Diffusion Models

Yutong (Kelly) He 3/26/25

Many figures adopted from Yang Song's "Generative Modeling by Estimating Gradients of the Data Distribution" tutorial & the lecture slides of CMU 10708 by Prof. Andrej Risteski, Prof. Albert Gu & Stanford CS 236 by Prof. Stefano Ermon

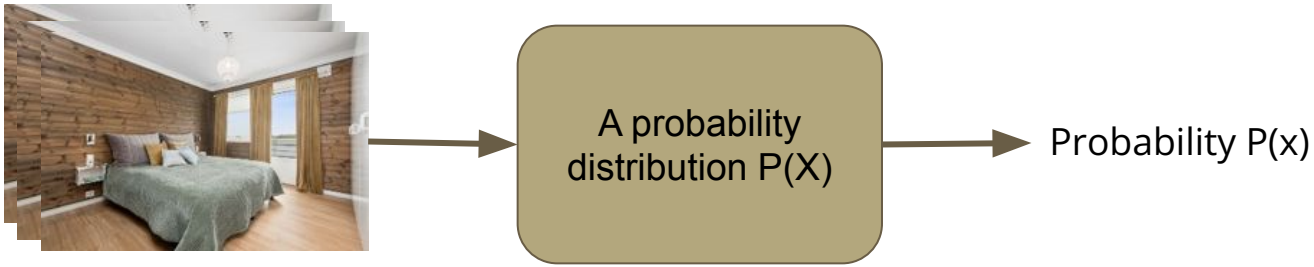
About Me

- Yutong (Kelly) He, 3rd year PhD in MLD advised by Zico & Russ
- Main things I have done in the past
  - H-Divergence: Comparing Distributions by Measuring Differences that Affect Decision Making [paper]
  - Generative Modeling:
    - Image-to-Image Translation [paper]
    - Super-Resolution [paper]
    - ✅ SDEdit: Guided Image Synthesis and Editing with *Stochastic Differential Equations* [paper]
    - ✅ CAC: Localized Text-to-Image Generation for Free via Cross Attention Control [paper]
    - ✅ MPGD: Manifold Preserving Guided *Diffusion* [paper]
    - Black-box Prompt Engineering for Personalized Text-to-Image Generation [paper]
    - ✅ LADiBI: Blind Inverse Problem Solving Made Easy by Text-to-Image Latent *Diffusion* [paper]
    - Diffusion applications in reinforcement learning [paper]

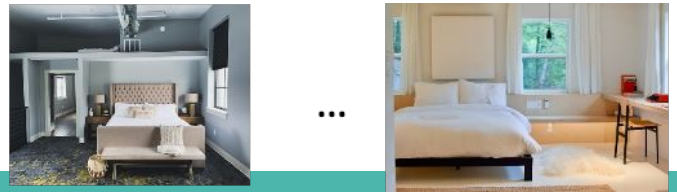
Generative Modeling

A (statistical) generative model is a **probability distribution P(X)** (as opposed to P(Y|X) in discriminative models)

- **Data:** samples (e.g., images of bedrooms)
- **Prior knowledge:** parametric form, loss function, optimization, etc.

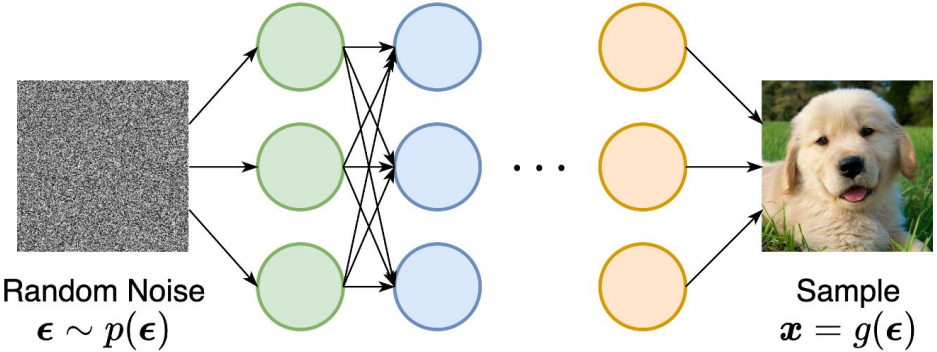


It is generative because sampling from p(x) generates new images



Generative Models

- **Likelihood Based:** Autoregressive models, variational autoencoders (VAE), normalizing flow, energy-based models (EBM)
- **Likelihood Free:** Generative adversarial networks (GAN)



Directly sampling from P(X) is usually hard because they are usually complicated! But **sampling from a simpler distribution** (eg. a Gaussian) is easy!

Generative Models

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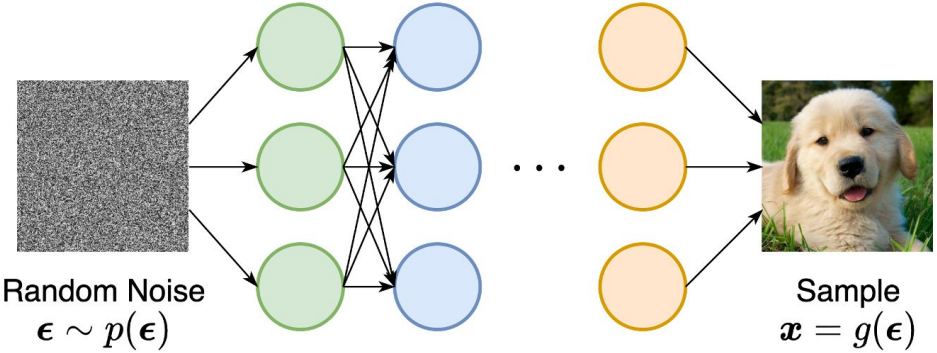

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But **sampling from a simpler distribution** (eg. a Gaussian) is easy!

GAN (Goodfellow et. al 2014) Overview

Generator

Tries to make the fake samples more and more realistic so that it can fool the discriminator

Discriminator

Tries to be better and better at distinguishing fake samples from real ones

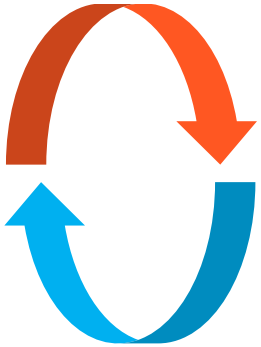
Ian J. Goodfellow, Jean Pouget-Abadie, Mehdi Mirza, Bing Xu, David Warde-Farley, Sherjil Ozair, Aaron Courville, Yoshua Bengio. “Generative Adversarial Networks”. NeurIPS 2014. https://arxiv.org/pdf/1406.2661

GAN (Goodfellow et. al 2014) Overview

$$\min_G \max_D V(D,G) = \mathbb{E}_{\boldsymbol{x}\sim p_{\text{data}}(\boldsymbol{x})}[\log D(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{z}\sim p_{\boldsymbol{z}}(\boldsymbol{z})}[\log(1 - D(G(\boldsymbol{z})))]$$

Generator

Tries to make the fake samples more and more realistic so that it can fool the discriminator



Discriminator

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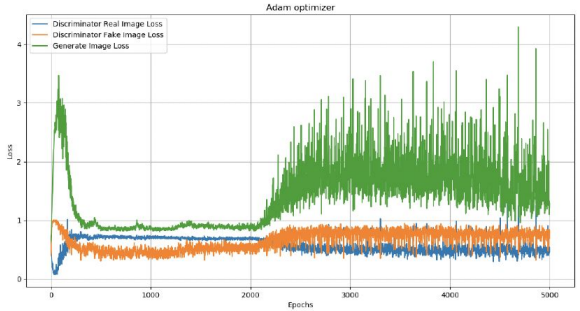
However...

- **Theorem (informal):** If the generator updates are made in function space and discriminator is optimal at every step, then the generator is guaranteed to converge to the data distribution
- **Unrealistic assumptions**!
- In practice, the generator and discriminator loss keeps oscillating during GAN training

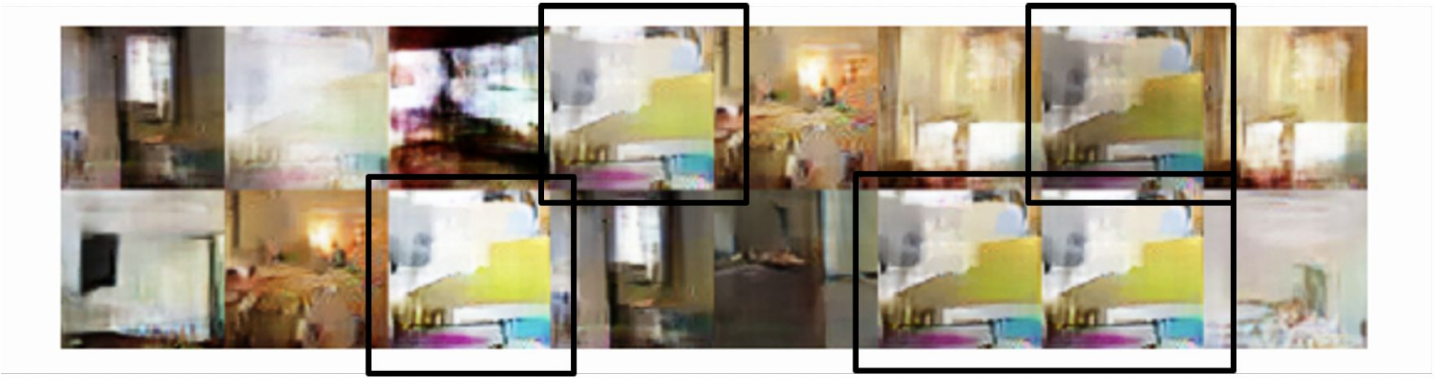


Source: Mirantha Jayathilaka

- No robust stopping criteria in practice (unlike MLE)

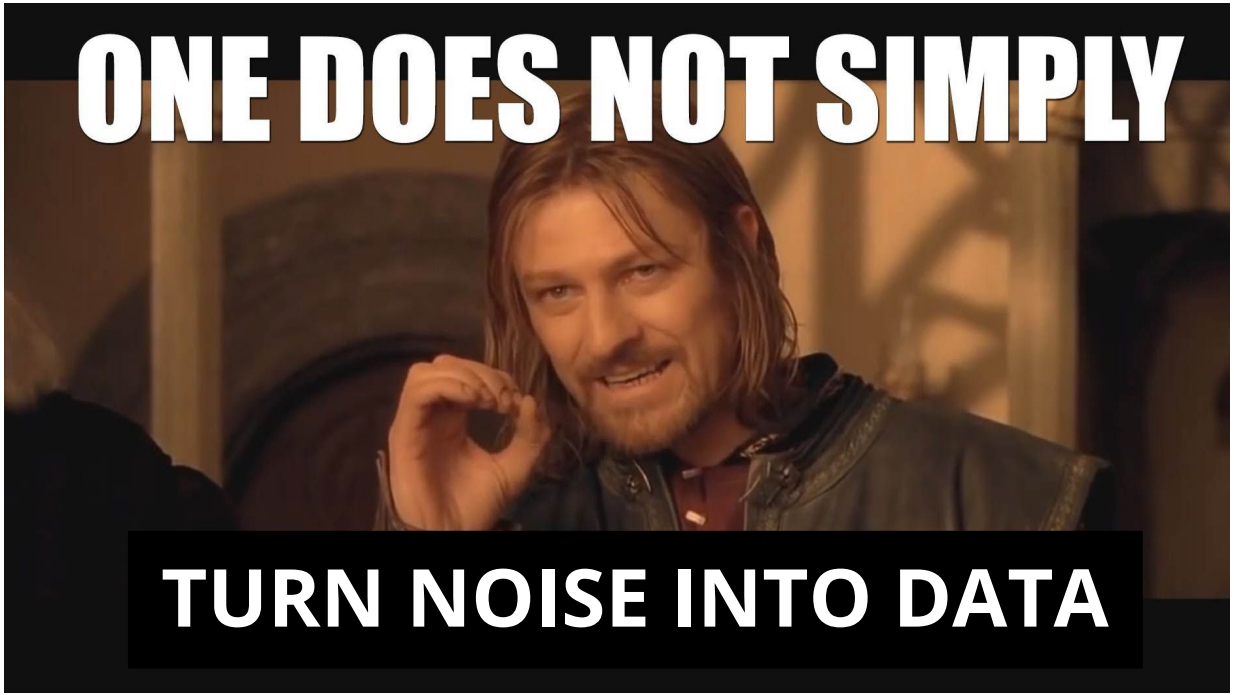
However...

- GANs are notorious for suffering from **mode collapse**
- Intuitively, this refers to the phenomena where the generator of a GAN collapses to one or few samples (dubbed as "modes")



Arjovsky et al., 2017

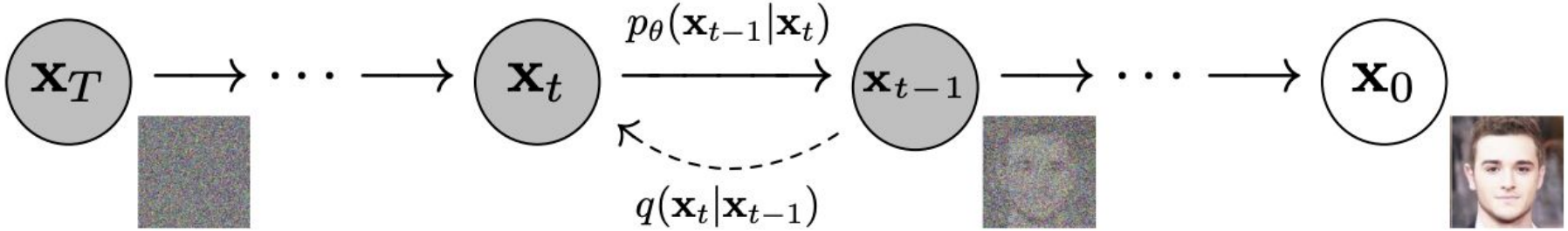
From Noise to Data



From Noise to Data

Now you get **Diffusion Model**!

Diffusion Model (Sohl-Dickstein, et al. 2015)



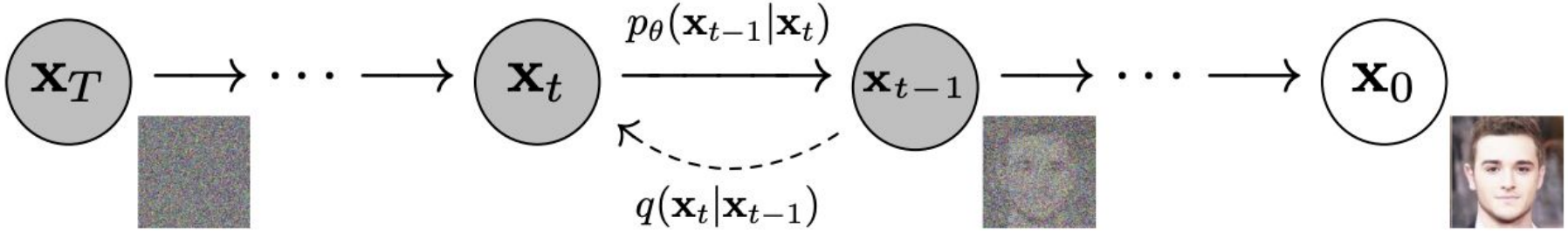
Forward (Adding Noise):

$$q(\mathbf{x}_{1:T}|\mathbf{x}_0) := \prod_{t=1}^{T} q(\mathbf{x}_t|\mathbf{x}_{t-1}), \qquad q(\mathbf{x}_t|\mathbf{x}_{t-1}) := \mathcal{N}(\mathbf{x}_t; \sqrt{1-\beta_t}\mathbf{x}_{t-1}, \beta_t\mathbf{I})$$

Backward (Denoising):

$$p_\theta(\mathbf{x}_{0:T}) := p(\mathbf{x}_T) \prod_{t=1}^{T} p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t), \qquad p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t) := \mathcal{N}(\mathbf{x}_{t-1}; \boldsymbol{\mu}_\theta(\mathbf{x}_t, t), \boldsymbol{\Sigma}_\theta(\mathbf{x}_t, t))$$

Diffusion Model (Sohl-Dickstein, et al. 2015)



Training Objective:

$$\mathbb{E}\left[-\log p_\theta(\mathbf{x}_0)\right] \leq \mathbb{E}_q\left[-\log \frac{p_\theta(\mathbf{x}_{0:T})}{q(\mathbf{x}_{1:T}|\mathbf{x}_0)}\right] = \mathbb{E}_q\left[-\log p(\mathbf{x}_T) - \sum_{t\geq 1} \log \frac{p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t)}{q(\mathbf{x}_t|\mathbf{x}_{t-1})}\right] =: L$$

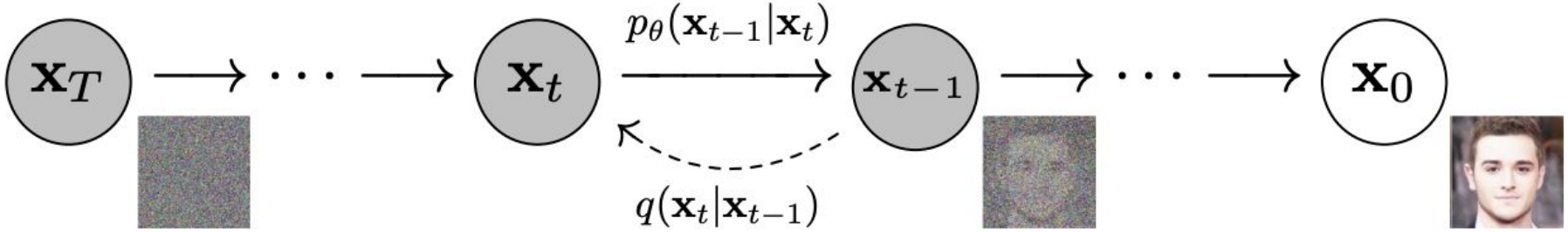
Or $$\mathbb{E}_q\Big[\underbrace{D_{\mathrm{KL}}(q(\mathbf{x}_T|\mathbf{x}_0) \parallel p(\mathbf{x}_T))}_{L_T} + \sum_{t>1} \underbrace{D_{\mathrm{KL}}(q(\mathbf{x}_{t-1}|\mathbf{x}_t, \mathbf{x}_0) \parallel p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t))}_{L_{t-1}} \underbrace{-\log p_\theta(\mathbf{x}_0|\mathbf{x}_1)}_{L_0}\Big]$$

Jascha Sohl-Dickstein, Eric Weiss, Niru Maheswaranathan, Surya Ganguli. “Deep Unsupervised Learning using Nonequilibrium Thermodynamics”. Proceedings of the 32nd International Conference on Machine Learning, PMLR 37:2256-2265, 2015. https://proceedings.mlr.press/v37/sohl-dickstein15.html

Diffusion Model (Sohl-Dickstein, et al. 2015)

Figure 3. The proposed framework trained on the CIFAR-10 (Krizhevsky & Hinton, 2009) dataset. *(a)* Example training data. *(b)* Random samples generated by the diffusion model.

Denoising Diffusion Probabilistic Model (DDPM) (Ho, et al. 2020)



So we know this thing is Markov – it just adds a small amount of noise at every time step. Then why don't we just learn a noise predictor to predict the noise at each time step and then gradually reduce the noise?

