from the contractual term and fire the worker sooner if it improves their expected payoff. Such deviations should not be possible in equilibrium, and the firing time must be such that it maximizes the firms' post-match expected payoff.

A. *Firms' Firing Decision*

Suppose that the match continues until time T. The expected payoff for the firm after the match is formed is derived following similar steps as in Section IV. The firm's expected payoff from the match is,

$$V(\mu, w, T) = \mu \cdot \frac{\underline{y} - w + \lambda(V_h + \overline{y})}{\rho + \delta + \lambda}\left(1 - e^{-(\rho+\delta+\lambda)T}\right) + (1-\mu)\frac{\underline{y} - w}{\rho + \delta}\left(1 - e^{-(\rho+\delta)T}\right) \tag{28}$$

The firm value given by (28) is analogous to the worker's value in (21). The first term is weighted by the belief that the match is productive and captures the firm's post-match payoff from a productive match. Similarly, the second term captures the post-match payoff conditional on being unproductive. We can solve for the firm's choice of $\overline{T}_f$ by assuming that the worker does not quit employment. We proceed as we did in Section IV. If $w < \underline{y}$ then the expected post-match payoff is increasing in T, and the firm never fires the worker ($\overline{T}_f \to \infty$). Similarly, if $w > \underline{y} + \lambda(V_h + \overline{y})$ then the expected payoff is decreasing in T and the firm chooses to fire the worker immediately ($\overline{T}_f = 0$). We focus on the the wage $w \in \left(\underline{y}, \underline{y} + \lambda(V_h + \overline{y})\right)$.

The firm chooses $T = \overline{T}_f$ to maximize $V(\mu, w, T)$.

$$\overline{T}_f(\mu, w) = \arg\max_T V(\mu, w, T)$$

For $w \in \left(\underline{y}, \underline{y} + \lambda(V_h + \overline{y})\right)$, the first expression in equation (28) is increasing in T and the second decreasing in T. If the worker works incrementally longer, the first part results in a marginal benefit to the firm if the match is productive, and the second part results in a marginal loss if the match is unproductive. The firm chooses $\overline{T}_f$ where the marginal loss $w - \underline{y}$ and the benefit $\lambda\mu(\overline{y} + V_h)$ are equalized.

$$\lambda \underline{\mu}_f(\overline{y} + V_h) = w - \underline{y}$$

Just like the belief threshold $\underline{\mu}(w)$ for the worker, we have a similar belief threshold such that the firm fires the worker immediately if $\mu \leq \underline{\mu}_f(w)$.
The first order condition with respect to T of (28) gives us,

$$\overline{T}_f(\mu, w) = \begin{cases} \infty & w \leq \underline{y} \\ \frac{1}{\lambda}\left[\ln\left(\frac{\mu}{1-\mu}\right) - \ln\left(\frac{\underline{\mu}_f(w)}{1-\underline{\mu}_f(w)}\right)\right] & w \in (\underline{y}, \underline{y} + \lambda\mu(V_h + \overline{y})) \\ 0 & w \geq \underline{y} + \lambda\mu(V_h + \overline{y}) \end{cases} \tag{29}$$

The firing time $\overline{T}_f$ is higher if the log odds ratio of the worker being productive is higher relative to the log odds ratio of the threshold belief, $\underline{\mu}_f(w)/(1 - \underline{\mu}_f(w))$ and this difference is weighted by the inverse of the learning rate λ. For the same difference in the log odds ratio, if the learning rate is faster the worker is fired sooner.

B. Endogenous Experimentation Duration

The endogenous experimentation duration is the shorter of the endogenous quitting time and the firms' choice of firing time.

$$\hat{T}(\mu, w) = \min\{\overline{T}_q(\mu, w), \overline{T}_f(\mu, w)\}$$

Notice that $\overline{T}_q(\mu, w)$ is increasing in w and $\overline{T}_f(\mu, w)$ is decreasing in w. Inequalities in (1) imply that $v_0(\rho + \delta) > \underline{y}$ and $v_0(\rho + \delta) - \lambda\mu(v_h - v_0) < \underline{y} + \lambda\mu(V_h + \overline{y})$[12]. For all $w < \max\{v_0(\rho + \delta) - \lambda\mu(v_h - v_0), \underline{y}\}$, $\overline{T}_q(\mu, w) < \overline{T}_f(\mu, w)$. For low enough wage, the workers are not compensated enough and they prefer quitting before the firm fires. For all $w > \min\{v_0(\rho+\delta), \underline{y}+\lambda\mu(V_h+\overline{y})\}$, $\overline{T}_f(\mu, w) < \overline{T}_q(\mu, w)$. For a high enough wage, the firms do not earn sufficient profit and they prefer firing the worker. Moreover, the continuity of $\overline{T}_q(\mu, w)$ and $\overline{T}_f(\mu, w)$ for $w \in \big(\max\{v_0(\rho + \delta) - \lambda\mu(v_h - v_0), \underline{y}\}, \min\{v_0(\rho + \delta), \underline{y} + \lambda\mu(V_h + \overline{y})\}\big)$ implies that the difference between the quitting time and the firing time, $(\overline{T}_q(\mu, w) - \overline{T}_f(\mu, w))$ is continuously increasing in w and is equal to 0 for some value of w. That is, there is some intermediate wage at which the quitting time for the worker and the firing time for the firm coincide. Interestingly, this wage is the same as $\tilde{w}$.

[12] The above follows from the second inequality in (1). To see this, substitute the total surplus $V_h + v_h = \dfrac{\lambda\overline{y} + \underline{y}}{\rho + \delta}$ into $v_0(\rho + \delta) + \lambda\mu v_0 - \lambda\mu(v_h + V_h) - (\underline{y} + \lambda\overline{y}) = \left(\dfrac{\rho + \delta + \lambda\mu}{\rho + \delta}\right)\left(v_0(\rho + \delta) - \dfrac{\lambda\overline{y} + \underline{y}}{\rho + \delta}\right) < 0$

To see this, note that

$$V(\mu, w, T) = V^p(\mu, T) - v(\mu, w, T)$$

and, at $w = \tilde{w}$, $\overline{T}(\mu) = \overline{T}_q(\mu, w)$

$$\frac{\partial V(\mu, w, T)}{\partial T} = \underbrace{\frac{\partial V^p(\mu, T)}{\partial T}}_{=0} - \underbrace{\frac{\partial v(\mu, w, T)}{\partial T}}_{=0} = 0$$

Therefore, it must be that

$$\overline{T}_f(\mu, \tilde{w}) = \overline{T}_q(\mu, \tilde{w}) = \overline{T}(\mu)$$

Figure (1) illustrates, how $\hat{T}(\mu, w)$ is determined. The endogenous quitting time increases with wage, whereas the firing time $\overline{T}_f$ decreases. They intersect at $\tilde{w}$.

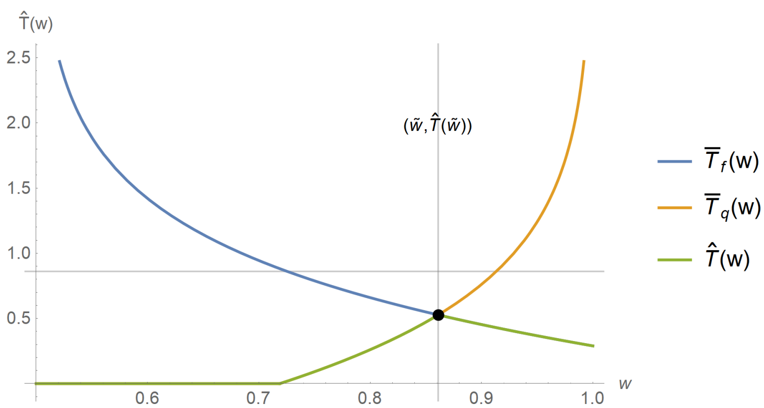


Figure 1. : Quitting/Firing time vs. offered fixed wage w

C. Expected Match Payoff

We can now state the expected post-match payoffs for the worker and the firm, given that the match dissolves endogenously at $\hat{T}(\mu, w)$.

$$v(\mu, w) = \begin{cases} v(\mu, w, \overline{T}_q(w)) & w \leq \tilde{w} \\ v(\mu, w, \overline{T}_f(w)) & w > \tilde{w} \end{cases}$$

and

$$V(\mu, w) = \begin{cases} V(\mu, w, \overline{T}_q(w)) & w \leq \tilde{w} \\ V(\mu, w, \overline{T}_f(w)) & w > \tilde{w} \end{cases}$$

