Volatility, Uncertainty, and Hotel Capacity

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Abstract

I show that firms facing demand uncertainty may adapt by changing their capacity investment. I distinguish between demand *volatility* as demand fluctuations to which capacity cannot respond but prices can, and demand *uncertainty* as demand fluctuations to which neither capacity nor prices can respond. In a simple model of capacity choice, converting uncertainty to volatility and vice versa has an ambiguous effect on capacity investment. Empirically, more demand uncertainty induces hotels to be larger, but leads to fewer hotels in a given market, whereas the effects of volatility are insignificant. These results suggest that hotels use capacity as insurance against mispricing, thereby substituting long-run capacity adjustments for short-run price adjustments.

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1 Introduction

By definition, capacity is fixed in the short run: firms cannot adjust capacity in response to fluctuations in demand. Given some capacity, a more feasible short-term response to a demand shift is to vary prices in order to ration scarce capacity. However, firms are often unable to adequately adjust both, prices *and* capacity, in response to changes in demand. Often, the inability to adjust prices stems from a lack of information. Firms may not be able to increase prices when demand increases because they could not foresee the demand increase when posting prices, and cannot adjust prices accordingly until after some goods are sold. Three characteristics— demand variation, uncertainty in when setting prices, and capacity constraints— are essential features of competition in markets that practice some form of "revenue management", including, but not limited to, airlines, hospitals, electricity markets, retail, and hotels.

The interaction between demand volatility, uncertainty, and capacity has been studied by industrial organization economists. Most studies treat capacity as the independent variable: given capacity constraints, stochastic demand has important implications for price volatility (Tishler *et al.*, 2008), pricing dynamics (Williams, 2020), capacity utilization and productivity (Butters, 2020), and aggregate output (Kalnins *et al.*, 2017). Generally speaking, this literature establishes that capacity constraints and demand fluctuations— two features of markets that are clearly of great importance on their own— can have especially pronounced effects on market outcomes when present at the same time. The reverse question— how demand fluctuations affect investment in capacity— has been investigated more recently in industries such as concrete (Collard-Wexler, 2013), shipping (Kalouptsidi, 2014), and oil drilling (Kellogg, 2014). These studies, however, do not explicitly model pricing and therefore cannot consider the separate implications of demand fluctuations to which prices can respond and to which prices cannot respond.

This paper does that: it studies the separate implications of demand fluctuations to which prices can and cannot respond to capacity investment. I use the term *volatility* to refer to demand fluctuations to which prices can respond but capacity cannot, and *uncertainty* to refer to demand fluctuations to which neither prices nor capacity can respond. I develop a model of a (residual) monopolist who chooses capacity in the presence of either volatility or uncertainty. I show that the conversion of volatility to uncertainty, and vice versa, has a theoretically ambiguous effect on capacity investment. Uncertainty-facing firms may have stronger incentives to invest in capacity as insurance against sellouts induced by failing to raise prices when demand is high, but they may weaken incentives to invest in capacity if they also fail to lower prices and fill this capacity when demand is low.

I begin with a simple model of a (residual) monopolist hotel choosing a level of capacity to build. I show that converting demand uncertainty into demand volatility has an ambiguous effect on hotel capacity investment. If the hotel is in a *slack* market, where capacity is cheap and sellouts are rare, uncertainty yields more investment in capacity than volatility. If the hotel is in a *tight* market, where capacity is expensive and sellouts are common, the opposite is true.

I then examine which side of this dichotomy hotels fall on. In the hotel industry, demand volatility and capacity interact in a particularly stark manner. Because hotels are nearly entirely unable to adjust capacity in response to short-run demand fluctuations, hotels often fail to fill capacity in especially volatile markets. Uncertainty adds another layer to these problems. If hotels struggle to predict demand when setting prices, they may run into capacity constraints when they mistakenly predicted demand to be low, or they may fail to fill capacity on nights when demand was predicted to be high. The effects of both demand volatility and uncertainty on capacity utilization have been studied in the hotel industry (Butters (2020), Kalnins *et al.* (2017)).