Jonathan Ho, Ajay Jain, Pieter Abbeel. "Denoising Diffusion Probabilistic Models". Advances in Neural Information Processing Systems 33 (NeurIPS 2020).
https://arxiv.org/pdf/2006.11239.pdf

Denoising Diffusion Probabilistic Model (DDPM) (Ho, et al. 2020)

Algorithm 1 Training

1: **repeat**
2: $\mathbf{x}_0 \sim q(\mathbf{x}_0)$
3: $t \sim \text{Uniform}(\{1, \dots, T\})$
4: $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
5: Take gradient descent step on
$$\nabla_\theta \left\| \boldsymbol{\epsilon} - \boldsymbol{\epsilon}_\theta(\sqrt{\bar{\alpha}_t}\mathbf{x}_0 + \sqrt{1-\bar{\alpha}_t}\boldsymbol{\epsilon}, t) \right\|^2$$
6: **until** converged

Algorithm 2 Sampling

1: $\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
2: **for** $t = T, \dots, 1$ **do**
3: $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ if $t > 1$, else $\mathbf{z} = \mathbf{0}$
4: $\mathbf{x}_{t-1} = \frac{1}{\sqrt{\alpha_t}} \left(\mathbf{x}_t - \frac{1-\alpha_t}{\sqrt{1-\bar{\alpha}_t}} \boldsymbol{\epsilon}_\theta(\mathbf{x}_t, t) \right) + \sigma_t \mathbf{z}$
5: **end for**
6: **return** $\mathbf{x}_0$

Jonathan Ho, Ajay Jain, Pieter Abbeel. “Denoising Diffusion Probabilistic Models”. Advances in Neural Information Processing Systems 33 (NeurIPS 2020).
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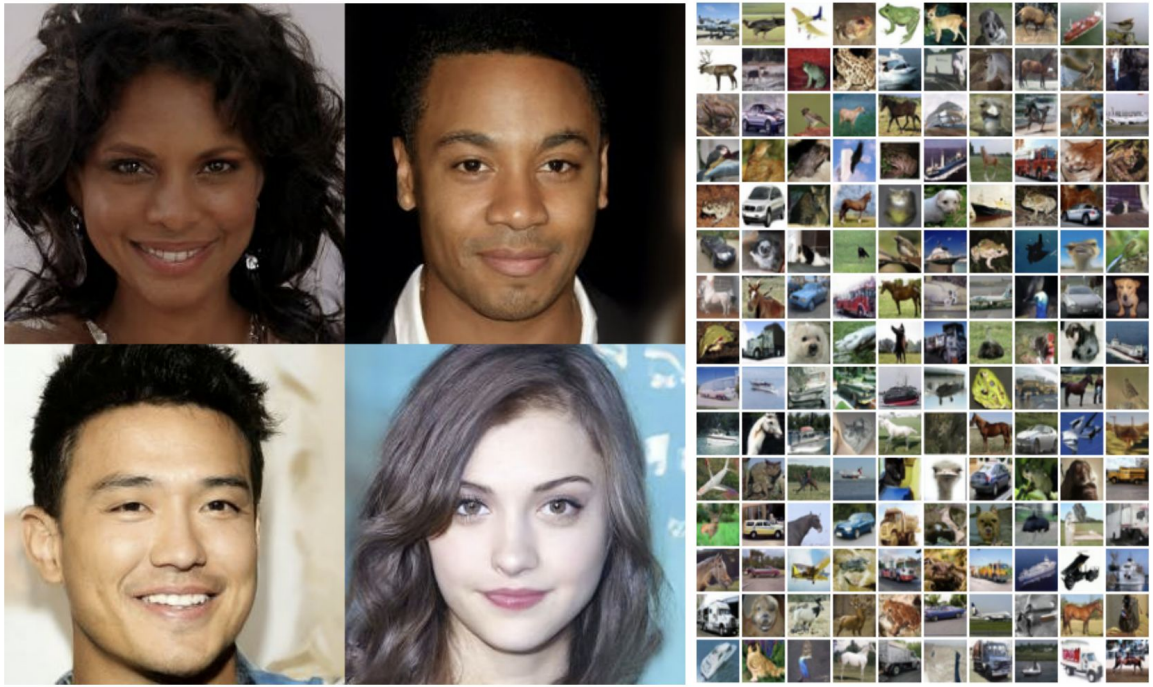
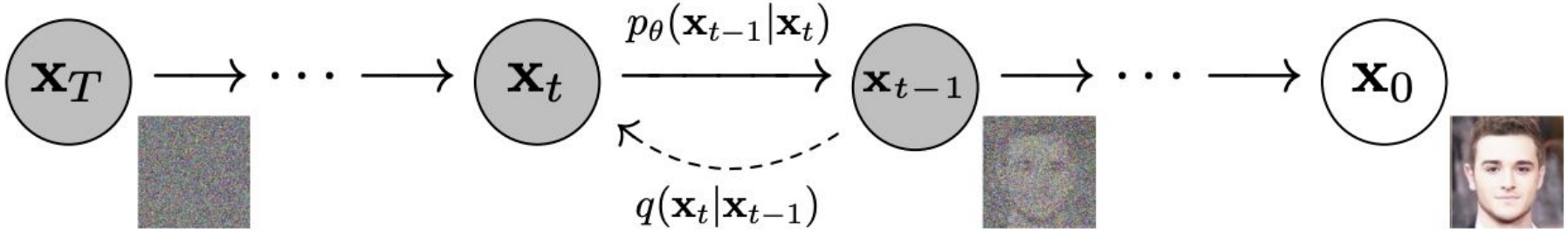


Figure 1: Generated samples on CelebA-HQ 256 × 256 (left) and unconditional CIFAR10 (right)

Denoising Diffusion Probabilistic Model (DDPM) (Ho, et al. 2020)



But how is this not dark magic? How do we know where/which direction the noise should go?

To explain this, we can take a probabilistic approach

A (statistical) generative model is a **probability distribution P(X)** (as opposed to P(Y|X) in discriminative models)

- **Likelihood Based:** Autoregressive models, variational autoencoders (VAE), normalizing flow, energy-based models (EBM)

  Maximizes the likelihood of the data under the model $\max_{\theta} \sum_{i=1}^{N} \log p_\theta(\mathbf{x}_i)$

- However, modeling $p_\theta(\mathbf{x})$ is hard
  - VAE: Surrogate loss
  - Normalizing Flow: Weird architecture
  - EBM: Intractable partition function
  - Autoregressive Models: Break it up with chain rule, works for some, doesn't make sense for other, can also take a long time and can't generate everything all at once

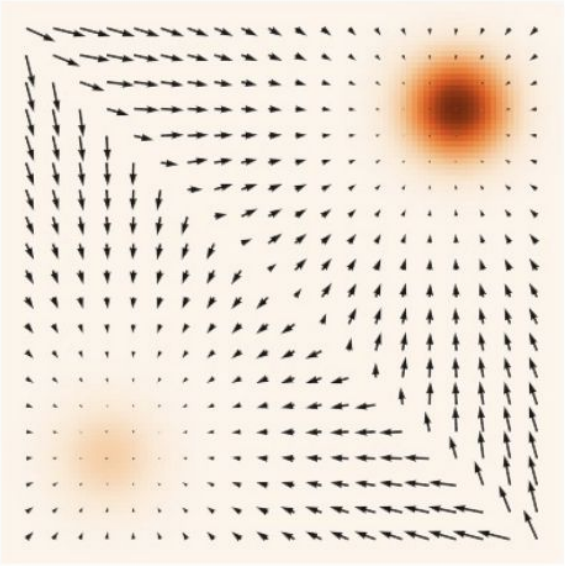
But ...

Do we really need to estimate $p_\theta(\mathbf{x})$ in order to train the model?

- How we usually train a model: Stochastic **gradient** descent (or ascent) with maximum **log likelihood**

- So we just need $\nabla_{\mathbf{x}} \log p(\mathbf{x})$

Score function



Score-based Model

Now we just need to train a model to estimate the score function

$$\mathbf{s}_\theta(\mathbf{x}) \approx \nabla_\mathbf{x} \log p(\mathbf{x})$$

by minimizing

$$\mathbb{E}_{p(\mathbf{x})}[\|\nabla_\mathbf{x} \log p(\mathbf{x}) - \mathbf{s}_\theta(\mathbf{x})\|_2^2]$$

- No more intractable partition function
- No more adversarial training
- No more weird architecture
- No breaking up with chain rule => can generate everything all at once

How to Train with Score Matching

But how do we get this $\nabla_{\mathbf{x}} \log p(\mathbf{x})$?

Turns out you don't have to!

We will rewrite the loss slightly:

$$\mathbb{E}_{p_{data}} ||\nabla_x \log p_\theta(x) - \nabla_x \log p_{data}(x)||^2$$

$$= \mathbb{E}_{p_{data}} ||\nabla_x \log p_\theta(x)||^2 + \mathbb{E}_{p_{data}} ||\nabla_x \log p_{data}(x)||^2$$

$$- 2\mathbb{E}_{p_{data}} \langle \nabla_x \log p_\theta(x), \nabla_x \log p_{data}(x) \rangle$$

$$\|x - y\|^2 = \langle x - y, x - y \rangle = \langle x, x \rangle + \langle y, y \rangle - 2\langle x, y \rangle$$

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Main trick: integration by parts (vector version):

$$\mathbb{E}_p \langle f(x), \nabla_x \log p(x) \rangle = -\mathbb{E}_p [div\, f(x)], \text{ where}$$

$$div\, f(x) = \sum_i \frac{\partial}{\partial x_i} f_i(x)$$

How to Train with Score Matching

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$$- 2\mathbb{E}_{p_{data}} \langle \nabla_x \log p_\theta(x), \nabla_x \log p_{data}(x) \rangle$$

Integrating by parts, the third term becomes,

$$-2\mathbb{E}_{p_{data}}[div\ \nabla_x \log p_\theta(x)] = 2\mathbb{E}_{p_{data}}[\mathrm{Tr}(\nabla_x^2 \log p_\theta(x))]$$

So, the loss can be equivalently written as:

$$\mathbb{E}_{p_{data}} ||\nabla_x \log p_\theta(x)||^2 + 2\,\mathbb{E}_{p_{data}}[\mathrm{Tr}(\nabla_x^2 \log p_\theta(x))]$$

Score Matching Loss

The training loss then becomes:

$$\frac{1}{N} \sum_{\text{training data } x_i} || s_\theta(x_i) ||^2 + 2 \left[\text{Tr}\left(D s_\theta(x_i) \right)\right]$$

Jacobian of s_θ

Aapo Hyvärinen. “Estimation of Non-Normalized Statistical Models by Score Matching”. JMLR 2005. https://jmlr.org/papers/volume6/hyvarinen05a/hyvarinen05a.pdf

However …

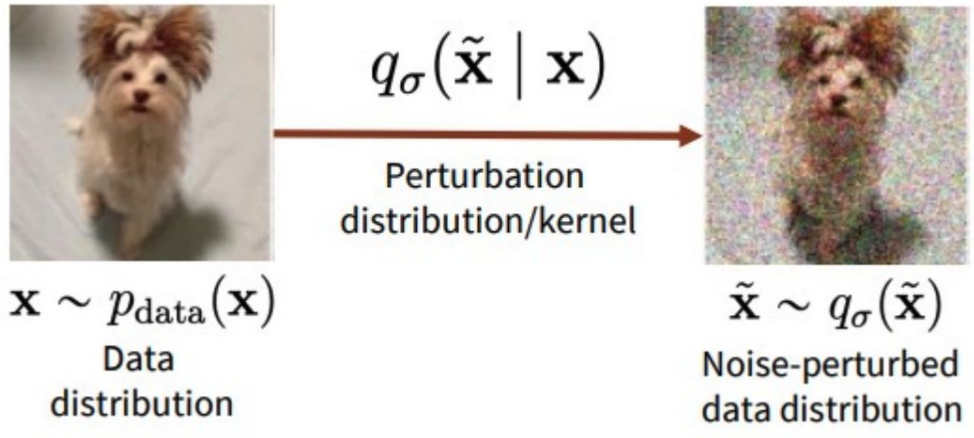
This Jacobian is super computationally expensive to calculate!

The training loss then becomes:

$$\frac{1}{N} \sum_{\text{training data } x_i} || s_\theta(x_i) ||^2 + 2 \left[\text{Tr}\left(D s_\theta(x_i) \right)\right]$$

Jacobian of s_θ

Denoising Score Matching

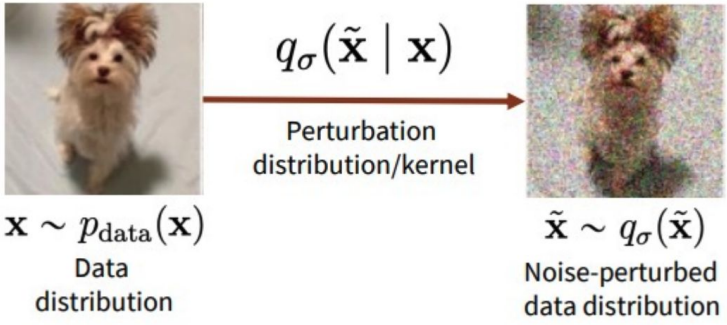


Now the objective has become

$$\frac{1}{2}\mathbb{E}_{q_\sigma(\tilde{\mathbf{x}}|\mathbf{x})p_{\text{data}}(\mathbf{x})}[\|\mathbf{s}_{\boldsymbol{\theta}}(\tilde{\mathbf{x}}) - \nabla_{\tilde{\mathbf{x}}} \log q_\sigma(\tilde{\mathbf{x}} \mid \mathbf{x})\|_2^2].$$

Pascal Vincent. “A Connection Between Score Matching and Denoising Autoencoders”. 2011. https://www.iro.umontreal.ca/~vincentp/Publications/smdae_techreport.pdf

Denoising Score Matching



Now the objective has become

$$\frac{1}{2}\mathbb{E}_{q_\sigma(\tilde{\mathbf{x}}|\mathbf{x})p_{\text{data}}(\mathbf{x})}[\|\mathbf{s}_{\boldsymbol{\theta}}(\tilde{\mathbf{x}}) - \nabla_{\tilde{\mathbf{x}}} \log q_\sigma(\tilde{\mathbf{x}} \mid \mathbf{x})\|_2^2].$$

If $\tilde{\mathbf{x}} = \mathbf{x} + \epsilon, \epsilon \sim \mathcal{N}(0, \sigma^2\mathbf{I})$, then

$$q_\sigma(\tilde{\mathbf{x}}|\mathbf{x}) = \frac{1}{(2\pi)^{d/2}\sigma^d} e^{-\frac{1}{2\sigma^2}\|\tilde{\mathbf{x}}-\mathbf{x}\|^2}$$

$$\nabla_{\tilde{\mathbf{x}}} \log q_\sigma(\tilde{\mathbf{x}} \mid \mathbf{x})\|_2^2 = \frac{1}{\sigma^2}(\mathbf{x} - \tilde{\mathbf{x}})$$

Pascal Vincent. “A Connection Between Score Matching and Denoising Autoencoders”. 2011. https://www.iro.umontreal.ca/~vincentp/Publications/smdae_techreport.pdf

Sampling with Langevin Dynamics

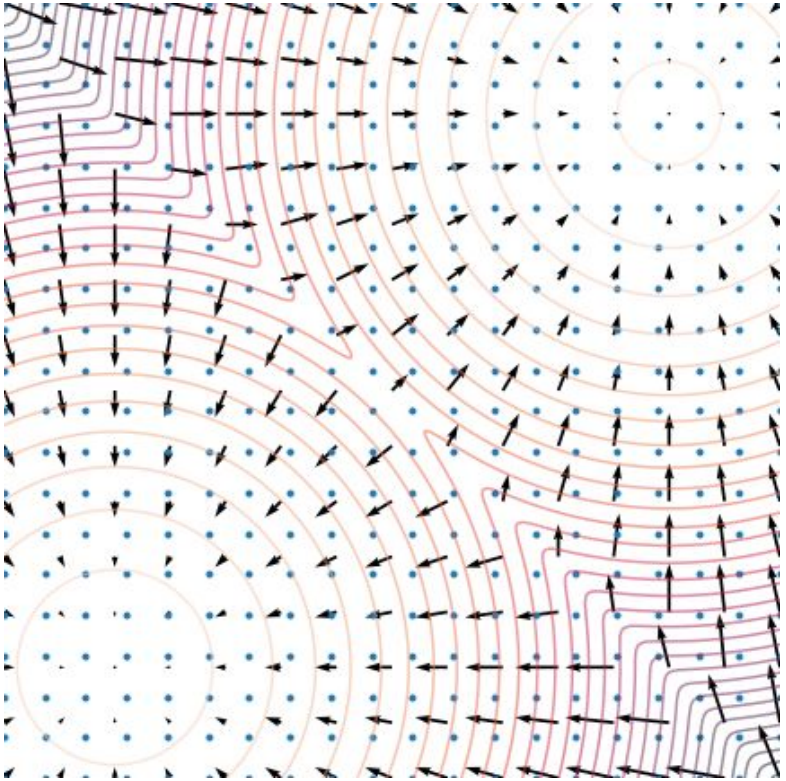
Once we have a trained score model, we can do “gradient ascent” in the data space to get a sample.

- First draw from a easier-to-sample prior distribution $\mathbf{x}_0 \sim \pi(\mathbf{x})$
- Then with small $\boldsymbol{\epsilon}$ and large K and $\mathbf{z}_i \sim \mathcal{N}(0, I)$, iterate

$$\mathbf{x}_{i+1} \leftarrow \mathbf{x}_i + \epsilon \nabla_{\mathbf{x}} \log p(\mathbf{x}) + \sqrt{2\epsilon}\, \mathbf{z}_i, \quad i = 0, 1, \cdots, K,$$

Or the learned $\mathbf{s}_\theta(\mathbf{x}) \approx \nabla_{\mathbf{x}} \log p(\mathbf{x})$

Sampling with Langevin Dynamics



However ...

