Assignment 2 Analysis **gth836x; 901929700**
Myles Lefkovitz **mlefkovitz@gatech.edu**

Part 1 – Local Random Search on a Neural Network

I. Dataset – Adult Income data:

On the University of California Irvine' Machine Learning Repository I found an Adult Income Data Set[i] that classifies adults into one of two income categories: '>50K', or '<=50K'. The '>50K' category identifies individuals that earned more than $50,000 in the given year, 1994. The '<=50K' category identifies individuals that earned less than or equal to $50,000. $50,000 in 1994 is approximately $81,000 in today's terms. The data has 13 attributes, 5 of which are real-valued, and 8 of which are categorical.

The 5 real-valued attributes and their minimum and maximum values are: age [17 – 90], education-num [1 – 16], capital-gain [0 – 99,999], capital-loss [0 – 4,356], hours-per-week [1 – 99].

The 8 categorical attributes and a brief description are: workclass (type of employer), education (level of education, duplicated in the education-num real-valued attribute), marital-status, occupation (type of job), relationship (family status), race, sex, native-country. See figures 1-8 below for an examination of these 8 attributes.

The data set's 2 categories, '>50K' and '<=50K', represent approximately 24% and 76% of the instances in the data set, respectively. To create a high-performing simplistic model, we could uniformly classify every individual in the '<=50K' category. We should consider a learner successful only if it categorizes adult incomes correctly more than 76% of the time.

II. Backpropagation

When completed backpropagation in assignment 1, cross validation settled on alpha=0.005 and hidden layer size = 3 as the best set of hyperparameters. See the model complexity chart in figure 1 below. There are a total of 47 input notes and one output node.

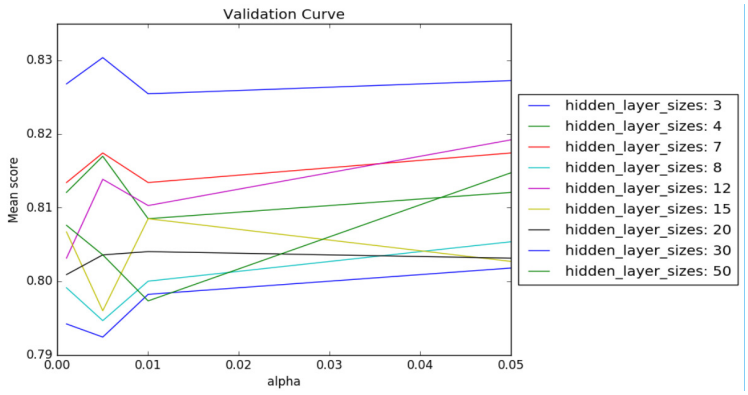


Figure 1. Neural Network Model Complexity Chart

This model's performance against training data is 86.8%
This model's performance against CV data is 83%
This model's performance against test data is 83.5%
This model took approximately 49 minutes to run

native-country	Count	Percent
United-States	29170	89.6%
Mexico	643	2.0%
?	583	1.8%
Philippines	198	0.6%
Germany	137	0.4%
Canada	121	0.4%
Puerto-Rico	114	0.4%
El-Salvador	106	0.3%
India	100	0.3%
Cuba	95	0.3%
England	90	0.3%
Jamaica	81	0.2%
South	80	0.2%
China	75	0.2%
Italy	73	0.2%
Dominican-Republic	70	0.2%
Vietnam	67	0.2%
Guatemala	64	0.2%
Japan	62	0.2%
Poland	60	0.2%
Columbia	59	0.2%
Taiwan	51	0.2%
Haiti	44	0.1%
Iran	43	0.1%
Portugal	37	0.1%
Nicaragua	34	0.1%
Peru	31	0.1%
Greece	29	0.1%
France	29	0.1%
Ecuador	28	0.1%
Ireland	24	0.1%
Hong	20	0.1%
Trinadad&Tobago	19	0.1%
Cambodia	19	0.1%
Laos	18	0.1%
Thailand	18	0.1%
Yugoslavia	16	0.0%
Outlying-US(Guam-USVI-etc)	14	0.0%
Honduras	13	0.0%
Hungary	13	0.0%
Scotland	12	0.0%
Holand-Netherlands	1	0.0%
Total	**32561**	**100.0%**

