Table A.1 Continuous distributions

Distribution	Notation	Parameters
Uniform	$\theta \sim \mathrm{U}(\alpha,\beta)$ $p(\theta) = \mathrm{U}(\theta\|\alpha,\beta)$	boundaries α,β with $\beta > \alpha$
Normal	$\theta \sim \mathrm{N}(\mu,\sigma^2)$ $p(\theta) = \mathrm{N}(\theta\|\mu,\sigma^2)$	location μ scale $\sigma > 0$
Lognormal	$\theta \sim \mathrm{lognormal}(\mu,\sigma^2)$ $p(\theta) = \mathrm{lognormal}(\theta\|\mu,\sigma^2)$	location μ log-scale $\sigma > 0$
Multivariate normal	$\theta \sim \mathrm{N}(\mu,\Sigma)$ $p(\theta) = \mathrm{N}(\theta\|\mu,\Sigma)$ (implicit dimension d)	symmetric, pos. definite, $d \times d$ variance matrix Σ
Gamma	$\theta \sim \mathrm{Gamma}(\alpha,\beta)$ $p(\theta) = \mathrm{Gamma}(\theta\|\alpha,\beta)$	shape $\alpha > 0$ inverse scale $\beta > 0$
Inverse-gamma	$\theta \sim \mathrm{Inv\text{-}gamma}(\alpha,\beta)$ $p(\theta) = \mathrm{Inv\text{-}gamma}(\theta\|\alpha,\beta)$	shape $\alpha > 0$ scale $\beta > 0$
Chi-square	$\theta \sim \chi^2_\nu$ $p(\theta) = \chi^2_\nu(\theta)$	degrees of freedom $\nu > 0$
Inverse-chi-square	$\theta \sim \mathrm{Inv\text{-}}\chi^2_\nu$ $p(\theta) = \mathrm{Inv\text{-}}\chi^2_\nu(\theta)$	degrees of freedom $\nu > 0$
Scaled inverse-chi-square	$\theta \sim \mathrm{Inv\text{-}}\chi^2(\nu,s^2)$ $p(\theta) = \mathrm{Inv\text{-}}\chi^2(\theta\|\nu,s^2)$	degrees of freedom $\nu > 0$ scale $s > 0$
Exponential	$\theta \sim \mathrm{Expon}(\beta)$ $p(\theta) = \mathrm{Expon}(\theta\|\beta)$	inverse scale $\beta > 0$
Laplace (double-exponential)	$\theta \sim \mathrm{Laplace}(\mu,\sigma)$ $p(\theta) = \mathrm{Laplace}(\theta\|\mu,\sigma)$	location μ scale $\sigma > 0$
Weibull	$\theta \sim \mathrm{Weibull}(\alpha,\beta)$ $p(\theta) = \mathrm{Weibull}(\theta\|\alpha,\beta)$	shape $\alpha > 0$ scale $\beta > 0$
Wishart	$W \sim \mathrm{Wishart}_\nu(S)$ $p(W) = \mathrm{Wishart}_\nu(W\|S)$ (implicit dimension $k \times k$)	degrees of freedom ν symmetric, pos. definite $k \times k$ scale matrix S
Inverse-Wishart	$W \sim \mathrm{Inv\text{-}Wishart}_\nu(S^{-1})$ $p(W) = \mathrm{Inv\text{-}Wishart}_\nu(W\|S^{-1})$ (implicit dimension $k \times k$)	degrees of freedom ν symmetric, pos. definite $k \times k$ scale matrix S
LKJ correlation	$\Sigma \sim \mathrm{LkjCorr}(\eta)$ $p(\Sigma) = \mathrm{LkjCorr}(\Sigma\|\eta)$ (implicit dimension $k \times k$)	shape $\eta > 0$