D. *Competitive Search Equilibrium*

The competitive search equilibrium in the case of fixed wage contracts without commitment to contract duration is as per the Definition 1 with $\boldsymbol{\omega} = w$. The offered wage endogenously pins down the experimentation duration, and therefore the contracts are fully characterized by only the wage. The firms' expected payoff is

$$\rho V^u(\mu, w, q) = -\kappa + \alpha_v(q)(V(\mu, w) - V^u(\mu, w, q))$$

and the workers' expected payoff is

$$(\rho + \delta)v^u(\mu, w, q) = v_0(\rho + \delta) + \alpha_u(q)(v(\mu, w) - v^u(\mu, w, q))$$

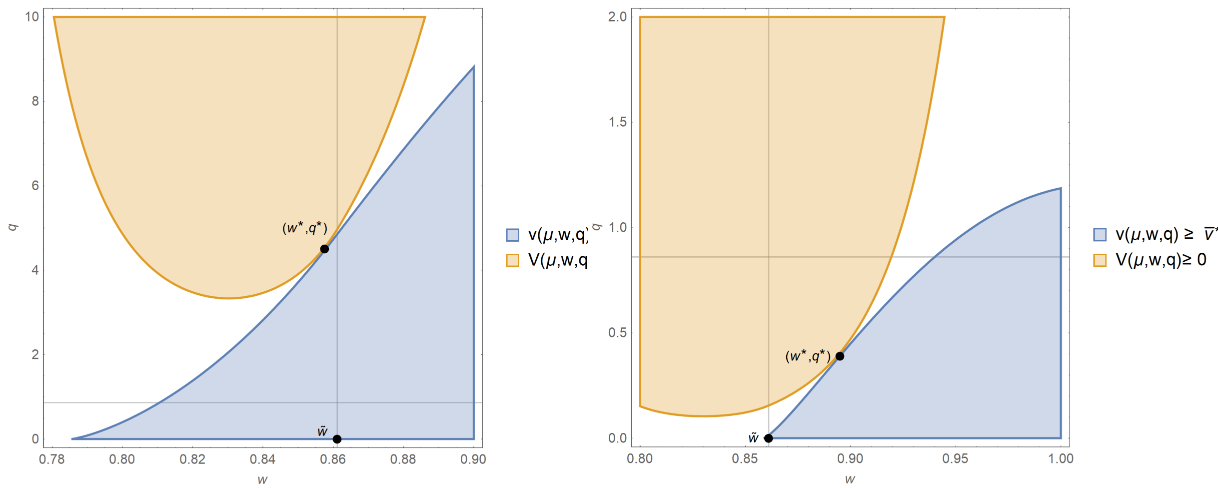
PROPOSITION 6: *The competitive search equilibrium $(w^*, q^*, u^*, \overline{v}^*)$ exists under a fixed-wage contract. The equilibrium contract entails two possibilities, (a) Quitting: workers quit employment (b) Tenure: workers are fired after a certain duration if they remain unproductive.*

The arguments for existence are similar to Proposition 4. Please refer to Appendix A for the detailed proof. Figure (2) illustrates the two possibilities for the equilibrium. Which type of equilibrium firms and workers end up with depends on the parameters of the model. For example, as already shown in Corollary (1), if $\kappa > \overline{\kappa}$, then there is quitting in equilibrium, i.e. $w^* < \tilde{w}$. At $\kappa = \overline{\kappa}$, $T^* = \overline{T}_f = \overline{T}_q = \overline{T}$ and $w^* = \tilde{w}$.

VII. Discussion

Interestingly, the fixed-wage contract without commitment to contract duration is efficient only for the edge case, when $w^* = \tilde{w}$. For any other value of the equilibrium wage, the experimentation is sub-optimal, and therefore, the equilibrium queue length is longer than what a planner prefers.

To understand why any wage other than $\tilde{w}$ results in sub-optimal experimentation, we can think of the firm and the worker as being involved in a two-sided two-armed bandit



(a) Quitting Equilibrium, $w^* \leq \tilde{w}$ ($\kappa = \kappa_0$) (b) Tenure Equilibrium, $w^* > \tilde{w}$ ($\kappa = 0.5\kappa_0$)

Figure 2. : Competitive Search Equilibrium under Fixed-wage contracts.

problem. Both sides, the firm, and the worker have the option to dissolve the match (quit or fire), the safe arm, or continue gambling to get a high output realization, the risky arm. Only if both parties choose the risky arm, does the experimentation continue, otherwise the match breaks down. With the workers and firms constrained to fixed wage contracts, when $w < \tilde{w}$ the split of the surplus is such that the firm gains too much compared to the worker, hence the worker quits earlier and vice versa for $w > \tilde{w}$. Only at $w = \tilde{w}$, the split is such that the time of separation coincides with the optimal experimentation duration, otherwise the match dissolves sub-optimally.

Under fully flexible wage contracts, the firm compensates the worker just sufficiently so that she doesn't quit. Then conditional on the expected compensation to the worker the firm's maximization problem coincides with that of the planner. Therefore, the firm chooses an optimal stopping time that maximizes the total expected post-match surplus. We can interpret the result as if under fixed wages, the firm and the worker are restricted in dividing the expected surplus between them which leads to sub-optimal experimentation. This implies that in instances where they can write flexible contracts based on the future output realizations, the job tenure will be longer, or the turnover will be lower. There is empirical evidence that shows that employee ownership contracts even for rank-and-file employees reduce turnover, and increase job tenure, Blasi et al. (2008). The ability to commit to a contract duration also mitigates this inefficiency. However, even in that case, it must be that the search friction is not too high, otherwise the worker quits in equilibrium.

VIII. Extensions

A. *Minimum wage or Worker Liquidity constraints*

Suppose that the firm faces a constraint that it cannot pay a wage below w_L. The reason for such a constraint could be due to an exogenously imposed minimum wage or the workers may face a liquidity constraint and they cannot accept a very low wage. To emphasize the role of commitment we continue to assume that the firm cannot commit to the contract duration, however, firms can offer fully flexible wages. In this case, the equilibrium experimentation duration may not be optimal depending on the value of the minimum wage relative to the optimal experimentation wage, $\tilde{w}$.

PROPOSITION 7: *If the firm cannot credibly commit to the tenure of the contract, there are two cases,*

1) *If the minimum wage $w_L \leq \tilde{w}$, then the firm offers a variable wage contract, and the match dissolves efficiently at time $\overline{T}$*

2) *If the minimum wage $w_L > \tilde{w}$, the equilibrium contract is such that the match dissolves inefficiently at $T^* < \overline{T}$.*

PROOF:
Case 1: *$w_L \leq \tilde{w}$: First, note that the contracts are $\boldsymbol{\omega} = (\hat{v}, T)$. We can focus on the set of contracts that pay the workers some minimum payoff $\overline{v}_L$ that prevents quitting given the minimum wage constraint, and pay the remaining $(\hat{v} - \overline{v}_L)(\geq 0)$ in cash bonus.*[13] *Note that the minimum payoff for the worker under this contract must be such that the worker is paid w_L until $w(t) = v_0(\rho + \delta) - \gamma(\mu, t)(v_h - v_0) \leq w_L$ and $w(t)$ thereafter. The continuation payoff for the worker at time $\hat{t}(w_L)$ is v_0 where $\hat{t}(w_L)$ is such that*

$$w(\hat{t}) = w_L = v_0(\rho + \delta) - \gamma(\mu, \hat{t})(v_h - v_0)$$

[13] Consider any other contract that promises a payoff $\hat{v}$, and the contract duration T that maximizes the firms' value. This contract can always be improved upon by the contract that pays $\overline{v}_L$ through wages and pays the remainder $\hat{v} - \overline{v}_L)$ in cash.

