Fourier Transform

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Definition For functions $f, g\colon \mathbb{R} \to \mathbb{C}$, their Hermitian form (i.e. inner product) is

$$\langle f, g \rangle = \int_{-\infty}^{\infty} f(\bar{x}) g(x) \mathrm{dx}$$

Definition The Fourier Transofrm is

$$\widetilde{f}(\omega) = \langle \frac{1}{\sqrt{2\pi}} e^{-i\omega x}, f =$$
$$= \langle \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\omega x} \mathrm{dx} \rangle$$

1 Properties of the Fourier Transform

1.1 Linearity

The Fourier transform of a sum of functions is equal to the sum of the Fourier transforms of the functions. Since we can obviously factor out constants, the Fourier Transofrm is a linear operator.
For functions $f, g\colon \mathbb{R} \to \mathbb{C}$,

$$\mathbb{F}\{f, g\}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} exp(-i\omega x))(f(x) + g(x)) \mathrm{dx}$$
$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} exp(-i\omega x) f(x) \mathrm{dx} + \frac{1}{2\pi} \int_{-\infty}^{\infty} exp(-i\omega x) g(x) \mathrm{dx}$$
$$= \widetilde{f}(\omega) + \widetilde{g}(\omega)$$