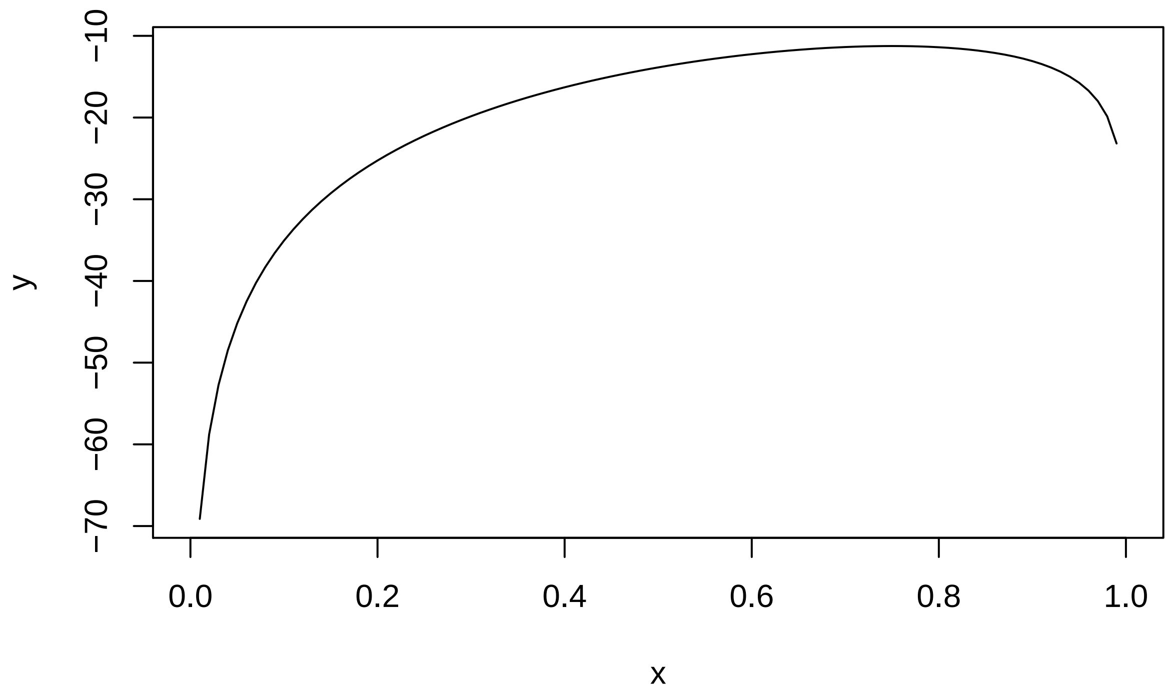
661hw1_Leibert

1. **Exercise 1: Comparison of drugs**

   In a crossover trial comparing a new drug to a standard, π denotes the probability that the new one is judged better. It is desired to estimate π and test $H_0 : \pi = 0.50$ agains $H_1 : \pi \neq 0.50$. The new drug is found to be better in 15 out of 20 independent observations.

   (a) Find and sketch the log-likelihood function. Is it close to the quadratic shape that large-sample normal approximations utilize?

```r
l = function(x){15*log(x)+5*log(1-x)}
curve(l, from=0, to=1,  xlab="x", ylab="y")
```



   It is close to the quadratic shape that large-sample normal approximations utilize.

   (b) Give the ML estimate of π.

$$
\begin{aligned}
f(15|\pi) &= \binom{20}{15}\ \pi^{15}\ \ (1-\pi)^5 \\
\mathcal{L}(\pi|15) &= \binom{20}{15}\ \pi^{15}\ \ (1-\pi)^5 \\
&\propto \pi^{15}\ \ (1-\pi)^5 \\
\ell(\pi|15) &= 15\log\pi\ \ + 5\log\pi
\end{aligned}
$$

$$\frac{\partial \ell}{\partial \pi} = \frac{15}{\pi} + \frac{5}{\pi - 1}$$

$$\begin{aligned}
0 &= \frac{15}{\pi} + \frac{5}{\pi - 1} \\
-\frac{5}{\pi - 1} &= \frac{15}{\pi} \\
\frac{\pi - 1}{\pi} &= -\frac{1}{3} \\
1 - \frac{1}{\pi} &= -\frac{1}{3} \\
-\frac{1}{\pi} &= -\frac{4}{3} \\
\hat{\pi}_{MLE} &= \frac{3}{4}
\end{aligned}$$

(c) Wald test

- Conduct a Wald test, report the p-value and state your conclusion.

Expected Information:

$$\begin{aligned}
I^{-1}(\pi) &= \left(-E\left[\frac{\partial^2 \ell}{\partial \pi^2} \right] \right)^{-1} \\
&= \left(-E\left[-\frac{15}{\pi^2} - \frac{5}{(\pi - 1)^2} \right]_{\pi = \hat{\pi}} \right)^{-1} \\
&= \left(-E\left[-\frac{15}{.75^2} - \frac{5}{(.75 - 1)^2} \right] \right)^{-1} \\
&= \frac{3}{320}
\end{aligned}$$

Wald test:

$$H_0 : \pi = 0.5$$

$$W = \frac{\left(\hat{\pi} - \pi_0\right)^2}{Var\left(\hat{\pi}\right)} = \frac{\left(\frac{3}{4} - \frac{1}{2}\right)^2}{\frac{3}{320}} = \frac{20}{3} \sim \chi_1^2$$