Figure 2. Adults by Native Country

workclass	Count	Percent
Private	22696	69.7%
Self-emp-not-inc	2541	7.8%
Local-gov	2093	6.4%
?	1836	5.6%
State-gov	1298	4.0%
Self-emp-inc	1116	3.4%
Federal-gov	960	2.9%
Without-pay	14	0.0%
Never-worked	7	0.0%
Total	**32561**	**100.0%**

Figure 3. Adults by work class

occupation	Count	Percent
Prof-specialty	4140	12.7%
Craft-repair	4099	12.6%
Exec-managerial	4066	12.5%
Adm-clerical	3770	11.6%
Sales	3650	11.2%
Other-service	3295	10.1%
Machine-op-inspct	2002	6.1%
?	1843	5.7%
Transport-moving	1597	4.9%
Handlers-cleaners	1370	4.2%
Farming-fishing	994	3.1%
Tech-support	928	2.9%
Protective-serv	649	2.0%
Priv-house-serv	149	0.5%
Armed-Forces	9	0.0%
Total	**32561**	**100.0%**

Figure 4. Adults by occupation

race	Count	Percent
White	27816	85.4%
Black	3124	9.6%
Asian-Pac-Islander	1039	3.2%
Amer-Indian-Eskimo	311	1.0%
Other	271	0.8%
Total	**32561**	**100.0%**

Figure 5. Adults by race

education	Count	Percent
HS-grad	10501	32.3%
Some-college	7291	22.4%
Bachelors	5355	16.4%
Masters	1723	5.3%
Assoc-voc	1382	4.2%
11th	1175	3.6%
Assoc-acdm	1067	3.3%
10th	933	2.9%
7th-8th	646	2.0%
Prof-school	576	1.8%
9th	514	1.6%
12th	433	1.3%
Doctorate	413	1.3%
5th-6th	333	1.0%
1st-4th	168	0.5%
Preschool	51	0.2%
Total	**32561**	**100.0%**

Figure 6. Adults by education

relationship	Count	Percent
Husband	13193	40.5%
Not-in-family	8305	25.5%
Own-child	5068	15.6%
Unmarried	3446	10.6%
Wife	1568	4.8%
Other-relative	981	3.0%
Total	**32561**	**100.0%**

Figure 7. Adults by relationship

marital-status	Count	Percent
Married-civ-spouse	14976	46.0%
Never-married	10683	32.8%
Divorced	4443	13.6%
Separated	1025	3.1%
Widowed	993	3.0%
Married-spouse-absent	418	1.3%
Married-AF-spouse	23	0.1%
Total	**32561**	**100.0%**

Figure 8. Adults by marital status

sex	Count	Percent
Male	21790	66.9%
Female	10771	33.1%
Total	**32561**	**100.0%**

Figure 9. Adults by sex

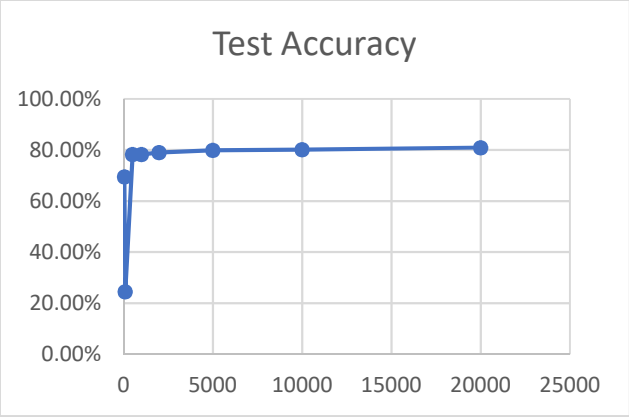
III. Randomized Optimization

Randomized Hill Climbing

Randomized Hill Climbing is an algorithm that selects a random starting point and iteratively finds the best point in its neighborhood and move to it. Using this process the algorithm 'climbs' the 'hill' to the optimum.