- The score is only defined in the whole data space, if data resides in a lower dimensional manifold (i.e. some area of the data space will have no support), then the score estimation is not consistent any more

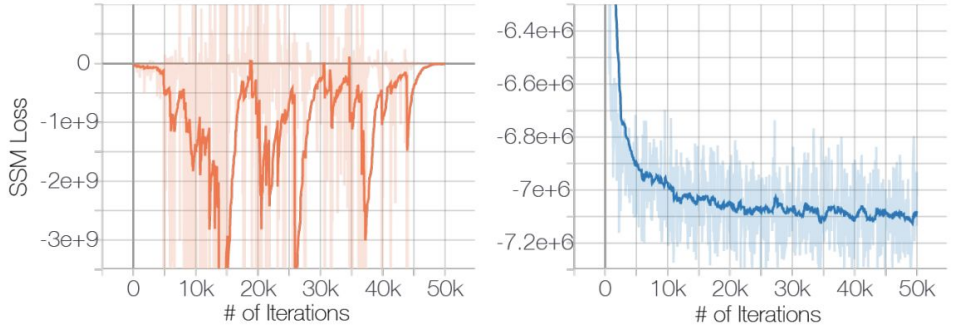
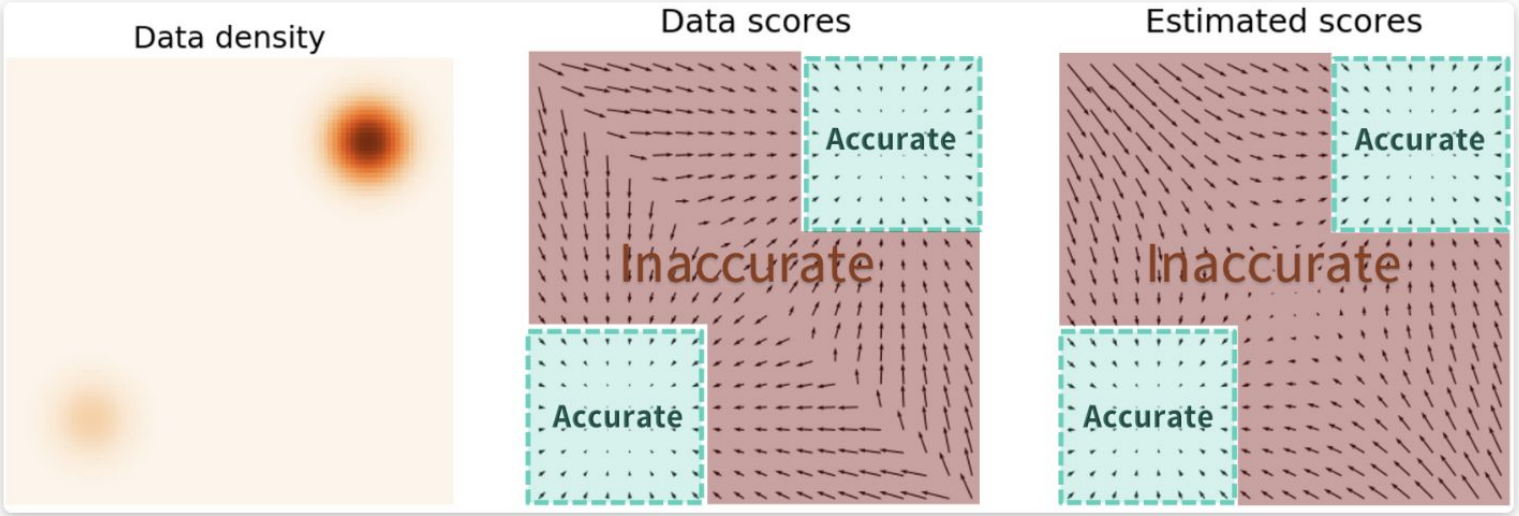


Figure 1: **Left**: Sliced score matching (SSM) loss w.r.t. iterations. No noise is added to data. **Right**: Same but data are perturbed with $\mathcal{N}(0, 0.0001)$.

However (2) ...

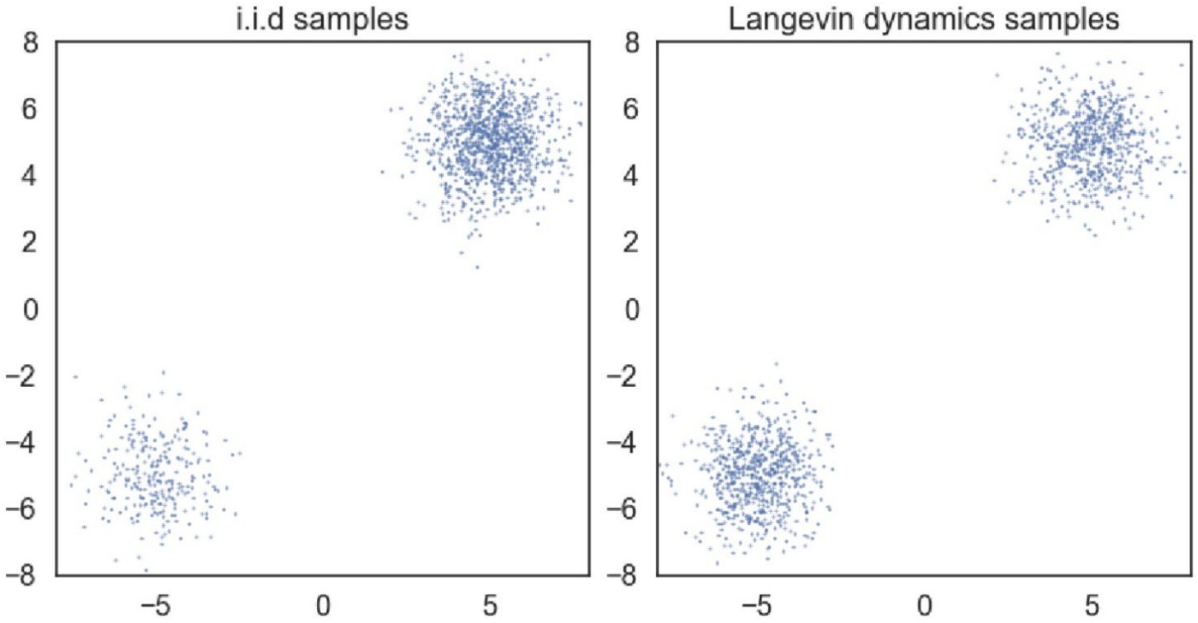
- Even when we do have full support data space, the score estimation is inaccurate in the low density regions

  And our initial sample is very likely to be in those low density regions!!!



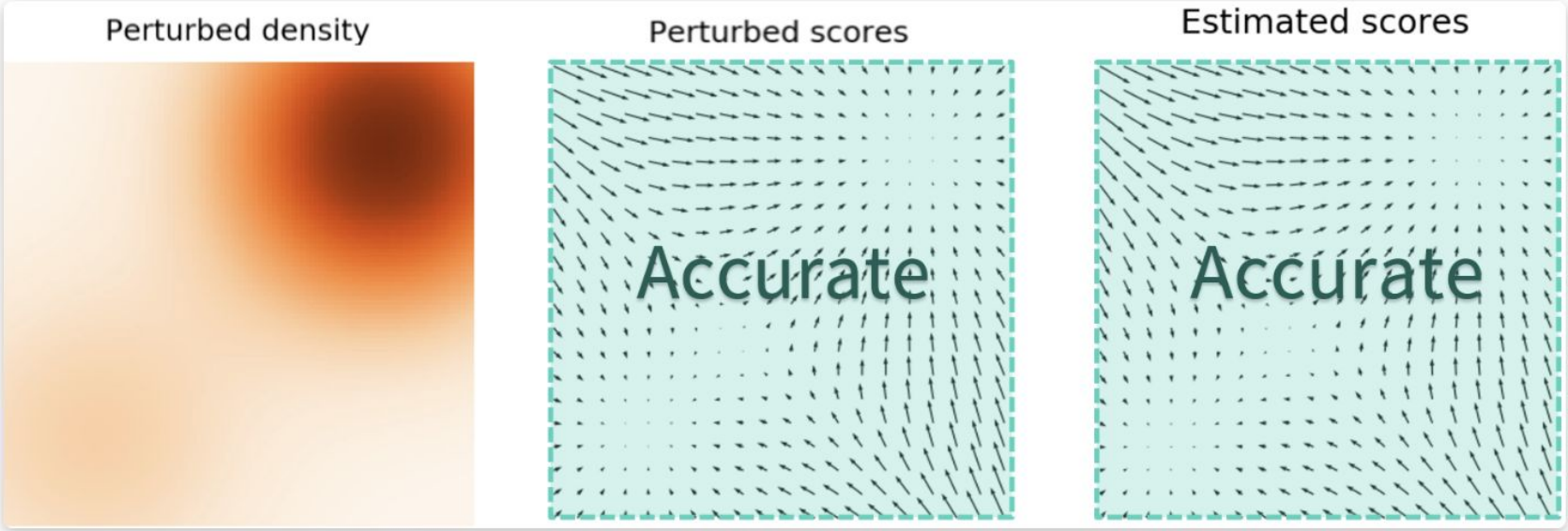
However (3) ...

- Even in high density region, we can still get inaccurate sample distributions

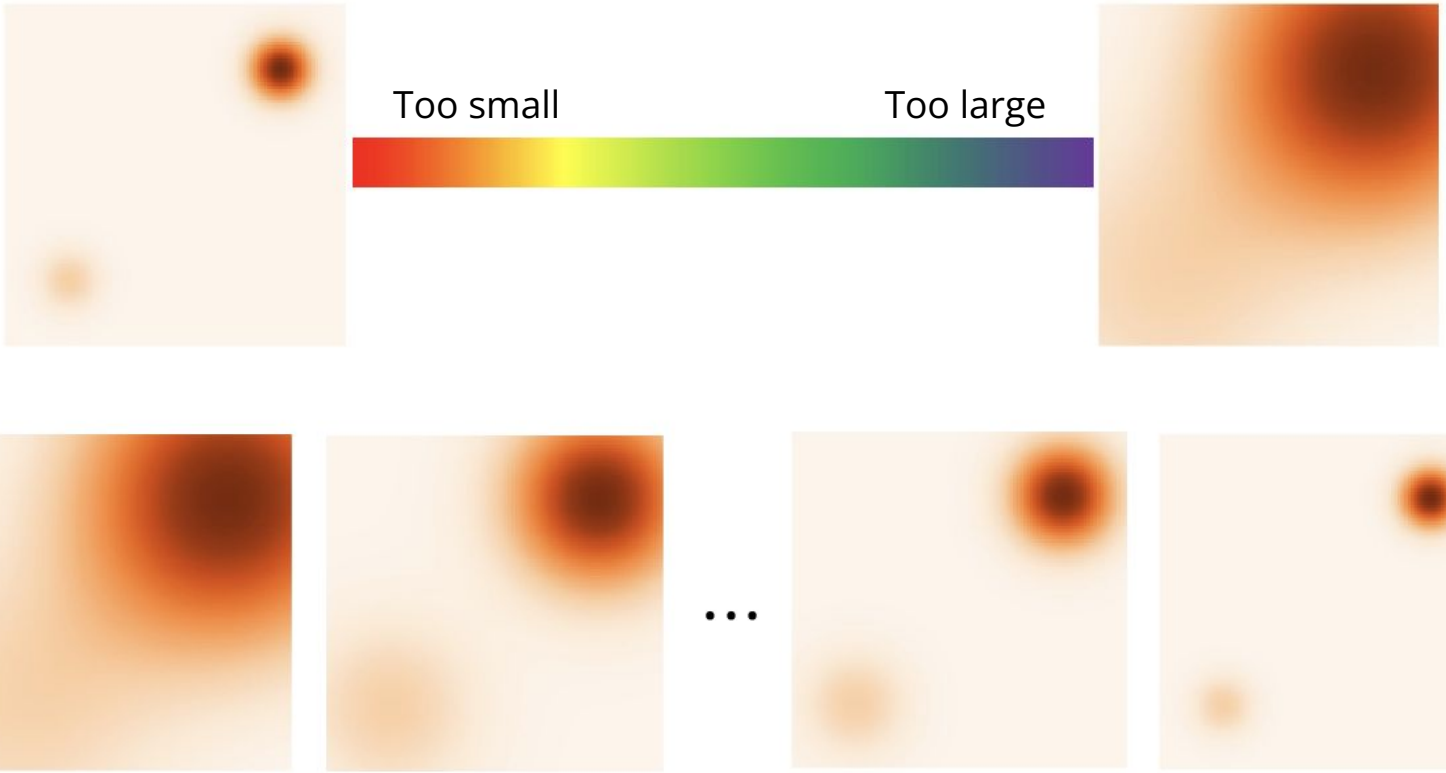


To solve all these problems

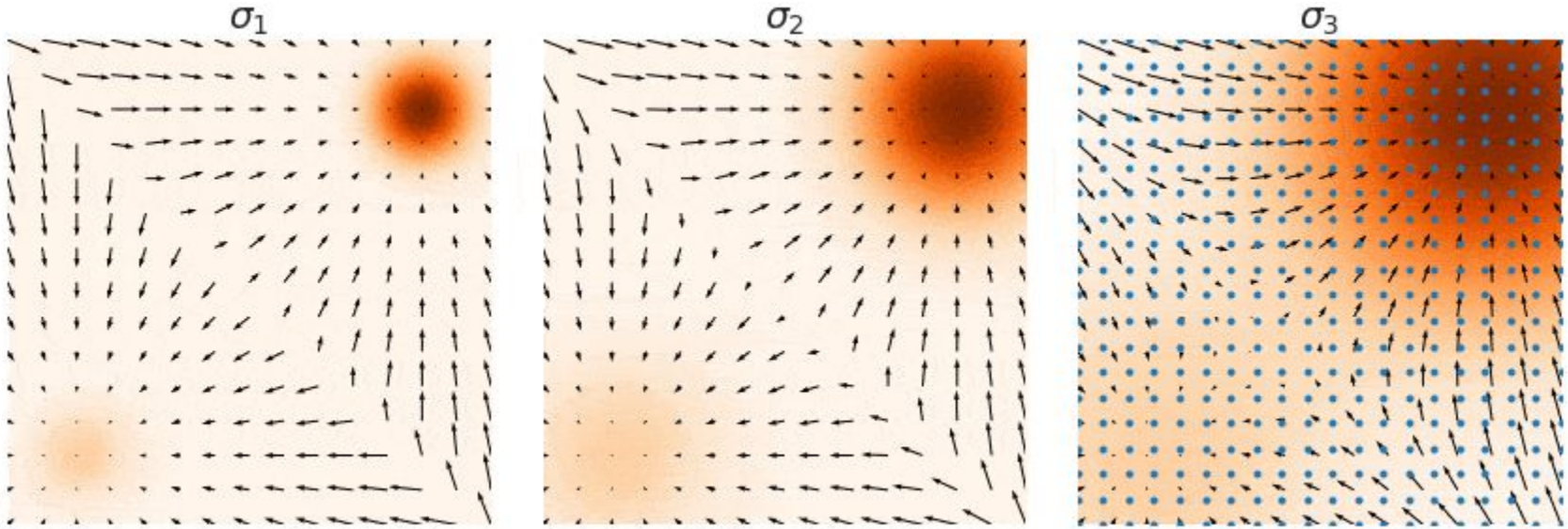
How about just **add noise** to the data?



But how much noise we should add?



Annealed Langevin Dynamics



Yang Song and Stefano Ermon. “Generative Modeling by Estimating Gradients of the Data Distribution”. NeurIPS 2019. https://arxiv.org/pdf/1907.05600.pdf

Annealed Langevin Dynamics

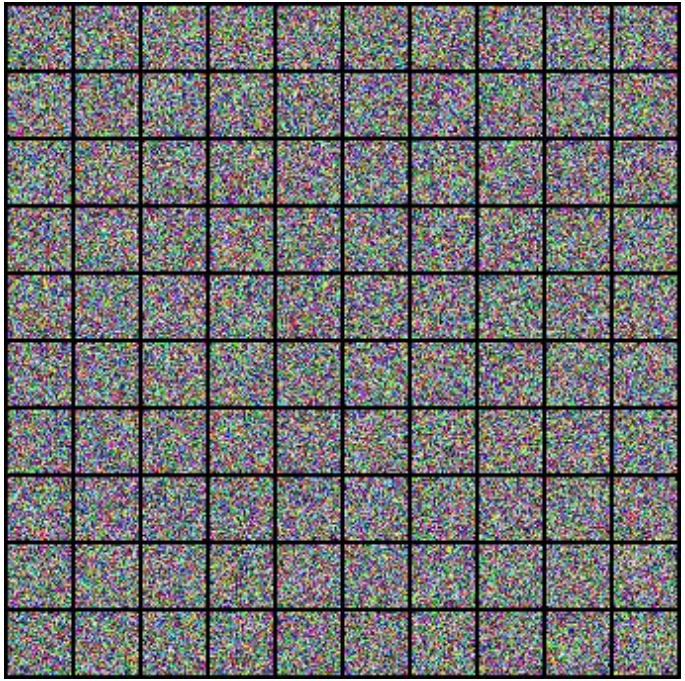
Algorithm 1 Annealed Langevin dynamics.

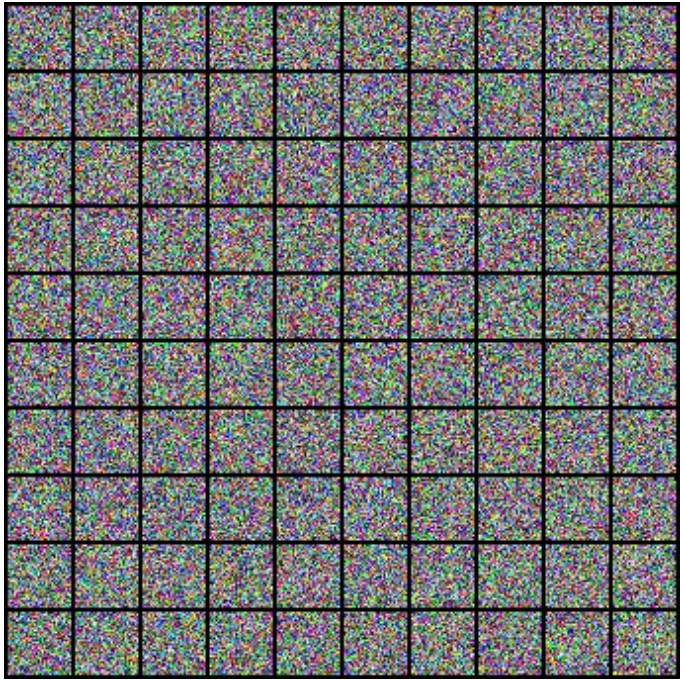
Require: $\{\sigma_i\}_{i=1}^{L}, \epsilon, T$.

Initialize $\tilde{\mathbf{x}}_0$
for $i \leftarrow 1$ to L **do**
  $\alpha_i \leftarrow \epsilon \cdot \sigma_i^2 / \sigma_L^2$ ▷ α_i is the step size.
  for $t \leftarrow 1$ to T **do**
    Draw $\mathbf{z}_t \sim \mathcal{N}(0, I)$
    $\tilde{\mathbf{x}}_t \leftarrow \tilde{\mathbf{x}}_{t-1} + \frac{\alpha_i}{2} \mathbf{s}_{\boldsymbol{\theta}}(\tilde{\mathbf{x}}_{t-1}, \sigma_i) + \sqrt{\alpha_i}\, \mathbf{z}_t$
  end for
  $\tilde{\mathbf{x}}_0 \leftarrow \tilde{\mathbf{x}}_T$
end for
return $\tilde{\mathbf{x}}_T$

Yang Song and Stefano Ermon. “Generative Modeling by Estimating Gradients of the Data Distribution.” NeurIPS 2019. https://arxiv.org/pdf/1907.05600.pdf

Noise Conditional Score Network (NCSN) (Song and Ermon 2019)



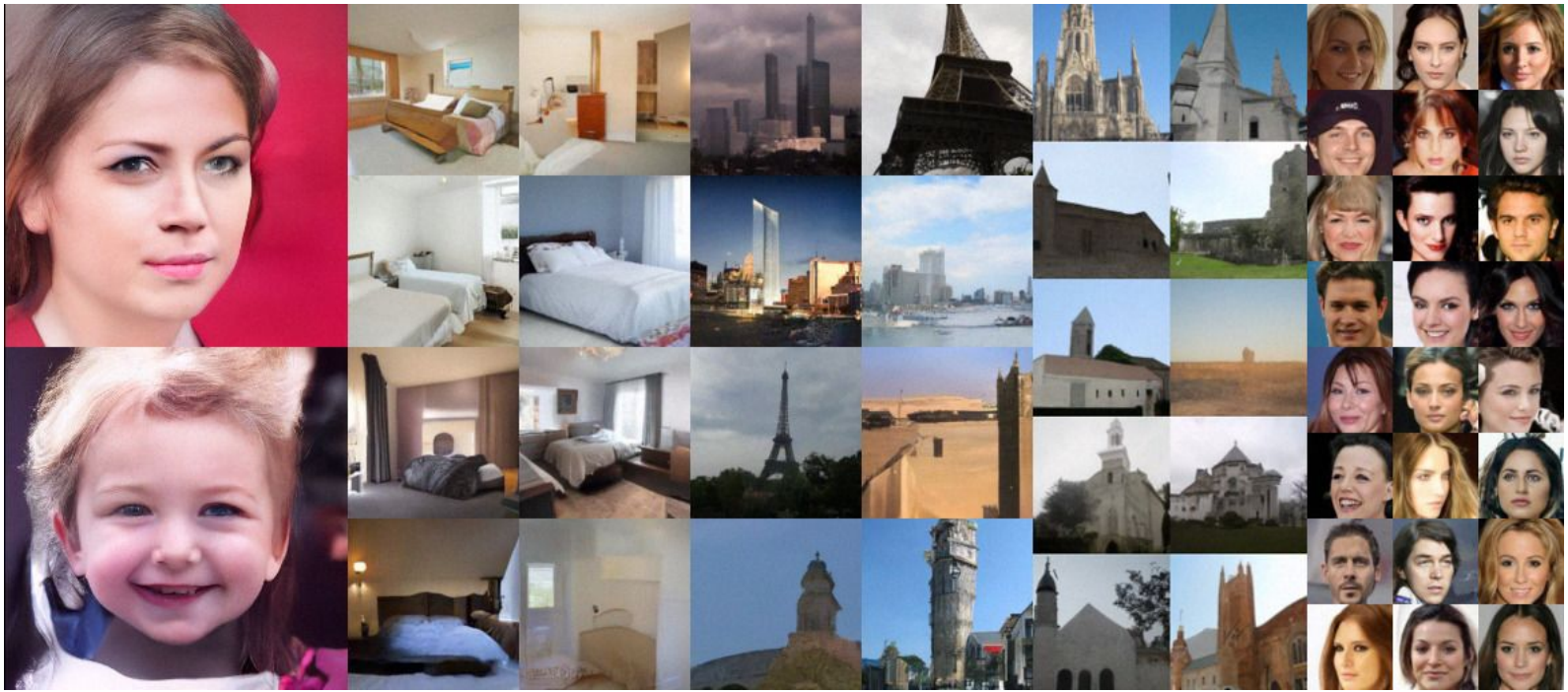


Yang Song and Stefano Ermon. “Generative Modeling by Estimating Gradients of the Data Distribution”. NeurIPS 2019. https://arxiv.org/pdf/1907.05600.pdf

Diffusion Model (Sohl-Dickstein, et al. 2015)

Figure 3. The proposed framework trained on the CIFAR-10 (Krizhevsky & Hinton, 2009) dataset. *(a)* Example training data. *(b)* Random samples generated by the diffusion model.

