10707
Deep Learning

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Convolutional Networks I

Computer Vision

- Design algorithms that can process visual data to accomplish a given task:

  - For example, object recognition: Given an input image, identify which object it contains

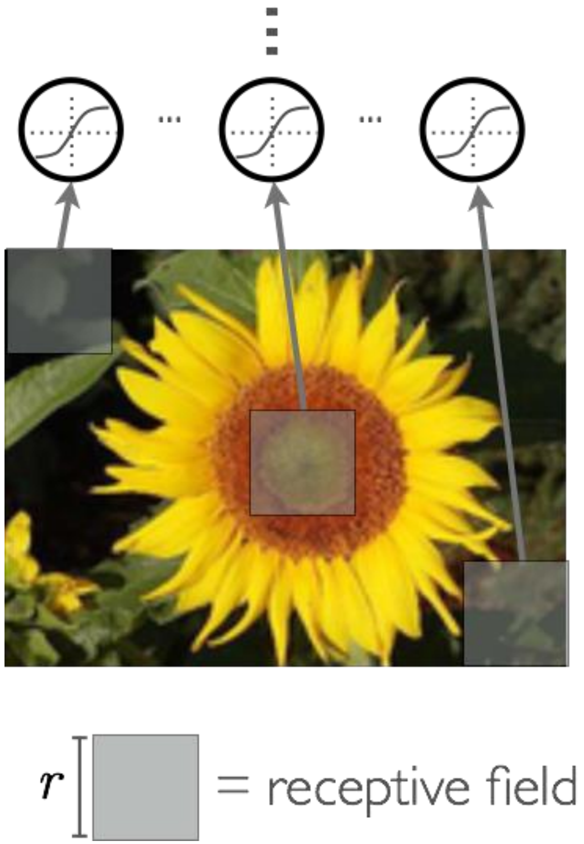
Computer Vision

- Our goal is to design neural networks that are specifically adapted for such problems
  - Must deal with very high-dimensional inputs: 150 x 150 pixels = 22500 inputs, or 3 x 22500 if RGB pixels
  - Can exploit the 2D topology of pixels (or 3D for video data)
  - Can build in invariance to certain variations: translation, illumination, etc.

- Convolutional networks leverage these ideas
  - Local connectivity
  - Parameter sharing
  - Convolution
  - Pooling / subsampling hidden units

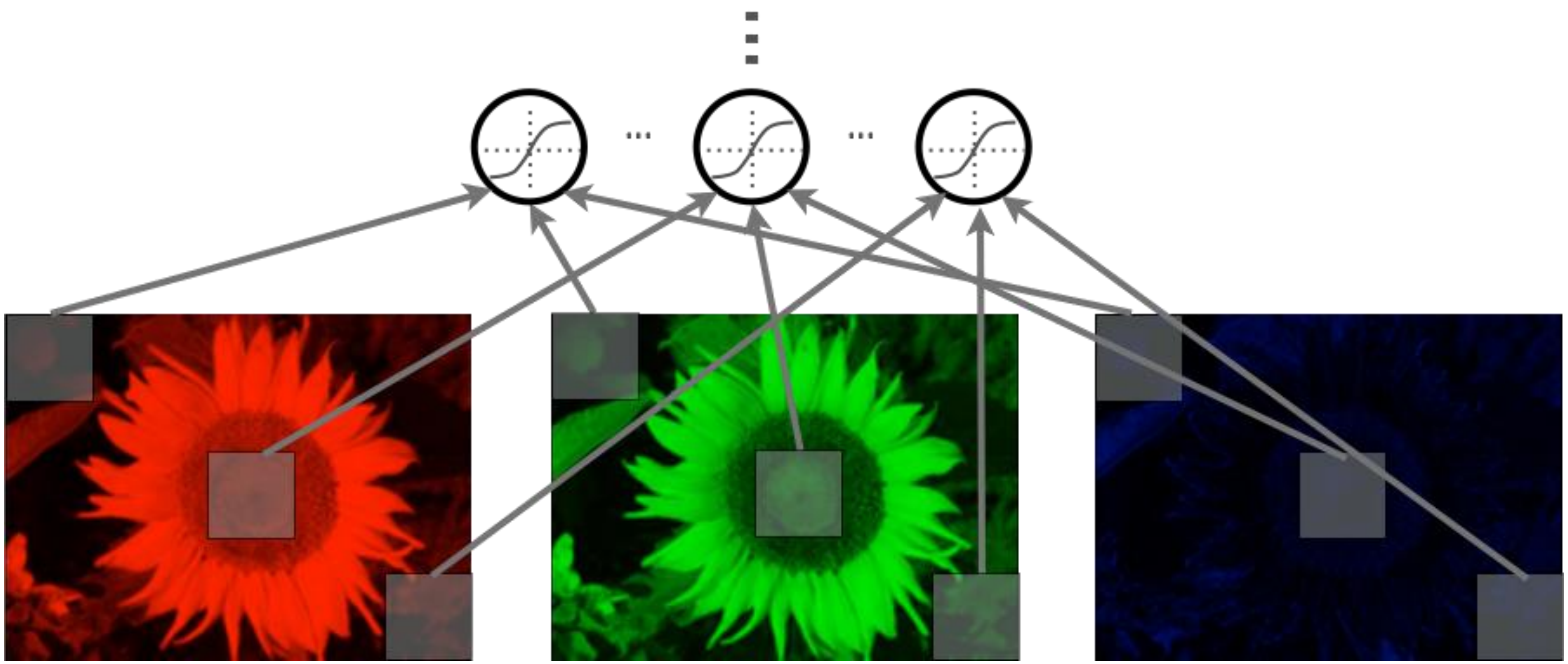
Local Connectivity

- Use a local connectivity of hidden units
  - ➢ Each hidden unit is connected only to a sub-region (patch) of the input image
  - ➢ It is connected to all channels: 1 if grayscale, 3 (R, G, B) if color image
- Why local connectivity?
  - ➢ Fully connected layer has a lot of parameters to fit, requires a lot of data
  - ➢ Spatial correlation is local



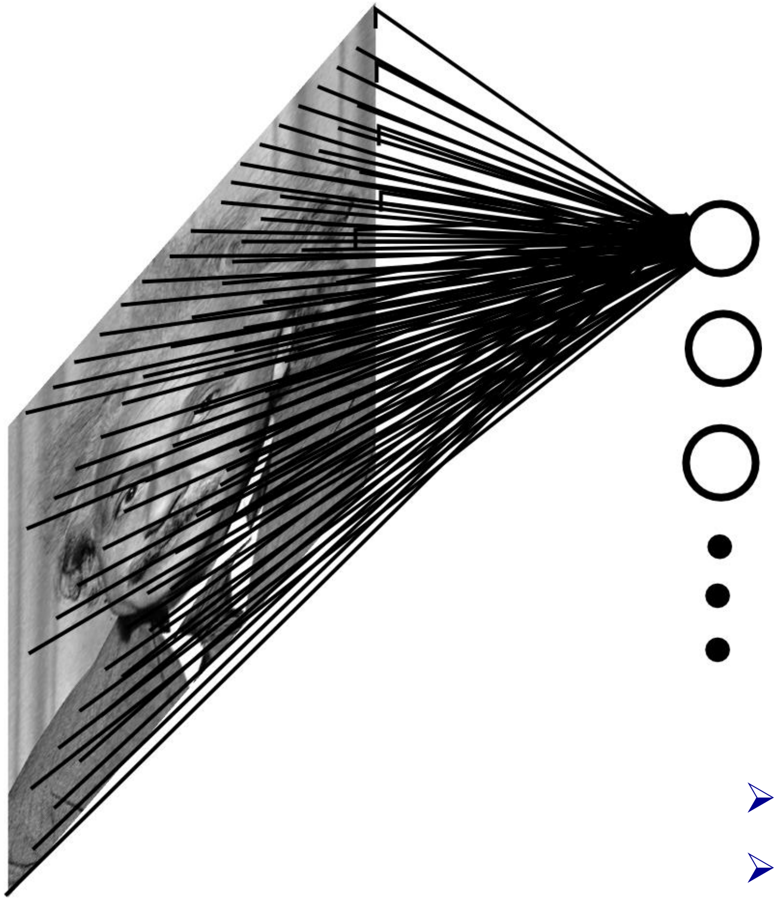
Local Connectivity

- Units are connected to all channels:
  - 1 channel if grayscale image,
  - 3 channels (R, G, B) if color image



Local Connectivity

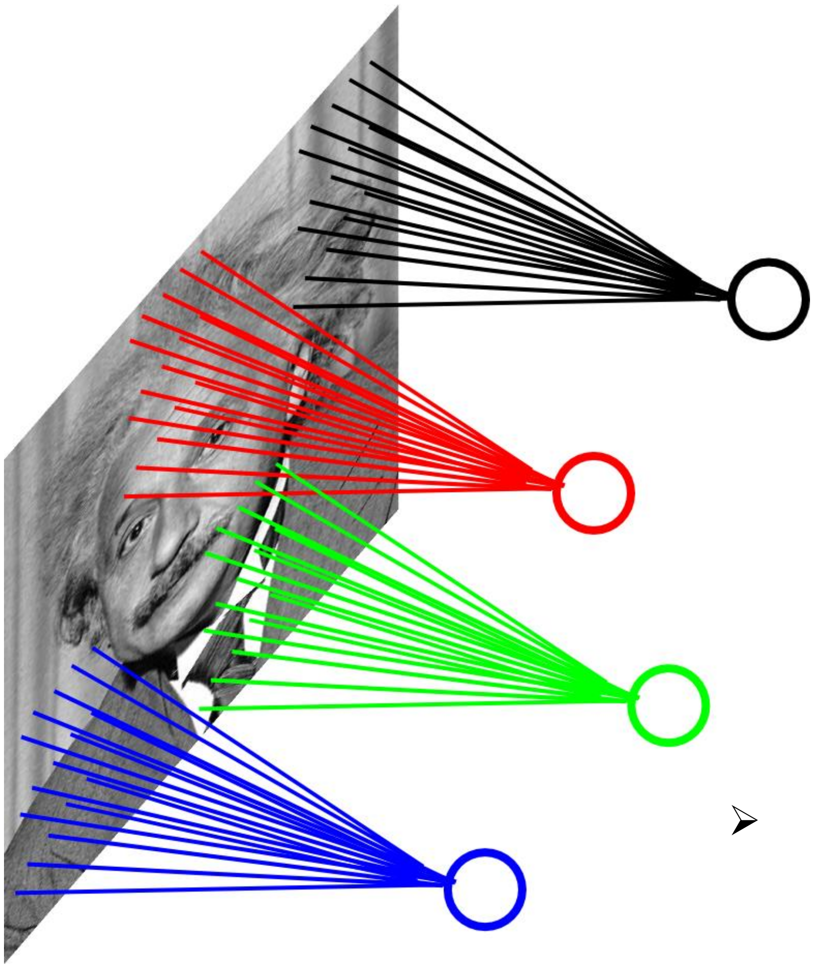
- Example: 200x200 image, 40K hidden units, ~2B parameters!