I use a panel dataset of hotel price and occupancy across a year of nights and twenty-five mid-size markets to test the theory. I estimate demand variation over time for each market and decompose this variation into a part that is easily predicted before prices are chosen and a part that may not be. The amount of total variation over time is market demand volatility, while the proportion of the variation that may not be responsive to prices is uncertainty. I then leverage cross-market variation in these measures to estimate whether hotels in more volatile (uncertain) markets are larger. I complement cross-sectional OLS estimates with estimates using growth in enrollment in college markets over time as an instrument for overall market demand. I show that, while volatility does not affect the size of hotels in a market, more uncertain demand yields larger hotels.

I then examine whether demand volatility and uncertainty have any aggregate effects on market structure. I find that markets with more uncertainty have fewer hotels. This suggests that demand uncertainty may have the effect of *defragmenting* a market: markets in which prices cannot respond adequately to demand shocks have fewer, larger firms. This is similar to what is found in the endogenous sunk costs literature (Sutton (1991), Ellickson (2007)). Investigating this further, I examine the relationship between *growth* in aggregate hotel demand, the evolution of market structure, and volatility and uncertainty. I find that markets with more uncertainty respond more to demand growth than other markets. This highlights a key difference from the endogenous sunk costs literature, in which endogenous sunk costs induce less growth in firms as the market expands.

Related Literature

This paper is related to several different strands of literature. The first is a theoretical literature on capacity choice (Kreps & Scheinkman (1983), Hviid (1990), Hviid (1991), De Frutos & Fabra (2011)). The models in this literature often feature two stages: a simultaneous-move capacity choice stage and a simultaneous-move pricing stage. Several of these papers have extended to include demand uncertainty at the capacity choice stage, and some have also explored demand uncertainty at the pricing stage. Unfortunately, many results in this literature are weak or non-existence results. I focus on a (residual) monopolist, which buys me simplicity, tractability, and the ability to draw sharper conclusions, at the expense of modeling competition in a more satisfying manner. The second strand of literature measures the effects of capacity or inventory constraints on pricing in markets such as baseball tickets (Sweeting, 2012), airlines (Williams, 2020), electricity (Tishler *et al.*, 2008), and hotels (Kalnins *et al.* (2017), Butters & Hubbard (2019), Cho *et al.* (2018)). The models here are rich and the results varied; the joint presence of capacity constraints and stochastic demand can make for interesting and irregular pricing patterns, especially when pricing is dynamic. I do not contribute anything *per se* to the literature of pricing with capacity and uncertainty; instead, I explore distortions in capacity choice as a natural next step given these studies.

This paper also relates to a classic theoretical literature (Abel (1983), Pindyck (1988), Caballero (1991)) and more recent empirical literature (Collard-Wexler (2013), Kellogg (2014), Kalouptsidi (2014)) studying how demand uncertainty at the time of investment affects the level of investment. The sign of the effect depends on several factors, including the convexity of profits with respect to demand shocks, reversibility of investment, and the costs of adjustment (either upwards or downwards). These papers do not consider the effect of the price response to demand shocks. Finally, this paper relates specifically to some papers that determine capacity and market structure in the hotel industry. Some of these papers (Butters & Hubbard (2019), Hubbard & Mazzeo (2019)) study the size composition of hotels in the context of endogenous sunk costs (Sutton, 1991), while others are more concerned with the total amount of capacity as it relates to the the "natural occupancy level" of a market (Gallagher & Corgel (2018), Lee & Jang (2012), deRoos (1999)).

2 Model

In this section, I develop a simple model to demonstrate how demand volatility and uncertainty differentially affect a hotel's optimal investment in capacity. Consider a (residual) monopolist hotel. On any particular night, residual demand for this hotel is $q(p, \lambda)$, where p is the hotel's price, and λ is a demand draw, distributed uniformly on the unit interval without loss of generality. Suppose $q(p, \lambda)$ is increasing in λ.