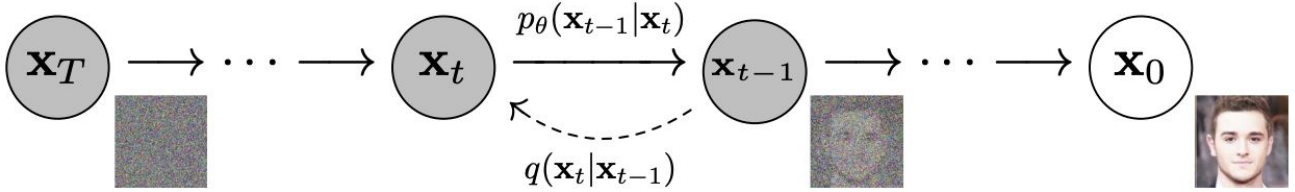
NCSN v2 (Song and Ermon 2020)



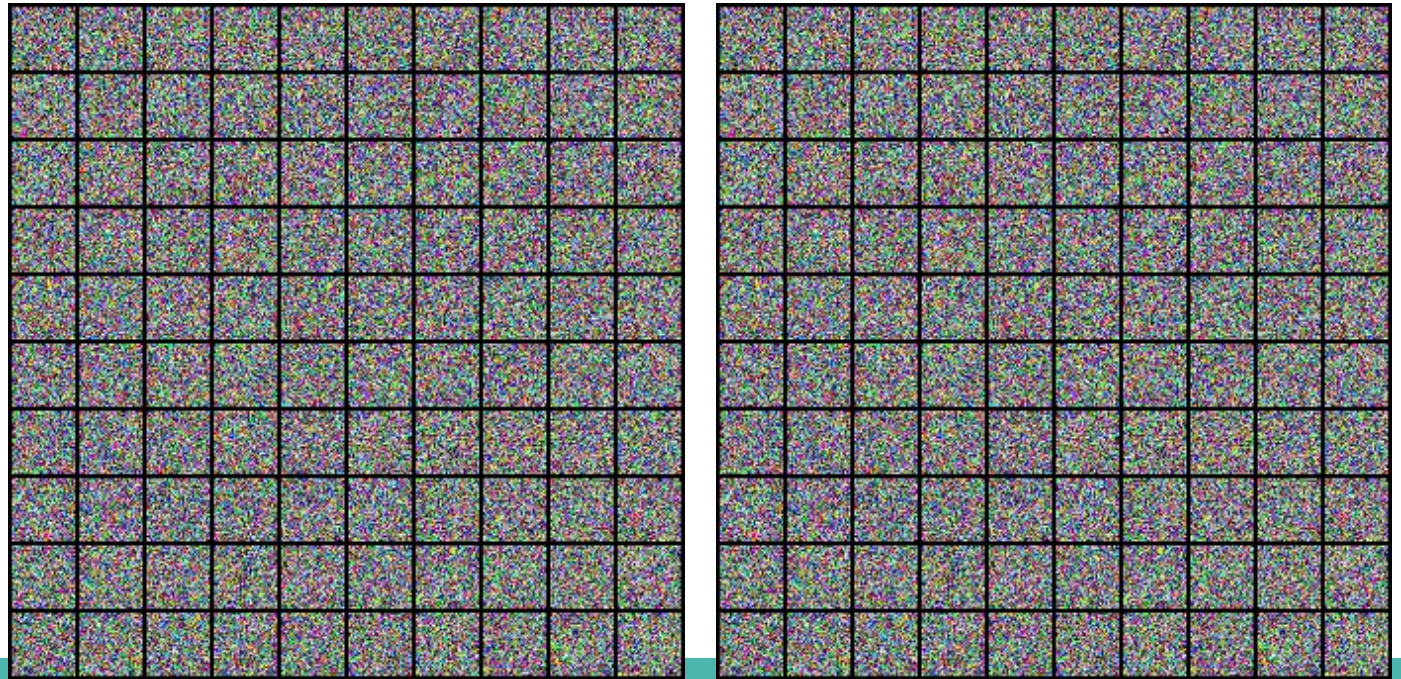
Yang Song and Stefano Ermon. “Improved Techniques for Training Score-Based Generative Models”. NeurIPS 2020. https://arxiv.org/pdf/2006.09011.pdf

Does this look familiar …

DDPM



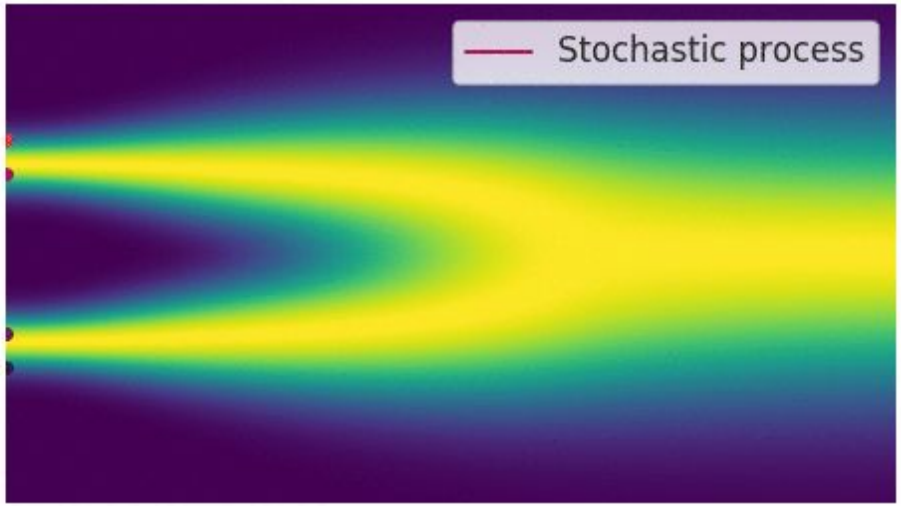
NCSN



When # of noise scales goes to infinity

It becomes a continuous-time stochastic process, many of which can be solved by stochastic differential equations (SDEs) (Diffusion is no exception)

$$\mathbf{dx} = \mathbf{f}(\mathbf{x}, t)\mathrm{d}t + g(t)\mathrm{d}\mathbf{w},$$



Score-SDE (Song et al. 2021)

Forward SDE (data → noise)

$\mathbf{x}(0)$ ——— $\mathrm{d}\mathbf{x} = \mathbf{f}(\mathbf{x}, t)\mathrm{d}t + g(t)\mathrm{d}\mathbf{w}$ ———→ $\mathbf{x}(T)$

score function

$\mathbf{x}(0)$ ←——— $\mathrm{d}\mathbf{x} = \left[\mathbf{f}(\mathbf{x}, t) - g^2(t)\nabla_{\mathbf{x}} \log p_t(\mathbf{x})\right] \mathrm{d}t + g(t)\mathrm{d}\bar{\mathbf{w}}$ ——— $\mathbf{x}(T)$

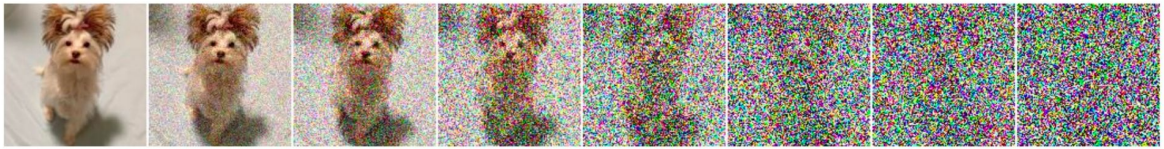
Reverse SDE (noise → data)

Yang Song, Jascha Sohl-Dickstein, Diederik P. Kingma, Abhishek Kumar, Stefano Ermon, and Ben Poole. “Score-Based Generative Modeling through Stochastic Differential Equations”. ICLR 2021. https://openreview.net/pdf?id=PxTIG12RRHS

Diffusion Model is Modeling a Score-SDE!

Forward SDE (data → noise)

$\mathbf{x}(0)$ — $\mathrm{d}\mathbf{x} = \mathbf{f}(\mathbf{x}, t)\mathrm{d}t + g(t)\mathrm{d}\mathbf{w}$ → $\mathbf{x}(T)$



score function

$\mathbf{x}(0)$ ← $\mathrm{d}\mathbf{x} = \left[\mathbf{f}(\mathbf{x}, t) - g^2(t)\nabla_{\mathbf{x}} \log p_t(\mathbf{x})\right] \mathrm{d}t + g(t)\mathrm{d}\bar{\mathbf{w}}$ — $\mathbf{x}(T)$

Reverse SDE (noise → data)

$$\mathbf{x}_i = \sqrt{1-\beta_i}\mathbf{x}_{i-1} + \sqrt{\beta_i}\mathbf{z}_{i-1}, \quad i = 1, \cdots, N$$

$$\mathrm{d}\mathbf{x} = -\frac{1}{2}\beta(t)\mathbf{x}\,\mathrm{d}t + \sqrt{\beta(t)}\,\mathrm{d}\mathbf{w}$$

How to sample from the reverse SDE

- First sample from the prior distribution $\mathbf{x}(T) \sim \pi$
- Then apply numerical SDE solver to solve for x(0)
  - Eg. Euler-Maruyama method
- Since we have the **score model** and we **only care about the final sample** (and not the trajectory), we can even improve the numerical solution by applying a MCMC-based approach called **Predictor-Corrector** samplers to fine-tune the trajectories
  - **Predictor:** Choose a proper step size, and then predict the next sample based on the current sample and trajectory (eg. any numerical SDE solver)
  - **Corrector:** improve the sample prediction with MCMC according to our score-based model so that becomes a higher-quality sample from the probability distribution of the next noise level (eg. Langevin dynamics)

Score-SDE (Song et al. 2021)

Yang Song, Jascha Sohl-Dickstein, Diederik P. Kingma, Abhishek Kumar, Stefano Ermon, and Ben Poole. “Score-Based Generative Modeling through Stochastic Differential Equations”. ICLR 2021. https://openreview.net/pdf?id=PxTIG12RRHS

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Yang Song, Jascha Sohl-Dickstein, Diederik P. Kingma, Abhishek Kumar, Stefano Ermon, and Ben Poole. “Score-Based Generative Modeling through Stochastic Differential Equations”. ICLR 2021. https://openreview.net/pdf?id=PxTIG12RRHS

Conditional Generation w/ Score-SDE (Song et al. 2021)

By Bayes' Rule we know that

$$\nabla_{\mathbf{x}} \log p(\mathbf{x} \mid \mathbf{y}) = \nabla_{\mathbf{x}} \log p(\mathbf{x}) + \nabla_{\mathbf{x}} \log p(\mathbf{y} \mid \mathbf{x})$$

Hence

$$\mathrm{d}\mathbf{x} = \{\mathbf{f}(\mathbf{x}, t) - g(t)^2[\nabla_{\mathbf{x}} \log p_t(\mathbf{x}) + \nabla_{\mathbf{x}} \log p_t(\mathbf{y} \mid \mathbf{x})]\}\mathrm{d}t + g(t)\mathrm{d}\bar{\mathbf{w}}$$

Time-dependent

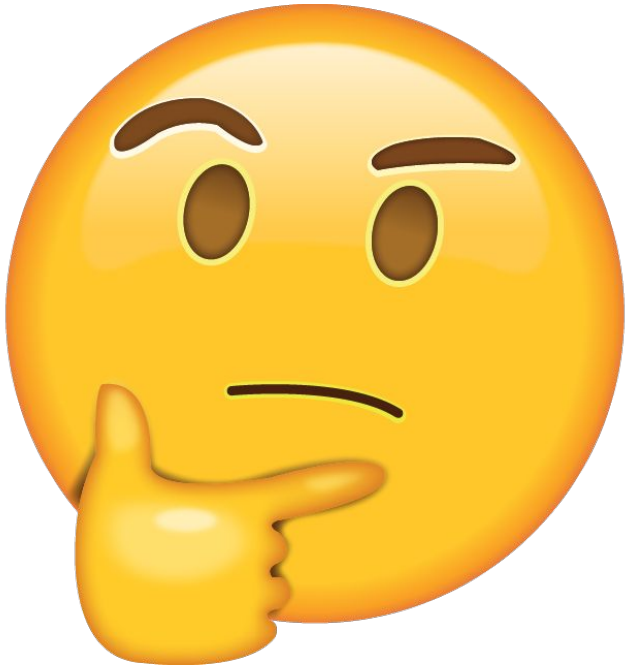
Yang Song, Jascha Sohl-Dickstein, Diederik P. Kingma, Abhishek Kumar, Stefano Ermon, and Ben Poole. "Score-Based Generative Modeling through Stochastic Differential Equations". ICLR 2021. https://openreview.net/pdf?id=PxTIG12RRHS

Conditional Generation w/ Score-SDE (Song et al. 2021)

Figure 4: *Left*: Class-conditional samples on 32 × 32 CIFAR-10. Top four rows are automobiles and bottom four rows are horses. *Right:* Inpainting (top two rows) and colorization (bottom two rows) results on 256 × 256 LSUN. First column is the original image, second column is the masked/gray-scale image, remaining columns are sampled image completions or colorizations.

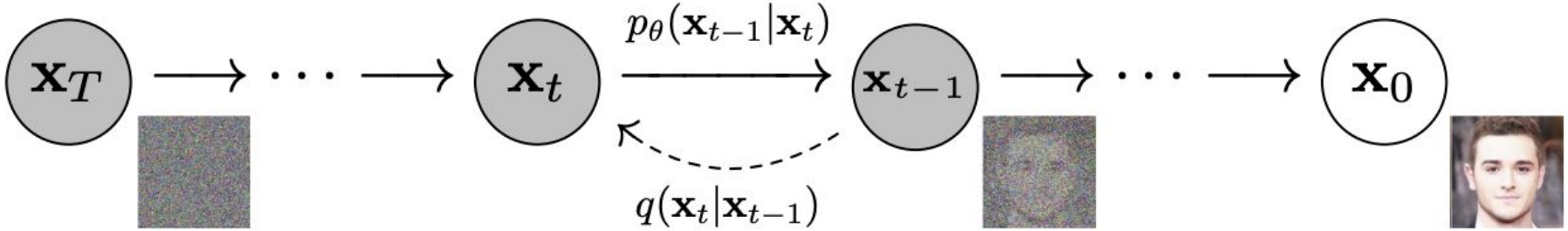
Yang Song, Jascha Sohl-Dickstein, Diederik P. Kingma, Abhishek Kumar, Stefano Ermon, and Ben Poole. “Score-Based Generative Modeling through Stochastic Differential Equations”. ICLR 2021. https://openreview.net/pdf?id=PxTIG12RRHS

Is diffusion model perfect?

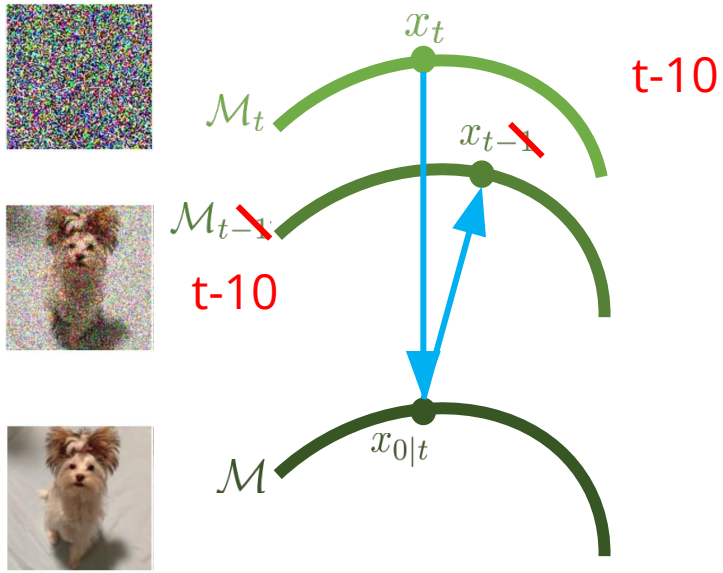


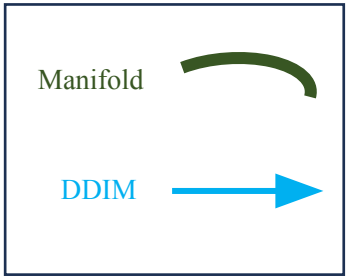
Big Problem: It's very very extremely slow...

Vanilla diffusion model can take 10~30 minutes to generate a single 256X256 image!