- Spatial correlation is local
- Too many parameters, will require a lot of training data!

Local Connectivity

- Example: 200x200 image, 40K hidden units, filter size 10x10, 4M parameters!



- This parameterization is good when input image is registered

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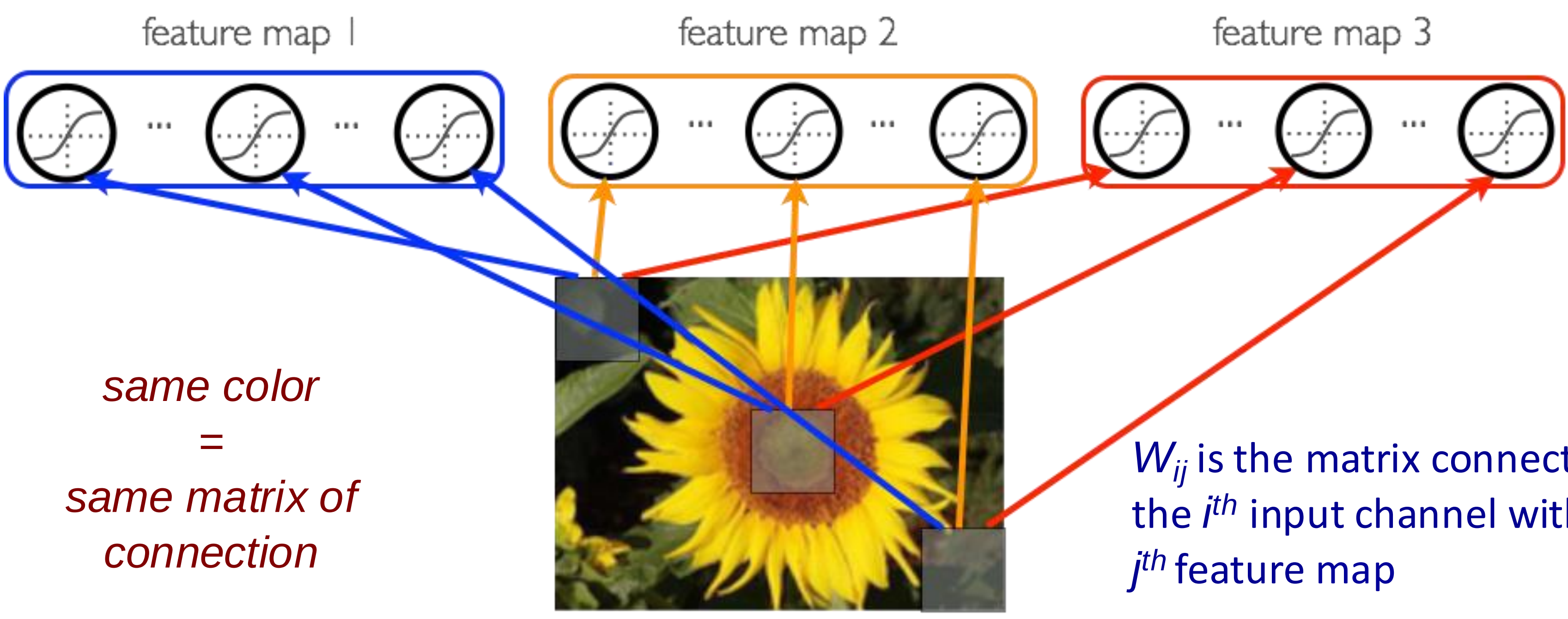
  - ➢ Parameter sharing
  - ➢ Convolution
  - ➢ Pooling / subsampling hidden units

Parameter Sharing

- Share matrix of parameters across some units
  - Units that are organized into the ‘feature map” share parameters
  - Hidden units within a feature map cover different positions in the image

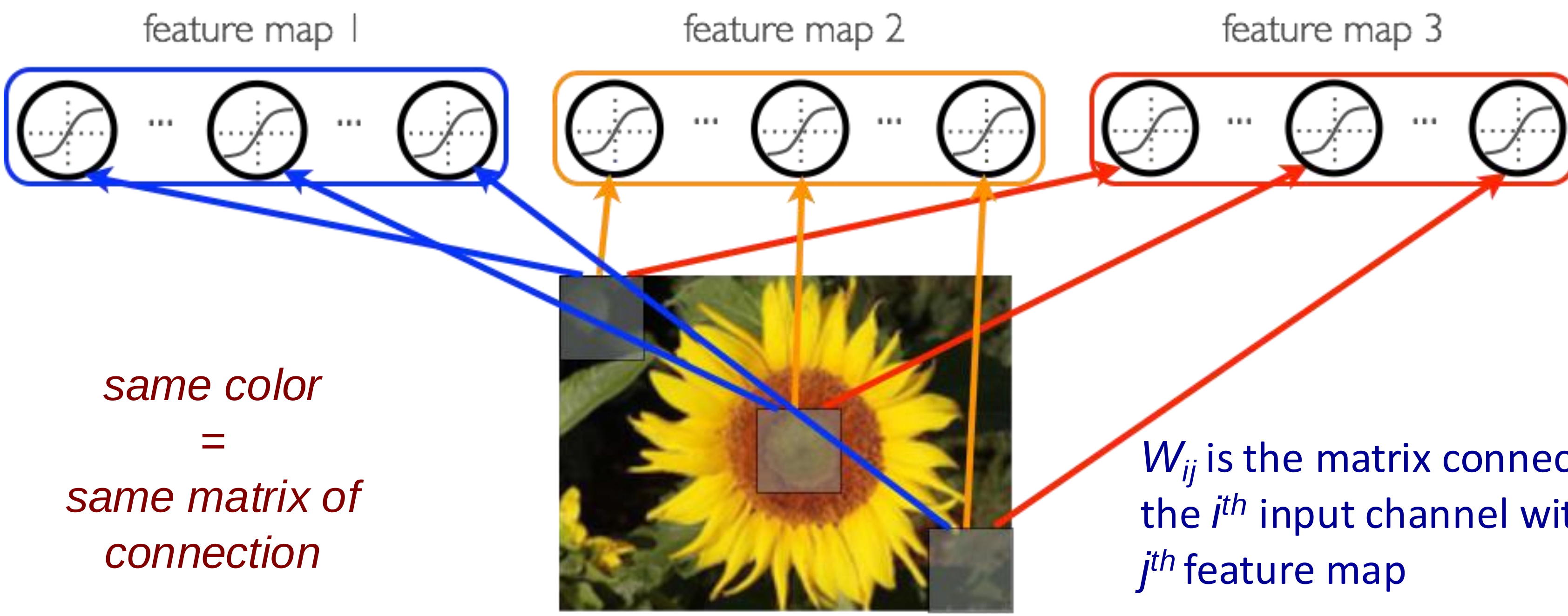


same color
=
same matrix of connection

W_{ij} is the matrix connecting the i^{th} input channel with the j^{th} feature map

Parameter Sharing

- Why parameter sharing?
  - Reduces even more the number of parameters
  - Will extract the same features at every position (features are "equivariant")



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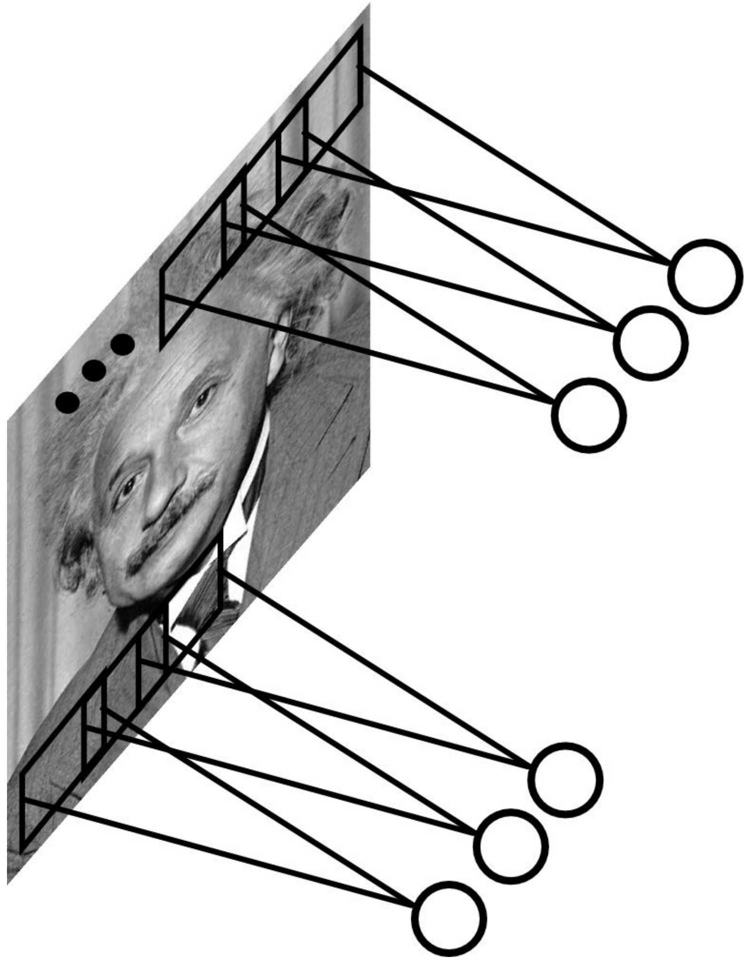
W_{ij} is the matrix connecting the i^{th} input channel with the j^{th} feature map