Before setting prices, the hotel chooses a capacity level $c \in \mathbb{R}$. I treat capacity as a continuous object. Given this capacity, the hotel's realized demand is

$$\tilde{q}(p, \lambda_c) = \min\{q(p, \lambda), c\}.$$

This hotel faces zero marginal costs and thus seeks to maximize its revenue, $\pi(p, \lambda, c) = p\tilde{q}(p, \lambda, c)$, or some expectation thereof. I consider two extreme environments between pure demand uncertainty and pure demand volatility. These environments effectively differ in their timing. For the volatility setting, the timing is as follows:

1. Hotel chooses capacity c.
2. Demand shifter λ is drawn.
3. Hotel chooses price $p_V(\lambda, c)$ according to

$$p_V(\lambda, c) = \text{argmax}_p \pi(p, \lambda, c).$$

4. Demand and revenues are realized.

In this setting, the hotel sets prices after observing the demand draw λ, so prices can vary with λ. The timing of the uncertainty setting is as follows:

1. Hotel chooses capacity c.
2. Hotel chooses price $p_U(c)$ according to

$$p_U(c) = \text{argmax}_p \int_0^1 \pi(p, \lambda, c)\ d\lambda.$$

3. Demand shifter λ is drawn.

4. Demand and revenues are realized.

In this setting, λ is only drawn after prices are set, so prices $p_U(c)$ cannot vary with λ. They do, however, vary with capacity. The primary object of interest is the marginal return to capacity. This is calculated as the derivative of *ex ante* profits $\Pi(c)$ with respect to c. For the volatility case this is

$$\frac{\partial \Pi_V(c)}{\partial c} = \frac{\partial}{\partial c} \int_0^1 \pi(p_V(\lambda, c), \lambda, c)\ d\lambda \tag{1}$$

and for the uncertainty case this is

$$\frac{\partial \Pi_U(c)}{\partial c} = \frac{\partial}{\partial c} \int_0^1 \pi(p_U(c), \lambda, c)\ d\lambda. \tag{2}$$

I make the relatively innocuous assumptions that profits are quasiconcave, the optimal volatility price, $p_V(\lambda, c)$ is weakly increasing in λ, and that the quantity at the optimal price $q(p_V(\lambda, c), \lambda)$ is increasing in λ so long as I begin with a simple model of a (residual) monopolist hotel choosing a level of capacity to build. I show that converting demand uncertainty into demand volatility has an ambiguous effect on hotel capacity investment. If the hotel is in a *slack* market, where capacity is cheap and sellouts are rare, uncertainty yields more investment in capacity than volatility. If the hotel is in a

6 $q(p_V(\lambda, c), \lambda) < c$, i.e., optimal quantity increases in λ so long as a sellout does not occur. This allows me to write *ex ante* volatility profits as

$$\Pi_v(c) = \int_0^{\bar{\lambda}_V(c)} p_V(\lambda, c) q(p_V(\lambda, c), \lambda)\ d\lambda + \int_{\bar{\lambda}_V(c)}^1 p_V(\lambda, c) c\ d\lambda$$

where $\bar{\lambda}_V(c)$ is some threshold value of $\lambda \in [0, 1]$ above which a sellout will optimally occur for this firm. The first term in the above equation is the expected revenues when λ is sufficiently low for a sellout to not occur, multiplied by the probability that a sellout does not occur. The second term is the probability that a sellout does occur multiplied by the expected revenues when a sellout does occur, which is simply price multiplied by capacity. Under the stated assumptions, we can also write *ex ante* uncertainty revenues as

$$\Pi_U(c) = \int\limits_0^{\bar{\lambda}_U(c)} p_U(c) q(p_U(c), \lambda) \; d\lambda + \int\limits_{\bar{\lambda}_U(c)}^{1} p_U(c) c \; d\lambda$$

where $\bar{\lambda}_U(c)$ is defined analogously to $\bar{\lambda}_V(c)$. The first proposition shows how the returns to capacity, defined in Equations 1 and 2, can be expressed more intuitively.

Proposition 3.1. Let $p_V(\lambda, \infty; z)$ be the optimal volatility price of a firm that faced no capacity constraint but marginal cost z (i.e., the monopoly price), and define $z(\lambda, c)$ by

$$p_V(\lambda, \infty; z(\lambda, c)) = p_V(\lambda, c).$$

Then

$$\frac{\partial \Pi_v(c)}{\partial c} = [1 - \bar{\lambda}_V(c)] \mathbb{E}(z(\lambda, c) | \lambda > \bar{\lambda}(c)) \tag{3}$$

and

$$\frac{\partial \Pi_U(c)}{\partial c} = [1 - \bar{\lambda}_U(c)] p_U(c). \tag{4}$$

