CS344 Fall 2023 Design and Analysis of Data Structures and Algorithms

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Due **September 21st** 9pm. Each problem, unless specified otherwise, has a maximum of 10 points. Avoid too many details. A succinct and clean proof is the best. You may use the algorithms we covered in class without referring to the details.

Homework 1

1. Suppose $f(n) \in \Theta(g(n))$ for two functions $f(n), g(n)$. Prove or disprove the following claims. To disprove a claim, provide a counter example.

   (a) $\log_2 f(n) \in \Theta(\log_2 g(n))$
   (b) $2^{f(n)} \in \Theta(2^{g(n)})$.

2. Prove that any connected acyclic graph with at least two vertices has at least two vertices of degree 1. Note that you cannot directly apply properties of trees. Your proof shall start with only the definitions of 'acyclic' and 'connected'. (Hint: use induction on the number of vertices)

3. Consider a directed graph G where each edge is colored either red, white or blue. A walk in G is called a French flag walk if its sequence of edge colors is red, white, blue, red, white, blue and so on. Describe an algorithm to find all vertices in G that can be reached from a given vertex v through a French flag walk.

4. A number maze is an $n \times n$ grid of positive integers. A token starts from the upper left corner and the goal is to move the token to the bottom right corner. On each turn, one can move the token by the number on the current square. For example, if a token is on a square labeled 3, one can move the token three steps up, three steps down, three steps left, or three steps right. The token cannot be moved off the board. Describe and analyze an efficient algorithm that returns the minimum number of moves needed to solve a given number maze, or report correctly when there is no solution.

5. Suppose we have n soccer teams and some pairs of teams competed against each other. For each game, one of the teams is a winner – there is no tie. Design and analyze an efficient algorithm that checks if these game results are consistent – for example, if there is a set of teams $A_1, A_2, \cdots A_k$ with A_i beats A_{i+1} and A_k beats A_1, then there is no clear winner in this set and there is inconsistency.

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