Parameter Sharing

- Share matrix of parameters across certain units



➢ Convolutions with certain kernels

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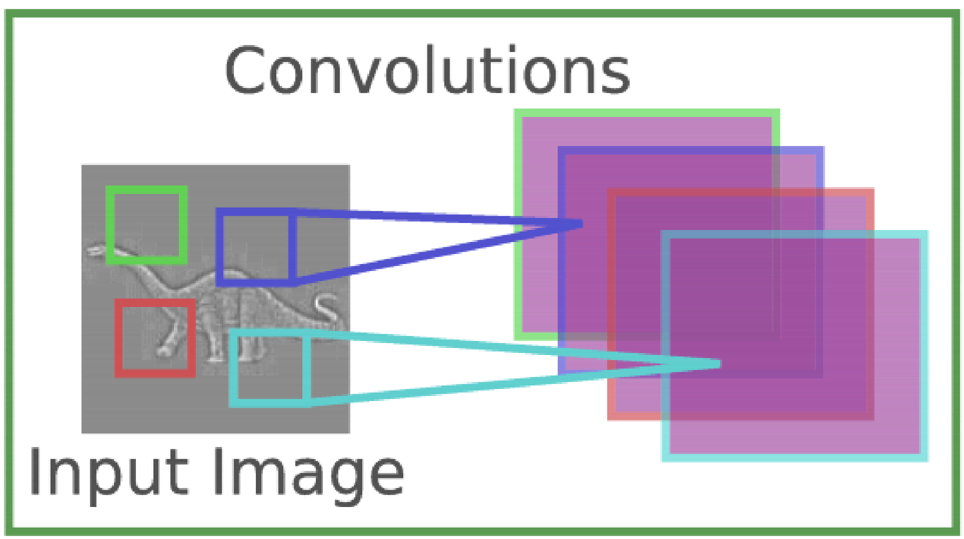
Parameter Sharing

- Each feature map forms a 2D grid of features
  - ➢ can be computed with a discrete convolution ($*$) of a kernel matrix k_{ij} which is the hidden weights matrix W_{ij} with its rows and columns flipped



Jarret et al. 2009

$$y_j = g_j \tanh\left(\sum_i k_{ij} * x_i\right)$$

- x_i is the i^th channel of input
- k_{ij} is the convolution kernel
- g_j is a learned scaling factor
- g_j is the hidden layer

can add bias

Discrete Convolution

$$(x * k)_{ij} = \sum_{pq} x_{i+p,j+q} k_{r-p,r-q}$$

- Example:

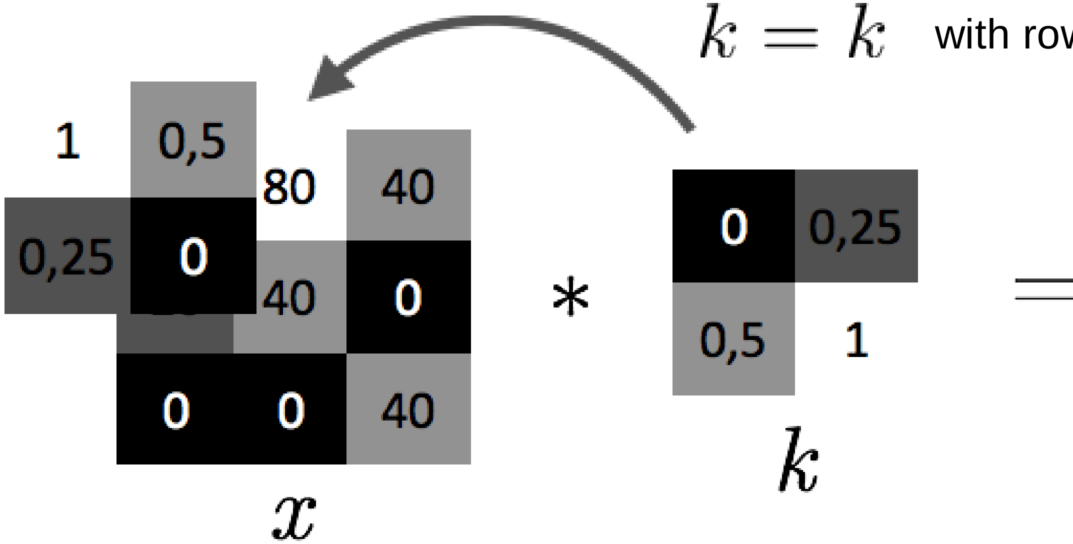
$$\underbrace{\begin{array}{|c|c|c|}\hline 0 & 80 & 40 \\ \hline 20 & 40 & 0 \\ \hline 0 & 0 & 40 \\ \hline \end{array}}_{x} * \underbrace{\begin{array}{|c|c|}\hline 0 & 0{,}25 \\ \hline 0{,}5 & 1 \\ \hline \end{array}}_{k} =$$

Discrete Convolution

$$(x * k)_{ij} = \sum_{pq} x_{i+p,j+q} k_{r-p,r-q}$$

- Example:

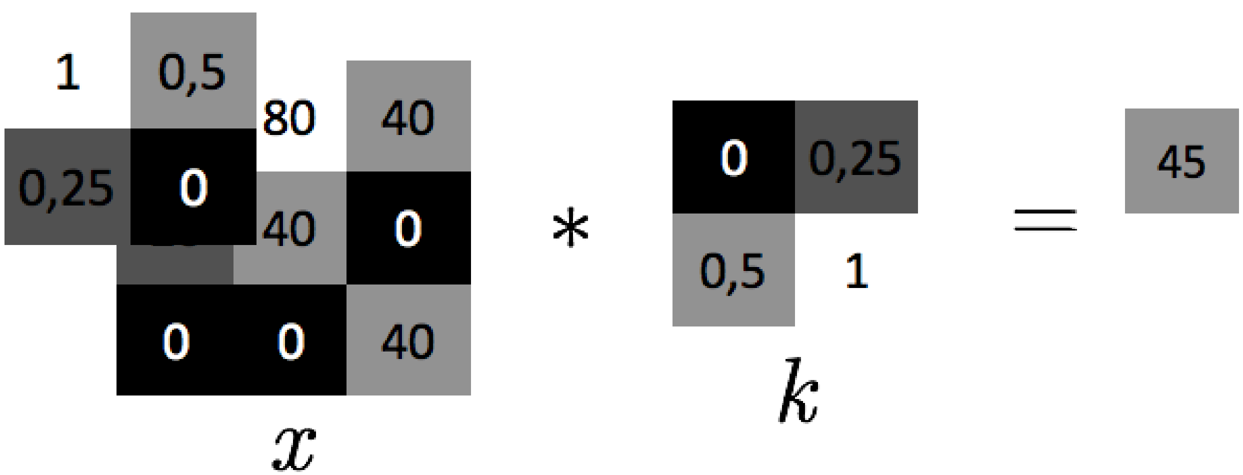
$\tilde{k} = k$ with rows and columns flipped



Discrete Convolution

$$(x * k)_{ij} = \sum_{pq} x_{i+p,j+q} k_{r-p,r-q}$$

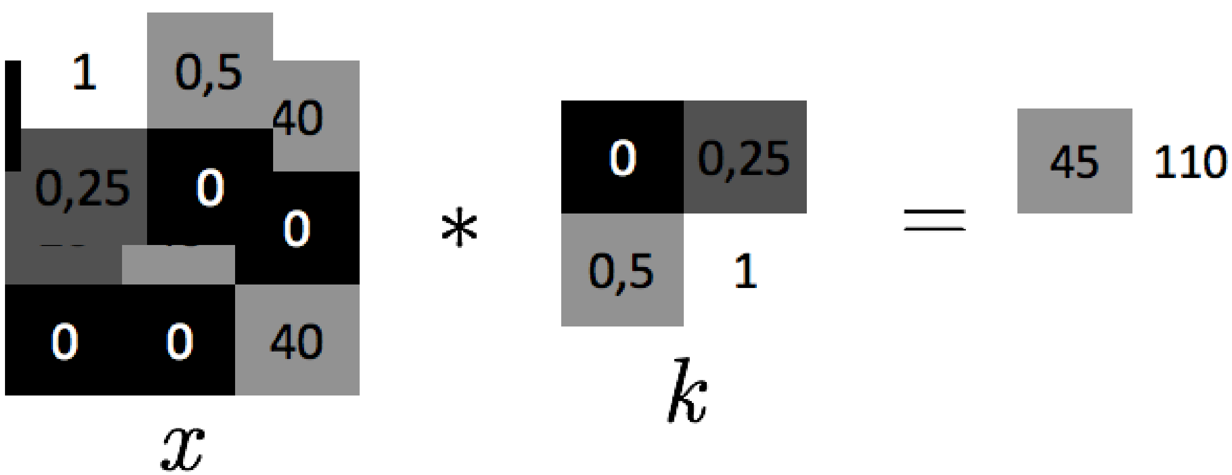
- Example: 1 x 0 + 0.5 x 80 + 0.25 x 20 + 0 x 40 = 45