Note that if $w_L \leq \tilde{w}$, then $\hat{t}(w_L) = \overline{T}_q(w_L)$. In other words, the time at which the firm starts to increase the flow wage of the worker is the same at which the worker would have quit if she was paid a fixed wage w_L. The minimum the firm has to promise the worker is the following wage schedule

$$\tilde{\boldsymbol{\omega}} = w(t) = \begin{cases} w_L & t < \hat{t} = \overline{T}(w_L) \\ v_0(\rho+\delta) - \lambda\gamma(\mu,t)(v_h - v_0) & t \geq \hat{t} \end{cases}$$

The value to the worker under this wage schedule is the same as under a fixed wage w_L. Therefore, the minimum the worker earns under the wage restriction w_L is,

$$\overline{v}_L = v(\mu, w_L, \overline{T}_q(w_L)) = v(\mu, w_L)$$

In a competitive search equilibrium, the firm has to promise the worker a continuation payoff $\hat{v}$ at least as high as $\overline{v}_L$ at the start of employment. In this case, the firm offers a bonus payment, $B = (\hat{v} - \overline{v}_L)$ at the start of employment, and then pays as per the wage schedule $\tilde{\boldsymbol{\omega}}$.

$$\tilde{\boldsymbol{\omega}} = w(t) = \begin{cases} w_L & t < \hat{t}(w_L) \\ v_0(\rho+\delta) - \lambda\gamma(\mu,t)(v_h - v_0) & t \geq \hat{t}(w_L) \end{cases}$$

The equilibrium is along similar lines as in the flexible wage case with the modification that $\hat{v} \geq \overline{v}_L$. The equilibrium experimentation duration is optimal, $\overline{T}$. The firms do not deviate from this tenure because it maximizes their value. The equilibrium $\hat{v}^* = B^* + \overline{v}_L$ is determined as outlined in Section III.

Case 2: *$w_L > \tilde{w}$: Suppose the firm pays according to the variable wage contract as described in the previou case, and will start increasing the wage so as to avoid the worker from quitting for $t \geq \hat{t}(w_L) = \overline{T}_q(w_L)$. However, note that because $w_L > \tilde{w}$, it follows that $\overline{T}_q(w_L) > \overline{T} > \overline{T}_f(w_L)$. Under this wage contract promising the minimum wage to the worker, the firm wants to fire the worker before starting to raise the wage. We have the following relationship,*

$$\underbrace{V(\mu, w_L, \overline{T}_f(w_L))}_{\text{Maximum value under fixed wage } w_L} > \underbrace{V(\mu, w_L, \overline{T})}_{\text{Value to firm when the match dissolved at } \overline{T}} > V(\mu, w_L, \overline{T}_q(w_L))$$

Therefore for $w_L > \tilde{w}$, the firm is better off firing the worker earlier than optimal experimentation duration, i.e. $\overline{T}_f(w_L) < \overline{T}$. The firm can credibly only offer a contract such *that* $\overline{T}_f(\mu, w) < \overline{T}$ *for all* $w_L > \tilde{w}$ *and in equilibrium the experimentation is sub-optimal.*

The proposition highlights how the minimum wage restrictions interact with the equilibrium contract. When allowing for flexible wages, if the firms cannot commit to the contract duration, then a very high minimum wage will result in inefficient experimentation and a lower expected payoff for the workers. This implies that if there are frictions in the economy that prevent the firms from credibly committing to a contract term, such as poor legal infrastructure, then setting a very high minimum wage would lead to inefficient experimentation.

B. Relaxing the Common Prior

So far we have assumed that the firm and the worker share a common prior over the initial match productivity. However, it could be that one of the parties is too optimistic or pessimistic. In this section, we highlight the implications for the experimentation duration if the worker and firms do not share a common prior, i.e. $\mu_f \neq \mu_w$.

We restrict attention to fully flexible wages to highlight the effect on tenure due to the mismatch in initial beliefs. Note that, following the arguments preceding Proposition (1), the contract offered under competitive search can still be summarized by $(\hat{v}, T)$, where $\hat{v}$ is the total promised expected payoff of the worker just after the match is formed, and T is the time until which the match continues. However, the firms post-match value is no longer simply $V^p(\mu, T) - \hat{v}$ because the variable wage that the firm needs to pay the worker now depends on the worker's belief about the probability of success which differs from that of the firm.
The workers are paid a time-varying $\boldsymbol{\omega} = \{w(t;T)\}_{t=0}^{T}$. The workers earn

$$\begin{aligned} v(\mu_w, \boldsymbol{\omega}) &= \int_0^T (w(z,T) + \lambda\gamma(\mu_w, z)v_h) \cdot (\mu_w + (1-\mu_w)e^{\lambda z})e^{-(\rho+\delta+\lambda)z}dz \\ &\quad + v_0 e^{-(\rho+\delta+\lambda)T}\left(\mu_w + (1-\mu_w)e^{\lambda T}\right) \\ &= \hat{v} \end{aligned}$$

Note that the wage path $w(t,T)$ also depends on T, because we constrain the total payoff to the worker to be $\hat{v}$. Therefore, if we differentiate the above equation by the contract duration T, we obtain the following condition,

$$
\begin{aligned}
&(w(T,T)+\lambda\gamma(\mu_w,T)v_h)(\mu_w e^{-(\rho+\delta+\lambda)T}+(1-\mu_w)e^{-(\rho+\delta)T}) \\
&\quad -\mu_w v_0(\rho+\delta+\lambda)e^{-(\rho+\delta+\lambda)T}-(1-\mu_w)(\rho+\delta)v_0 e^{-(\rho+\delta)T}= \\
&\qquad -\int_0^T \frac{\partial w(z,T)}{\partial T}\cdot(\mu_w+(1-\mu_w)e^{\lambda z})\cdot e^{-(\rho+\delta+\lambda)z}dz
\end{aligned}
\tag{30}
$$

On the other hand, the firms' payoff is

$$
\begin{aligned}
&V(\mu_f,\boldsymbol{\omega})=V^p(\mu_f,T) \\
&\quad \underbrace{-\int_0^T (w(z)+\lambda\gamma(\mu_f,z)v_h)\cdot(\mu_f+(1-\mu_f)e^{\lambda z})\cdot e^{-(\rho+\delta+\lambda)z}dz-v_0 e^{-(\rho+\delta+\lambda)T}\left(\mu_f+(1-\mu_f)e^{\lambda T}\right)}_{\text{expected payment}} \\
&=V^p(\mu_f,T)-\hat{v}+\int_0^T\left(\lambda v_h(\gamma(\mu_w,z)-\gamma(\mu_f,z))+w(z,T)(\mu_w-\mu_f)\cdot(1-e^{\lambda z})\right)e^{-(\rho+\delta+\lambda)z}dz \\
&\quad +v_0(\mu_w-\mu_f)(1-e^{\lambda T})e^{-(\rho+\delta+\lambda)T}
\end{aligned}
$$

where $\boldsymbol{\omega}$ must be such that $v(\mu_w,\boldsymbol{\omega},t)\geq v_0$ for all $t\in[0,T]$, i.e the worker's continuation payoff must exceed her outside option. If the firms and the worker had a common prior, the above reduces to $V^p(\mu,T)-\hat{v}$. Therefore, when the firm chose the tenure to maximize its value, the wage path was immaterial. However, with mismatched priors the choice of tenure

$$
\begin{aligned}
&\frac{\partial V^p(\mu_f,T)}{\partial T} \\
&\quad+\left(\lambda v_h(\gamma(\mu_w,T)-\gamma(\mu_f,T))+w(T,T)(\mu_w-\mu_f)\cdot(1-e^{\lambda T})\right)e^{-(\rho+\delta+\lambda)T} \\
&\quad+\int_0^T\frac{\partial w(z,T)}{\partial T}(\mu_w-\mu_f)\cdot(1-e^{\lambda z})e^{-(\rho+\delta+\lambda)z}dz \\
&\quad-v_0(\mu_w-\mu_f)(\rho+\delta+\lambda-(\rho+\delta)e^{\lambda T})e^{-(\rho+\lambda+\delta)T}=0
\end{aligned}
\tag{31}
$$

depends on the wage path and specifically $w(T,T)$ and $\partial w(t,T)/\partial T$. To highlight how the equilibrium tenure is affected, we restrict attention to a simple contract where the firm pays the worker the minimum payoff to prevent quitting and pays the remainder $(\hat{v}-v_0)$ in cash bonus. The firm pays the worker as per the following wage schedule,

$$
w(t)=v_0(\rho+\delta)-\lambda\gamma(\mu_w,t)(v_h-v_0)
$$

and $\hat{v}-v_0$ is paid as a bonus at recruitment.[14] This simplifies the expression that determines the equilibrium tenure length (31),

$$\frac{\partial V^p(\mu_f,T)}{\partial T} = -\lambda \Big(v_h(\gamma(\mu_w,T) - \gamma(\mu_f,T)) + (\mu_w - \mu_f)(\gamma(\mu_w,T)(v_h - v_0)(e^{\lambda T} - 1) + v_0) \Big) e^{-(\rho+\delta+\lambda)T} \tag{32}$$

PROPOSITION 8: *If the workers are optimistic, i.e.* $\mu = \mu_f < \mu_w$*, then the equilibrium tenure is longer than the efficient, and if the workers are pessimistic, i.e.* $\mu = \mu_f > \mu_w$*, then the equilibrium tenure is shorter than efficient.*

The proposition follows from the inspection of (32). First, firms have the true prior so that if they set the LHS to 0, then the tenure is efficient. However, if $\mu_f < \mu_w$, the RHS is negative, and therefore the tenure chosen by the firms is longer than the efficient tenure. If the workers are optimistic then the firms need to increase their wages at a slower rate to keep the workers indifferent between working and quitting, which allows the firms to experiment longer, and vice versa if the workers are pessimistic.

