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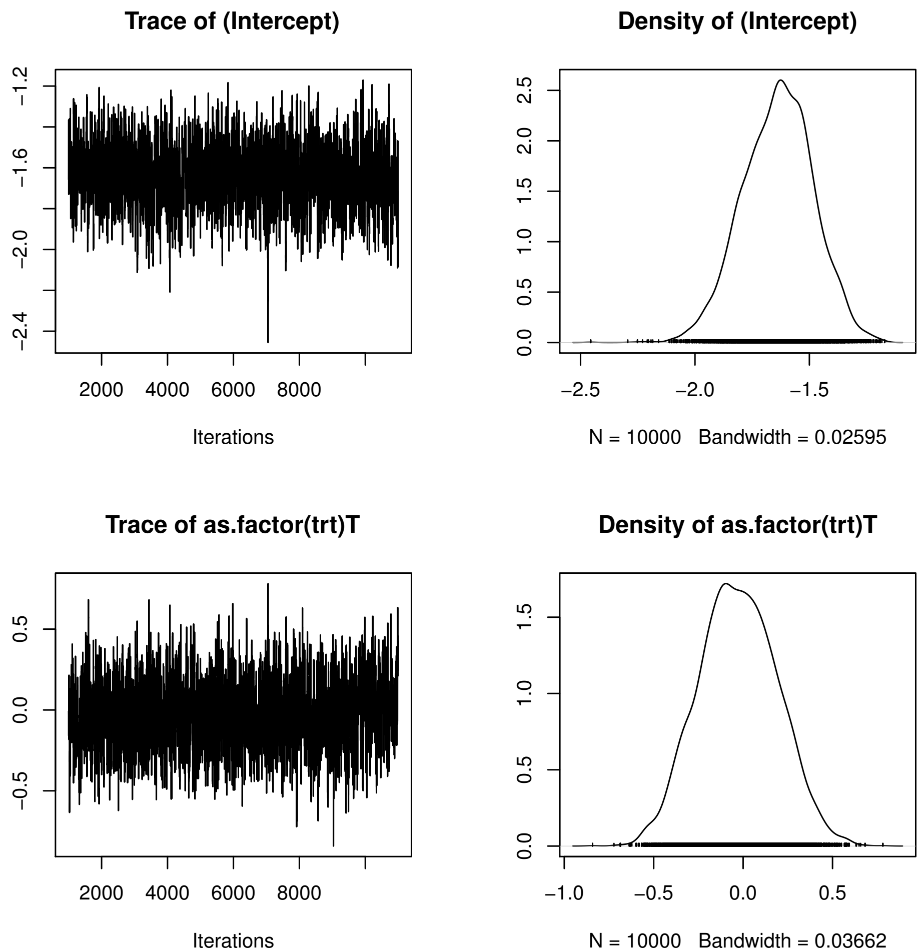
Math 611
Homework 10

1. For the Beta Blockers data set, consider the observations from center (clinical site) 1 only. Use the appropriate transformations to the data frame to create a binary random variable (1=Death, 0=No Death). Use 'MCMCpack' to generate a sample from the posterior distribution of a logistic regression model using a random-walk Metropolis Hastings algorithm and a multivariate normal prior of your choice. Present the plots of this model.

```
bb<-read.csv("BetaBLOCKERS.csv",header=T)
bblist<-list()

for( i in 1:1){ #£

        CD<-bb[which(bb$center == i & bb$trt== "C" & bb$value== "Death" ) , ]
        CD<-CD[1,][rep(seq_len(nrow(CD[1,])), each=sum(CD[,1])),]

        TD<-bb[which(bb$center == i & bb$trt== "T" & bb$value== "Death" ) , ]
        TD<-TD[1,][rep(seq_len(nrow(TD[1,])), each=sum(TD[,1])),]

        CT<-bb[which(bb$center == i & bb$trt== "C" & bb$value== "Total" ) , ]
        CT<-CT[1,][rep(seq_len(nrow(CT[1,])), each=(sum(CT[,1])-
                nrow(CD)),]

        TT<-bb[which(bb$center == i & bb$trt== "T" & bb$value== "Total" ) , ]#£
        TT<-TT[1,][rep(seq_len(nrow(TT[1,])), each=(sum(TT[,1])-
                nrow(TD)),]

        bblist[[i]]<-rbind(CD,TD,CT,TT)}
   bblist<- do.call("rbind", bblist);rownames(bblist)<-NULL
   bblist$death<-ifelse(bblist$value == "Death" , 1,0)
   table(bblist[,5])

##
##   0   1
## 508  98
```

```
require("MCMCpack")
posterior <- MCMClogit(death~as.factor(trt) , b0=0, B0=.001,data=bblist)
summary(posterior)

Iterations = 1001:11000
Thinning interval = 1
Number of chains = 1
Sample size per chain = 10000

1. Empirical mean and standard deviation for each variable,
   plus standard error of the mean:

                     Mean     SD Naive SE Time-series SE
(Intercept)      -1.63735 0.1544 0.001544       0.004622
as.factor(trt)T -0.02719 0.2180 0.002180       0.006417

2. Quantiles for each variable:
```

```
                    2.5%    25%       50%      75%   97.5%
(Intercept)      -1.9484 -1.743 -1.63246 -1.5327 -1.3470
as.factor(trt)T -0.4363 -0.179 -0.03035  0.1245  0.4002

plot(posterior)
```