Discrete Convolution

$$(x * k)_{ij} = \sum_{pq} x_{i+p,j+q} k_{r-p,r-q}$$

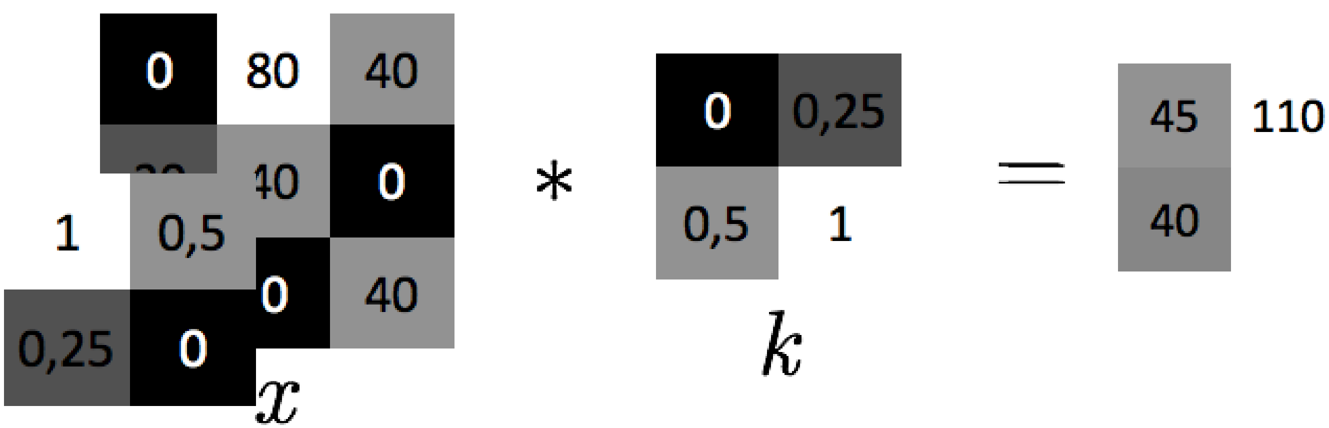
- Example: 1 x 80 + 0.5 x 40 + 0.25 x 40 + 0 x 0 = 110



Discrete Convolution

$$(x * k)_{ij} = \sum_{pq} x_{i+p,j+q} k_{r-p,r-q}$$

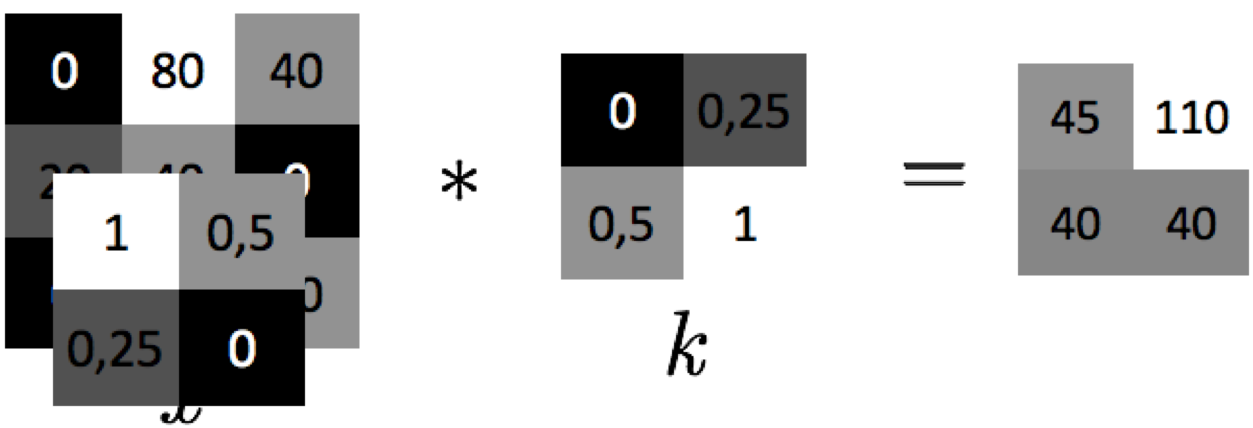
- Example: $1 \times 20 + 0.5 \times 40 + 0.25 \times 0 + 0 \times 0 = 40$



Discrete Convolution

$$(x * k)_{ij} = \sum_{pq} x_{i+p,j+q} k_{r-p,r-q}$$

- Example: 1 x 40 + 0.5 x 0 + 0.25 x 0 + 0 x 40 = 40



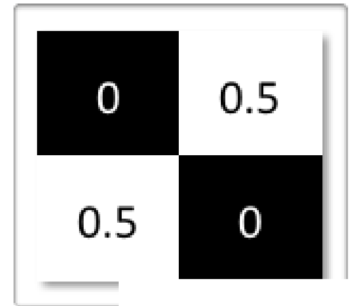
Discrete Convolution

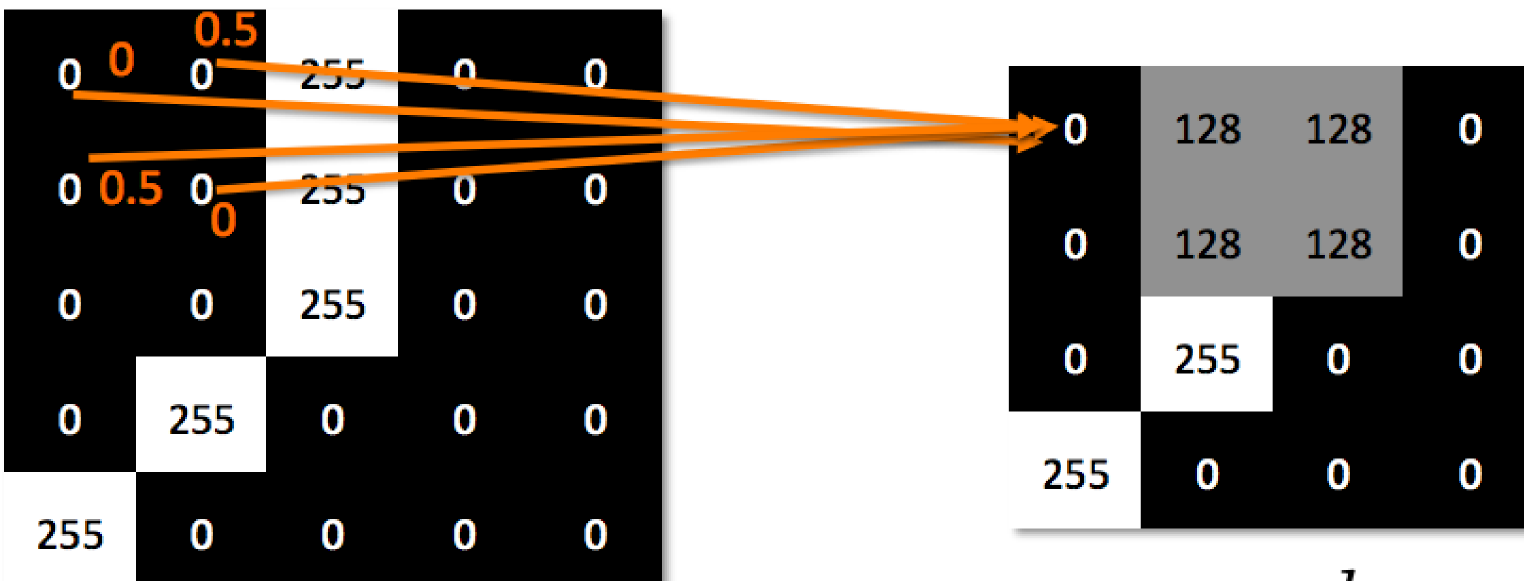
- Pre-activations from channel x_i into feature map y_j can be computed by:
  - getting the convolution kernel where $k_{ij} = \widetilde{W}_{ij}$ from the connection matrix W_{ij}
  - applying the convolution $x_i * k_{ij}$
- This is equivalent to computing the discrete correlation of x_i with W_{ij}

Example

- Illustration:

$$x * k_{ij}, \quad \text{where } W_{ij} = \tilde{W}_{ij}$$





x_i $\qquad$ $x_i * k_{ij}$

Example

- With a non-linearity, we get a detector of a feature at any position in the image:

$$x * k_{ij}, \quad \text{where } W_{ij} = \tilde{W}_{ij}$$

0	0.5
0.5	0

0	0	255	0	0
0	0	255	0	0
0	0	255	0	0
0	255	0	0	0
255	0	0	0	0

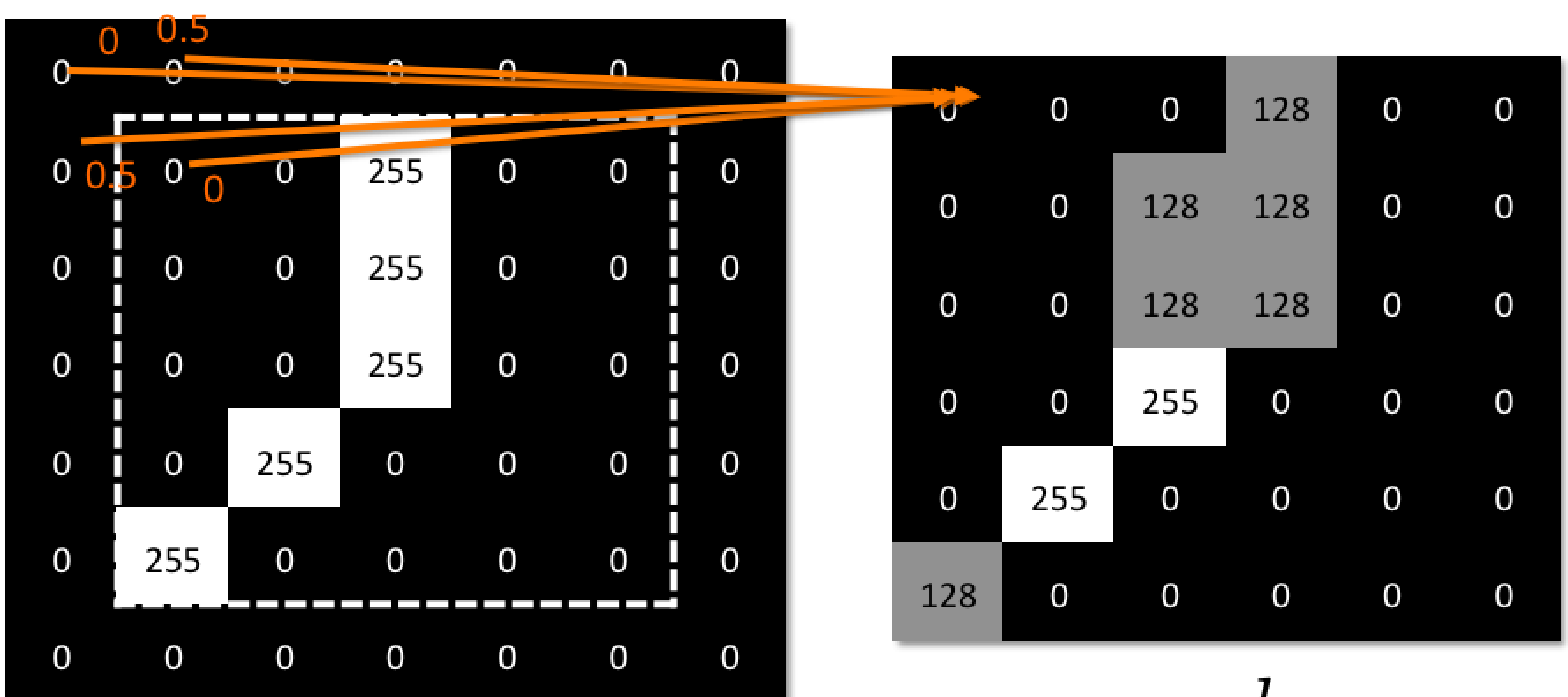
x_i

0.02	0.19	0.19	0.02
0.02	0.19	0.19	0.02
0.02	0.75	0.02	0.02
0.75	0.02	0.02	0.02

$\text{sigm}(0.02\ x_i * k_{ij}\ \text{-}4)$

Example

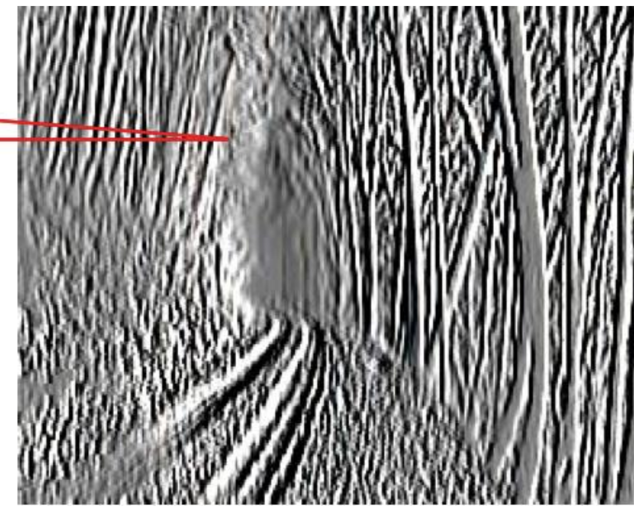
- Can use "zero padding" to allow going over the borders (*)



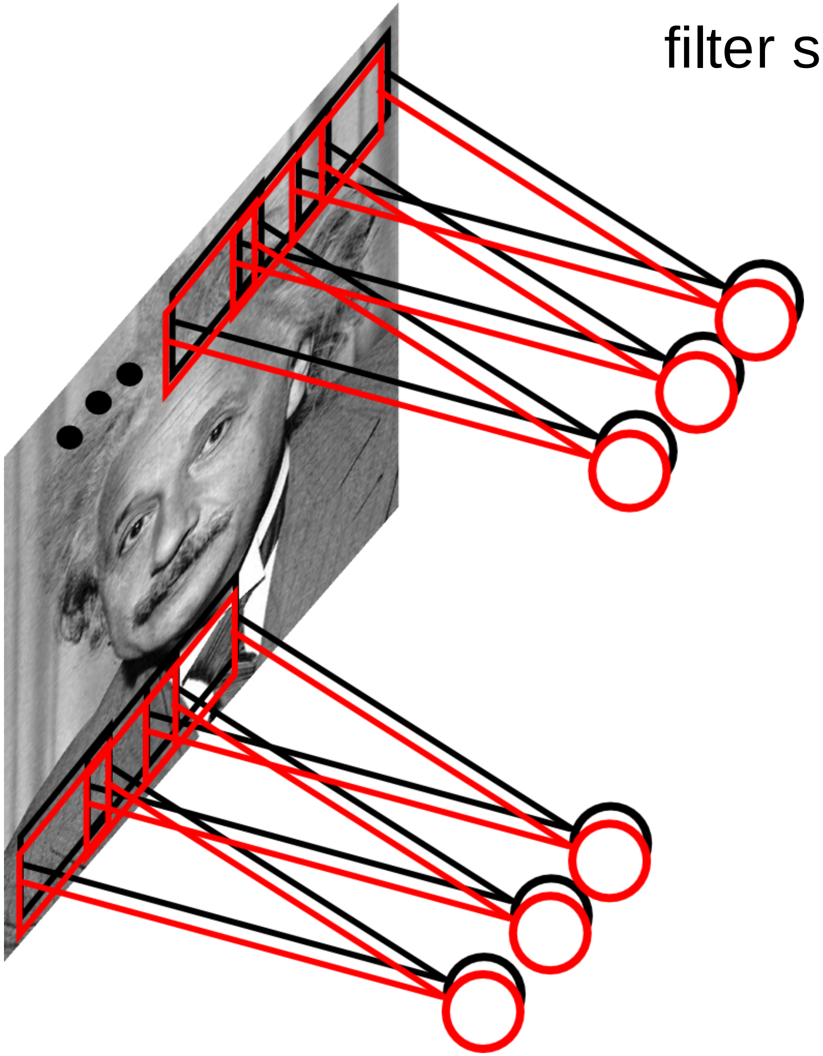
x_i $\qquad$ $x_i \underline{*} k_{ij}$

Example

$$* \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} =$$



Multiple Feature Maps



- Example: 200x200 image, 100 filters, filter size 10x10, 10K parameters

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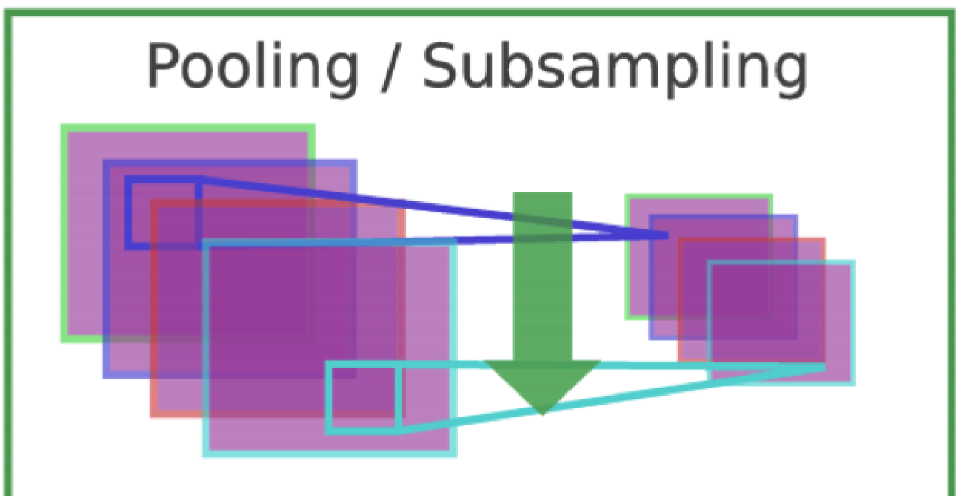
Pooling

- Pool hidden units in same neighborhood
  - pooling is performed in non-overlapping neighborhoods (subsampling)

$$y_{ijk} = \max_{p,q} x_{i,j+p,k+q}$$



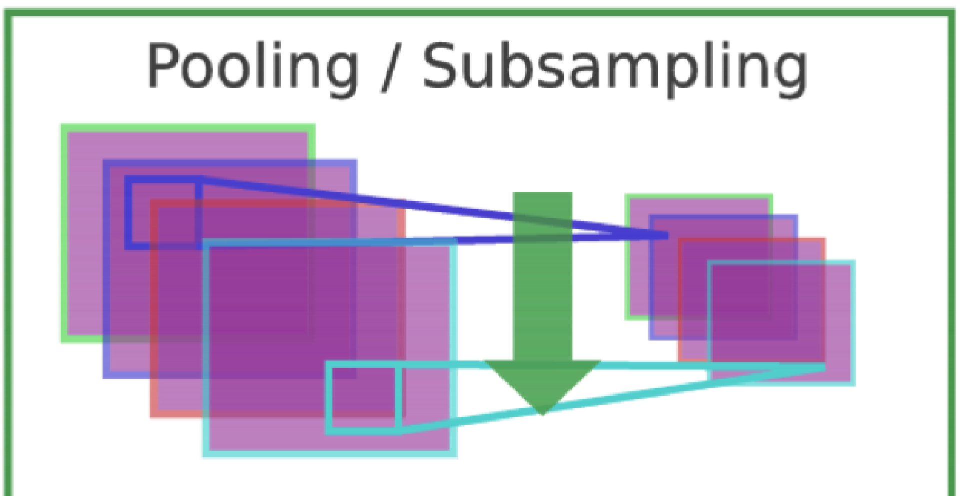
Jarret et al. 2009

- x_i is the i^{th} channel of input
- $x_{i,j,k}$ is value of the i^{th} feature map at position j,k
- p is vertical index in local neighborhood
- q is horizontal index in local neighborhood
- y_{ijk} is pooled / subsampled layer