Note that if $\mu = \mu_w > \mu_f$ (or $\mu = \mu_w < \mu_f$) then whether the equilibrium tenure is longer or shorter than efficient is not necessarily clear, and it depends on the parameters of the model. Let us consider one such case for illustration. Suppose that $\mu = \mu_w > \mu_f$, i.e. the firms are pessimistic about the workers' productivity. In this case, the firms want to fire the workers sooner, i.e. if the firms set the LHS of (32) to 0, they choose a tenure length shorter than the efficient experimentation duration. However, the RHS is also negative. Therefore, this pushes the firms to choose a longer experimentation duration than what they would have set if they set the LHS to 0. Whether the equilibrium experimentation duration chosen in this manner is longer or shorter than the efficient experimentation duration is ambiguous and depends on the parameters of the model. There could be policies that change v_h, i.e. the post-success payoff share of the workers (and firms) in such a way that the experimentation is equal or closer to the efficient experimentation duration.

IX. Conclusion

In this paper, we propose a theoretical framework to analyze endogenous experimentation and tenure contracts under uncertainty about match productivity and search frictions. We

[14] We could consider other wage paths, that give a continuation payoff $> v_0$, but satisfy $\partial w(t,T)/\partial T = 0$, i.e. the wage at any time t does not depend on the duration of the contract. The contract considered here covers the case when the firm cannot commit to contract duration.

rationalize the commonly observed tenure contracts awarded to researchers and show that such contracts are efficient. However, if the search frictions are too severe, the equilibrium wages decline and result in sub-optimal experimentation and inefficiency due to early quits by workers. We also highlight the importance of firms' ability to commit to contract tenure, which may depend on the quality of institutions enforcing employment contracts or reputational concerns for the firm. If the firms cannot commit to the tenure, the equilibrium is generally inefficient. Using our model, we can speak to the implications of several policies, such as minimum wage for apprentices or entry-level workers negotiated by trade unions. We highlight that a minimum wage may increase the surplus that accrues to the worker, however, if the minimum wage is set too high it may result in sub-optimal experimentation, lower surplus for the workers, and lower social welfare. We are also able to speak to the inefficient experimentation when the parties do not share a common prior over match productivity. Our model applies not just to labor markets in performance-driven professions such as entrepreneurship, and scientific research, but also to other settings with bilateral experimentation such as firms licensing patents from patent owners, patients experimenting with physicians. The framework can also accommodate multiple types of workers where the type is workers' private information, as well as team production and experimentation.

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Proofs

PROOF OF PROPOSITION 4:
From Definition 1, the equilibrium contract must satisfy,

$$V^*(\mu;\overline{v}) := \max_{\boldsymbol{\omega},q} V^u(\mu,\boldsymbol{\omega},q)$$

subject to $v^u(\mu,\boldsymbol{\omega},q)=\overline{v}$. The firms enter the sub-markets until the value is driven to 0, i.e. $V^*(\mu;\overline{v})=0$. First, let us show that the firm's maximization is well-defined. Define the function $\mathcal{W}:\mathbb{R}^+\to\mathbb{R}^+$ such that $V(\mu,\mathcal{W}(T),T)=0$. This function corresponds to the wage that gives the firm 0 profit for a given contract duration T. Since $\overline{T}_q(\mu,w)$ is monotonic in w, let its inverse be $t_q^{-1}(T;\mu)$. The feasible set for the firm to choose $\boldsymbol{\omega}$ is $\boldsymbol{\Omega}=\{(w,T):T\in[0,\overline{T}_q(\mu,w)], w\in[t_q^{-1}(T;\mu),\mathcal{W}(T;\mu)]\}$. Given $q(\mu,\boldsymbol{\omega};\overline{v})$ such that for $\boldsymbol{\omega}\in\boldsymbol{\Omega}$, the value to the worker $v^u(\mu,\boldsymbol{\omega},q(\mu,\boldsymbol{\omega};\overline{v}))=\overline{v}$, the maximization problem

$$\max_{\boldsymbol{\omega}\in\boldsymbol{\Omega}} V^u(\mu,\boldsymbol{\omega},q(\mu,\boldsymbol{\omega};\overline{v}))$$

is well-defined as $V^u(\mu,\boldsymbol{\omega},q(\mu,\boldsymbol{\omega};\overline{v}))$ is bounded in the domain $\boldsymbol{\omega}\in\boldsymbol{\Omega}$. Since $\boldsymbol{\Omega}$ is compact-valued, and $V^u(\mu,\boldsymbol{\omega},q(\mu,\boldsymbol{\omega};\overline{v}))$ is continuous in $\boldsymbol{\omega}$, $V^*(\mu;\overline{v})$ is continuous in $\overline{v}$ by the Theorem of Maximum. By the Envelope Theorem, the $V^*(\mu;\overline{v})$ is decreasing in $\overline{v}$. Clearly for $\overline{v}=V^p(\mu,\overline{T})$, $q(\mu,\boldsymbol{\omega};\overline{v})=0$ is singleton and $V^*(\mu;\overline{v})=-\kappa/\rho<0$. Moreover for $\overline{v}=v_0$, if $v(\mu,\boldsymbol{\omega})>v_0$ then $q(\mu,\boldsymbol{\omega};\overline{v})\to\infty$ and if $v(\mu,\boldsymbol{\omega})=v_0$, then $q(\mu,\boldsymbol{\omega};\overline{v})\in[0,\infty)$. In this case, $V^*(\mu;\overline{v})\to\max_{w,T}(V^p(\mu,T)-v(\mu,w,T))>0$. Therefore, there exists some $\overline{v}^*$ such that $V^*(\mu,\overline{v}^*)=0$. This gives us equilibrium contract, $\omega^*=\arg\max V^u(\mu,\boldsymbol{\omega},q(\mu,\boldsymbol{\omega};\overline{v}^*))$, the queue length $q^*=q(\mu,\boldsymbol{\omega}^*;\overline{v}^*)$ and the equilibrium unemployment $u^*=\delta/(\delta+\alpha_u(q^*))$.
PROOF OF THEOREM 1:
From Definition 1, the equilibrium contract must satisfy,

$$V^*(\mu;\overline{v}) := \max_{\boldsymbol{\omega},q} V^u(\mu,\boldsymbol{\omega},q)$$

subject to $v(\mu, \boldsymbol{\omega}, q) \geq \overline{v}$. The firms enter the sub-markets until the value is driven to 0, i.e. $V^*(\mu; \overline{v}) = 0$. We can re-write this optimization problem in its dual form as below,

$$\max_{\boldsymbol{\omega}, q} v^u(\mu, \boldsymbol{\omega}, q)$$

subject to $V(\mu, \boldsymbol{\omega}, q) \geq V^*(\mu; \overline{v})$. Combining this with the free-entry condition, the constraint set is modified to,

$$(FE) \qquad V^u(\mu, \boldsymbol{\omega}, q) \geq V^*(\mu; \overline{v}) = 0$$

Note that the above inequality constraint must bind. If not, then the optimizing $(\boldsymbol{\omega}, q)$ are in the interior of the constraint set. In this case, more firms will enter this sub-market (as there is positive value to be made) so that q declines, and the workers are better off, implying that $(\boldsymbol{\omega}, q)$ cannot be the solution to the optimization problem.
We now write the optimization problem where $\boldsymbol{\omega} = (w, T)$.