A random hill climbing algorithm was used to train the weights of the adult income neural network described above. Below, you can find the accuracy of the algorithm at different numbers of iterations. At 20,000 iterations the algorithm correctly predicted 83% of the instances in the training set and 80.9% of the instances in the test set and took 834s (around 14 minutes).

Iterations	Train Accuracy	Test Accuracy	Time (s)
50	67.81%	69.48%	1.599
100	24.96%	24.48%	3.322
500	76.96%	78.23%	15.954
1000	77.01%	78.23%	32.823
2000	78.21%	79.06%	69.573
5000	78.80%	79.90%	155.691
10000	80.31%	80.10%	364.867
20000	83.08%	80.94%	834.851



Simulated Annealing

Simulated Annealing is an algorithm that selects a random starting point and iteratively selects a point at random in its neighborhood to potentially move to. The move is made if the point is closer to the optimum, or probabilistically dependent on the temperature otherwise.

A simulated annealing algorithm was used to train the weights of the adult income neural network described above. Two hyperparameters were tuned:

1. Initial Temperature (10^11, 10^8, 10^5)
2. Cooling rate (0.99, 0.95, 0.90)

Iterations were capped at 10,000 for simplicity. Some combinations of initial temperature and cooling rate were thrown out. In the end 4 were evaluated:

1. T = 10^11, Cooling rate = 0.95
2. T = 10^8, cooling rate = 0.95
3. T = 10^5, cooling rate = 0.99
4. T = 10^11, cooling rate = 0.90

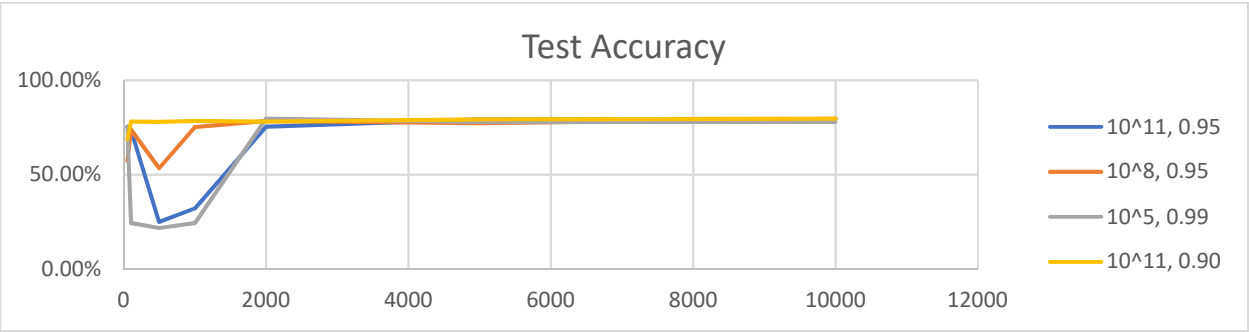
For higher cooling rates, you'd expect to need more iterations before converging to the optimum (the temperature stays higher for more iterations). For lower cooling rates, you accept some risk that the algorithm will cool too quickly and miss the global optimum.

The evaluation output below shows that at 10,000 iterations, the algorithm with initial temperature 10^8 and cooling rate 0.95 performed just as well as the algorithm with initial temperature 10^11 and cooling rate 0.90. They each classified 79.2% of training instances and 79.69% of test instances correctly. The algorithms took 383 and 359 seconds to find the weights respectively (around 6.5 and 6 minutes).