Pooling

- Pool hidden units in same neighborhood
  - an alternative to "max" pooling is "average" pooling

$$y_{ijk} = \frac{1}{m^2} \sum_{p,q} x_{i,j+p,k+q}$$



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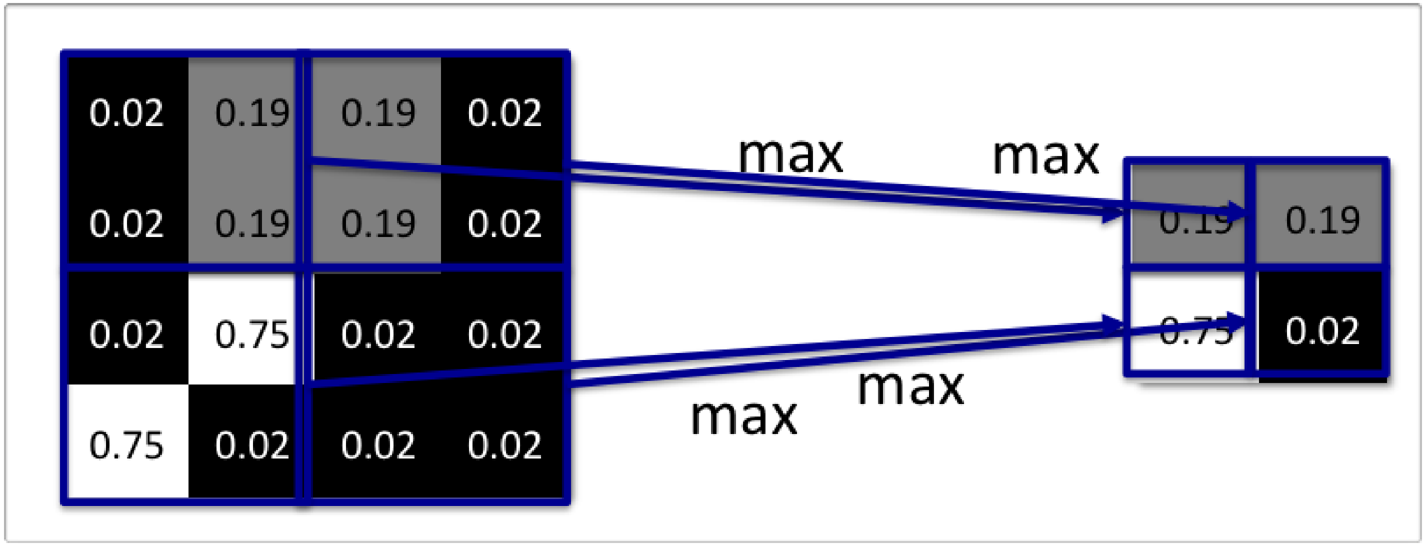
- p is vertical index in local neighborhood
- q is horizontal index in local neighborhood
- y_{ijk} is pooled / subsampled layer
- m is the neighborhood height/width

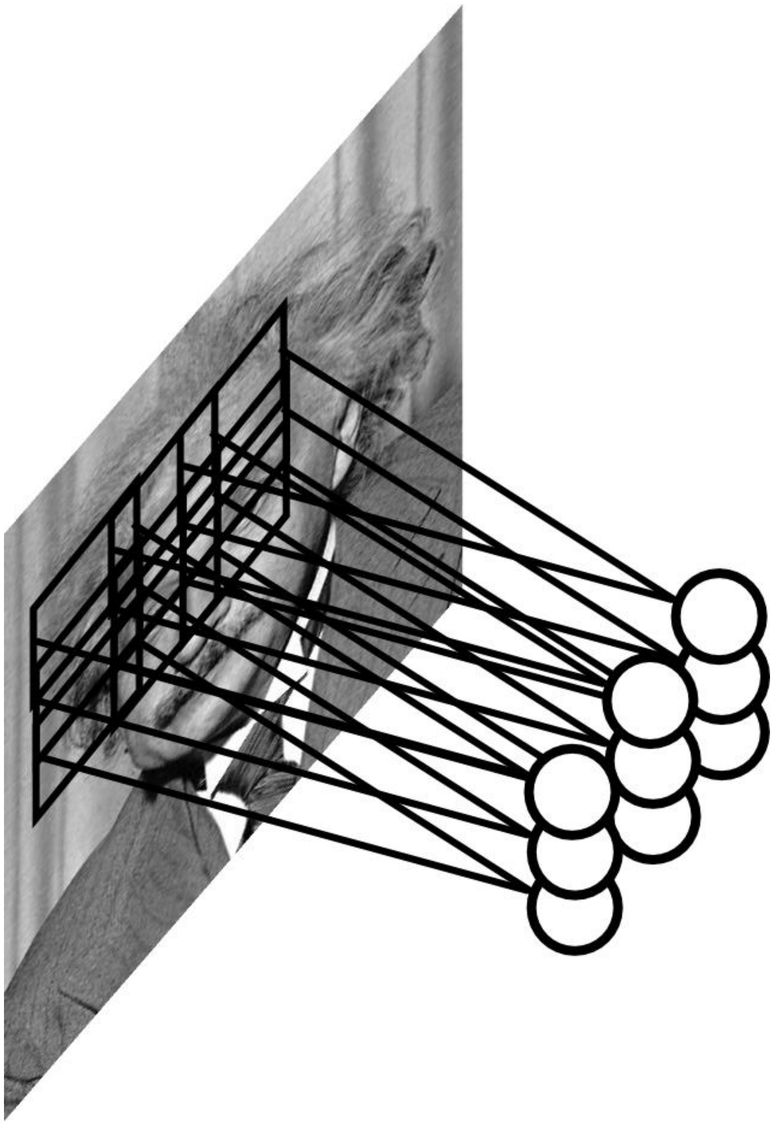
Example: Pooling

- Illustration of pooling/subsampling operation



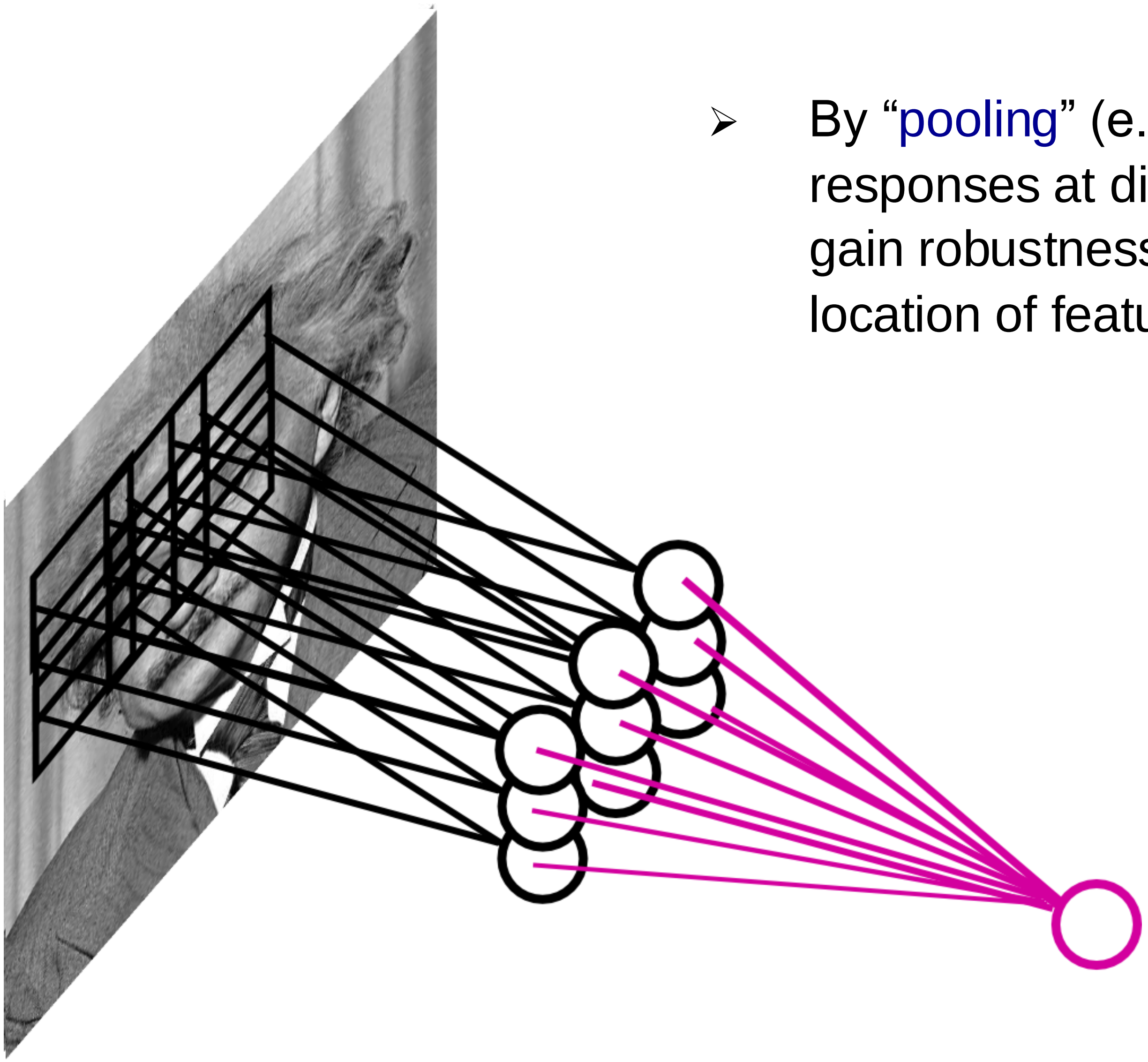
- Why pooling?
  - ➢ Introduces invariance to local translations
  - ➢ Reduces the number of hidden units in hidden layer

Example: Pooling



- can we make the detection robust to the exact location of the eye?

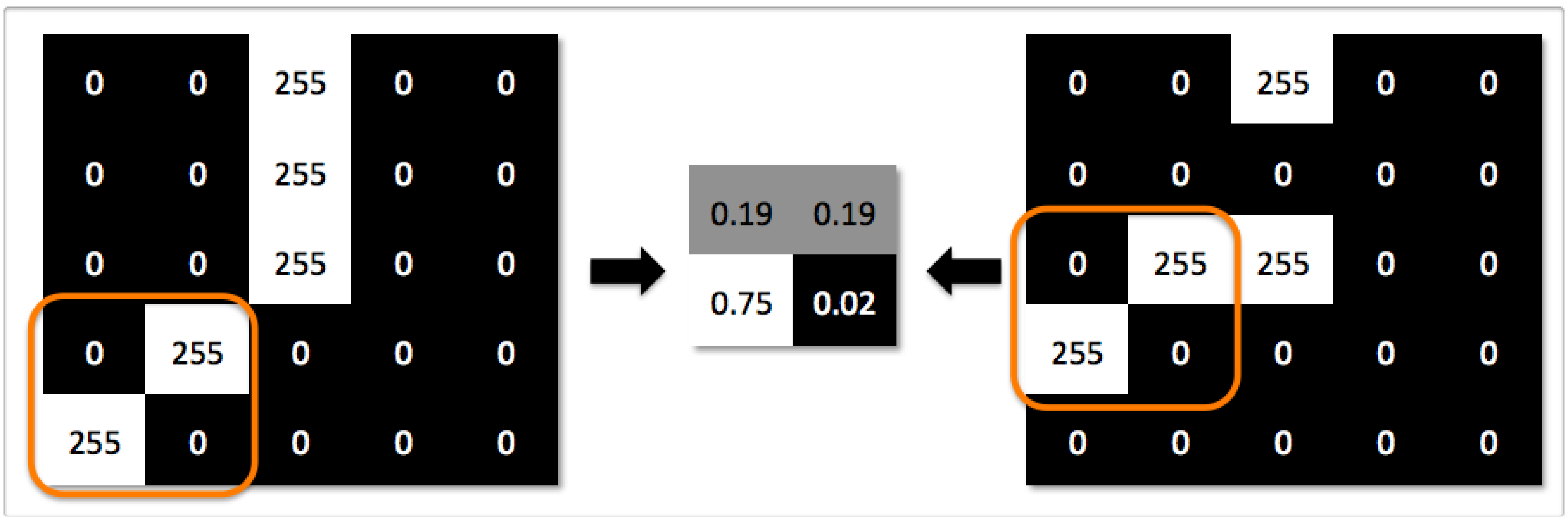
Example: Pooling



- By "pooling" (e.g., taking max) filter responses at different locations we gain robustness to the exact spatial location of features.