DDIM (Song et. al 2021): Fast Sampling by Skipping Time Steps





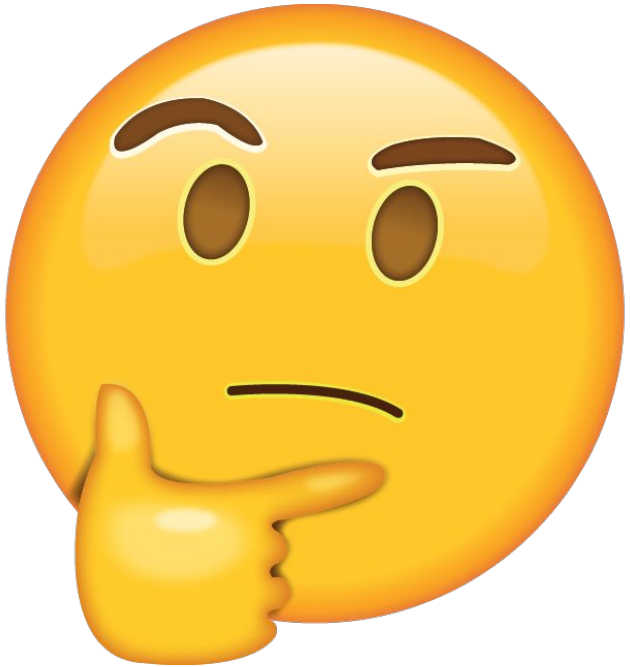
Jiaming Song, Chenlin Meng, Stefano Ermon. “Denoising Diffusion Implicit Models”. ICLR 2021. https://arxiv.org/pdf/2010.02502

Diffusion Distillation

Learning another model such that 1 step in this new model is equivalent to N steps in the teacher diffusion model

1. Progressive Distillation
2. Consistency Models
3. Consistency Trajectory Models
4. Adversarial Distillation

Is diffusion model perfect? How about conditional sampling?



Can we use off-the-shelf classifiers?

By Bayes' Rule we know that

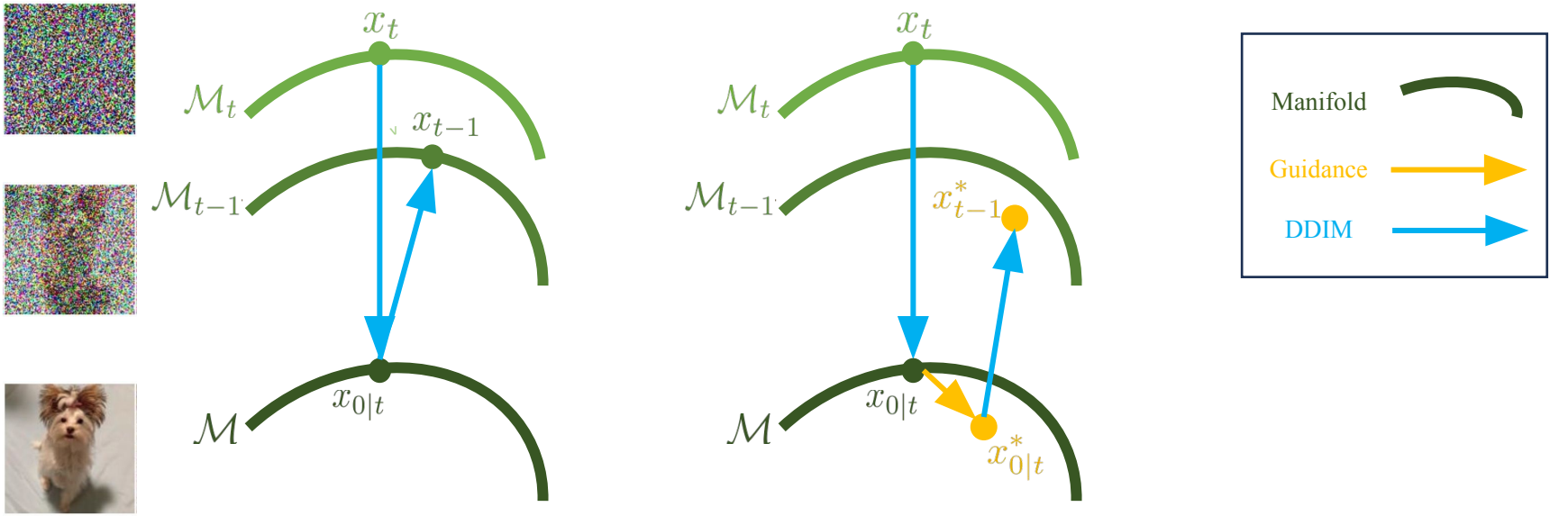
$$\nabla_{\mathbf{x}} \log p(\mathbf{x} \mid \mathbf{y}) = \nabla_{\mathbf{x}} \log p(\mathbf{x}) + \nabla_{\mathbf{x}} \log p(\mathbf{y} \mid \mathbf{x})$$

Hence

$$\mathrm{d}\mathbf{x} = \{\mathbf{f}(\mathbf{x}, t) - g(t)^2[\nabla_{\mathbf{x}} \log p_t(\mathbf{x}) + \nabla_{\mathbf{x}} \log p_t(\mathbf{y} \mid \mathbf{x})]\}\mathrm{d}t + g(t)\mathrm{d}\bar{\mathbf{w}}$$

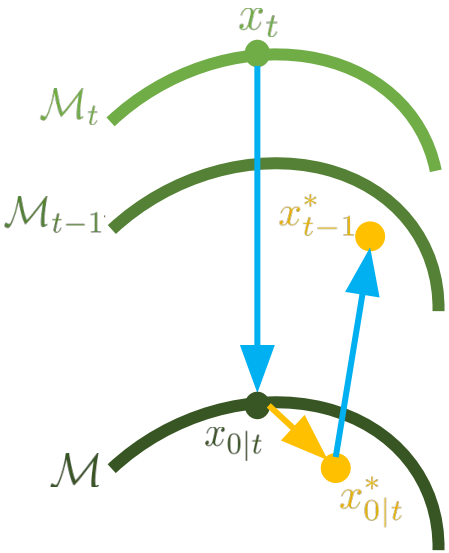
NO! Because this needs to be **time-dependent!**

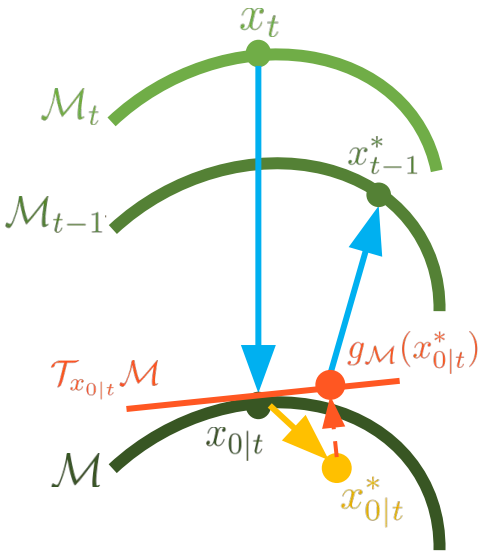
Diffusion Posterior Sampling (Chung et. al 2023): Enable Diffusion Guidance from Off-the-Shelf Classifiers

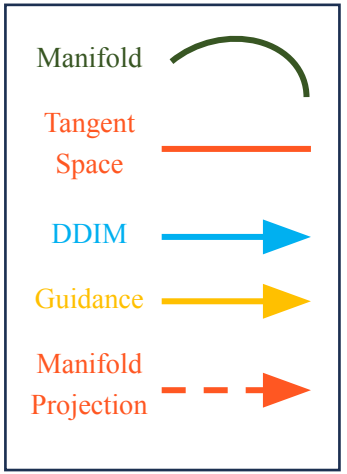


Hyungjin Chung, Jeongsol Kim, Michael T. Mccann, Marc L. Klasky, Jong Chul Ye. “Diffusion Posterior Sampling for General Noisy Inverse Problems”. ICLR 2023.
https://arxiv.org/pdf/2209.14687

Manifold Preserving Guided Diffusion (He et. al 2024): Making sure that the guidance gradients are well behaved



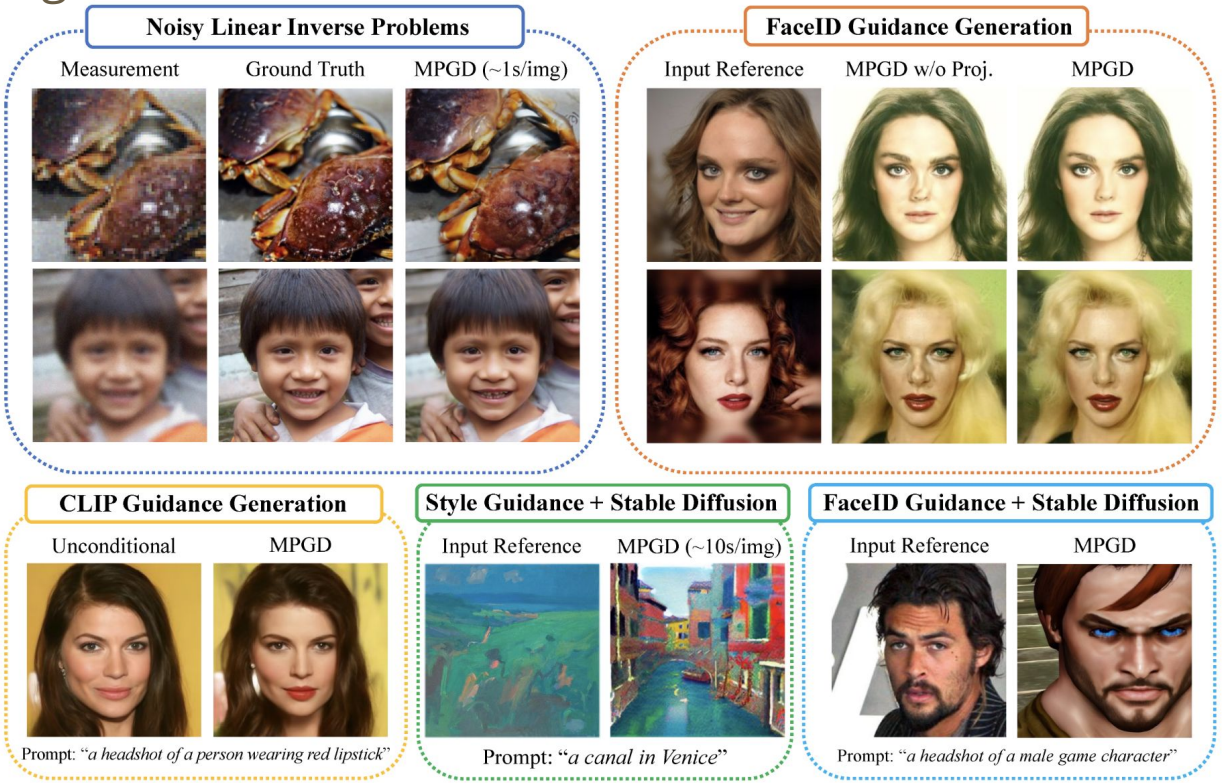




Yutong He, Naoki Murata, Chieh-Hsin Lai, Yuhta Takida, Toshimitsu Uesaka, Dongjun Kim, Wei-Hsiang Liao, Yuki Mitsufuji, J. Zico Kolter, Ruslan Salakhutdinov, Stefano Ermon. “Manifold Preserving Guided Diffusion”. https://arxiv.org/pdf/2311.16424

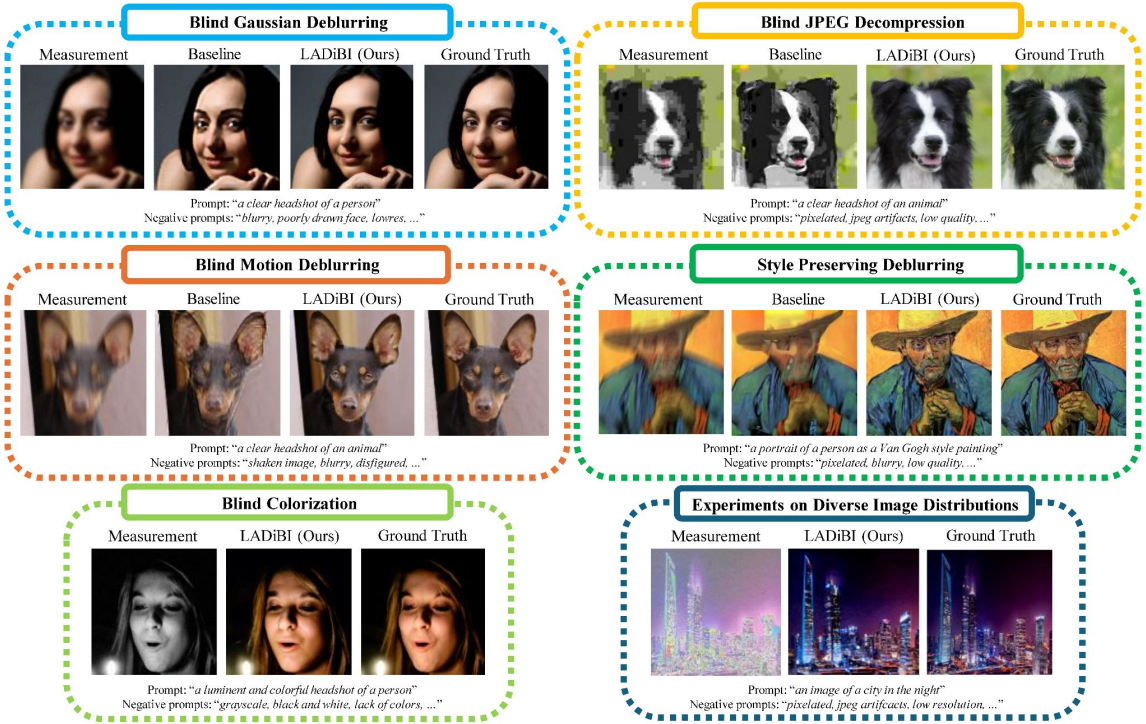
MPGD (He et. al 2024)

The same diffusion model can now be used for many applications without additional training!

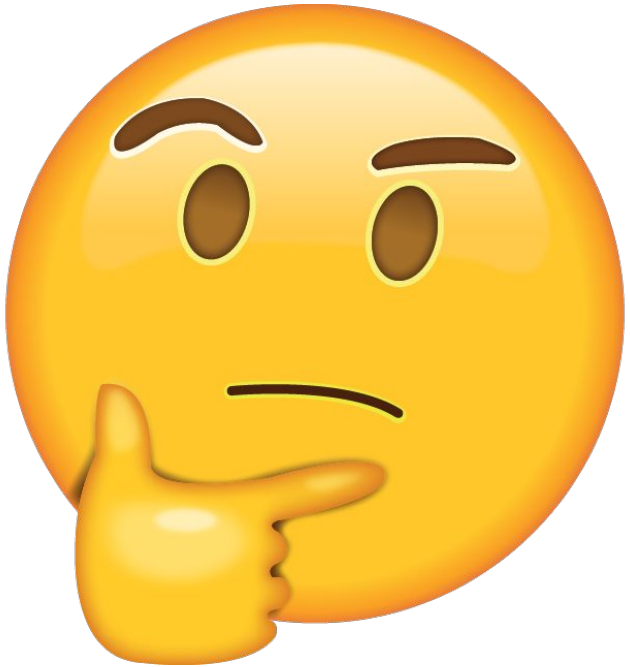


LaDiBI (Dontas et. al 2024)

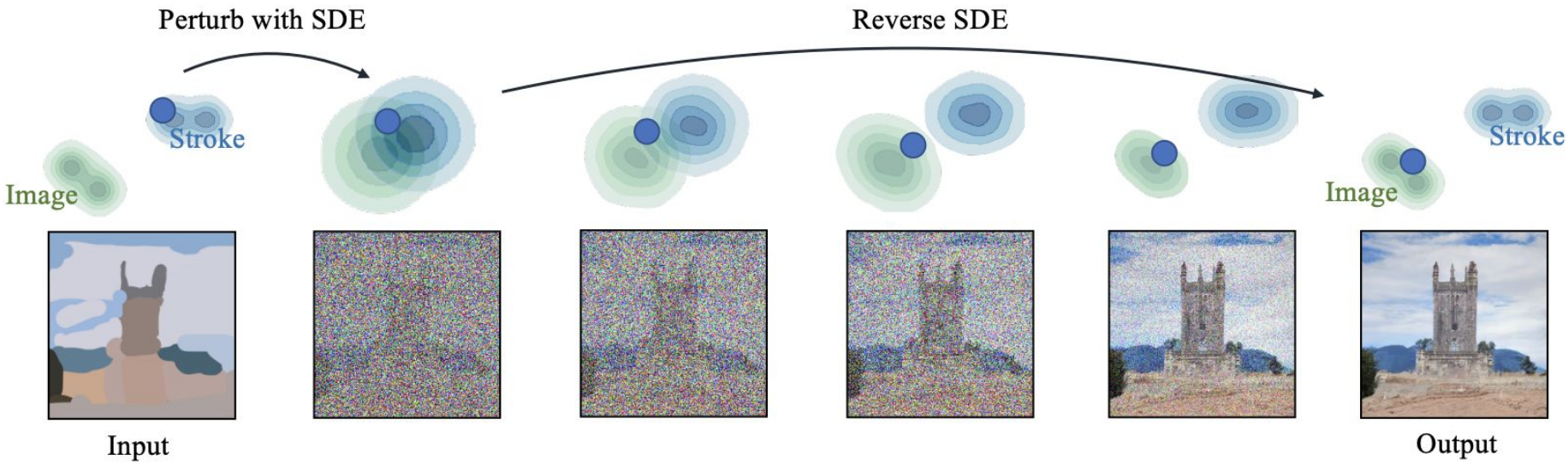
The same diffusion model can now be used for many applications without additional training!



Any other ways to apply condition to diffusion models?

