Notes from Sheldon Ross' A First Course in Probability

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1 Chapter 1: Combinatorial Analysis

Formula There are $\binom{n-1}{r-1}$ distinct positive integer-valued vectors $(X_1, X_2, \ldots, X_r)$ satsify $X_1 + X_2 + \ldots + X_r = n$.

Formula There are $\binom{n+r-1}{n}$ distinct non-negative integer-valued vectors $(X_1, X_2, \ldots, X_r)$ satsify $X_1 + X_2 + \ldots + X_r = n$.

2 Axioms of Probability

2.1 Sample Space and Events

Definition This set of all possible outcomes of an experiement is known as the *sample space* of the experiment and is denoted by **S**. If the experiment consists of n independent events of m possibilities, then the sample space consists of $n \ldots m$ points. **S** is denoted by a set characterised by the vector n variables, followed by the possible values of n. Any subset E of the sample space is known as an *event*.

Definition The event $E \cup F$ is called the *union* of the even E and the event F. The event $E \cap F$ or EF is called the *intersection* of events E and F to consist of all outcomes that are both in E and F.

Definition The null event $\emptyset$ refers to the event consisting of no points. If $EF = \emptyset$, then E and F sare said to be *mutually exclusive*.

Definition If events $E_1, E_2, \ldots, E_n$ are events, the union of these events, denoted by $\cup_{n=1}^{\infty}$ is defined to be that event which consists of all points that in E_n for at least one value of $n = 1, 2, \ldots$. Similarly, the intersection of the events E_n denoted by $\cap_{n=1}^{\infty}$ is defined to be the event consisting fo those points that are in all of the events $E_n, n = 1, 2, \ldots$.

Definition For any event E, we define the new event E^c as the *complement* of E to cosnist of all points in the sample space S that are not in E. E^c occurs if and only if E does not occur.

Definition For any two events E and F, if all of the points E and F, if all of the points in E are also in F, then we say that E is cotnained in F and write $E \subset F$ or$F \supset E$. Therefore, if $E \subset F$, the occurrence of E will necessarily imply the occurrence of F. IF $E \subset F$ and $F \subset E$, $E = F$.

Formula
Set Theory Laws
Commutative Law: $E \cup F = F \cup$ AND $EF = FE$
Associative Law $(E \cup F) \cup G = E \cup (F \cup G)$ AND $(EF)G = E(FG)$
Distributive Law $(E \cup F)G = EG \cup FG$ AND $EF \cup G = (E \cup G) = (E \cup G)(F \cup G)$

DeMorgan's Laws
$(\cup_{i=1}^{n} E_i)^c = \cap_{i=1}^{n} {E_i}^c$
$(\cap_{i=1}^{n} E_i)^c = \cup_{i=1}^{n} {E_i}^c$

2.2 Axioms of Probability

Definition One way of defining the probabilityo f an event is in terms of its relative frequency. That is $P(E)$ is defined as the limiting percentage of time that E occurs.

Axiom $0 \leq P(E) \leq 1$
The probability that the outcome of the experiment is a point in E is some number between 0 and 1.

Axiom $P(S) = 1$
The outcome will be a point in the sample space S.

Axiom For any sequence of mutually exclusive events $E_1, E_2, \ldots$ (that is events for which $E_i E_j = \emptyset$ when $i \neq j$)

$$P(\cup_{i=1}^{\infty}) = \Sigma_{i=1}^{\infty} P(E_i)$$

P(E) is the probability of the event E. For ar any sequence of mutually exclusive events the probability of at least one of these events occurring is just the sum of their respective probabilities.