	Train Accuracy			
Iterations	**10^11, 0.95**	**10^8, 0.95**	**10^5, 0.99**	**10^11, 0.90**
50	75.1%	55.7%	75.0%	68.7%
100	74.2%	73.5%	25.0%	76.8%
500	25.4%	55.4%	23.2%	76.8%
1000	34.9%	75.0%	25.0%	77.1%
2000	75.0%	77.7%	78.8%	76.8%
5000	78.7%	80.6%	76.9%	78.6%
10000	78.2%	79.2%	76.8%	79.2%

	Test Accuracy			
Iterations	**10^11, 0.95**	**10^8, 0.95**	**10^5, 0.99**	**10^11, 0.90**
50	75.42%	57.40%	75.52%	68.75%
100	73.96%	73.85%	24.48%	78.23%
500	25.00%	53.44%	21.77%	77.92%
1000	32.19%	75.21%	24.48%	78.54%
2000	75.52%	78.44%	79.69%	78.23%
5000	79.38%	77.40%	77.92%	79.27%
10000	79.38%	79.69%	77.92%	79.69%

	Time (s)			
Iterations	**10^11, 0.95**	**10^8, 0.95**	**10^5, 0.99**	**10^11, 0.90**
50	1.639	1.603	1.665	1.625
100	3.253	3.377	3.576	3.236
500	15.5	15.971	16.207	16.587
1000	31.59	31.891	32.026	33.889
2000	72.631	67.173	66.496	64.847
5000	166.806	172.218	232.083	166.185
10000	376.698	383.365	399.799	359.033



Genetic Algorithm

Genetic Algorithms combine instances in populations to attempt to reach the global optimum.

A genetic algorithm was used to train the weights of the adult income neural network described above. Two hyperparameters were tuned:

1. # of instances mated
2. # of instances mutated

With population size held constant at 200 (around 10% of the training set) two sets of hyperparameters were evlautes:

1. # of instances mated = 100; # instances mutated = 10
2. # of instances mated = 200; # instances mutated = 200

The evaluation output below shows that the parameters don't seem to change much. Neither does adding additional iterations (other than processing time consumed). Genetic algorithms seem to perform very well with few iterations, but don't seem to improve with additional computation time.

	Train Accuracy	
Iterations	200, 100, 10	200, 200, 200
50	76.92%	76.79%
100	76.83%	77.19%
500	76.83%	76.83%
1000	76.83%	

	Test Accuracy	
Iterations	200, 100, 10	200, 200, 200
50	78.44%	77.92%
100	78.23%	78.54%
500	78.23%	78.23%
1000	78.23%	

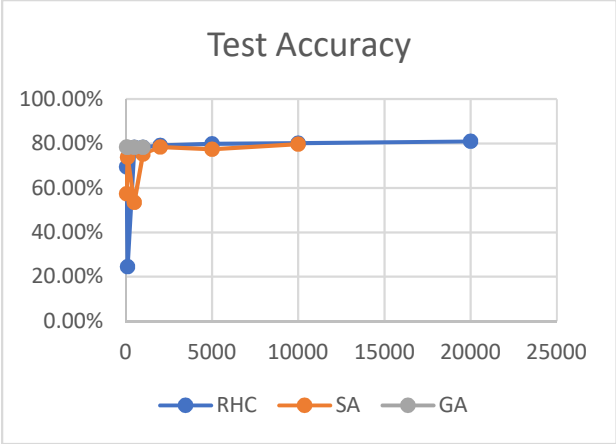
	Time (s)	
Iterations	200, 100, 10	200, 200, 200
50	28.61	53.882
100	58.511	107.897
500	344.068	601.419
1000	706.556	