Proof: See Appendix A. □

The two expressions 3 and 4 show that a key determinant of the returns to higher capacity are the probability that capacity constraints will bind, i.e., the probability of a sellout. This is the first multiplicative term of each expression. The second term in Equation 3 is the expected fictitious marginal cost $z(\lambda)$ that would be needed for a non-capacity constrained monopolist to increase its price to the capacity-constrained price. This term should become larger as capacity constraints become more binding (i.e., as capacity shrinks) as well. The second term in 4 is simply the uniform price the uncertainty-facing firm would set given some capacity c. This is also higher for smaller capacities.

In light of the importance of the probability of a sellout in determining the returns to capacity, the following proposition provides some intuition for when the returns might be higher for the volatility facing firm or for the uncertainty-facing firm.

Proposition 3.2. Suppose there exists a choke price $p_{choke}(\lambda)$ and that $qq_{p\lambda} - q_p q_\lambda > 0$, where subscripts denote derivatives. Let $\bar{c}$ be the level of capacity above which sellouts never occur in either setting, U or V, and let $0 < c^* < c^{**} < 1$. Then

$$\lambda_V(c) - \lambda_U(c) < 0 \text{ if } c \in [0, c^*)$$

and

$$\lambda_V(c) - \lambda_U(c) > 0 \text{ if } c \in (c^{**}, \bar{c}]$$

Proof: See Appendix A. □

In other words, for sufficiently small capacity levels, the volatility-facing firm is more likely to sell out, and for sufficiently high capacity levels, the uncertainty-facing firm is more likely to sell out. The intuition is fairly straightforward. The uncertainty facing firm cannot vary prices as λ changes, which means that its non-capacity constrained optimal quantity, $q(p_U(c), \lambda)$ will vary more than $q(p_V(\lambda, c), \lambda)$. For sufficiently high levels of capacity, the volatility-facing firm may optimally "target" a quantity below capacity for all possible λ, but the uncertainty-facing firm may still sell out at high values of λ because prices cannot adjust accordingly. On the other hand, if c is sufficiently low, the volatility-facing firm may desire to sell out all of the time, but the uncertainty-facing firm may fail to sell out on low-λ draws, because it cannot decrease prices accordingly.

This result, combined with the previous proposition, suggests that the returns to capacity will be higher for the volatility-facing firm when capacity levels are sufficiently low, because the volatility-facing firm is more likely to sell out. If capacity levels are sufficiently high, the opposite would seem to be true, and uncertainty-facing firms would have higher returns from capacity. The following result confirms this intuition, so long as a choke price exists when λ is in a neighborhood of 0.

Theorem 3.3 Suppose there exists a choke price $p_{choke}(\lambda)$ and that $qq_{p\lambda} - q_p q_\lambda > 0$, where subscripts denote derivatives. Then there exists some c^* and c^{**} such that

$$\frac{\partial \Pi_V(c)}{\partial c} > \frac{\partial \Pi_U(c)}{\partial c}$$

if $c < c^*$, and

$$\frac{\partial \Pi_V(c)}{\partial c} < \frac{\partial \Pi_U(c)}{\partial c}$$

if $c > c^{**}$ and $c < \bar{c}$.