2. The following R-function uses the Metropolis-Hastings algorithm and an exponential proposal density to draw from the logarithm of the density of interest. Modify this function such that it uses the random walk Metropolis Hastings and a normal proposal density, to draw observations from the logarithm of a density of interest (only the code is needed-no output).

```
MHexp<-function(logf,currentvalue,B)
     {# B: number of iterations
          S<-rep(0,B)
          n_accept<-0
          for(i in 1:B)
          {proposal<-rexp(1,currentvalue)
          probacc<-exp(logf(proposal)-logf(currentvalue)
               dexp(currentvalue,proposal,log=TRUE)-
               dexp(proposal,currentvalue,log=TRUE))
          accept<-ifelse(runif(1)<probacc,1,0)
          currentvalue<-ifelse(accept==1,proposal,currentvalue)
          S[i]<-currentvalue
          n.accept<-n.accept+(accept==1)}
          c(S,n.accept/B)
}
```

```
MHexp<-function(logf,currentvalue,B,SD=1)
       {# B: number of iterations
       S<-rep(0,B)
       n_accept<-0

       for(i in 1:B)
               {proposal<-rnorm(1,currentvalue,SD)

               probacc<-exp( (logf(proposal) )-(logf(currentvalue) ) )

               accept<-ifelse(runif(1)<probacc,1,0)

               currentvalue<-ifelse(accept==1,proposal,currentvalue)
               S[i]<-currentvalue

       n.accept<-n.accept+(accept==1)}
       c(S,n.accept/B)
}
```

3. The number of failures N(t), which occur in a computer network over the time interval $[0, t)$, can be described by a homogeneous Poisson process $\{N(t), t \geq 0\}$. On an average, there is a failure after every 4 hours, i.e. the intensity (rate) of the process per hour-interval is equal to $\lambda = 0.25$.

a. What is the probability of at most 1 failure in $[0, 8)$, at least 2 failures in $[8, 16)$, and at most 1 failure in $[16, 24)$ (time unit: hour)?

Let $N(t) \sim \lambda t$, where $\lambda = 0.25$

$$\begin{aligned}
&P\Big(\text{at most 1 failure in } [0,8) \textit{ and } \text{at least 2 failures in } [8,16) \textit{ and } \text{at most 1 failure in } [16,24)\Big)\\
&= P\Big(N(8) - N(0) \leq 1,\ N(16) - N(8) \geq 2,\ N(24) - N(16) \leq 1\Big)\\
&= P\Big(N(8) \leq 1\Big)\ P\Big(N(8) \geq 2\Big)\ P\Big(N(8) \leq 1\Big)
\end{aligned}$$

$$P\Big(N(8) \leq 1\Big) = \sum_{x=0}^{1} \frac{2^x\ e^{-2}}{x!} = 0.4060058$$

$$P\Big(N(8) \geq 2\Big) = 1 - P\Big(N(8) \leq 1\Big) = 1 - \left[\sum_{x=0}^{1} \frac{2^x\ e^{-2}}{x!}\right] = 0.5939942$$

$$\begin{aligned}
&P\Big(N(8) \leq 1\Big)\ P\Big(N(8) \geq 2\Big)\ P\Big(N(8) \leq 1\Big)\\
&= (0.4060058)\,(0.5939942)\,(0.4060058)\\
&= 0.09791443
\end{aligned}$$

b. What is the probability that the third failure occurs after 8 hours?

Let T_3 be the time of the third failure. We want to know the probability that the third failure occurs after 8 hours. Alternatively, we want to know the probability of 0, 1, or 2 failures within the first 8 hours.

$$P(T_3 > 8) = P\Big(N(8) \leq 2\Big) = \sum_{x=0}^{2} \frac{2^x\ e^{-2}}{x!} = 0.6766764$$