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Church-Turing Thesis Summary

Effective Method

Before understanding the Church-Turing Thesis, one must understand what effective method is when used by both Church and Turing to describe their thesis. Effective method is described to be a procedure that can be accomplished by a human with just a paper and pencil. The human task doesn't need any intuition or insight and is rather thought to be an unintelligent task. It is obviously assumed that the human in question is not limited by contingencies such as boredom, death, or lack of paper (the amount of paper is assumed to be always sufficient).

This point of a task or procedure being able to be accomplished by a human is extremely critical in understanding the Church-Turing Thesis as without this knowledge, one is prone to misunderstand the scope of what this thesis encapsulates. Turing mentions this fact numerous times in his writings to signify its importance. It is also important to note that the technical term of *mechanical* is interchangeable with "being calculable by effective methods."

Church-Turing Thesis

Alonzo Church and Alan Turing worked independently during the 1930s to formulate their theses on how to determine what is calculable through an effective method. Church worked on his lambda-definable thesis and arrived upon, "A function of positive integers is effectively calculable only if lambda-definable." A function is lambda-definable if "...the values of the function can be obtained by a certain process of repeated substitution." On the other hand, Turing worked on his Turing Machine (also defined as Logical Computing Machines, or L.C.M.) and arrived upon, "L.C.M.s can do anything that could be described as 'rule of thumb' or 'purely mechanical.'" Again, the mechanical can be interchanged with being calculable by effective methods.

It was later found that these two theses are equivalent in the sense that the set of all mathematical functions where values can be obtained through effective methods are *computable* (the definition of computability is of importance and later mentioned) by both Turing machines and Lambda-definability, when discussing positive integers. It concerns positive integers as Church's Lambda-definability is concerned with the substitution of positive integers. This equivalence did not only occur between Church's and Turing's thesis, but also other many analyses of effective calculable functions. Thus, Church-Turing Thesis was formed which claimed that any effectively calculable function is computable by a Turing machine. The strongest support for this claim is the equivalence factor, where all *current* analyses of what is an effectively calculable function was able to be computed by a Turing machine.

Computability

Due to the Church-Turing Thesis, it is widely believed that the idea of mechanical machines is defined by whether or not it is under the scope of a Turing machine. As such, it is believed that a function is computable if and only if it is Turing computable. This makes sense as Turing defines computability as a task that can be performed by a human with a paper and pencil (a.k.a. an effective method) and Church-Turing Thesis claims to encapsulate all effectively calculable functions under a Turing machine. In a way, the idea of what is computable or not lies in one's faith in the Church-Turing Thesis as to how truthful the thesis actually is.

Until now, I assumed Turing's definition of computability (human computability) to define what is computable. However, if the term computable is more abstractly interpreted as something that is simply calculable within the physical limitations, then it is not Turing computable. Such misunderstandings have been apparent, such as this quote: "Every finitely realizable physical system can be perfectly simulated by a universal model computing machine operating by finite means." The universal model computing machine (Turing machine) isn't able to compute every "finitely realizable physical system," only systems of effective methods. It is common for media to misinterpret the term computability to encapsulate Turing machines capabilities bigger than what it's supposed to.

Maximality Thesis

An extension to the Church-Turing Thesis, the Maximality Thesis states that "All functions that can be generated by machines (working in accordance with a finite program of instructions) are computable by effective methods." This thesis would assume that all machines, past, present, or future, fall under the scope of a Turing machine. It would also assume that all functions generated by machines are human computable. This differs from the Church-Turing thesis as the Church-Turing thesis assumes that there may be functions uncomputable by humans but computable still with other means.

The Maximality Thesis is composed of two forms: the stronger form and the weaker form. The stronger form is: functions generated by machines that are not limited to physical laws of the universe are computable. It is known that the stronger form is false as the article gives an example of the Accelerating Turing machine (ATM) and proves how an ATM can generate the halting function but a Turing machine cannot. Since the explanations of ATM and the halting functions are in great detail, I won't mention them, but it is well known that Turing machines cannot generate the halting function since Turing proved this fact in the famous halting problem.

The weaker form of the Maximality thesis is: functions generated by machines that conform to the physical laws are computable. This weaker form would assume that all functions generated by a physical machine are able to be tasked by a human given infinite time and paper. However, this is unknown. A hypercomputer is said to be a machine that can achieve more than the Turing machine. Since, as of now, a hypercomputer doesn't exist, the validity of the weaker maximality thesis is yet to be verified.

Source: https://plato.stanford.edu/entries/church-turing/#MaxiThes