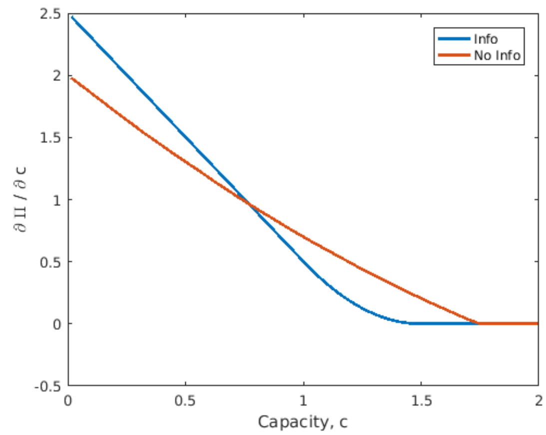
Proof: See Appendix A. □

This theorem states that the returns to higher capacity are larger for a volatility-facing firm than for an uncertainty facing firm if capacity is sufficiently low, and the opposite is true if capacity is sufficiently high. The assumption that there exists a choke price, while clearly satisfied for many functional forms including linear demand, is a sufficient but not necessary condition for this to be true. This is confirmed in Figure 1, which shows the returns to capacity for volatility- and uncertainty-facing firms in three different scenarios. In panel (a), the firm faces downward-sloping demand with a slope of -1 and an uncertain intercept that varies uniformly between 2 and 3. In panel (b), everything remains the same, but volatility is increased: the intercept now ranges uniformly between 1 and 4. Perhaps not surprisingly, the gap between the returns to capacity for the volatility- and uncertainty-facing firms becomes larger as the amount of volatility/uncertainty grows. Panel (c) shows the case of logit demand. Though logit demand does not have a choke price, the result still holds that the returns to capacity are higher for the volatility-facing firm so long as capacity is low, and higher for the uncertainty-facing firm otherwise.

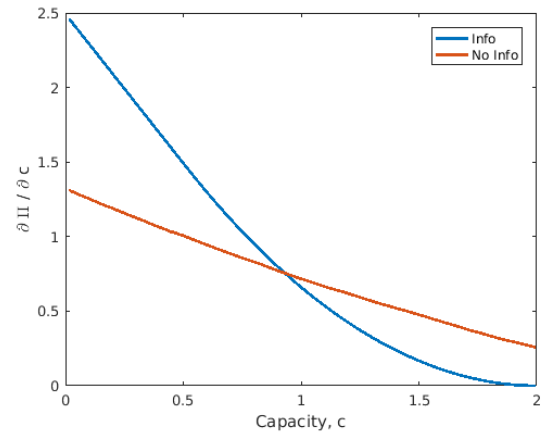
In the presence of constant marginal costs of capacity, all of these scenarios imply that the hotel will be larger when facing volatility than when facing uncertainty, if and only if capacity is sufficiently costly. The question is thus which side of this dichotomy hotels are on. If capacity is sufficiently costly relative to the amount of residual demand, hotels are a "constrained market" where uncertainty decreases hotel size, and if capacity is cheap, hotels are a "slack market" where uncertainty increases hotel size.

Figure 1: Payoffs from capacity under volatility versus uncertainty

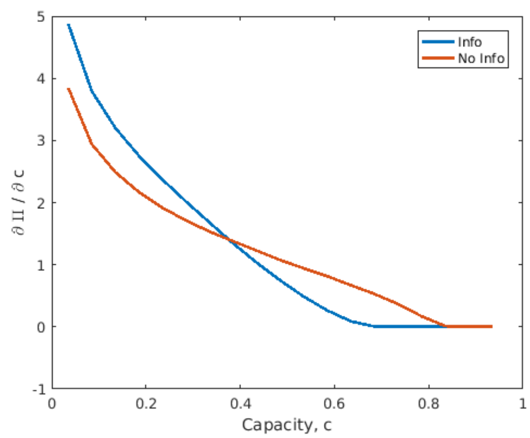
(a) Linear demand, $\lambda \sim U[2,3]$



(b) Linear demand, $\lambda \sim U[1,4]$



(c) Logit demand



3 Empirical Strategy

3.1 Data

My main data source is STR, Inc., a hotel benchmarking company that collects price and occupancy data from its member hotels and offers members competitive reports on pricing and performance. An observation in my dataset is a hotel on a given night. My dataset includes the average daily rate (or ADR, which I hereafter use interchangeably with “price”) and occupancy rate for each hotel-night across eighteen different U.S. markets for a calendar year, between July 2016 and June 2017. I also have information about hotel characteristics, namely the anonymized franchisee, brand, management company, and parent company, as well as quality tier and capacity. I do not observe individual booking data. As a result, I do not know how far in advance rooms were booked, through what channel they were booked, and the degree to which price dispersion exists for a specific hotel on a specific night.

The data cover twenty-five markets. Eighteen of these markets are United States college towns with relatively large student populations, prominent Division IA football teams, and a degree of isolation from other, larger metropolitan areas. The other seven are not traditionally college towns but are of similar size and degree of isolation. In college markets, there are particular nights on which demand will be predictably high. These nights are the nights before and after undergraduate move-in, graduation, and home football games, which I identified using online search. I flag these as “high demand days” in my data. The hotels in my data are all within fifteen miles of the college’s stadium, though most of these are actually within a five-mile radius of the stadium.