$$\text{(A1)} \qquad \max_{w,T,q} v^u(\mu, w, T, q) = v_0 + \max_{w,T,q} \frac{\alpha_u(q)(v(\mu, w, T) - v_0)}{\rho + \delta + \alpha_u(q)}$$

subject to

$$\text{(A2)} \qquad V^u(\mu, w, T, q) = \frac{-\kappa + \alpha_v(q)(V^p(\mu, T) - v(\mu, w, T))}{\rho + \alpha_v(q)} = 0$$

and

$$\text{(A3)} \qquad T \leq \overline{T}_q(\mu, w)$$

The final constraint follows from the requirement that only the firm has commitment power when it comes to the duration of the contract and the worker can choose to quit anytime she pleases. Solving the optimization,

$$\mathcal{L}(w, T, q, \theta_1, \theta_2) = \frac{\alpha_u(q)(v(\mu, w, T) - v_0)}{\rho + \delta + \alpha_u(q)}$$

$$+ \theta_1(-\kappa + \alpha_v(q)(V^p(\mu, T) - v(\mu, w, T))) + \theta_2(\overline{T}_q(\mu, w) - T)$$

The solution (w, T, q) must satisfy ,

(A4) $$\left(\frac{\alpha_u(q)}{\rho+\delta+\alpha_u(q)} - \theta_1\alpha_v(q)\right) v_w(\mu, w, T) + \theta_2 \overline{T}_{qw}(\mu, w) = 0$$

(A5) $$\left(\frac{\alpha_u(q)}{\rho+\delta+\alpha_u(q)} - \theta_1\alpha_v(q)\right) v_T(\mu, w, T) + \theta_1\alpha_v(q) V^p_T(\mu, T) - \theta_2 = 0$$

(A6) $$\frac{\varepsilon(q)\rho}{(1-\varepsilon(q))(\alpha_u(q)+\rho+\delta)} = \frac{V^p(\mu, T) - v(\mu, w, T)}{(v(\mu, w, T) - v_0)}$$

the free-entry condition $\kappa = \alpha_v(q)(V^p(\mu, T) - v(\mu, w, T))$ and $\overline{T}_q(\mu, w) \geq T$. In a tenure equilibrium, the constraint $\overline{T}_q(\mu, w) > T$ does not bind (no quitting). Therefore, $\theta_2 = 0$. Since $v_w(\mu, w, T) > 0$ for $\overline{T}_q(\mu, w) > T$, from Equation (A4) it must be that $\left(\frac{\alpha_u(q)}{\rho+\delta+\alpha_u(q)} - \theta_1\alpha_v(q)\right) = 0$. Substituting into Equation (A5) and noting that the free-entry condition must bind so that $\theta_1 > 0$, gives us $V^p_T(\mu, T) = 0$. That is, $T^* = \overline{T}$, the optimal tenure that maximizes the total expected value from the match $V^p(\mu, T)$.
On the other hand, if the quitting time constraint binds, i.e. $T = \overline{T}_q(\mu, w)$, then $\theta_2 > 0$ and it follows from (A4) that $\left(\frac{\alpha_u(q)}{\rho+\delta+\alpha_u(q)} - \theta_1\alpha_v(q)\right) < 0$. Substituting into (A5) gives us $V^p_T(\mu, T) > 0$. Therefore, it must be that $T^* < \overline{T}$.
PROOF OF PROPOSITION 5:
In equilibrium, the free-entry condition, along with the tangency implies that we can equate the implicit derivatives of the iso-profit curve corresponding to 0 profit and the indifference curve corresponding to the payoff $\overline{v}^*$ in the q-w space. The implicit derivative of the iso-profit curve corresponding to the free-entry condition,

$$\frac{dq}{dw} = -\frac{\frac{\partial(V^p(\mu,T^*)-v(\mu,w^*,T^*))}{\partial w}}{-\frac{\partial \frac{\kappa}{\alpha_v(q)}}{\partial q}} = -\frac{\frac{dV^p(\mu,T^*)}{dT^*}\frac{dT^*}{dw} - \frac{dv(\mu,w^*,T^*)}{dw}}{\frac{\kappa\alpha_v'(q)}{(\alpha_v(q))^2}}$$

and the implicit derivative of the indifference curve corresponding to $\overline{v}^*$,

$$\frac{dq}{dw} = -\frac{\frac{\partial}{\partial w}\frac{(v_0(\rho+\delta)+\alpha_u(q)v(\mu,w^*,T^*))}{(\rho+\delta+\alpha_u(q))}}{\frac{\partial}{\partial q}\frac{(v_0(\rho+\delta)+\alpha_u(q)v(\mu,w^*,T^*))}{(\rho+\delta+\alpha_u(q))}} = \frac{(\rho+\delta+\alpha_u(q))\alpha_u(q)\frac{dv(\mu,w^*,T^*)}{dw}}{(\rho+\delta)(v(\mu,w^*,T^*)-v_0)\alpha_u'(q)}$$

We first equate the two slopes, and make the following substitution from the worker's indifference condition,

$$\frac{(\rho+\delta)(v(\mu,w^*,T^*)-v_0)}{(\rho+\delta+\alpha_u(q))} = v(\mu,w^*,T^*)-\overline{v}^*$$

and the free-entry condition, $\kappa = \alpha_v(q)(V^p(\mu,T^*)-v(\mu,w^*,T^*))$. Finally, we substitute

$$1-\Theta(\mu,w^*) = \frac{\frac{\partial V^p(\mu,T^*)}{\partial T}\cdot\frac{dT^*}{dw}}{\frac{dv(\mu,w^*,T^*)}{dw}}$$

which gives us the following condition,

$$\frac{(v(\mu,w^*,T^*)-\overline{v}^*)}{(V^P(\mu,T^*)-v(\mu,w^*,T^*))} = \frac{(1-\varepsilon(q))}{\varepsilon(q)\Theta(\mu,w^*)}$$

Rearranging the terms, and dividing by $(1-\varepsilon(q)(1-\Theta(\mu,w^*)))$

$$v(\mu,w^*,T^*) = \overline{v}^* + \left(\frac{1-\varepsilon(q^*)}{1-\varepsilon(q^*)(1-\Theta(\mu,w^*))}\right)\cdot(V^p(\mu,T^*)-\overline{v}^*)$$

PROOF OF THEOREM 2:

We denote the planner's value function by $\overline{V}(\mu,u)$. The planner's solution satisfies the following HJB equation,

$$\text{(A7)}\quad \rho\overline{V}(\mu,u) = \max_{q,T}\left\{u\left(v_0(\rho+\delta)+\alpha_u(q)\cdot(\overline{V}(\mu,T)+v_0)-\frac{\kappa}{q}\right)+\frac{d\overline{V}(\mu,u)}{du}\cdot\frac{du}{dt}\right\}$$

where $\dot{u} = \delta - u(\delta+\alpha_u(q))$.

We can see that the value function $\overline{V}(\mu,u) = A + Bu$, i.e. it is linear in u. Using the FOCs w.r.t. $(q,\overline{T})$ and the Envelope Theorem,

$$\frac{\partial}{\partial T} = 0 \implies \frac{dV^p(\mu,\overline{T})}{d\overline{T}} = 0 \implies \overline{T} = \frac{1}{\lambda}\left[\ln\left(\frac{\mu}{1-\mu}\right)+\ln\left(1+\frac{\lambda}{\rho+\delta}\right)+\ln\left(\frac{\lambda\overline{y}}{v_0(\rho+\delta)-\underline{y}}-1\right)\right]$$

$$\frac{\partial}{\partial q} = 0 \implies B = \frac{\alpha_u'(q)V^p(\mu,\overline{T})+\frac{\kappa}{q^2}}{\alpha_u'(q)} = V^p(\mu,\overline{T})+\frac{\kappa}{q^2\alpha_u'(q)}$$

Using the Envelope Theorem,

$$B = v_0 + \frac{\alpha_u(q)(V^p(\mu, \overline{T}) - v_0) - \frac{\kappa}{q}}{(\rho + \delta + \alpha_u(q))}$$

The efficient queue length q_p satisfies,

$$\frac{(\rho + \delta)(V^p(\mu, \overline{T}) - v_0)}{\kappa} = -\frac{q\alpha_u'(q) + \alpha_u(q) + \rho + \delta}{q^2\alpha_u'(q)}$$

Note that the LHS is independent of q, and therefore, q_p is independent of u and only depends on the parameters and μ.