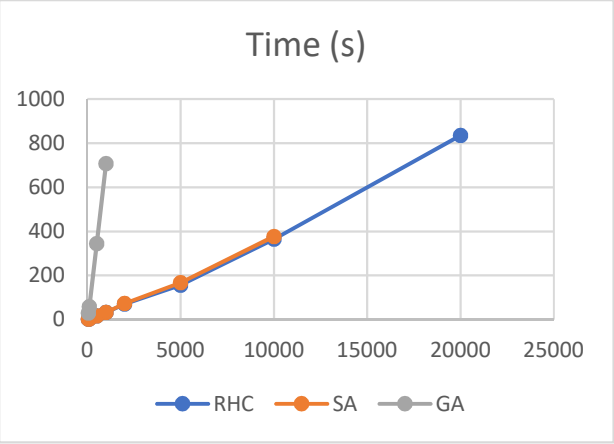
IV. Summary

As shown above different algorithms have different performance rates when training neural networks. In this example, backpropagation was most closely approximated by using random hill climbing. The algorithm performs only marginally worse on the test and training set and took less than 1/3rd the training time. Simulated annealing and the genetic algorithm also performed better than chance on the test set (76% as shown in the introduction) and all have shorter training times than backpropagation.

In the additional graphs below, note the test accuracy convergence around 80% and the time different algorithms take with different iterations. Genetic algorithms seem to take much more time per iteration, which makes sense considering the multiple steps required to compare and match instances.

Algorithm	Backpropagation	RHC	SA	GA
Training Accuracy	86.8%	83.1%	79.2%	76.8%
Test Accuracy	83.5%	80.9%	79.7%	78.2%
Duration	49 min	14 min	6 min	12 min


Test Accuracy
100.00%
80.00%
60.00%
40.00%
20.00%
0.00%
0
5000
10000
15000
20000
25000
RHC
SA
GA


Time (s)
1000
800
600
400
200
0
0
5000
10000
15000
20000
25000
RHC
SA
GA

Part 2 – 3 Problem Domains

I. Count Ones

Count Ones is a well understood problem where the parameter space is a bit string (a string of 1s and 0s) and the optimal result is the string of all 1s.

We can use randomized optimization to solve this problem quickly and reliably. The evaluation function is simply the sum of the bits in the bit string (000011101 sums to 4). By starting with a random string, the optimization algorithms can search through the neighborhood of strings to find the single global maximum.

Algorithm performance:

With enough iterations and a sufficiently small parameter space, any optimization algorithm should eventually be able to solve the count ones problem. Since the domain is unstructured, we would expect RHC and SA to perform better than GA and MIMIC.

With an initial N (bit string size) = 8, we see that any of the algorithms (RHC, SA, GA, MIMIC) can solve this problem in under .1 second with 200 or fewer iterations.

Paring N up to 80, we increase the complexity. We should expect GA and MIMIC to take more iterations to achieve the same results. It takes RHC and SA 1000 iterations to converge on the optimum. It only takes MIMIC 500 iterations to converge. GA does not come close to converging after 1000 iterations. It takes MIMIC >2 seconds of clock time to run 500 iterations, vs <0.006 seconds for RHC and SA.

Increasing the N to 800, we further increase complexity. We could expect GA and MIMIC to fail to converge in reasonable time, while RHC and SA should still succeed. Indeed, we see convergence in 20,000 iterations for both RHC and SA. Total clock time for either algorithm is under .25 seconds. MIMIC approaches 5 minutes of runtime at 500 iterations and is far from convergence at that point.

See detailed output below:

N = 8	Accuracy			
Iterations	RHC	SA	GA	MIMIC
100	8			8
200		8	8	

N = 8	Time (s)			
Iterations	RHC	SA	GA	MIMIC
100	0			0.031
200		0	0	

N = 80	Accuracy			
Iterations	RHC	SA	GA	MIMIC
100				79
200	68	59	44	78
500	77	77	46	80
1000	80	80	46	

N = 80	Time (s)			
Iterations	RHC	SA	GA	MIMIC
100				0.457
200	0.005	0.001	0.017	0.861
500	0.006	0.001	0.028	2.034
1000	0.005	0.002	0.04	

N = 800	Accuracy			
Iterations	RHC	SA	GA	MIMIC
100				611
200	478	433	394	644
500	508	494	404	688
1000	586	573	401	
5000	784	776	410	
10000	799	799	429	
20000	800	800	410	

