HW1

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2(a)

$$u = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix}, \qquad v = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \qquad \text{Definition: } \mathbf{a} \cdot \mathbf{b} = \sum_{i=1}^{n} a_i b_i = a_1 b_1 + a_2 b_2 + \ldots a_n b_n$$

$$\begin{aligned} u^T v &= \begin{pmatrix} u_1 & u_2 = \cdots & u_n \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \\ &= u_1 v_1 + u_2 v_2 + \cdots + u_n v_n \\ &= \sum_{i=1}^{n} u_i v_i \\ &= \mathbf{u} \cdot \mathbf{v} \end{aligned}$$

2(b)

$$u = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix}, \qquad v = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}, \qquad M = \begin{pmatrix} m_{1,1} & m_{1,2} & \cdots & m_{1,n'} \\ m_{2,1} & m_{2,2} & \cdots & m_{2,n'} \\ \vdots & \vdots & \ddots & \vdots \\ m_{n,1} & m_{n,2} & \cdots & m_{n,n'} \end{pmatrix}$$

$$
\begin{aligned}
u \cdot Mv &= u \cdot \begin{pmatrix} m_{1,1} & m_{1,2} & \cdots & m_{1,n'} \\ m_{2,1} & m_{2,2} & \cdots & m_{2,n'} \\ \vdots & \vdots & \ddots & \vdots \\ m_{n,1} & m_{n,2} & \cdots & m_{n,n'} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \\
&= u \cdot \begin{pmatrix} \sum_{j=1}^{n'} m_{1j} v_j \\ \sum_{j=1}^{n'} m_{2j} v_j \\ \vdots \\ \sum_{j=1}^{n'} m_{nj} v_j \end{pmatrix} & \text{Let } \mu = \begin{pmatrix} \sum_{j=1}^{n'} m_{1j} v_j \\ \sum_{j=1}^{n'} m_{2j} v_j \\ \vdots \\ \sum_{j=1}^{n'} m_{nj} v_j \end{pmatrix} \\
&= \mathbf{u} \cdot \mu \\
&= \sum_{i=1}^{n} u_i \mu_i \\
&= \sum_{i=1}^{n} u_i \left(\sum_{j=1}^{n'} m_{ij} v_j \right) \\
&= \sum_{i=1}^{n} \sum_{j=1}^{n'} m_{ij} u_i v_j
\end{aligned}
$$

2(c)

$$
\begin{aligned}
\nabla \left(b^T x \right) &= \nabla \left(\sum_{i=1}^{n} b_i x_i \right) \\
&= \nabla \left(b_1 x_1 + b_2 x_2 + \cdots + b_n x_n \right) \\
&= \begin{pmatrix} \frac{\partial}{\partial x_1} b_1 x_1 + b_2 x_2 + \cdots + b_n x_n \\ \frac{\partial}{\partial x_2} b_1 x_1 + b_2 x_2 + \cdots + b_n x_n \\ \vdots \\ \frac{\partial}{\partial x_n} b_1 x_1 + b_2 x_2 + \cdots + b_n x_n \end{pmatrix} \\
&= \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} \\
&= \mathbf{b}
\end{aligned}
$$

2(d)

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, \qquad A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}, \qquad A^T = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{nn} \end{pmatrix}$$

$$\begin{aligned} x^T A x &= \begin{pmatrix} x_1 & x_2 & \cdots & x_n \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \\ &= \begin{pmatrix} x_1 & x_2 & \cdots & x_n \end{pmatrix} \begin{pmatrix} \sum_{j=1}^{n} a_{1j} x_j \\ \sum_{j=1}^{n} a_{2j} x_j \\ \vdots \\ \sum_{j=1}^{n} a_{1j} x_j \end{pmatrix} \\ &= x_1 \sum_{j=1}^{n} a_{1j} x_j + x_2 \sum_{j=1}^{n} a_{2j} x_j + ... + x_n \sum_{j=1}^{n} a_{nj} x_j \end{aligned}$$

$$
\nabla\left(x^T A x\right) = \begin{pmatrix} \frac{\partial}{\partial x_1}\left[x_1 \sum_{j=1}^{n} a_{1j}x_j + x_2 \sum_{j=1}^{n} a_{2j}x_j + ... + x_n \sum_{j=1}^{n} a_{nj}x_j\right] \\ \frac{\partial}{\partial x_2}\left[x_1 \sum_{j=1}^{n} a_{1j}x_j + x_2 \sum_{j=1}^{n} a_{2j}x_j + ... + x_n \sum_{j=1}^{n} a_{nj}x_j\right] \\ \vdots \\ \frac{\partial}{\partial x_n}\left[x_1 \sum_{j=1}^{n} a_{1j}x_j + x_2 \sum_{j=1}^{n} a_{2j}x_j + ... + x_n \sum_{j=1}^{n} a_{nj}x_j\right] \end{pmatrix}
$$