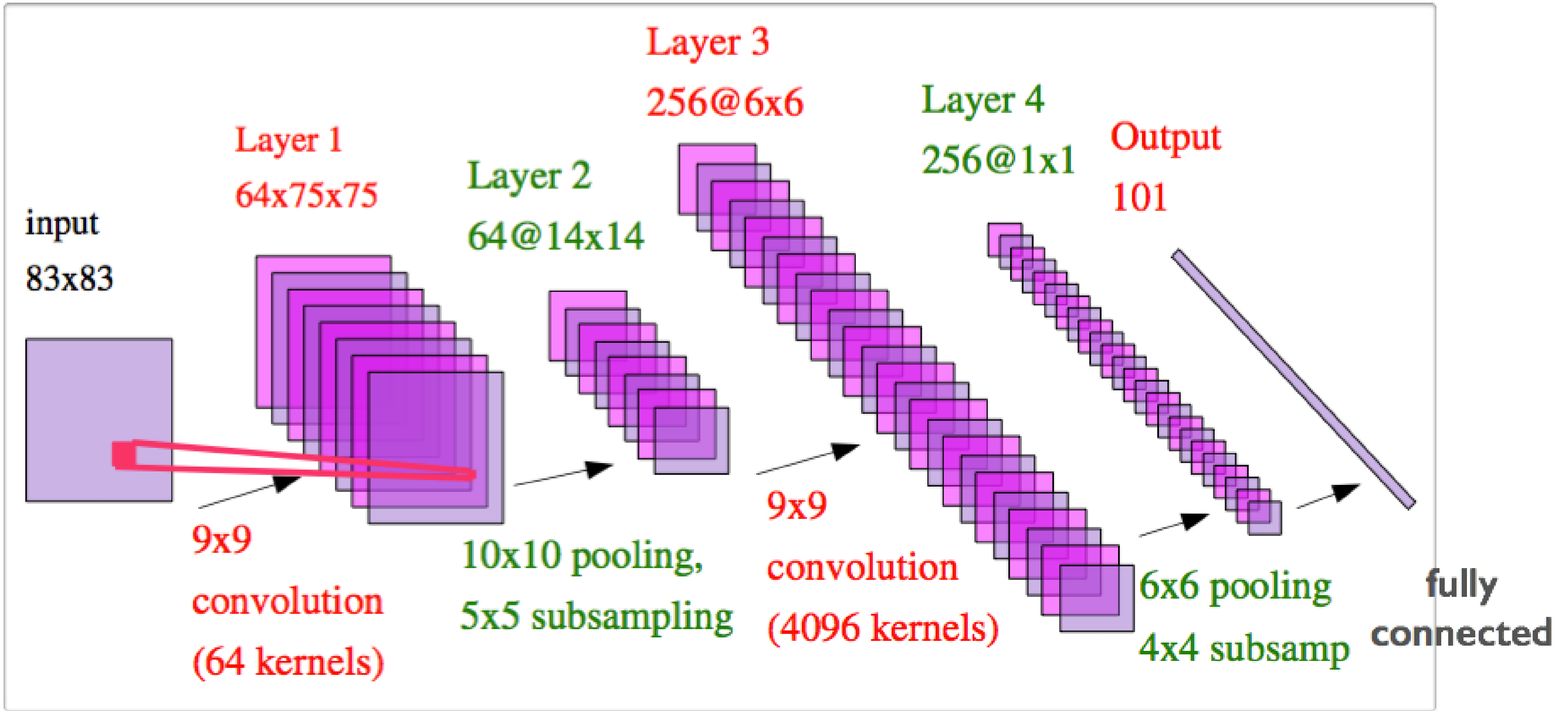
Translation Invariance

- Illustration of local translation invariance
  - ➢ both images result in the same feature map after pooling/subsampling



Convolutional Network

- Convolutional neural network alternates between the convolutional and pooling layers



From Yann LeCun's slides

Convolutional Network

- For classification: Output layer is a regular, fully connected layer with softmax non-linearity
  - Output provides an estimate of the conditional probability of each class

- The network is trained by stochastic gradient descent
  - Backpropagation is used similarly as in a fully connected network
  - We have seen how to pass gradients through element-wise activation function
  - We also need to pass gradients through the convolution operation and the pooling operation

Gradient of Convolutional Layer

- Let l be the loss function
  - For max pooling operation $y_{ijk} = \max_{p,q} x_{i,j+p,k+q}$, the gradient for x_{ijk} is

$$\nabla_{x_{ijk}} l = 0, \text{ except for } \nabla_{x_{i,j+p',k+q'}} l = \nabla_{y_{ijk}} l$$

where p', q' = argmax $x_{i,j+p,k+q}$

  - In other words, only the "winning" units in layer x get the gradient from the pooled layer

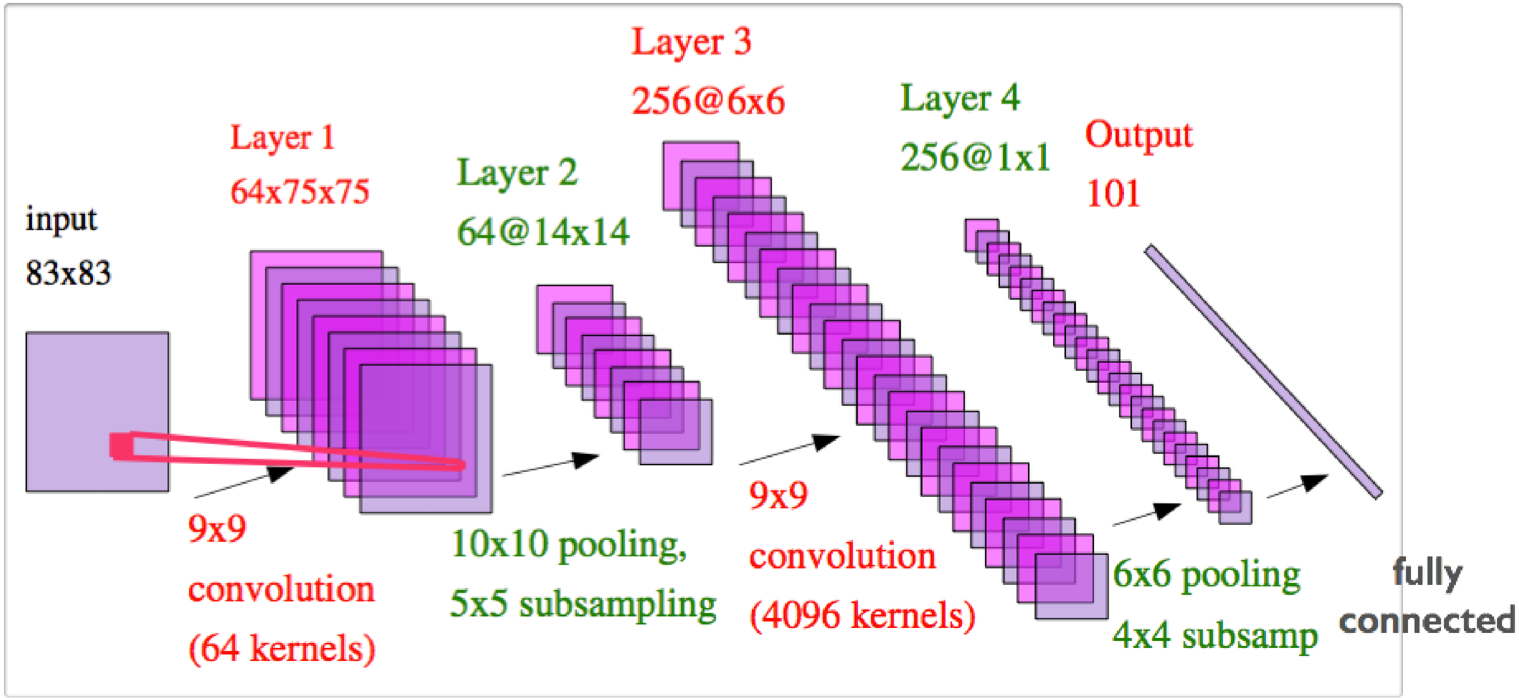
  - For the average operation $y_{ijk} = \frac{1}{m^2} \sum_{p,q} x_{i,j+p,k+q}$, the gradient for x_{ijk} is

$$\nabla_x l = \frac{1}{m^2} \text{upsample}(\nabla_y l)$$

where upsample inverts subsampling

Convolutional Network

• Convolutional neural network alternates between the convolutional and pooling layers



• Need to introduce other operations that can improve object recognition.