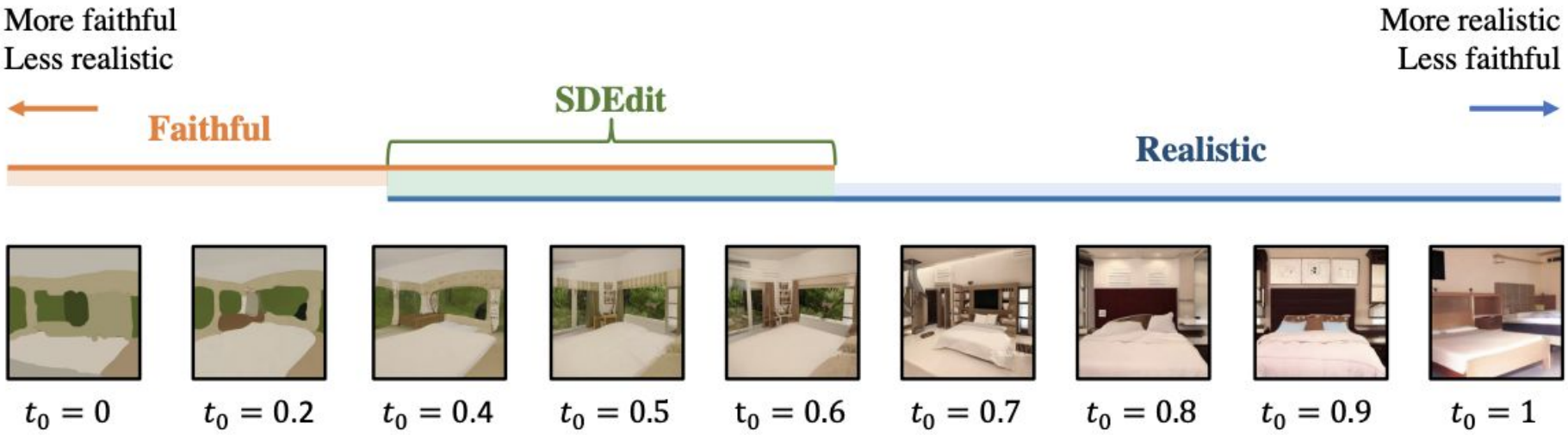


SDEdit: Conditional Generation w/o Training (Meng et al. 2022)



Chenlin Meng, Yutong He, Yang Song, Jiaming Song, Jiajun Wu, Jun-Yan Zhu, and Stefano Ermon. “SDEdit: Guided Image Synthesis and Editing with Stochastic Differential Equations”. ICLR 2022. https://arxiv.org/pdf/2108.01073.pdf

SDEdit: Conditional Generation w/o Training ([Meng et al. 2022])



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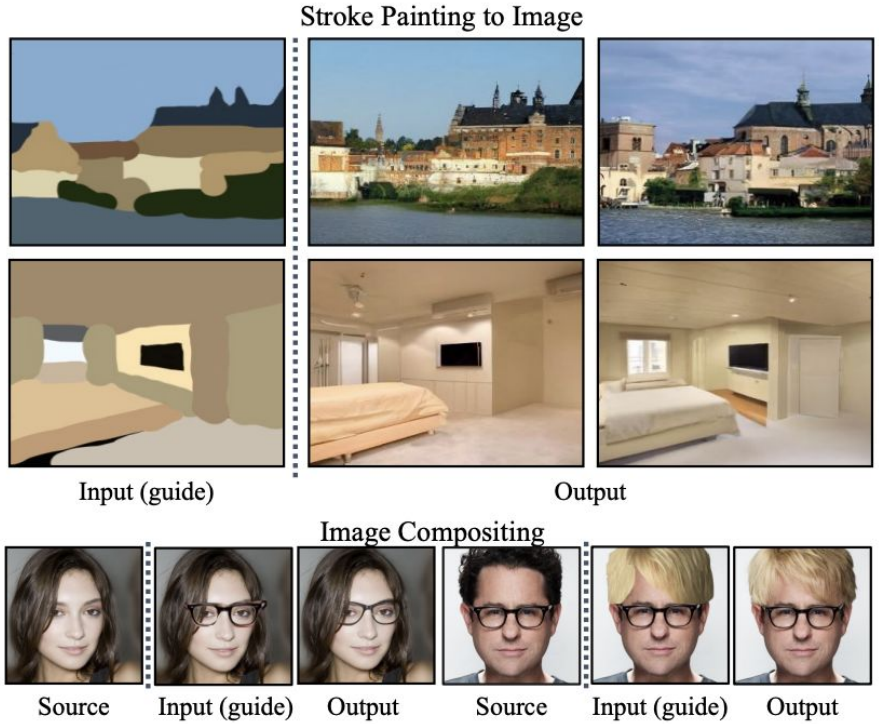
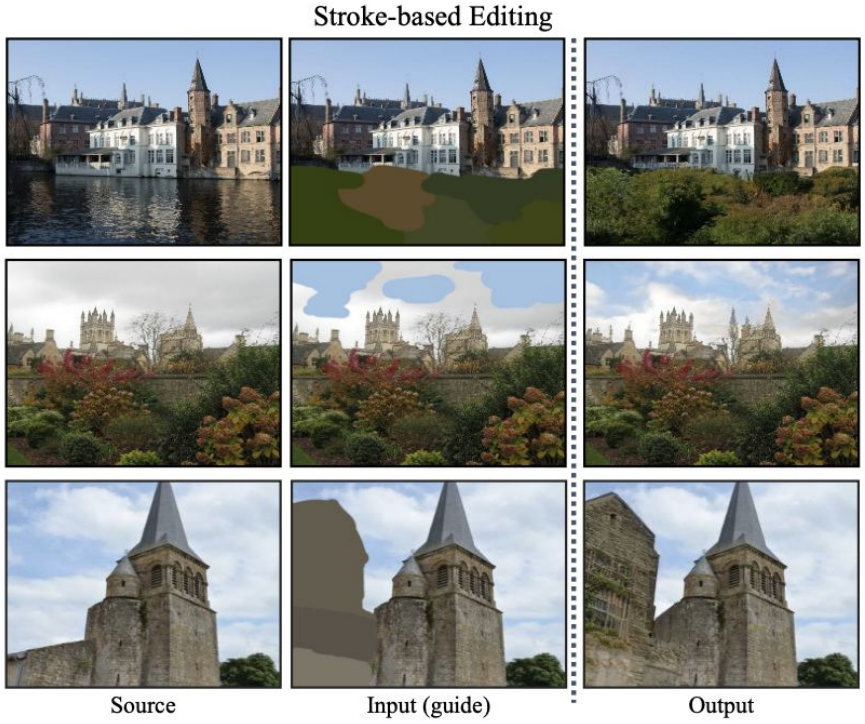
SDEdit: Conditional Generation w/o Training ([Meng et al. 2022])





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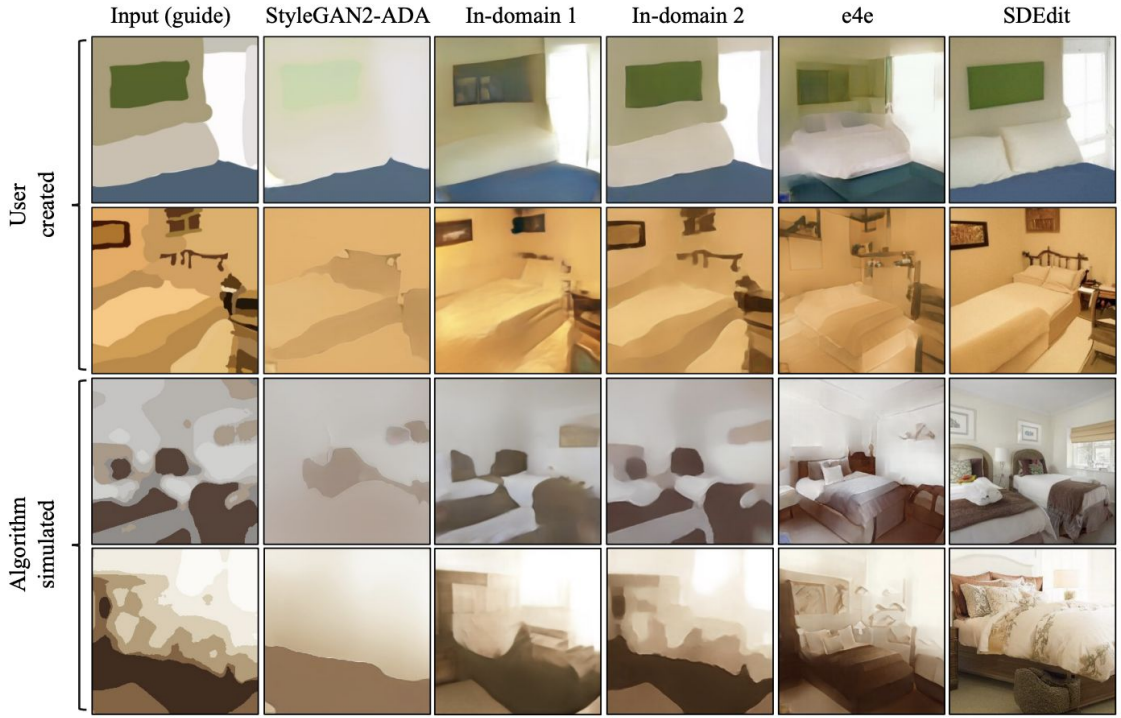

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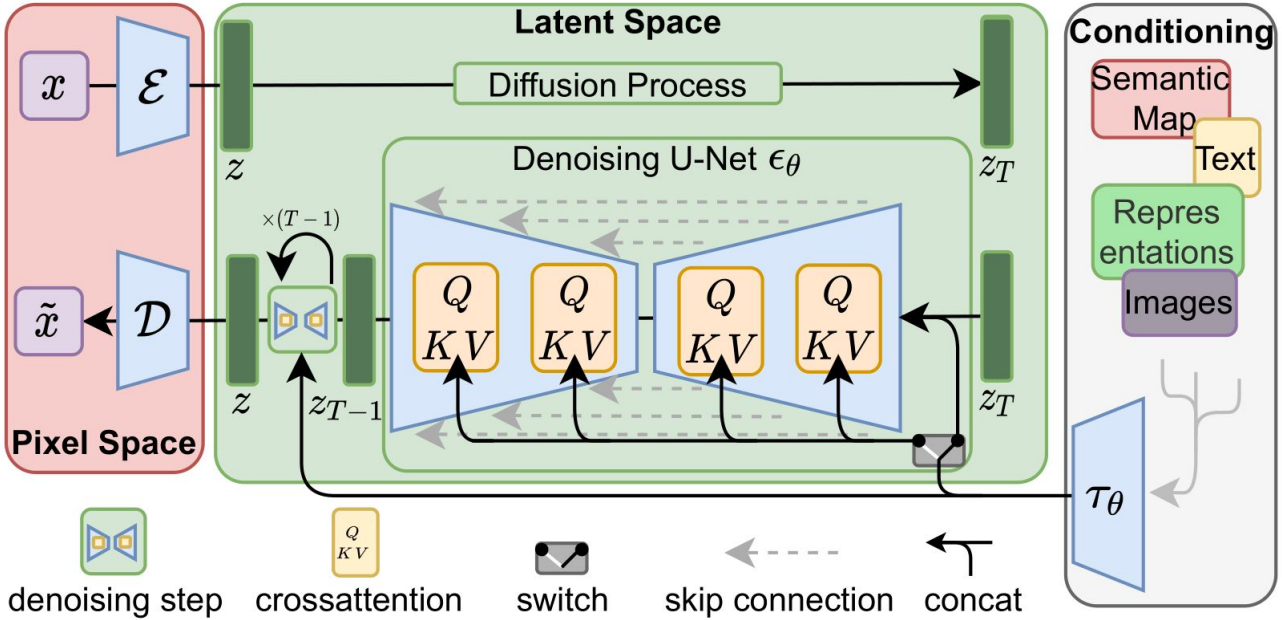
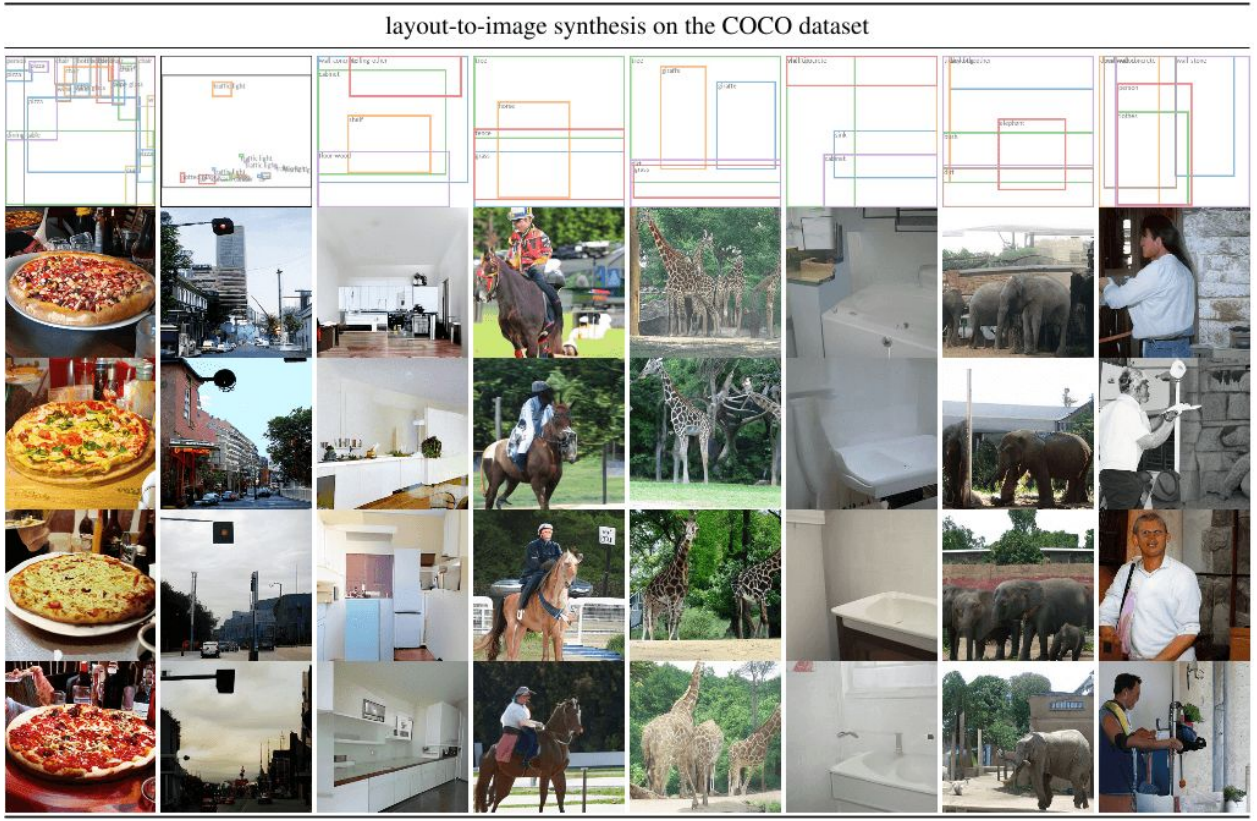
Stable Diffusion: Conditional Generation w/ diffusion in the latent space (Rombach et al. 2022)



Robin Rombach*, Andreas Blattmann*, Dominik Lorenz, Patrick Esser, Björn Ommer. “High-Resolution Image Synthesis with Latent Diffusion Models”. CVPR 2022.
https://arxiv.org/pdf/2112.10752.pdf

Stable Diffusion (Rombach et al. 2022)



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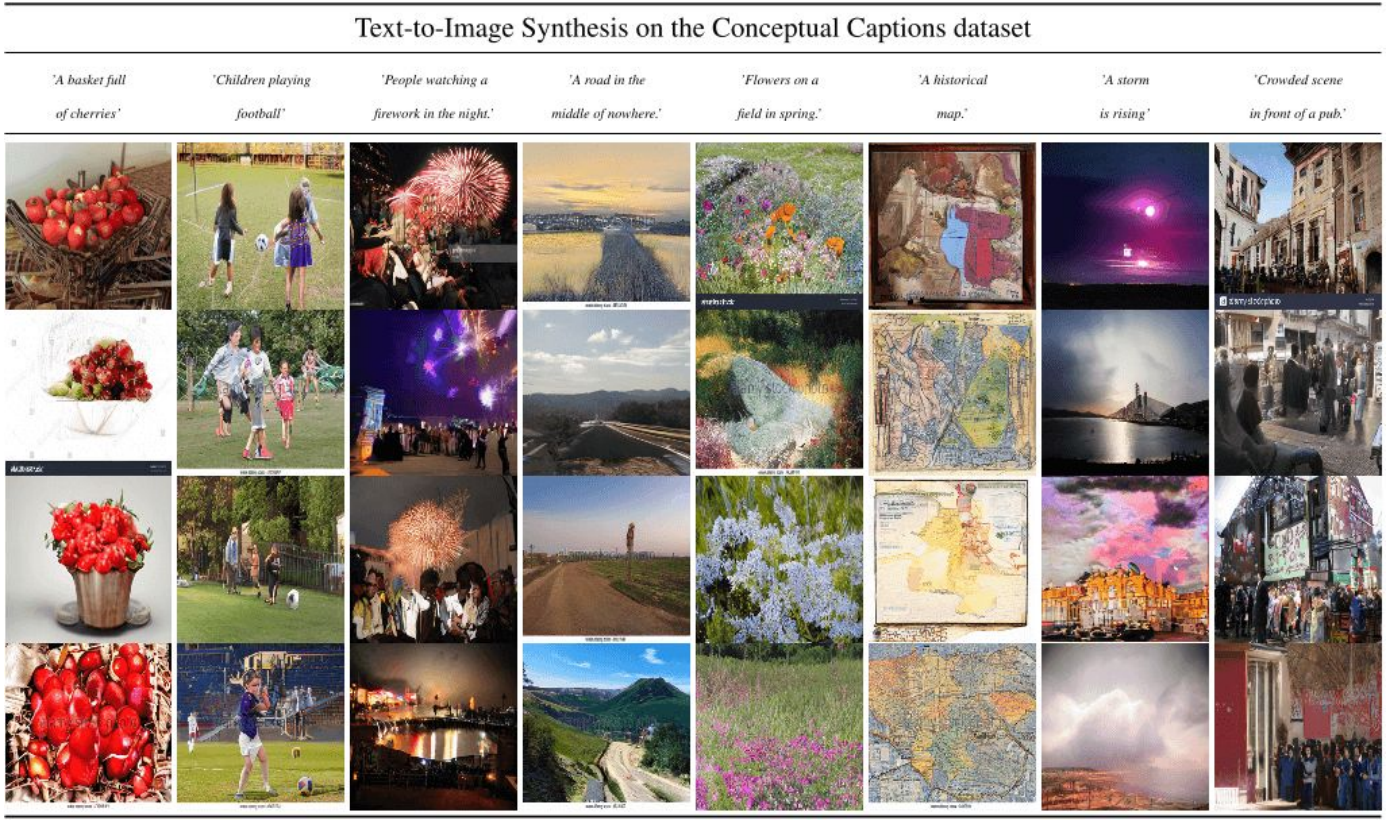

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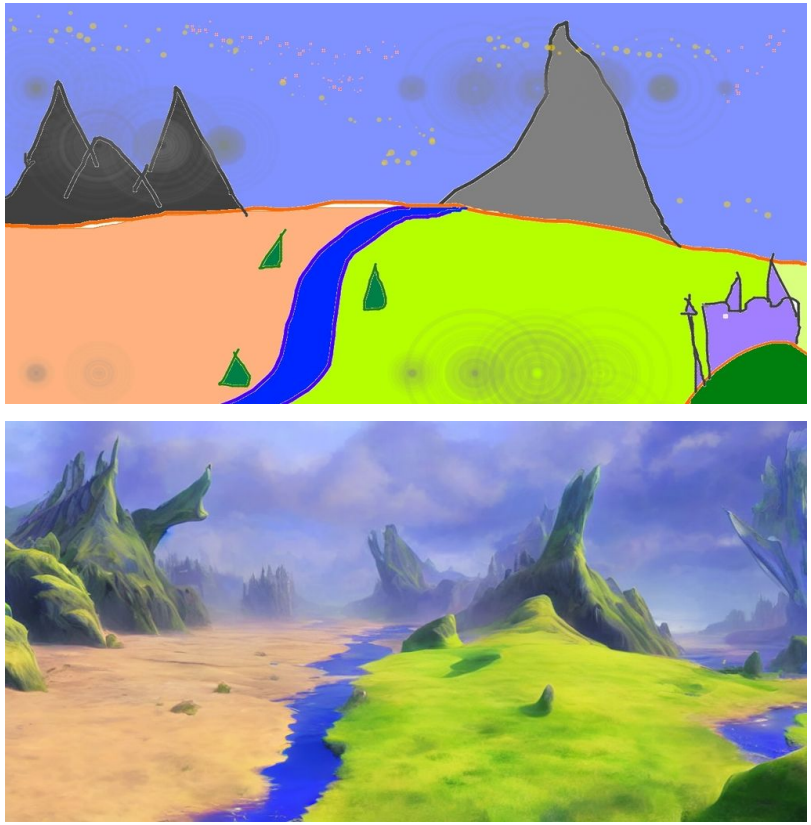
Stable Diffusion (Rombach et al. 2022)

Semantic Synthesis on Flickr-Landscapes [21]

Robin Rombach*, Andreas Blattmann*, Dominik Lorenz, Patrick Esser, Björn Ommer. “High-Resolution Image Synthesis with Latent Diffusion Models”. CVPR 2022.
https://arxiv.org/pdf/2112.10752.pdf

Stable Diffusion (Rombach et al. 2022)

"A fantasy landscape, trending on artstation"

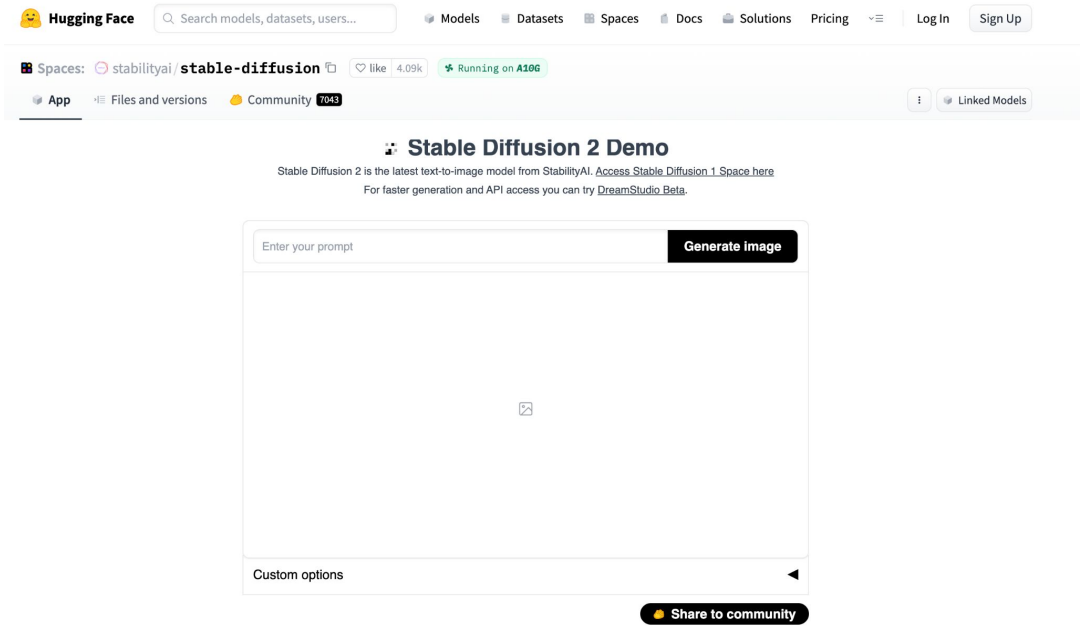


Uses SDEdit!