Density function	Mean, variance, and mode
$p(\theta) = \frac{1}{\beta-\alpha},\ \theta \in [\alpha,\beta]$	$\mathrm{E}(\theta) = \frac{\alpha+\beta}{2}$ $\mathrm{var}(\theta) = \frac{(\beta-\alpha)^2}{12}$ no mode
$p(\theta) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2\sigma^2}(\theta-\mu)^2\right)$	$\mathrm{E}(\theta) = \mu$ $\mathrm{var}(\theta) = \sigma^2$ $\mathrm{mode}(\theta) = \mu$
$p(\theta) = (\sqrt{2\pi}\sigma\theta)^{-1}\exp(-\frac{1}{2\sigma^2}(\log\theta-\mu)^2)$	$\mathrm{E}(\theta) = \exp(\mu+\frac{1}{2}\sigma^2)$, $\mathrm{var}(\theta) = \exp(2\mu+\sigma^2)(\exp(\sigma^2)-1)$ $\mathrm{mode}(\theta) = \exp(\mu-\sigma^2)$
$p(\theta) = (2\pi)^{-d/2}\|\Sigma\|^{-1/2}$ $\times\exp\left(-\frac{1}{2}(\theta-\mu)^T\Sigma^{-1}(\theta-\mu)\right)$	$\mathrm{E}(\theta) = \mu$ $\mathrm{var}(\theta) = \Sigma$ $\mathrm{mode}(\theta) = \mu$
$p(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\beta\theta},\ \theta>0$	$\mathrm{E}(\theta) = \frac{\alpha}{\beta}$ $\mathrm{var}(\theta) = \frac{\alpha}{\beta^2}$ $\mathrm{mode}(\theta) = \frac{\alpha-1}{\beta}$, for $\alpha \geq 1$
$p(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{-(\alpha+1)}e^{-\beta/\theta},\ \theta>0$	$\mathrm{E}(\theta) = \frac{\beta}{\alpha-1}$, for $\alpha > 1$ $\mathrm{var}(\theta) = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)}, \alpha>2$ $\mathrm{mode}(\theta) = \frac{\beta}{\alpha+1}$
$p(\theta) = \frac{2^{-\nu/2}}{\Gamma(\nu/2)}\theta^{\nu/2-1}e^{-\theta/2},\ \theta>0$ same as $\mathrm{Gamma}(\alpha=\frac{\nu}{2},\beta=\frac{1}{2})$	$\mathrm{E}(\theta) = \nu$ $\mathrm{var}(\theta) = 2\nu$ $\mathrm{mode}(\theta) = \nu-2$, for $\nu \geq 2$
$p(\theta) = \frac{2^{-\nu/2}}{\Gamma(\nu/2)}\theta^{-(\nu/2+1)}e^{-1/(2\theta)},\ \theta > 0$ same as $\mathrm{Inv\text{-}gamma}(\alpha=\frac{\nu}{2},\beta=\frac{1}{2})$	$\mathrm{E}(\theta) = \frac{1}{\nu-2}$, for $\nu > 2$ $\mathrm{var}(\theta) = \frac{2}{(\nu-2)^2(\nu-4)}, \nu>4$ $\mathrm{mode}(\theta) = \frac{1}{\nu+2}$
$p(\theta) = \frac{(\nu/2)^{\nu/2}}{\Gamma(\nu/2)}s^\nu\theta^{-(\nu/2+1)}e^{-\nu s^2/(2\theta)},\ \theta>0$ same as $\mathrm{Inv\text{-}gamma}(\alpha=\frac{\nu}{2},\beta=\frac{\nu}{2}s^2)$	$\mathrm{E}(\theta) = \frac{\nu}{\nu-2}s^2$ $\mathrm{var}(\theta) = \frac{2\nu^2}{(\nu-2)^2(\nu-4)}s^4$ $\mathrm{mode}(\theta) = \frac{\nu}{\nu+2}s^2$
$p(\theta) = \beta e^{-\beta\theta},\ \theta>0$ same as $\mathrm{Gamma}(\alpha=1,\beta)$	$\mathrm{E}(\theta) = \frac{1}{\beta}$ $\mathrm{var}(\theta) = \frac{1}{\beta^2}$ $\mathrm{mode}(\theta) = 0$
$p(\theta) = \frac{1}{2\sigma}\exp\left(-\frac{\|x-\mu\|}{\sigma}\right)$	$\mathrm{E}(\theta) = \mu$ $\mathrm{var}(\theta) = 2\sigma^2$ $\mathrm{mode}(\theta) = \mu$
$p(\theta) = \frac{\alpha}{\beta^\alpha}\theta^{\alpha-1}\exp(-(\theta/\beta)^\alpha),\ \theta>0$	$\mathrm{E}(\theta) = \beta\Gamma(1+\frac{1}{\alpha})$ $\mathrm{var}(\theta) = \beta^2[\Gamma(1+\frac{2}{\alpha})-(\Gamma(1+\frac{1}{\alpha}))^2]$ $\mathrm{mode}(\theta) = \beta(1-\frac{1}{\alpha})^{1/\alpha}$
$p(W) = \left(2^{\nu k/2}\pi^{k(k-1)/4}\prod_{i=1}^k\Gamma\left(\frac{\nu+1-i}{2}\right)\right)^{-1}$ $\times\|S\|^{-\nu/2}\|W\|^{(\nu-k-1)/2}$ $\times\exp\left(-\frac{1}{2}\mathrm{tr}(S^{-1}W)\right)$, W pos. definite	$\mathrm{E}(W) = \nu S$
$p(W) = \left(2^{\nu k/2}\pi^{k(k-1)/4}\prod_{i=1}^k\Gamma\left(\frac{\nu+1-i}{2}\right)\right)^{-1}$ $\times\|S\|^{\nu/2}\|W\|^{-(\nu+k+1)/2}$ $\times\exp\left(-\frac{1}{2}\mathrm{tr}(SW^{-1})\right)$, W pos. definite	$\mathrm{E}(W) = (\nu-k-1)^{-1}S$
$p(\Sigma) = \det(\Sigma)^{\eta-1}$ $\times 2^{\sum_{i=1}^k(2\eta-2+k-i)(k-i)}$ $\times\prod_{i=1}^k\left(\mathrm{B}\left(\frac{i+1}{2},\frac{i+1}{2}\right)\right)^k$	$\mathrm{E}(\Sigma) = \mathrm{I}_k$,