N = 800	Time (s)			
Iterations	RHC	SA	GA	MIMIC
100				56.811
200	0.007	0.007	0.101	107.005
500	0.014	0.009	0.149	264.48
1000	0.015	0.016	0.352	
5000	0.079	0.053	1.386	
10000	0.132	0.184	2.821	
20000	0.238	0.232	5.609	

Summary:

We see that on simple versions of this problem, or with very limited iterations, MIMIC is able to more closely approximate the optimum, but as we increase the difficulty of the problem, we see that, as expected, SA reaches the optimum in less clock time (and possibly fewer iterations).

II. The Traveling Salesman Problem

The traveling salesman problem defines the optimization problem where an algorithm must traverse every node in a connected network while traveling the minimum distance in that network.

Algorithm performance:

With enough iterations and a sufficiently small parameter space, any optimization algorithm should eventually be able to solve the traveling salesman problem. Since the domain is structured, we would expect GA and MIMIC to perform better than SA and RHC.

As we can see in the output below, with only 5 nodes, any of our algorithms converge on the solution for the traveling salesman problem and do so relatively quickly (<1 second). For problems with many nodes, genetic algorithms stand out with the ability to find much better solutions without too much cost.

	Inverse of Distance			
N	RHC	SA	GA	MIMIC
5	0.415422	0.415422	0.415422	0.415422
50	0.111754	0.116182	0.155463	0.116501
100	0.073061	0.071545	0.111254	0.053612

	Time (s)			
N	RHC	SA	GA	MIMIC
5	0.17	0.201	2.475	0.964
50	0.453	0.653	14.298	63.466
100	0.924	1.064	29.152	481.098

Summary:

We see that on simple versions of this problem any algorithm can converge on the optimum solution, but as we increase the complexity of the problem, we see that, as expected, GA is able to use the domain's structure to find a better path than its competitors.

III. Knapsack Problem

The knapsack problem defines the optimization problem where an algorithm must add items to a knapsack such that the total value of included items is maximized and the weight constraint is not exceeded. An algorithm must identify which items to include and which to exclude.

Algorithm performance:

With enough iterations and a sufficiently small parameter space, any optimization algorithm should eventually be able to solve the knapsack problem. Since the domain is structured, we would GA and MIMIC to perform better than SA and RHC.

To compare the algorithms, I first selected a simple problem space: 2 available items, 1 copy of each, with a knapsack that holds 1 unit of weight and has 1 unit of volume. As you can see in the analysis below, SA, GA, and MIMIC all performed similarly and had the same total value with similar runtimes. The available items were increased to increase the complexity of the problem. It's easy to see MIMIC succeeding with Genetic Algorithms not far behind.

	Total Value of Knapsack items			
Available Items	RHC	SA	GA	MIMIC
2	0.23169	0.95089	0.95089	0.95089
20	9.123346	9.970638	10.39449	10.47648
200	79.2045	83.44919	86.33455	93.15977

	Time(s)			
Available Items	RHC	SA	GA	MIMIC
2	0.036	0.152	0.128	0.15
20	0.129	0.248	0.252	2.057
200	0.816	0.925	1.382	98.728

Summary:

We see that on simple versions of this problem, or with very limited iterations, many algorithms are able to closely approximate the optimum, but as we increase the difficulty of the problem (even slightly) we see that, as expected, MIMIC reaches higher maxima. This supports the initial hypothesis that MIMIC and GA perform better on structured problem domains.

IV. Summary

These three problem domains, Count Ones, Traveling Salesman, and Knapsack, can all be optimized using the randomized optimization techniques discussed in this course, but each algorithm performs differently on each problem domain. The simulated annealing algorithm performs best on unstructured problems like count ones. Genetic algorithms perform best on a structured problem like the traveling salesman problem. MIMIC is able to perform best on a complex structured problem like the knapsack problem.

[i] http://archive.ics.uci.edu/ml/datasets/Adult