Robin Rombach*, Andreas Blattmann*, Dominik Lorenz, Patrick Esser, Björn Ommer. "High-Resolution Image Synthesis with Latent Diffusion Models". CVPR 2022.
https://arxiv.org/pdf/2112.10752.pdf

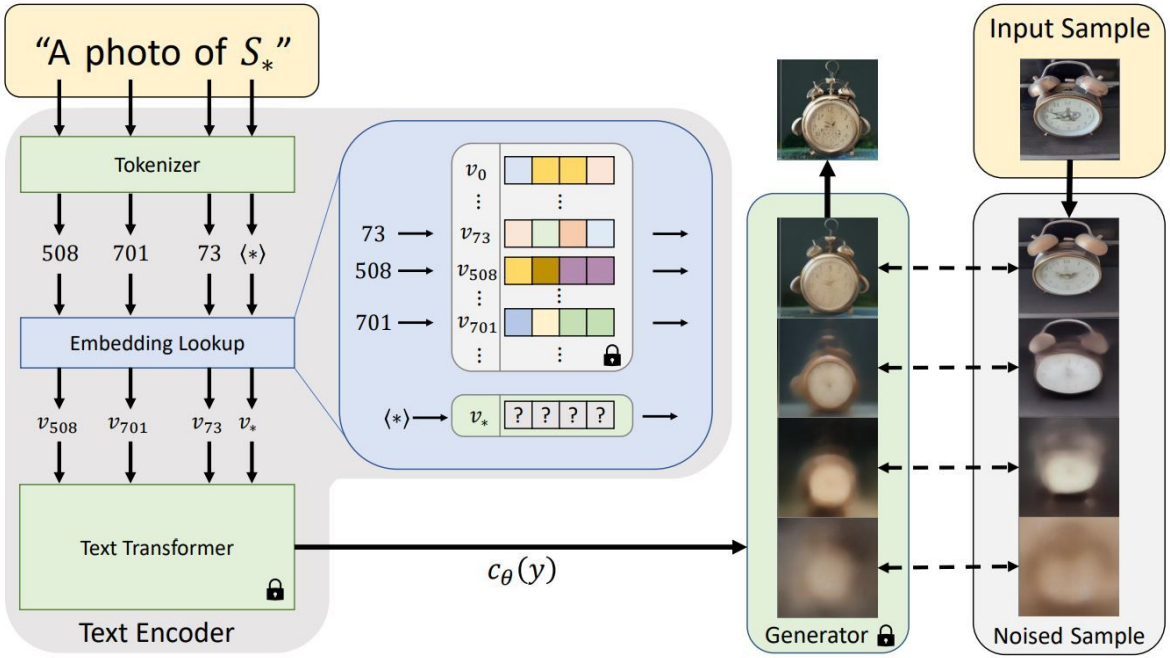
Stable Diffusion (Rombach et al. 2022)

Demo: https://huggingface.co/spaces/stabilityai/stable-diffusion (v2)
https://huggingface.co/spaces/stabilityai/stable-diffusion-1 (v1)



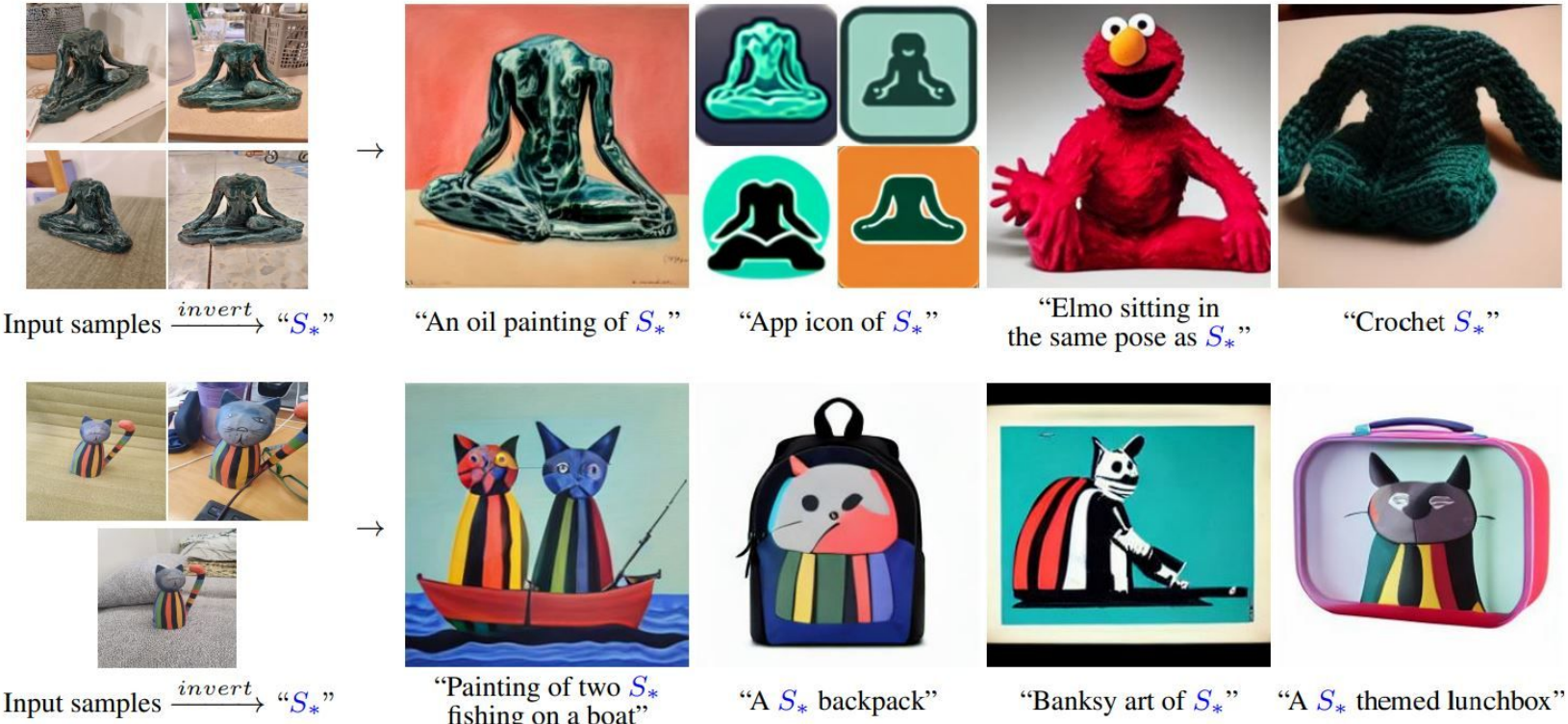
Robin Rombach*, Andreas Blattmann*, Dominik Lorenz, Patrick Esser, Björn Ommer. “High-Resolution Image Synthesis with Latent Diffusion Models”. CVPR 2022.
https://arxiv.org/pdf/2112.10752.pdf

Textual Inversion (Gal et al. 2022)



Rinon Gal, Yuval Alaluf, Yuval Atzmon, Or Patashnik, Amit H. Bermano, Gal Chechik, Daniel Cohen-Or. "An Image is Worth One Word: Personalizing Text-to-Image Generation using Textual Inversion". arXiv 2022. https://textual-inversion.github.io/

Textual Inversion (Gal et al. 2022)



Rinon Gal, Yuval Alaluf, Yuval Atzmon, Or Patashnik, Amit H. Bermano, Gal Chechik, Daniel Cohen-Or. "An Image is Worth One Word: Personalizing Text-to-Image Generation using Textual Inversion". arXiv 2022. https://textual-inversion.github.io/

DreamBooth (Ruiz et. al 2022)

Input images

in the Acropolis

swimming

sleeping

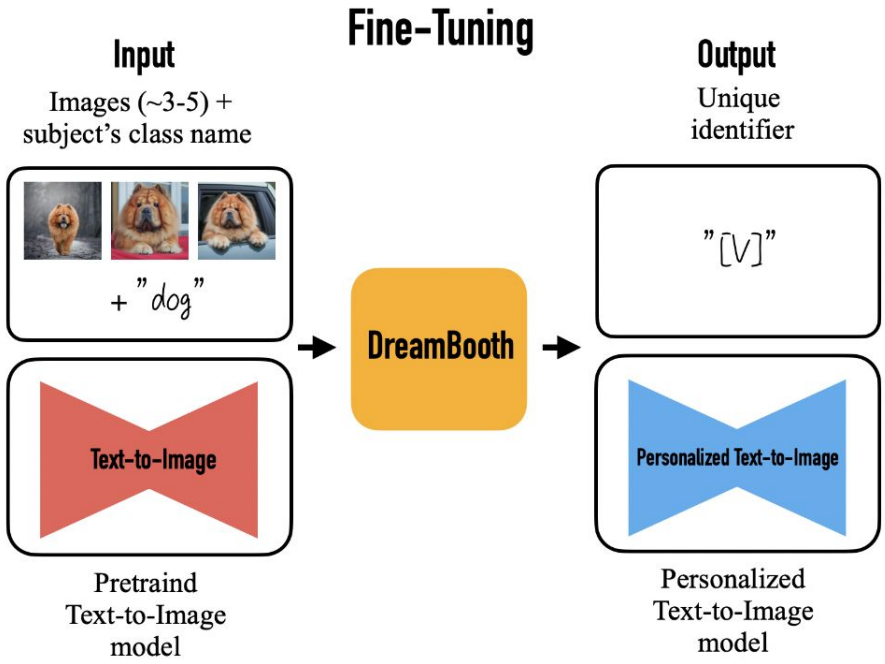
in a doghouse

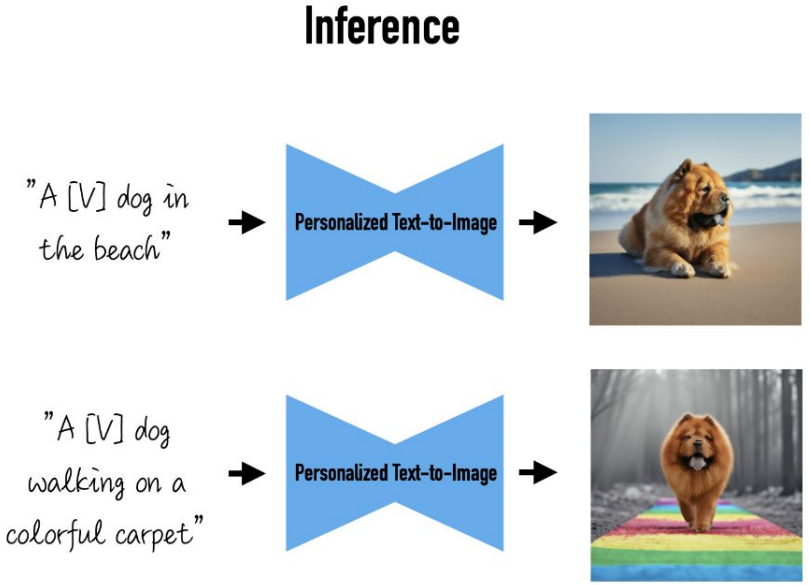
in a bucket

getting a haircut

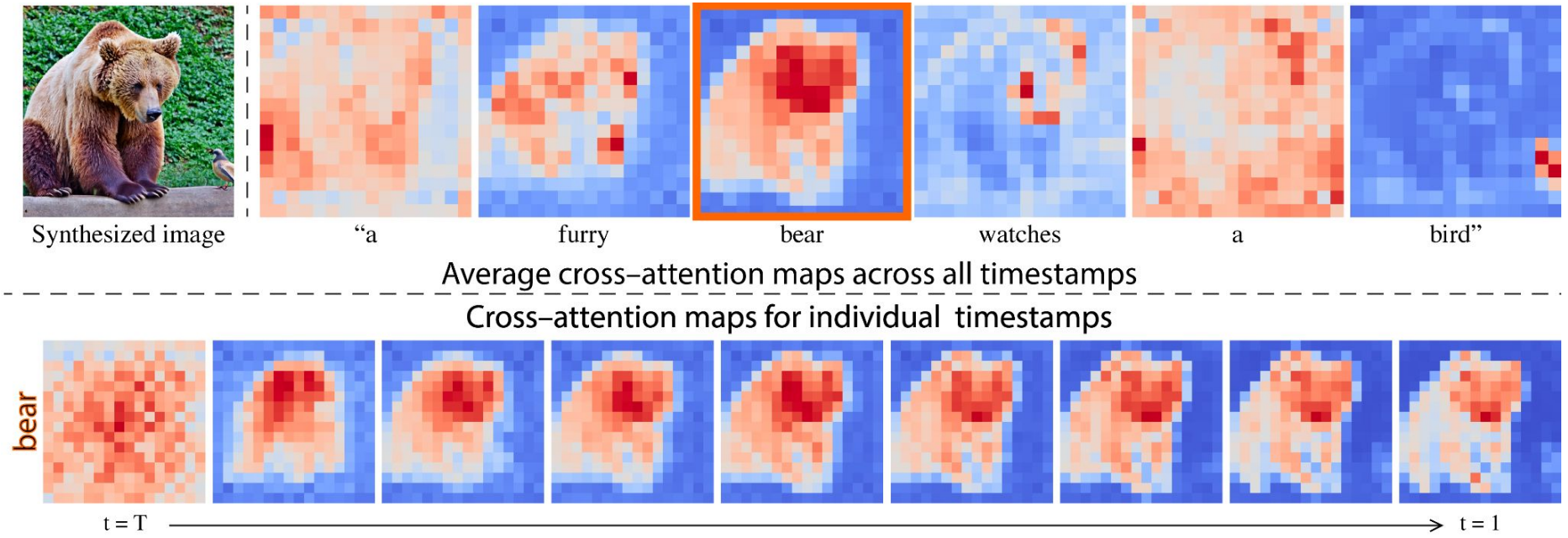
Nataniel Ruiz, Yuanzhen Li, Varun Jampani, Yael Pritch, Michael Rubinstein, Kfir Aberman. DreamBooth: Fine Tuning Text-to-Image Diffusion Models for Subject-Driven Generation. https://dreambooth.github.io/

DreamBooth (Ruiz et. al 2022)



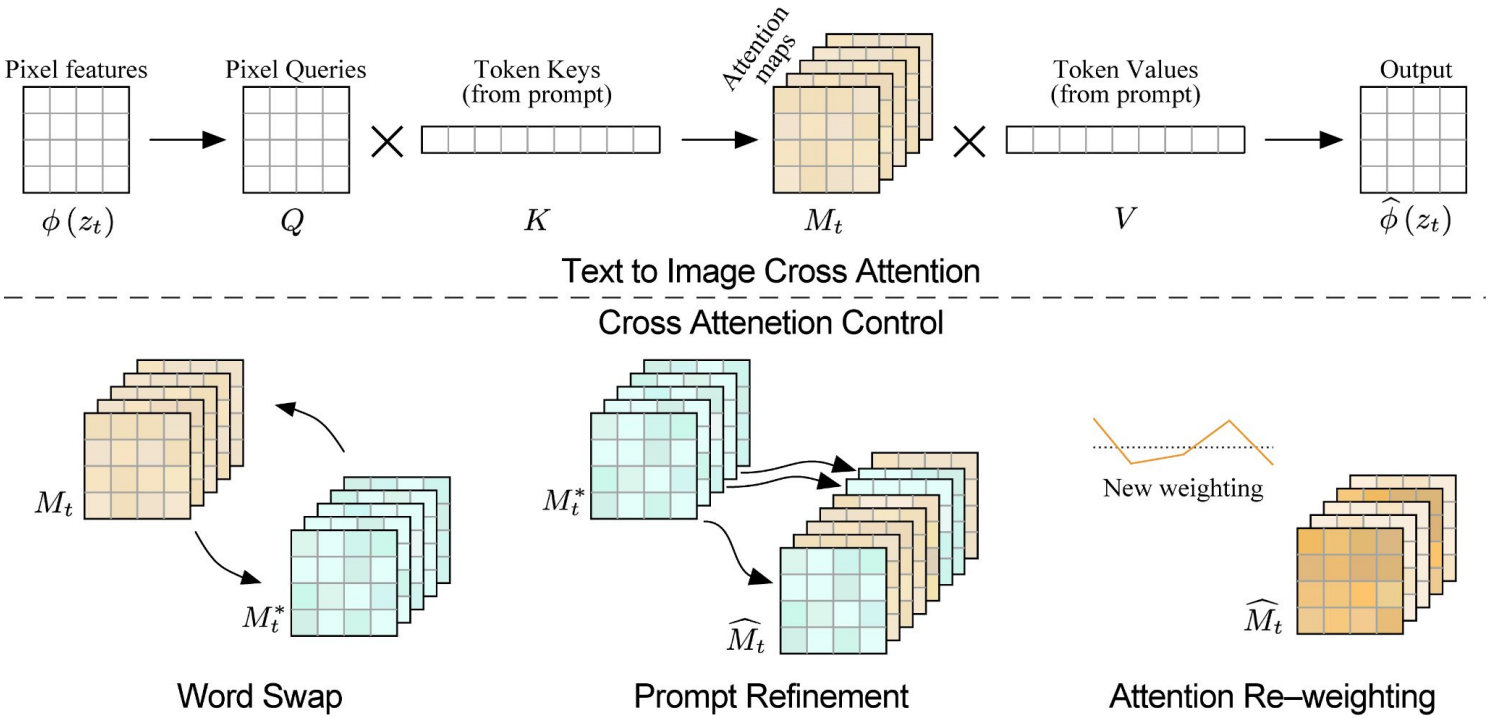


Prompt-to-Prompt Image Editing (Hertz et al. 2022)