My markets are summarized in Figure 2. The non-college town markets are generally similarly sized to the college towns I use, but because they are less college-oriented, they are likely to have different demand characteristics in two important ways. First, demand in these markets is less volatile, because it is less affected by seasonality and key events, such as graduation and football games, than the college town markets. For instance, much of the hotel traffic in Rochester, MN, is driven by the Mayo Clinic, which likely has a more constant level of activity year-round than a college campus with a large seasonal undergraduate population. Second, demand in these markets may be more uncertain, because what day-to-day variation exists in these markets’ demand is less likely to be associated with salient, easily-predicted events like graduation and football games.

As seen in Figure 2, there is significant heterogeneity in the number of hotels in a market, which is expected given the heterogeneity in market size, but there is also heterogeneity in the average and median hotel size in a particular market. At first glance, hotel size does not seem to be obviously correlated with any salient market features. These scatterplots reveal a weak correlation between the number of hotels and average hotel size (correlation coefficient = .2013), and college-centric and non-college-centric markets are fairly evenly distributed throughout the scatterplot.

Furthermore, as shown in Figure 3, there is considerable heterogeneity within a particular brand. Though franchisee disclosure documents[1] provide guidelines on how a particular branded hotel should look and suggest room counts, properties are allowed to vary considerably in their room counts according to market needs and the wishes of the franchisee/developer.

3.2 Empirical Strategy: Hotel Capacity

Generally speaking, my empirical strategy is to compare the size of hotels that face a high degree of demand uncertainty and/or volatility to hotels that face less. My main specification relies on cross-market variation in these demand characteristics. That is, some markets have more volatile demand or a greater percentage of variation in day-to-day demand that stems from difficult-to-predict sources.

In order to measure volatility and uncertainty, I estimate a nested logit demand system. For hotel j in nest $g(j) \in \{downscale,\ midscale,\ upscale\}$ on city-night t, consumer i's utility is given by

$$u_{ijt} = -\alpha \log(p)_{jt} + \beta \tilde{x}_t + \lambda_t + \lambda_{b(j),w(t)} + \xi_{jt} + \varepsilon_{i,g(j),t}(\nu) + \varepsilon_{ijt}$$

where γ is a parameter that affects how strong substitution is within-nest versus across-nests. $\tilde{x}_t$ contains market-wide, common-knowledge demand shifters such as day-of-week and month-of-year interacted with city fixed effects. λ_t contains all other market-wide demand variation that is not attributable to these demand shifters. Operationally, it is a fixed effect for all hotels (i.e., the inside good) on night t. The outside good, 0, is not staying at a hotel that night, and has a mean utility of zero. $\lambda_{b(j),w(t)}$ is a fixed effect for hotel brand $b(j)$ on $w(t) \in \{weekend,\ weekday\}$, and ξ_{jt} is a hotel-specific nightly demand shock.

[1] The state of Wisconsin mandates that all chains that operate franchises within the state must make their franchisee disclosure documents public. They are accessible at `https://www.wdfi.org/fi/securities/franchise/default.htm`.

Shares s_{jt} can be expressed as follows:

$$\log(s_{jt}) - \log(s_{0t}) = -\alpha \log(p)_{jt} + \beta \tilde{x}_t + \lambda_t + \lambda_{b(j),w(t)} + \xi_{jt} + \gamma \log(s_{g(j),t}) \tag{5}$$

where $s_{g(j),t}$ is the aggregate share of j's nest on night t. I use the estimates from this specification to construct my measures of demand volatility and uncertainty. Letting $x_t = \beta\tilde{x}_t$, and $m(t)$ be the city associated with city-night t, market-wide volatility in city m is measured as

$$\text{Volatility}_m = \text{var}(x_t + \lambda_t | m(t) = m).$$

That is, volatility is the amount of night-to-night variation in market-wide demand. This is because ξ_{jt}, the only other demand shifter that varies from night to night, is a residual in Equation 5 and is assumed mean-zero and independent across hotels in market m. As a result, for a market with a sufficiently large number of hotels, by the law of large numbers, these terms should not induce significant market-wide variation in demand.