We now compare the equilibrium (q^*, T^*) with the planner's choice. The equivalent condition for the competitive search equilibrium is,

$$-\frac{(\rho + \delta + \alpha_u(q))}{\alpha_u(q)} \left(\frac{\alpha_u(q) + q\alpha_u'(q)}{q^2\alpha_u'(q)} \right) = \frac{(\overline{v} - v_0)}{\kappa}(\rho + \delta)$$

$$\frac{(\overline{v} - v_0)}{\kappa}(\rho + \delta) = -\frac{\rho + \delta + \alpha_u(q) + q\alpha_u'(q)}{q^2\alpha_u'(q)} - \frac{(\rho + \delta)}{q\alpha_u(q)}$$

Note that $q\alpha_u(q) = \alpha_v(q)$, and from the free entry

$$V(\mu, \boldsymbol{\omega}) = V^p(\mu, T^*) - \overline{v} = \frac{\kappa}{\alpha_v(q)}$$

Substituting and rearranging,

$$\frac{(\rho + \delta)(V^p(\mu, T^*) - v_0)}{\kappa} = -\frac{q\alpha_u'(q) + \alpha_u(q) + \rho + \delta}{q^2\alpha_u'(q)}$$

If we are in the fully-flexible wages benchmark, then $T^* = \overline{T}(\mu) = \overline{T}$, and $q_p = q^*$. This proves that the competitive search equilibrium allocation under the fully-flexible wages benchmark is efficient.

Under the fixed-wage competitive search equilibrium, if $w^* \geq \tilde{w}$, then $T^* = \overline{T}$. Therefore, $q_p = q^*$, i.e. under fixed-wage contracts if the equilibrium is one where the offered contract duration is shorter than the quitting time, then the equilibrium is efficient.

Note that for $w^* < \tilde{w}$, $T^* = \overline{T}_q(\mu, w^*) < \overline{T}$. Note that the RHS of Equation (A) is decreasing in q. Therefore, $q_p < q^*$ as $V^p(\mu, \overline{T}_q(\mu, w^*) < V^p(\mu, \overline{T})$. It follows that the decentralized equilibrium under fixed-wages is not efficient if workers quit in equilibrium, or $w^* < \tilde{w}$.

PROOF OF PROPOSITION 6:

Define

$$\tilde{V}(\mu, w; \overline{v}) = V^u(\mu, w, q(w; \overline{v}))$$

which is the firm's post-match payoff given the workers' indifference condition ($v^u(\mu, w, q) = \overline{v}$) and it is continuous in w. We can pin down the two extreme wages in the feasible set of wages for which the queue length must go to 0 to keep the worker indifferent and receive a payoff $\overline{v}$. For these wages, $w \in \{\underline{w}(\overline{v}), \overline{w}(\overline{v})\}$, $v(\mu, w) = \overline{v}$ (so that $q(w; \overline{v}) \to 0$) and $\tilde{V}(\mu, w; \overline{v}) = V^u(\mu, w, q(w; \overline{v})) = -\kappa/\rho$.[15]. $\tilde{V}(\mu, w; \overline{v})$ is continuous in this domain, and firms maximization

$$V^*(\mu; \overline{v}) = \max_{w \in [\underline{w}(\overline{v}), \overline{w}(\overline{v})]} \left\{ \tilde{V}(\mu, w; \overline{v}) \right\}$$

is well-defined. (Weirstrass Theorem).

$V^*(\mu, \overline{v})$ is continuous (Maximum Theorem) and decreasing (Envelope Theorem) in $\overline{v}$ and the free-entry condition pins down $\overline{v}^*$. To see this, consider $\overline{v} = v_0$. In this case as long as firm offers a wage, such that the worker gets $v(\mu, w) > v_0$ and $V(\mu, w) > 0$, the indifference condition implies that $q(w; v_0) \to \infty$. The firm's payoff, $V^*(\mu; v_0) > 0$. Next, consider $\overline{v} = \max_w \{V(\mu, w)\}$, i.e. the workers get the highest possible surplus available to the firm. In this case, $q(w; \overline{v}) \to 0$ and therefore, $V^*(\mu; \overline{v}) < 0$. By the Intermediate Value Theorem there must exist a $\overline{v}^*$ such that $V^*(\mu; \overline{v}^*) = 0$.

$$w^* = \arg \max_{w \in [\underline{w}(\overline{v}^*), \overline{w}(\overline{v}^*)]} \left\{ \tilde{V}(\mu, w, \overline{v}^*) \right\}$$

and $q^* = q(\mu, w^*; \overline{v}^*)$. The workers' entry condition pins down the $u^* = \delta/(\delta + \alpha_u(q^*))$.

[15]Note that, $\underline{w}(\overline{v}) > \underline{y}$ and $\overline{w}(\overline{v}) < \underline{y} + \lambda(\overline{y} + V_h)$

B1. *Worker's Payoff and Quitting Decision*

PROOF OF PROPOSITION 3:
Worker's productivity is $\theta \in \{p, l\}$. Worker's type is $\mu \in (0,1)$ where $\mu = Pr(\theta = p)$ is the belief that the worker has high productivity. The firm has a prior over the types of the workers, $\mu \sim F[0,1]$.

The state variables can be summarized in time τ, the time for which the worker has been unproductive and the output in the previous instant. Therefore, the state variables are (τ, y_τ). If the worker after producing $\underline{y}$ until time t, and at time t produces $y_t = \overline{y}$, then the firm and the worker both update their belief $Pr(\theta = p|y_t = \overline{y}) = 1$. If the worker continues to produce $\underline{y}$ the worker and the firm update their belief about her productivity,

$$\mu(\tau) = Pr(\theta = p|\underline{y}, \tau) = \frac{\mu e^{-\lambda\tau}}{\mu e^{-\lambda\tau} + 1 - \mu}$$

The worker has an outside option that pays b_0 per unit time. Hence, the worker's lifetime value from taking the outside option is

$$v_0 = \frac{b_0}{\rho + \delta}$$

The worker's choice is the decision whether to quit and take her outside option. At each point of time τ, the worker after observing the state (τ, y_τ) will make a decision whether to continue or quit her employment in order to take the outside option. Let $d_q \in \{0,1\}$ where $d_q = 1$ refers to the decision to quit and $d_q = 0$ to continue with the current employment. The worker's optimal decision can be written as follows.

$$v(\mu, \tau) = \max_{d_q \in \{0,1\}} \begin{cases} v(\mu, \tau) & \text{if } d_q = 0 \\ v_0 & \text{if } d_q = 1 \end{cases}$$

$v(\mu, \tau)$ is the continuation payoff of a worker who has produced a low output and has been unproductive for time τ. Note that at any instant the worker believes that she has high productivity with probability $\gamma(\mu, \tau)$. Let $v(\theta, \tau)$ denote the continuation value for a worker of type $\theta \in \{p, l\}$, then

$$v(\mu,\tau) = \gamma(\mu,\tau) \cdot v(p,\tau) + (1-\gamma(\mu,\tau)) \cdot v(l,\tau)$$

If the worker produces $\overline{y}$ then $\gamma(\mu,\tau) = 1$, i.e the uncertainty over her type is resolved. The worker now renegotiates wage, so that the worker's continuation value after producing a high output is v_h.

$$v_h = v_p + \beta\left(\frac{\underline{y}+\lambda\overline{y}}{\rho+\delta} - v_p\right)$$

Since, the worker updates her belief each instant depending on the output produced, we can write the expression for it as below,

$$\begin{aligned} v(\mu,\tau) = w\Delta t + e^{-(\rho+\delta)\Delta t}\gamma(\mu,\tau) \cdot (&Pr(y_{\tau+\Delta t} = \underline{y}|p) \cdot v(p,\tau+\Delta t) \\ &+ (1 - Pr(y_{\tau+\Delta t} = \overline{y}|p)) \cdot v_h) \\ &+ e^{-(\rho+\delta)\Delta t} \cdot (1-\gamma(\mu,\tau)) \cdot v(l,\tau+\Delta t) \end{aligned}$$

The first expression is the flow payoff $w\Delta t$. The remaining expression is the expected continuation value depending on the belief over the worker's type and the output produced. $e^{-(\rho+\delta)\Delta t}$ is the discounted value (rate ρ) given the worker does not die (rate δ) in the period Δt.