Amir Hertz, Ron Mokady, Jay Tenenbaum, Kfir Aberman, Yael Pritch and Daniel Cohen-Or. “Prompt-to-Prompt Image Editing with Cross-Attention Control”. arXiv 2022. https://prompt-to-prompt.github.io/

Prompt-to-Prompt Image Editing (Hertz et al. 2022)



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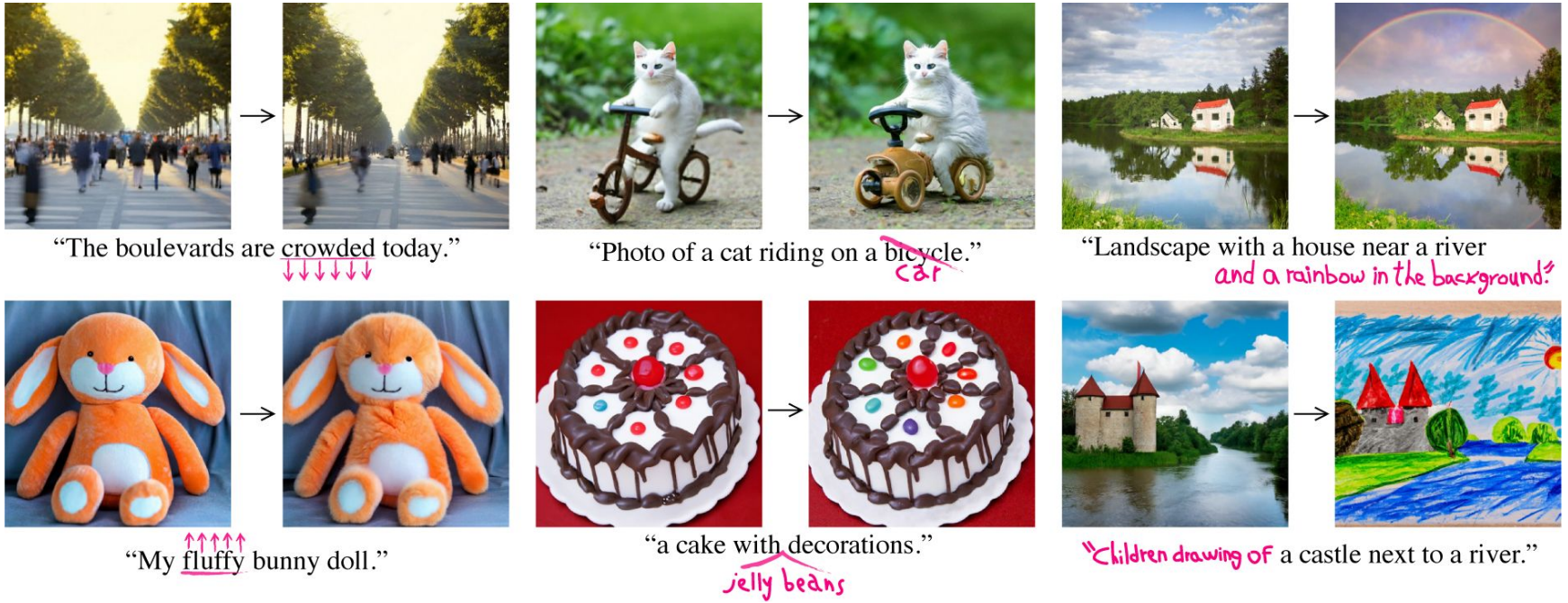
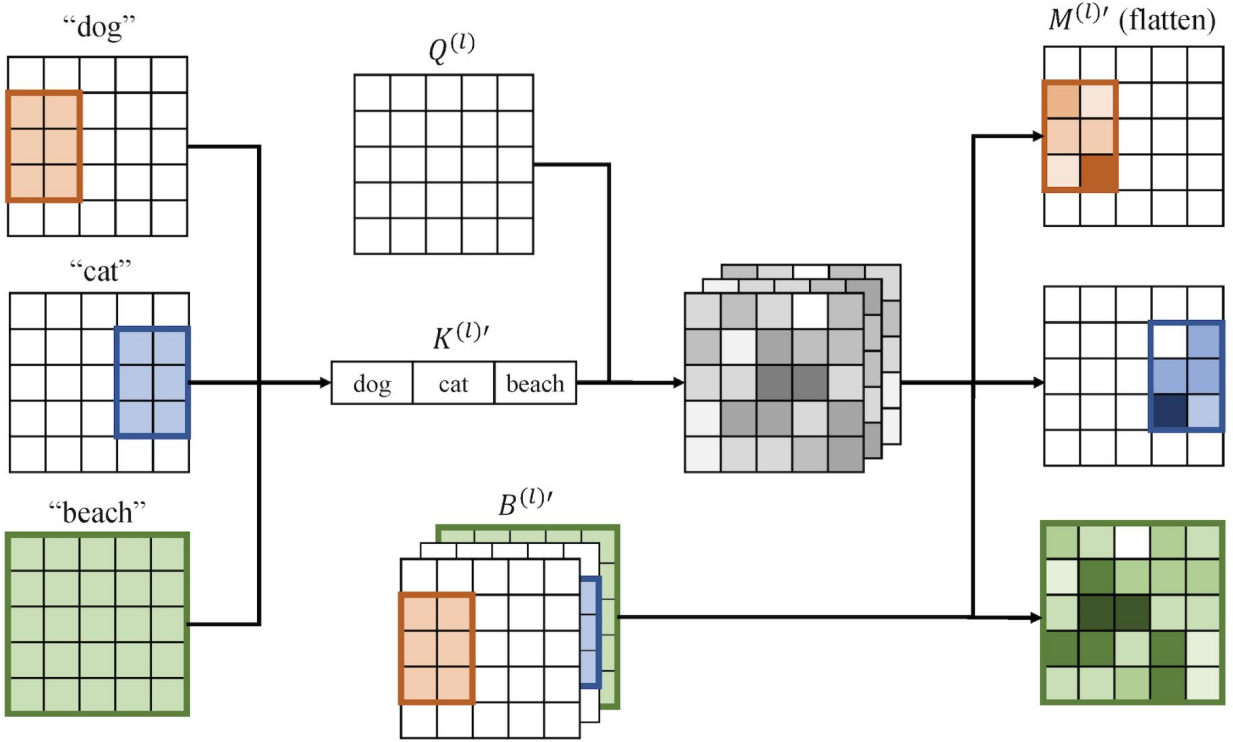
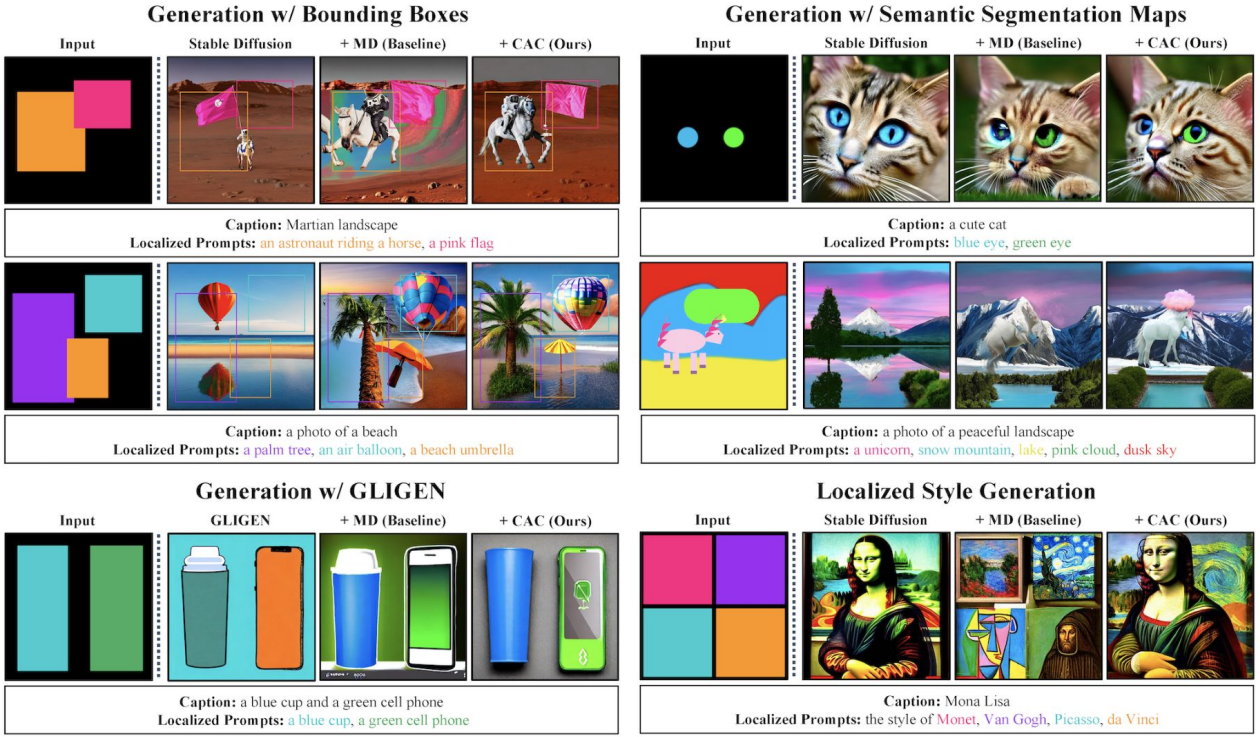


Amir Hertz, Ron Mokady, Jay Tenenbaum, Kfir Aberman, Yael Pritch and Daniel Cohen-Or. "Prompt-to-Prompt Image Editing with Cross-Attention Control". arXiv 2022. https://prompt-to-prompt.github.io/

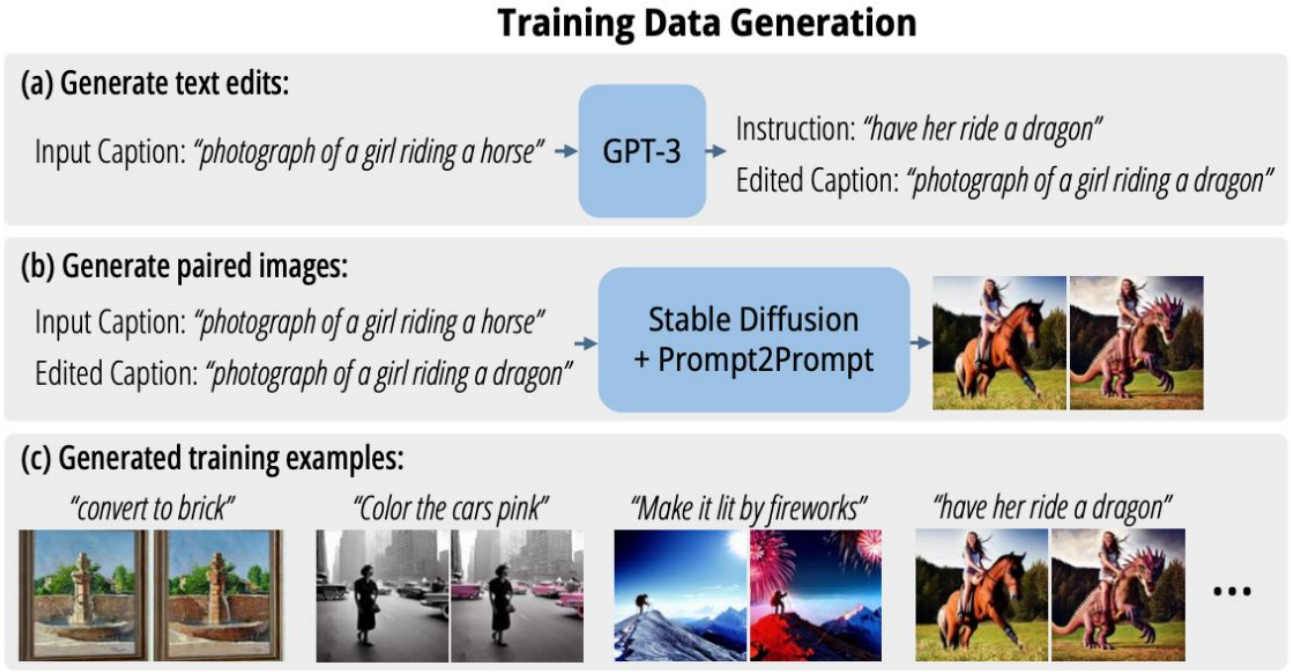
CAC (He et. al 2023)



CAC (He et. al 2023)

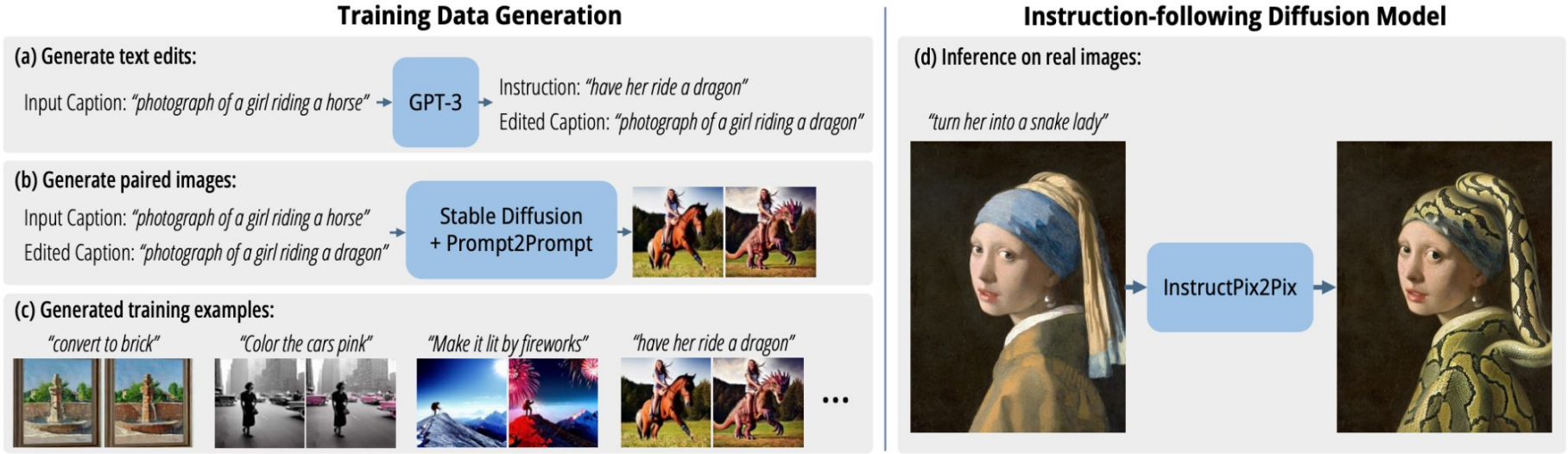


InstructPix2Pix (Brooks et al. 2023)



Tim Brooks, Aleksander Holynski, Alexei A. Efros. "InstructPix2Pix: Learning to Follow Image Editing Instructions". CVPR 2023.
https://www.timothybrooks.com/instruct-pix2pix

InstructPix2Pix (Brooks et al. 2023)



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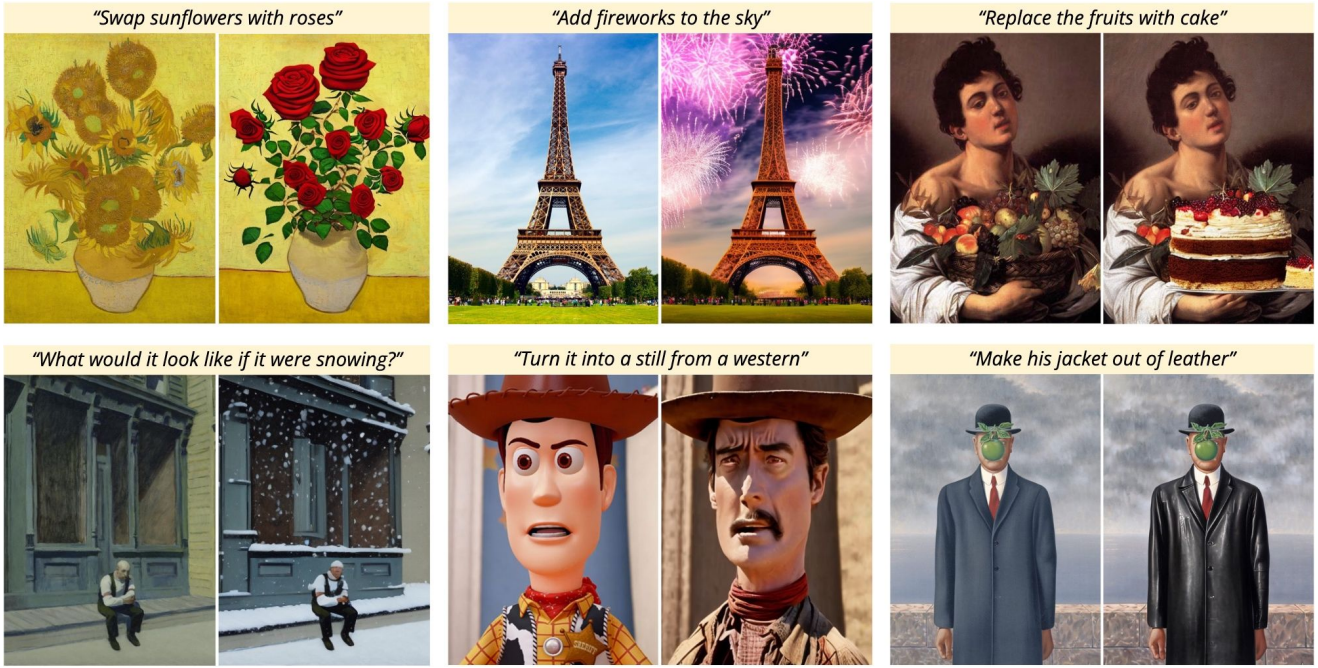
InstructPix2Pix (Brooks et al. 2023)

Demo



Tim Brooks, Aleksander Holynski, Alexei A. Efros. "InstructPix2Pix: Learning to Follow Image Editing Instructions". CVPR 2023.
https://www.timothybrooks.com/instruct-pix2pix

Questions I have been thinking about

- How to best incorporate auxiliary information in conditional generation of diffusion models so that it minimizes the additional training
  - How can we incorporate pretrained non-time-dependent models in conditional generation of diffusion models (in the data space)?
- Instead of the latent space, can we learn a diffusion model in the function space?
  - And since we already know how to represent data in the function space, can we in turn (better) sample data from this space?
  - Also denoising diffusion really looks like SGD, what is their relationship?

Other Fun Papers

- Faster Sampling
  - EDM
  - Distillation Papers
- Guided Diffusion
  - Classifier-guided diffusion
  - Classifier-free diffusion guidance
  - ControlNet
- Other Modalities
  - Video
  - Point cloud
  - Music
  - Discrete
  - Reinforcement Learning

Resources

- Lil' Log. "What are Diffusion Models?" https://lilianweng.github.io/posts/2021-07-11-diffusion-models/
- Yang Song. "Generative Modeling by Estimating Gradients of the Data Distribution". https://yang-song.net/blog/2021/score/
- Niels Rogge and Kashif Rasul. "The Annotated Diffusion Model". https://huggingface.co/blog/annotated-diffusion
- Calvin Luo. "Understanding Diffusion Models: A Unified Perspective". https://calvinyluo.com/2022/08/26/diffusion-tutorial.html

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  My website: https://kellyyutonghe.github.io/