

CSE 303 Final Exam

1 a) $S \rightarrow 0S012S21210$

This language produces a palindrome of odd length
b) $(01 + 20)^* (02 + 002)^+$

2 $S \rightarrow 0S0 \mid 1S1 \mid \epsilon$ Eliminate ϵ $S \rightarrow 0S0 \mid 1S1 \mid 00 \mid 11$
 $A \rightarrow 0B1 \mid 1B0$ \rightarrow $A \rightarrow 0B1 \mid 1B0 \mid 01 \mid 10$
 $B \rightarrow 0B \mid 1B \mid \epsilon$ $B \rightarrow 0B \mid 1B \mid 0 \mid 1$

Eliminate Unit \rightarrow no unit productions

3.

State	input	TOS	Action	Comment
q_0	ϵ	Z_0	(q_F, Z_0)	Accept empty string
q_0	0	Z_0	$(q_0, 00Z_0)$	push two zeroes
q_0	0	0	$(q_0, 000)$	push all subsequent zeroes as 0
q_0	1	0	(q_1, ϵ)	pop a zero for 1
q_1	1	0	(q_1, ϵ)	pop subsequent 1's as zeroes
q_1	ϵ	Z_0	(q_F, Z_0)	Accept when stack empty

Q. 4. Given the pumping Lemma constant N , assume the String $0^N 1^{N+1} 2^{N+2}$ is context free. Then, String S , $|S| = 3N + 3 \geq N$.
 Given $S = uvwx$ and $|vwx| \leq N$ where $v, x \neq \epsilon$

Cases 1) if vwx is the String of zeroes, pumping it up will increase zeroes more than N and is no longer part of the language

2) if vwx is the String of ones, pumping it up will increase ones more than N and is no longer part of the language.

3) if vwx is the String of 2's, pumping it up will increase 2's more than N and is no longer part of the language

4) if vwx is the String of 0's and 1's, v has at least one 0 and x has at least one 1.

i) v has only zeroes and pumping up will make $i \geq j$ and no longer part of language

ii) v has zeroes and ones and pumping up will make $i > i \geq k$ and is no longer part of language

5) vwx is the String of ones and twos, v has at least one 1 and x has at least one 2.

i) v has only 2's and pumping up will make $j \geq k$ and no longer part of the language

ii) v has 1's and 2's pumping it down will make $k \leq i$ and is no longer part of the language

Due to contradictions, there are no cases where language $\{0^i 1^j 2^k \mid i < j < k\}$ is context-free

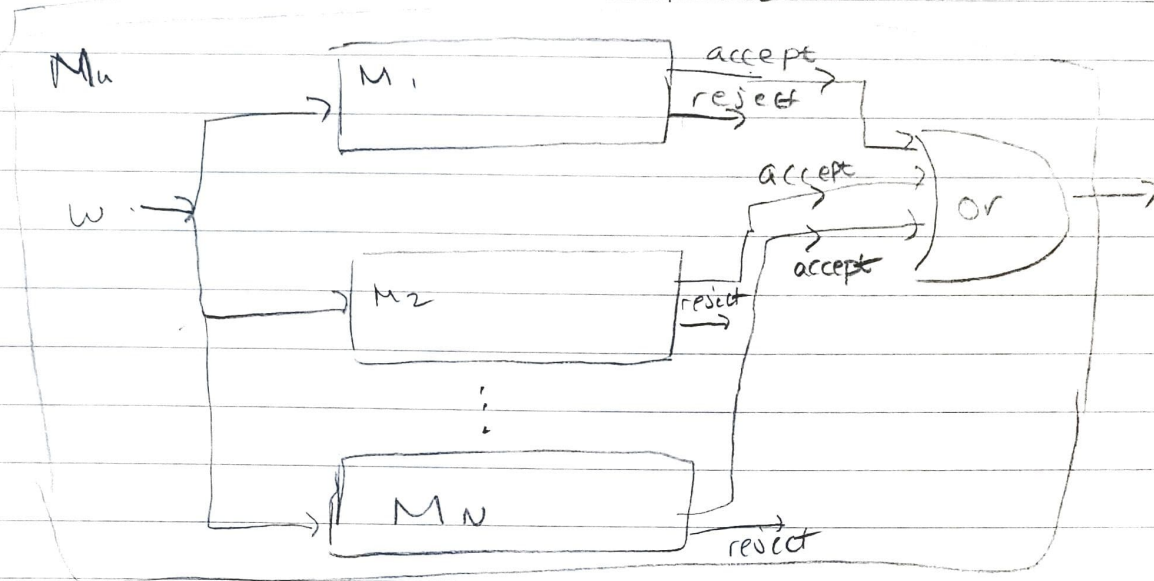
5.

State	Tape Symbol				
	B	0	1	X	Y
q_0	(q_F, B, L)	(q_1, X, R)	(q_2, X, R)	(q_0, X, R)	
q_1		$(q_1, 0, R)$	(q_3, X, L)	(q_1, X, R)	
q_2		(q_3, X, L)	$(q_2, 1, R)$	(q_2, X, R)	
q_3	(q_0, B, R)	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_3, X, L)	
q_F					

Algorithm: - Mark first number read with X

- If input was 0, go right until 1 and mark X
- If input was 1, go right until 0 and mark X
- go left until B
- repeat from step 1
- If input is only X's while scanning for number and hits B, accept

6. a) Yes, L is recursive. If L is the union of L_1, L_2, \dots .
A TM could be made $L_1 \cup L_2 \cup L_3 \dots$.



Where,

M_u = TM for $L_1 \cup L_2 \cup L_3 \dots L_n$. Using or logic, at least one accept state is needed for M_u to accept. Else, it rejects.

b) It is undecidable. There exists no machines that could determine whether a program will perform as advertised based on all inputs. One example is a program is advertised to determine the halting problem for any input. Such statement cannot be proven since the halting problem itself is undecidable.