Table A.1 Continuous distributions *continued*

Distribution	Notation	Parameters
t	$\theta \sim t_\nu(\mu,\sigma^2)$ $p(\theta) = t_\nu(\theta\|\mu,\sigma^2)$ t_ν is short for $t_\nu(0,1)$	degrees of freedom $\nu > 0$ location μ scale $\sigma > 0$
Multivariate t	$\theta \sim t_\nu(\mu,\Sigma)$ $p(\theta) = t_\nu(\theta\|\mu,\Sigma)$ (implicit dimension d)	degrees of freedom $\nu > 0$ location $\mu = (\mu_1,\ldots,\mu_d)$ symmetric, pos. definite $d \times d$ scale matrix Σ
Beta	$\theta \sim \mathrm{Beta}(\alpha,\beta)$ $p(\theta) = \mathrm{Beta}(\theta\|\alpha,\beta)$	'prior sample sizes' $\alpha > 0, \beta > 0$
Dirichlet	$\theta \sim \mathrm{Dirichlet}(\alpha_1,\ldots,\alpha_k)$ $p(\theta) = \mathrm{Dirichlet}(\theta\|\alpha_1,\ldots,\alpha_k)$	'prior sample sizes' $\alpha_j > 0$; $\alpha_0 \equiv \sum_{j=1}^k \alpha_j$
Logistic	$\theta \sim \mathrm{Logistic}(\mu,\sigma)$ $p(\theta) = \mathrm{Logistic}(\theta\|\mu,\sigma)$	location μ scale $\sigma > 0$
Log-logistic	$\theta \sim \mathrm{Log\text{-}logistic}(\alpha,\beta)$ $p(\theta) = \mathrm{Log\text{-}logistic}(\theta\|\alpha,\beta)$	scale $\alpha > 0$ shape $\beta > 0$

Table A.2 Discrete distributions

Distribution	Notation	Parameters
Poisson	$\theta \sim \mathrm{Poisson}(\lambda)$ $p(\theta) = \mathrm{Poisson}(\theta\|\lambda)$	'rate' $\lambda > 0$
Binomial	$\theta \sim \mathrm{Bin}(n,p)$ $p(\theta) = \mathrm{Bin}(\theta\|n,p)$	'sample size' n (positive integer) 'probability' $p \in [0,1]$
Multinomial	$\theta \sim \mathrm{Multin}(n;p_1,\ldots,p_k)$ $p(\theta) = \mathrm{Multin}(\theta\|n;p_1,\ldots,p_k)$	'sample size' n (positive integer) 'probabilities' $p_j \in [0,1]$; $\sum_{j=1}^k p_j = 1$
Negative binomial	$\theta \sim \mathrm{Neg\text{-}bin}(\alpha,\beta)$ $p(\theta) = \mathrm{Neg\text{-}bin}(\theta\|\alpha,\beta)$	shape $\alpha > 0$ inverse scale $\beta > 0$
Beta-binomial	$\theta \sim \mathrm{Beta\text{-}bin}(n,\alpha,\beta)$ $p(\theta) = \mathrm{Beta\text{-}bin}(\theta\|n,\alpha,\beta)$	'sample size' n (positive integer) 'prior sample sizes' $\alpha > 0, \beta > 0$