The probability that a worker of type p, will produce a low output is,

$$Pr(y_{\tau+\Delta t} = \underline{y}|p) = e^{-\lambda\Delta t}$$

Therefore the continuation value to the worker,

$$\begin{aligned} \text{(B1)} \qquad v(\mu,\tau) = w\Delta t + e^{-(\rho+\delta)\Delta t}\gamma(\mu,\tau) \cdot \left(e^{-\lambda\Delta t} \cdot v(p,\tau+\Delta t) + (1-e^{-\lambda\Delta t}) \cdot v_h\right) \\ + e^{-(\rho+\delta)\Delta t} \cdot (1-\gamma(\mu,\tau)) \cdot v(l,\tau+\Delta t) \end{aligned}$$

Subtract $v(\mu,\tau+\Delta t) = \gamma(\tau+\Delta t) \cdot v(p,\tau+\Delta t) + (1-\gamma(\tau+\Delta t)) \cdot v(l,\tau+\Delta t)$ from both sides, add and subtract $\gamma(\mu,\tau)v(p,\tau+\Delta t) + (1-\gamma(\mu,\tau))v(l,\tau+\Delta t)$, divide throughout by Δt, and take the limit $\Delta t \to 0$, the left hand side (B1) is $\lim_{\Delta t \to 0} -\dfrac{(v(\mu,\tau+\Delta t) - v(\mu,\tau))}{\Delta t} = -\dfrac{dv(\mu,\tau)}{d\tau}$.

(B2)

$$-\frac{dv(\mu,\tau)}{d\tau} = w - \lim_{\Delta t\to 0}\left(\frac{1-e^{-(\rho+\delta+\lambda)\Delta t}}{\Delta t}\gamma(\mu,\tau) + \frac{\gamma(\tau+\Delta t)-\gamma(\mu,\tau)}{\Delta t}e^{-(\rho+\delta+\lambda)\Delta t}\right)v(p,\tau$$

$$+\Delta t) - \lim_{\Delta t\to 0}\left(\frac{1-e^{-(\rho+\delta)\Delta t}}{\Delta t}(1-\gamma(\mu,\tau)) - \frac{\gamma(\tau+\Delta t)-\gamma(\mu,\tau)}{\Delta t}\right)v(l,\tau+\Delta t)$$

$$+\lim_{\Delta t\to 0}\frac{(1-e^{-\lambda\Delta t})}{\Delta t}\gamma(\mu,\tau)e^{-(\rho+\delta)\Delta t}v_h$$

Computing the expression at the limit $\Delta t \to 0$,

(B3)

$$\begin{aligned}-\frac{dv(\mu,\tau)}{d\tau} = w &- \left((\rho+\delta+\lambda)\gamma(\mu,\tau) + \frac{d\gamma(\mu,\tau)}{d\tau}\right)v(p,\tau)\\ &- \left((\rho+\delta)(1-\gamma(\mu,\tau)) + \frac{d(1-\gamma(\mu,\tau))}{d\tau}\right)v(l,\tau)\\ &+ \lambda\gamma(\mu,\tau)v_h\end{aligned}$$

Note that

(B4)

$$\frac{d\gamma(\mu,\tau)}{d\tau} = -\lambda\frac{\mu(1-\mu)}{(\mu+(1-\mu)e^{\lambda\tau})^2}e^{\lambda\tau} = -\lambda\gamma(\mu,\tau)(1-\gamma(\mu,\tau))$$

Substituting (B4) in Equation (B3),

$$\begin{aligned}-\frac{dv(\mu,\tau)}{d\tau} &= w + \lambda\gamma(\mu,\tau)v_h - (\rho+\delta)(\gamma(\mu,\tau)v(p,\tau) + (1-\gamma(\mu,\tau))v(l,\tau))\\ &\quad - \lambda\gamma(\mu,\tau)\left(\gamma(\mu,\tau)v(p,\tau) + (1-\gamma(\mu,\tau))v(l,\tau)\right)\end{aligned}$$

From the definition of $v(\mu,\tau) = \gamma(\mu,\tau)v(p,\tau) + (1-\gamma(\mu,\tau))v(l,\tau)$,

(B5)

$$-\frac{dv(\mu,\tau)}{d\tau} = w + \lambda\gamma(\mu,\tau)v_h - (\rho+\delta+\lambda\gamma(\mu,\tau))v(\mu,\tau)$$

Equation (B5) is a linear differential equation in $v(\mu,\tau)$. Rearranging,

$$\frac{dv(\mu,\tau)}{d\tau} - (\rho+\delta+\lambda\gamma(\mu,\tau))v(\mu,\tau) = -(w+\lambda\gamma(\mu,\tau)v_h)$$

Multiply throughout by $e^{-\int_0^\tau (\rho+\delta+\lambda\gamma(s))ds}$

$$\frac{d}{d\tau}\left(v(\mu,\tau)e^{-\int_0^\tau (\rho+\delta+\lambda\gamma(s))ds}\right) = -(w+\lambda\gamma(\mu,\tau)v_h)e^{-\int_0^\tau (\rho+\delta+\lambda\gamma(s))ds}$$

$$e^{-\int_0^\tau (\rho+\delta+\lambda\gamma(s))ds} = (\mu+(1-\mu)e^{\lambda\tau})\cdot e^{-(\rho+\delta+\lambda)\tau}$$

Suppose, the worker quits at time t_q. The worker then earns b_0 until perpetuity. Therefore,

$$v(\mu,t_q) = v_0 = \frac{b_0}{\rho+\delta}$$

This provides us with the boundary conditions,

$$\int_t^{t_q} d\left(v(\mu,\tau)e^{-\int_0^\tau (\rho+\delta+\lambda\gamma(s))ds}\right) = -\int_t^{t_q}(w+\lambda\gamma(\mu,\tau)v_h)(\mu+(1-\mu)e^{\lambda\tau})\cdot e^{-(\rho+\delta+\lambda)\tau}d\tau$$

The solution to the differential equation is,

$$\begin{aligned} v(\mu,t,t_q)-v_0 &= \gamma(\mu,t)\left(\frac{w+\lambda v_h}{\rho+\delta+\lambda}-v_0\right)\left(1-e^{-(\rho+\delta+\lambda)(t_q-t)}\right) \\ &+(1-\gamma(\mu,t))\left(\frac{w}{\rho+\delta}-v_0\right)\left(1-e^{-(\rho+\delta)(t_q-t)}\right) \end{aligned} \tag{B6}$$

LEMMA 3: *Any worker of type μ, maximizes value at time $t\in[0,t_q]$ by quitting at time*

$$\overline{T}_q(\mu) = \frac{1}{\lambda}\cdot\left[\ln\left(1+\frac{\lambda}{\rho+\delta}\right)+\ln\left(\frac{\mu}{1-\mu}\right)+\ln\left(\frac{\lambda}{\rho+\delta+\lambda}\cdot\left(\frac{v_h-\frac{w}{\rho+\delta}}{v_0-\frac{w}{\rho+\delta}}\right)-1\right)\right]$$

PROOF:
Differentiate with respect to t_q and rearrange,

$$\begin{aligned}\frac{\partial v(\mu,t,t_q)}{\partial t_q} = \frac{(1-\mu)(\rho+\delta)e^{-(\rho+\delta+\lambda)(t_q-t)}}{\mu+(1-\mu)e^{\lambda t}}&\left[\left(1+\frac{\lambda}{\rho+\delta}\right)\left(\frac{\mu}{1-\mu}\right)\left(\frac{w+\lambda v_h}{\rho+\delta+\lambda}-v_0\right)\right. \\ &\left.-e^{\lambda t_q}\left(v_0-\frac{w}{\rho+\delta}\right)\right]\end{aligned}$$

$$\frac{\partial v(\mu,t,t_q)}{\partial t_q} = \begin{cases} \geq 0 & t_q \leq \overline{T}_q(\mu) \\ < 0 & t_q > \overline{T}_q(\mu) \end{cases}$$

Therefore, the worker will prefer $\overline{T}_q$ over any other $t_q \in [0,\infty)$ to maximize value at any time t.

