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# MATH 640: Bayesian Statistics

## Homework 2, due Monday, February 4

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Please submit a PDF or .doc version of your homework to Blackboard by 11:59pm on the due date. Please type *all* responses. You are encouraged to use R for all calculations.

### Theoretical Exercises

1. Let  $y_i, i = 1, \dots, n$  be iid normally distributed with mean  $\mu$  and known variance  $\sigma^2$ . Assuming a conjugate prior for  $\mu$  with parameters  $\mu_0$  and  $\tau_0^2$ , fully specify the posterior distribution. (Hint: in lecture, we considered the case of  $n = 1$ . Here you will generalize that example.)
2. Let  $x_i, i = 1, \dots, n$  be iid exponential with rate  $\lambda$ . Find a conjugate prior for the exponential likelihood.
3. Let  $Z$  be a geometric random variable with probability of success  $\theta$ . Here note that  $Z$  is the number of failures until the first success. Thus the parameterization of the geometric we'll use is

$$p(Z|\theta) = (1 - \theta)^Z \theta.$$

Find Jeffreys' prior for the geometric likelihood.

### Analysis Exercises

1. Table (1) contains data collected from three recent surveys regarding whether or not Americans approve of the job Congress is doing. Each sample was taken from registered voters (data courtesy of [realclearpolitics.com](http://realclearpolitics.com)).

Table 1: Recent Congressional Approval Polls

Poll	Date	Sample Size	Number who Approve
Economist/YouGov	1/12 - 1/15	1289	142
CNN	12/6 - 12/9	919	175
Monmouth University	11/9 - 11/12	716	165

- (a) For the Monmouth University poll above, conduct an analysis of the proportion of registered voters who approve of the job Congress is doing using the standard uniform prior. In your analysis, provide a brief discussion of the results along with an estimate of the posterior mean, 95% credible interval, and plot of the posterior distribution.
  - (b) Now analyze the CNN poll using the posterior distribution from part (a) as the new prior. As before, your analysis should give a brief discussion of the results along with an estimate of the posterior median, 95% credible interval, and plot of the posterior distribution.
  - (c) First analyze the Economist/YouGov poll using a standard uniform prior. Determine the posterior mean, 95% credible interval, and plot of the posterior distribution. Next, analyze the Economist/YouGov poll using the posterior distribution from part (b) as the prior. As before, your analysis should give a brief discussion of the results along with an estimate of the posterior mean, 95% credible interval, and plot of the posterior distribution. Compare the results of the analysis using a noninformative prior to the results using an informative prior be sure to present a graph showing both posteriors.
2. Consider data from a skin cancer prevention study found in the file `skin.txt` alongside this assignment. Let  $x_i$  denote the number of skin cancers found on subject  $i$  from the study. We can assume that  $x_i$  are iid  $Pois(\lambda)$  for  $i = 1, \dots, n$ . Find Jeffrey's prior for  $\lambda$  and use it to determine the posterior distribution of  $\lambda|x_1, \dots, x_n$ . The variable `numsc` contains the count of skin cancers found on each subject. Using this, calculate the posterior mean and 95% credible interval of the rate of skin cancers. Also provide a plot of the posterior distribution.