Density function	Mean, variance, and mode
$p(\theta) = \frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)\sqrt{\nu\pi}\sigma}(1+\frac{1}{\nu}(\frac{\theta-\mu}{\sigma})^2)^{-(\nu+1)/2}$	$\mathrm{E}(\theta) = \mu$, for $\nu>1$ $\mathrm{var}(\theta) = \frac{\nu}{\nu-2}\sigma^2$, for $\nu>2$ $\mathrm{mode}(\theta) = \mu$
$p(\theta) = \frac{\Gamma((\nu+d)/2)}{\Gamma(\nu/2)\nu^{d/2}\pi^{d/2}}\|\Sigma\|^{-1/2}$ $\times(1+\frac{1}{\nu}(\theta-\mu)^T\Sigma^{-1}(\theta-\mu))^{-(\nu+d)/2}$	$\mathrm{E}(\theta) = \mu$, for $\nu>1$ $\mathrm{var}(\theta) = \frac{\nu}{\nu-2}\Sigma$, for $\nu>2$ $\mathrm{mode}(\theta) = \mu$
$p(\theta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1}$ $\theta \in [0,1]$	$\mathrm{E}(\theta) = \frac{\alpha}{\alpha+\beta}$ $\mathrm{var}(\theta) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$ $\mathrm{mode}(\theta) = \frac{\alpha-1}{\alpha+\beta-2}$
$p(\theta) = \frac{\Gamma(\alpha_1+\cdots+\alpha_k)}{\Gamma(\alpha_1)\cdots\Gamma(\alpha_k)}\theta_1^{\alpha_1-1}\cdots\theta_k^{\alpha_k-1}$ $\theta_1,\ldots,\theta_k \geq 0; \sum_{j=1}^k\theta_j = 1$	$\mathrm{E}(\theta_j) = \frac{\alpha_j}{\alpha_0}$ $\mathrm{var}(\theta_j) = \frac{\alpha_j(\alpha_0-\alpha_j)}{\alpha_0^2(\alpha_0+1)}$ $\mathrm{cov}(\theta_i,\theta_j) = -\frac{\alpha_i\alpha_j}{\alpha_0^2(\alpha_0+1)}$ $\mathrm{mode}(\theta_j) = \frac{\alpha_j-1}{\alpha_0-k}$
$p(\theta) = \frac{\exp\left(-\frac{x-\mu}{\sigma}\right)}{\sigma\left(1+\exp\left(-\frac{x-\mu}{\sigma}\right)\right)}$	$\mathrm{E}(\theta) = \mu$ $\mathrm{var}(\theta) = \frac{1}{3}\sigma^2\pi^2$ $\mathrm{mode}(\theta) = \mu$
$p(\theta) = \frac{\frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1}}{\left[1+\left(\frac{x}{\alpha}\right)^\beta\right]^2},\ \theta > 0$	$\mathrm{E}(\theta) = \frac{1}{1+\left(\frac{x}{\alpha}\right)^{-\beta}}$ $\mathrm{var}(\theta) = \alpha^2\frac{2\pi/\beta}{\sin(2\pi/\beta)},\ \beta > 2$ $\mathrm{mode}(\theta) = \alpha\left(\frac{\beta-1}{\beta+1}\right)^{\frac{1}{\beta}},\ \beta > 1$

Density function	Mean, variance, and mode
$p(\theta) = \frac{1}{\theta!}\lambda^\theta\exp(-\lambda)$ $\theta = 0,1,2,\ldots$	$\mathrm{E}(\theta) = \lambda$, $\mathrm{var}(\theta) = \lambda$ $\mathrm{mode}(\theta) = \lfloor\lambda\rfloor$
$p(\theta) = \binom{n}{\theta}p^\theta(1-p)^{n-\theta}$ $\theta = 0,1,2,\ldots,n$	$\mathrm{E}(\theta) = np$ $\mathrm{var}(\theta) = np(1-p)$ $\mathrm{mode}(\theta) = \lfloor(n+1)p\rfloor$
$p(\theta) = \binom{n}{\theta_1\ \theta_2\cdots\theta_k}p_1^{\theta_1}\cdots p_k^{\theta_k}$ $\theta_j = 0,1,2,\ldots,n; \sum_{j=1}^k\theta_j = n$	$\mathrm{E}(\theta_j) = np_j$ $\mathrm{var}(\theta_j) = np_j(1-p_j)$ $\mathrm{cov}(\theta_i,\theta_j) = -np_ip_j$
$p(\theta) = \binom{\theta+\alpha-1}{\alpha-1}\left(\frac{\beta}{\beta+1}\right)^\alpha\left(\frac{1}{\beta+1}\right)^\theta$ $\theta = 0,1,2,\ldots$	$\mathrm{E}(\theta) = \frac{\alpha}{\beta}$ $\mathrm{var}(\theta) = \frac{\alpha}{\beta^2}(\beta+1)$
$p(\theta) = \frac{\Gamma(n+1)}{\Gamma(\theta+1)\Gamma(n-\theta+1)}\frac{\Gamma(\alpha+\theta)\Gamma(n+\beta-\theta)}{\Gamma(\alpha+\beta+n)}$ $\times\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)},\quad \theta = 0,1,2,\ldots,n$	$\mathrm{E}(\theta) = n\frac{\alpha}{\alpha+\beta}$ $\mathrm{var}(\theta) = n\frac{\alpha\beta(\alpha+\beta+n)}{(\alpha+\beta)^2(\alpha+\beta+1)}$