However, we need to verify whether $v(\mu,t,\overline{T}_q(\mu)) - v_0 \geq 0$ for all $t \in [0,\overline{T}_q(\mu)]$. We write $v(\mu,t,\overline{T}_q)$ as,

$$\begin{aligned} v(\mu,t,\overline{T}_q) - v_0 = \frac{1}{\mu+(1-\mu)e^{\lambda t}} \Bigg[& \mu\left(\frac{w+\lambda v_h}{\rho+\delta+\lambda} - v_0\right) + (1-\mu)e^{\lambda\overline{T}_q}\left(\frac{w}{\rho+\delta} - v_0\right) e^{-\lambda(\overline{T}_q - t)} \\ & - e^{-(\rho+\delta+\lambda)(\overline{T}_q - t)}\left(\mu\left(\frac{w+\lambda v_h}{\rho+\delta+\lambda} - v_0\right) + (1-\mu)e^{\lambda\overline{T}_q}\left(\frac{w}{\rho+\delta} - v_0\right)\right)\Bigg] \end{aligned}$$

Substituting, $\overline{T}_q(\mu)$ so that $(1-\mu)e^{\lambda\overline{T}_q}\left(v_0 - \frac{w}{\rho+\delta}\right) = \left(1+\frac{\lambda}{\rho+\delta}\right)\mu\left(\frac{w+\lambda v_h}{\rho+\delta+\lambda} - v_0\right)$,

$$\begin{aligned} v(\mu,t,\overline{T}_q) - v_0 &= \frac{\mu}{\mu+(1-\mu)e^{\lambda t}}\left(\frac{w+\lambda v_h}{\rho+\delta+\lambda} - v_0\right)\Bigg[1 + \frac{\lambda}{\rho+\delta}e^{-(\rho+\delta+\lambda)(\overline{T}_q - t)} \\ &\qquad - \left(1+\frac{\lambda}{\rho+\delta}\right)e^{-\lambda(\overline{T}_q - t)}\Bigg] \\ &\geq 0 \end{aligned}$$

The inequality follows from the fact that the function,

$$h(x) = \frac{\lambda}{\rho+\delta}e^{-(\rho+\delta+\lambda)x} - \left(1+\frac{\lambda}{\rho+\delta}\right)e^{-\lambda x}$$

is increasing in x for all $x \geq 0$. Hence, for all $t \leq \overline{T}_q$,

$$v(\mu,t,\overline{T}_q) - v_0 = \frac{\mu}{\mu+(1-\mu)e^{\lambda t}}\left(\frac{w+\lambda v_h}{\rho+\delta+\lambda} - v_0\right)(1 + h(\overline{T}_q - t)) \geq 0$$

The worker earns a value strictly higher than v_0 for all time $t \in [0,\overline{T}_q(\mu))$ and quits at time $\overline{T}_q(\mu)$.

B2. Post High-output Bargaining

Once the worker produces a high output, there is no uncertainty about the worker's type. Assume that the disagreement payoff for the firm is V^u and that for the worker is v_p.

We impose free entry condition on the firm and therefore, $V^u = 0$. The Nash-Bargaining solution is

$$\max_w \left\{ \left(\frac{w}{\rho+\delta} - v_p\right)^\beta \left(\frac{\lambda\overline{y} + \underline{y} - w}{\rho+\delta} - V^u\right)^{1-\beta} = \left(\frac{w}{\rho+\delta} - v_p\right)^\beta \left(\frac{\lambda\overline{y} + \underline{y} - w}{\rho+\delta}\right)^{1-\beta} \right\}$$

The FOC and rearrangement gives us the value to the worker $W(w)$,

$$W(w^*) = \frac{w^*}{\rho+\delta} = \beta\left(\frac{\lambda\overline{y} + \underline{y}}{\rho+\delta} - v_p\right) + v_p$$

and that to the firm,

$$V(w^*) = \frac{\lambda\overline{y} + \underline{y} - w^*}{\rho+\delta} = (1-\beta)\left(\frac{\lambda\overline{y} + \underline{y}}{\rho+\delta} - v_p\right)$$

In the main paper we abstract away from the mechanics that determine v_p. However, here we provide a microfoundation for the same. For ease of exposition, we assume fixed wages, however, the arguments generalize to the fully flexible wages easily. Let us assume that when the worker disagrees, she quits and the wage is determined in competitive search for productive workers with free entry. The arrival rate of jobs to the worker is α_u and that of the workers to the firm is α_v.

(B7) $$(\rho+\delta)v_p(w, q_p) = v_0(\rho+\delta) + \alpha_u(q_p)\left(\frac{w}{\rho+\delta} - v_p(w, q_p)\right)$$

(B8) $$\rho V^u(w, q_p) = \max_{w, q_p}\left\{-\kappa + \alpha_v(q_p)\left(\frac{\lambda\overline{y} + \underline{y} - w}{\rho+\delta} - V^u(w, q_p)\right)\right\}$$

subject to

$$q_p \in Q(\overline{v}, w) \equiv \{q : v_p(w, q) \geq \overline{v}\}$$

and $V^u(w, q_p) = 0$ (free entry). We can write the dual of the above problem after imposing the free entry condition,

$$(\rho+\delta)v_p = \max_{q_p}\left\{v_0(\rho+\delta) + \alpha_u(q_p)\left(\frac{w}{\rho+\delta} - v_p\right)\right\} \tag{B9}$$

subject to,

$$V^u = 0 = -\kappa + \alpha_v(q_p)\frac{\lambda\overline{y} + \underline{y} - w}{\rho+\delta} \tag{B10}$$

Substituting $q\alpha_u = \alpha_v$ and using the first order condition with respect to q_p,

$$\begin{aligned}\left(\frac{\alpha_v'}{q_p} - \frac{\alpha_v}{q_p^2}\right)\left(\frac{w}{\rho+\delta} - v_p\right) &= -\frac{\alpha_v'\kappa}{\alpha_v} \\ &= \frac{\alpha_v'(w - \lambda\overline{y} - \underline{y})}{q_p(\rho+\delta)}\end{aligned} \tag{B11}$$

Let $\dfrac{q\alpha_v'(q)}{\alpha_v(q)} = \varepsilon(q)$. Rearranging,

$$\frac{w^*}{\rho+\delta} = \varepsilon(q_p)\left(\frac{\lambda\overline{y} + \underline{y}}{\rho+\delta} - v_p\right) + v_p \tag{B12}$$

Solution to (B9) subject to (B10), and the equation, (B12) determine the equilibrium (q_p^*, v_p, w^*). Moreover, note that the arrival rate of workers to the recruitment pool of productive workers simply scales the number of firms v, so that $q_p^* = u/v$ holds. Therefore, the prior wage and arrival rate of workers has no effect on v_p which we take as exogenous in the main section. This allows us to subsume these values into (v_h, V_h) and focus on the wage contracts that will arise in equilibrium.

B3. Microfounding Post-High-Output Valuations:

So far we have assumed that q_p^* is determined in equilibrium. However, note that the corresponding u depends on the arrival rate of workers to the recruitment pool for productive workers. With the Nash-Bargaining protocol described earlier, in equilibrium no worker who produces a high output disagrees so that $u = 0$ and therefore, no sub-market for productive workers recruitment exists. In this sub-section we justify the formulation of equations (2) and (3) from the main paper.

Suppose that when the worker produces a high output, the match dissolves with probability π. This can also be rationalized as the workers adopting a mixed strategy and choosing to leave employment to search in the productive pool with probability π. The firm must offer the worker at least her outside option value v_p in order to retain the worker when renegotiating. We assume that the firm gives the worker lowest possible value so that she remains indifferent between quitting and continuing. The value to the worker is,

$$v_h = v_p$$

and the firm,

$$V_h = (1-\pi)\left(\frac{\lambda\overline{y} + \underline{y}}{\rho+\delta} - v_p\right)$$

The above follows because if the match dissolves the firms go back to the recruitment pool where they earn 0 profit. As is clear from the derivation in the previous section,

$$v_h = v_p = v_0 + \left(\frac{\alpha_u(q_p^*)}{\alpha_u(q_p^*) + \rho + \delta}\right)\left(\frac{\lambda\overline{y} + \underline{y}}{\rho+\delta} - v_0\right)$$

and

$$V_h = (1-\pi)\left(1 - \frac{\alpha_u(q_p^*)}{\alpha_u(q_p^*) + \rho + \delta}\right)\left(\frac{\lambda\overline{y} + \underline{y}}{\rho+\delta} - v_0\right)$$

Let $\pi = \epsilon$ where $\epsilon \to 0^+$ and $\left(\frac{\alpha_u(q_p^*)}{\alpha_u(q_p^*) + \rho + \delta}\right) = \tilde{\beta}$,

$$v_h = v_0 + \tilde{\beta}\left(\frac{\lambda\overline{y} + \underline{y}}{\rho+\delta} - v_0\right)$$

$$V_h = (1-\tilde{\beta})\left(\frac{\lambda\overline{y} + \underline{y}}{\rho+\delta} - v_0\right)$$

This is akin to the formulation with Nash Bargaining in Equations (2) and (3) except we replace $\beta \to \tilde{\beta}$, $v_p \to v_0$. The share of the surplus that the productive workers can get depends on the conditions in the productive workers' market. For example, a longer queue length q_p^* implies a lower matching rate and therefore reduces their share of surplus.