Rectification

- Rectification layer: $y_{ijk} = |x_{ijk}|$

  ➢ introduces invariance to the sign of the unit in the previous layer

  ➢ for instance, loss of information of whether an edge is black-to-white or white-to-black

Local Contrast Normalization

- Perform local contrast normalization

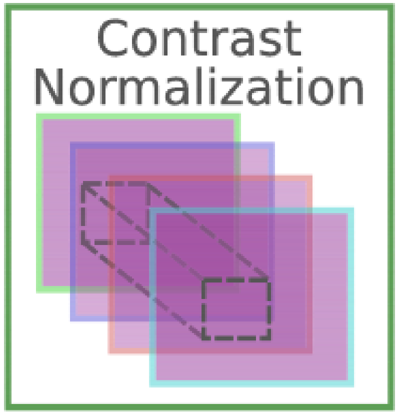
Local average

$$v_{ijk} = x_{ijk} - \boxed{\textstyle\sum_{ipq} w_{pq} x_{i,j+p,k+q}}$$

$$y_{ijk} = v_{ijk} / \max(c, \sigma_{jk})$$

Local stdev

$$\sigma_{jk} = \boxed{\left(\textstyle\sum_{ipq} w_{pq} v^2_{i,j+p,k+q}\right)^{1/2}} \qquad \sum_{pq} w_{pq} = 1$$



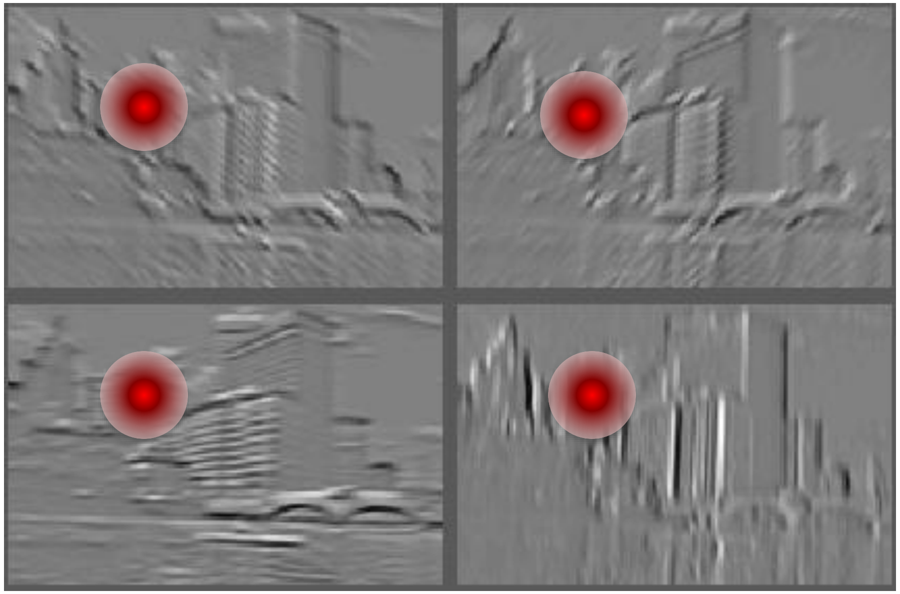
where c is a small constant to prevent division by 0

- reduces unit's activation if neighbors are also active
- creates competition between feature maps
- scales activations at each layer better for learning

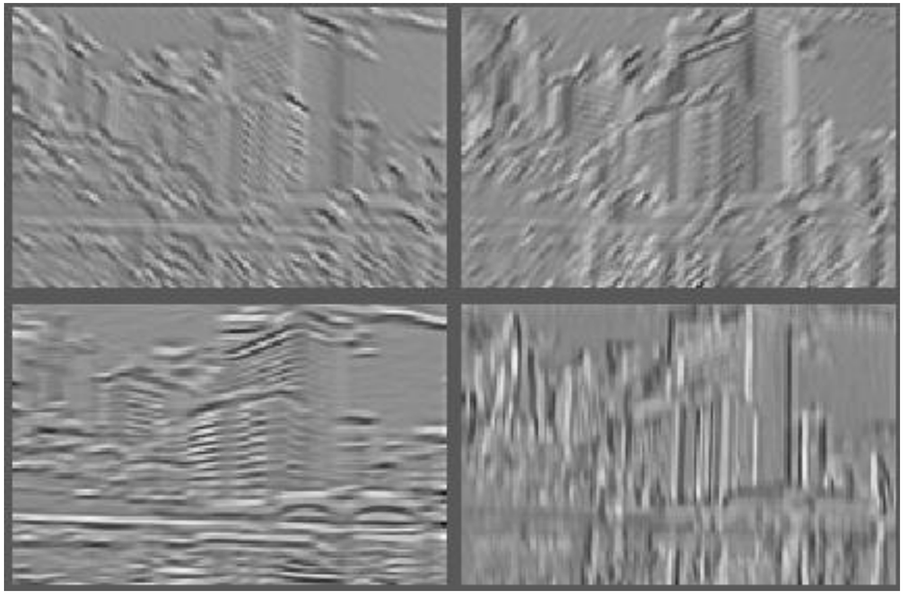
Local Contrast Normalization

- Perform local contrast normalization
  - ➢ Local mean=0, Local std. = 1, “Local” is 7x7 Gaussian

Feature Maps

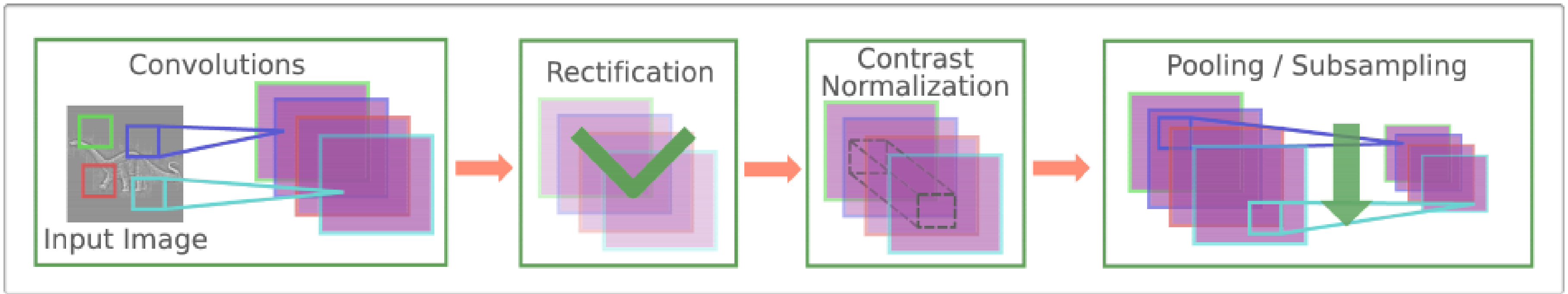


Feature Maps after Contrast Normalization

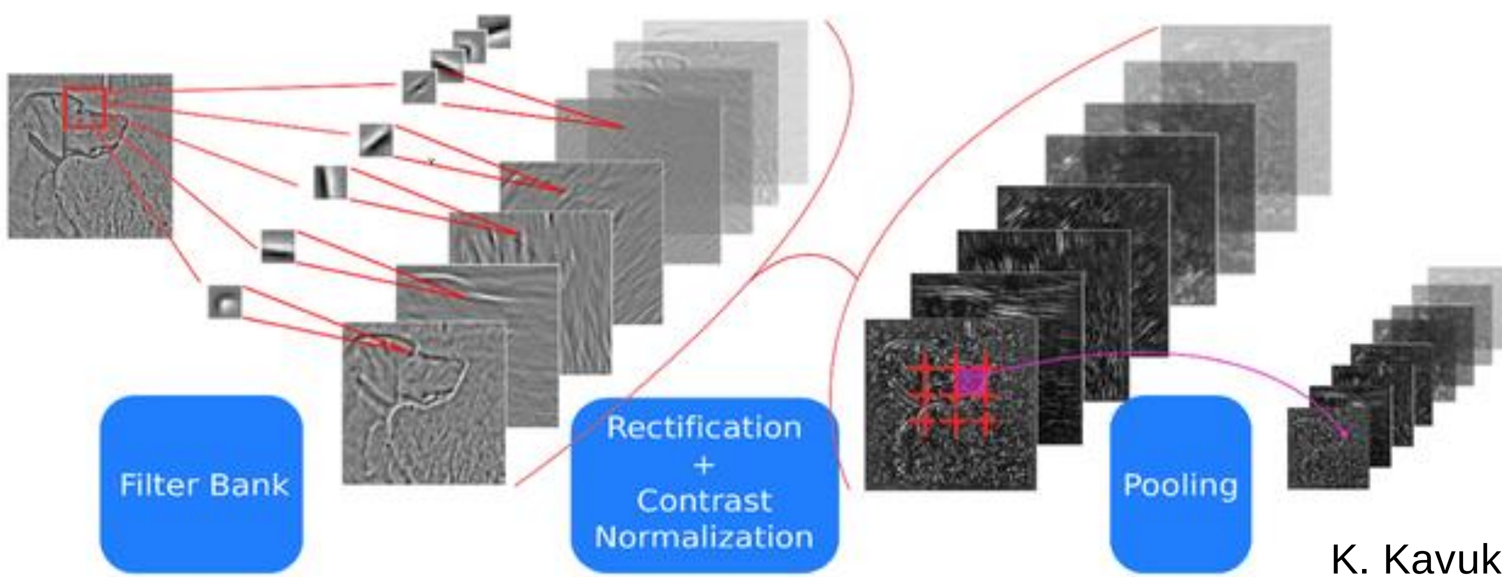


Convolutional Network

- These operations are inserted after the convolutions and before the pooling



Jarret et al. 2009



K. Kavukcuoglu

Remember Batch Normalization

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1...m}\}$;
Parameters to be learned: γ, β

Output: $\{y_i = \text{BN}_{\gamma,\beta}(x_i)\}$

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i \qquad \text{// mini-batch mean}$$

$$\sigma_{\mathcal{B}}^2 \leftarrow \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_{\mathcal{B}})^2 \qquad \text{// mini-batch variance}$$

$$\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} \qquad \text{// normalize}$$

$$\boxed{y_i \leftarrow \gamma\widehat{x}_i + \beta \equiv \text{BN}_{\gamma,\beta}(x_i)} \qquad \text{// scale and shift}$$

Learned linear transformation to adapt to non-linear activation function (γ and β are trained)