$$
= \begin{pmatrix} 2a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n + a_{21}x_2 + ... + a_{n1}x_n \\ a_{12}x_1 + a_{21}x_1 + 2a_{22}x_2 + ... + a_{2n}x_n + ... + a_{n2}x_n \\ \vdots \\ a_{1n}x_1 + a_{2n}x_2 + ... + a_{n1}x_1 + a_{n2}x_2 + ... + 2a_{nn}x_n \end{pmatrix}
$$

$$
= \begin{pmatrix} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n + a_{11}x_1 + a_{21}x_2 + ... + a_{n1}x_n \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n + a_{12}x_1 + a_{22}x_2 + ... + a_{n2}x_n \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n + a_{1n}x_1 + a_{2n}x_2 + ... + a_{nn}x_n \end{pmatrix}
$$

$$
= \begin{pmatrix} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n \end{pmatrix} + \begin{pmatrix} a_{11}x_1 + a_{21}x_2 + ... + a_{n1}x_n \\ a_{12}x_1 + a_{22}x_2 + ... + a_{n2}x_n \\ \vdots \\ a_{1n}x_1 + a_{2n}x_2 + ... + a_{nn}x_n \end{pmatrix}
$$

$$
= \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}
$$

$$
= \left(A + A^T\right) x
$$

2(e)

??? something about symmetry

2(f)

$$A = \begin{pmatrix} 1 & 4 \\ 2 & 1 \end{pmatrix}, \quad A^T = \begin{pmatrix} 1 & 2 \\ 4 & 1 \end{pmatrix}$$

$$\begin{aligned}
\begin{pmatrix} x_1 & x_2 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &= \begin{pmatrix} x_1 & x_2 \end{pmatrix} \begin{pmatrix} x_1 + 4x_2 \\ 2x_1 + x_2 \end{pmatrix} \\
&= x_1(x_1 + 4x_2) + x_2(2x_1 + x_2) \\
&= x_1^2 + 4x_1x_2 + 2x_1x_2 + x_2^2 \\
&= x_1^2 + 6x_1x_2 + x_2^2
\end{aligned}$$

$$\begin{aligned}
\begin{pmatrix} x_1 & x_2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &= \begin{pmatrix} x_1 & x_2 \end{pmatrix} \begin{pmatrix} x_1 + 2x_2 \\ 4x_1 + x_2 \end{pmatrix} \\
&= x_1(x_1 + 2x_2) + x_2(4x_1 + x_2) \\
&= x_1^2 + 2x_1x_2 + 4x_1x_2 + x_2^2 \\
&= x_1^2 + 6x_1x_2 + x_2^2
\end{aligned}$$

3

```r
setwd("g:/504")
dat<-read.table("economic_data.txt",header=T)
options(scipen = 999)

y<-dat[,ncol(dat)]
B<-as.matrix(dat[,2:(ncol(dat)-1)])
B<-cbind(1,B)

#solve(t(B) %*% B ) %*% t(B) %*% y

cor(B[,-1])
```

```
##           A1        A2         A3         A4        A5        A6
## A1 1.0000000 0.9915892  0.6205874  0.4647442 0.9791634 0.9911492
## A2 0.9915892 1.0000000  0.6042089  0.4464368 0.9910901 0.9952735
## A3 0.6205874 0.6042089  1.0000000 -0.1775504 0.6865068 0.6682035
## A4 0.4647442 0.4464368 -0.1775504  1.0000000 0.3644163 0.4172451
## A5 0.9791634 0.9910901  0.6865068  0.3644163 1.0000000 0.9939528
## A6 0.9911492 0.9952735  0.6682035  0.4172451 0.9939528 1.0000000
```

```r
BB<-B[,-c(7,6)]
cor(BB[,-1])
```

```
##           A1        A2         A3         A4
## A1 1.0000000 0.9915892  0.6205874  0.4647442
## A2 0.9915892 1.0000000  0.6042089  0.4464368
## A3 0.6205874 0.6042089  1.0000000 -0.1775504
## A4 0.4647442 0.4464368 -0.1775504  1.0000000
```

```r
solve(t(BB) %*% BB ) %*% t(BB) %*% y
```

```
##               [,1]
##    50135.42355558
## A1    55.34521216
## A2     0.03537306
## A3    -0.85377061
## A4    -0.54975517
```

??? something about multicollinearity

```r
dat<-dat[,-1]
( lm(B~.,data=dat) )
```

```
##
## Call:
## lm(formula = B ~ ., data = dat)
##
## Coefficients:
##    (Intercept)              A1              A2              A3
## -3475440.82413        14.78948        -0.03575        -2.02020
##             A4              A5              A6
##       -1.03277        -0.04912      1825.54365
```

4

```r
 times<-rep(NA,10)

for ( i in 1:10) {

  times[i]<-system.time( for ( j in 1:(10^i) ) { 1*1 } )[1]
  if ( times[i] > 1 ) { break}}
times
```

```
##  [1] 0.00 0.00 0.00 0.00 0.04 0.28 2.57   NA   NA   NA
```

```r
system.time( for( i in 1: (1/times[7] * 10^7) ) { 1*1} )
```

```
##    user  system elapsed
##    0.99    0.00    1.04
```

```r
1/times[7] * 10^7
```

```
## [1] 3891051
```