I then measure uncertainty as the fraction of this market-wide demand variation that is not explained by common-knowledge demand shifters. By independence of x_t and λ_t, which is true by construction, this equals

$$\text{Uncertainty}_m = \frac{\text{var}(\lambda_t | m(t) = m)}{\text{Volatility}_m}.$$

My main empirical specification is

$$\text{Capacity}_j = \beta_1 \text{Market Size}_m + \beta_2 \text{Volatility}_m + \beta_3 \text{Uncertainty}_m + \sum_g \beta_g \mathbf{1}[g(j) = g] + u_j.$$

This specification includes two controls in addition to the main independent variables of interest. I include fixed effects for the quality segment to which hotel j belongs, because upscale hotels tend to be larger.[2] I also include measures of market size as a control, because larger markets may have larger hotels.

The error term of this regression, u_j, contains all factors that affect capacity that are not included as regressors. If these factors are correlated with market demand characteristics, this

[2]One reason for this is that the luxury hotels' costs do not scale with capacity as much as midscale hotels. For instance, upscale resorts provide hotel-wide amenities like dining, event space, and outdoor recreation, while business hotels focus more heavily on in-room amenities. See Butters & Hubbard (2019) for more.

will yield biased estimates. The included regressor for which endogeneity poses the greatest concern is market size, depending on the way it is defined. I take a few different approaches with regard to market size:

1. **Excluding market size.** If market size is uncorrelated with demand volatility and uncertainty, excluding it, or any proxy for it, may not induce bias in the estimates. However, to the extent that volatility and uncertainty affect rival hotels' entry or capacity decisions, a hotel may adjust its own capacity investment in response. As a result, the measured effect of uncertainty on hotel size may be the reduced form of a more complicated response that is difficult to interpret as causal without strong modeling assumptions. I perform this regression mainly as a base case.

2. **Using number of rivals' rooms as market size proxy.** This approach would let market size be the total capacity of all rivals in the same city:

$$\text{Market Size}_{jm} = \sum_{j':m(j)=m, j \neq j} \text{Capacity}_{j'}.$$

   This approach directly addresses the previous approach's concern, that rivals' capacity may affect j's capacity. The major drawback of this approach is that rivals' capacity is the outcome of rivals' choices. For instance, if costs of building capacity, which are unobserved and likely market-specific, are high, this may induce both a hotel and its rivals to build less capacity. As a result, including market size as a regressor may introduce endogeneity into the model.

   There is also the concern of simultaneity. Assuming capacity is a strategic substitute, large hotels may exist in a market because rivals are small, or large hotels cause rivals to be small later on. To address any concerns, I supplement these regressions with regressions using the aggregate capacity of rivals constructed before the year 2000 as an independent variable, and only using hotels opened since 2000 as observations.

3. **Using university enrollment as a market size proxy.** College enrollment is a more plausibly exogenous market size proxy. Larger universities require more hotel rooms for visitors. The problem with using enrollment as a proxy for market size is that some college towns are not strictly college towns. Some of the larger towns in my data, such as Knoxville and Madison, have other important contributors to market demand for hotels. The non-university contribution to market size would be contained in the residual, which would now likely be correlated with volatility: towns with more sources of demand are

perhaps more likely to smooth demand over time. For instance, non-university affiliated events will be scheduled specifically on dates that expected university-affiliated demand will be low for cost or availability reasons.

4. **Using university enrollment as an instrument for rivals' capacity.** A solution to the above problem is to use rivals' capacity in the main estimating regression, but to instrument for rivals' capacity using university enrollment. This eliminates sources of market size that are not university-affiliated from the error term of the main regression, because rivals' market capacity is included in the regression. At the same time, it alleviates the issue of treating rivals' market capacity as endogenous. The main drawback of this specification is power. In particular, university enrollment proves to be a weak instrument, because the largest universities in my dataset often are located in small towns. As first-stage F statistics are consistently less than 3 for this approach, I leave it out of my analysis.

5. **Leveraging changes in university enrollment.** In this specification, I use

$$\text{Rooms Per Student in } 2000_m = \frac{\text{Total Capacity in } 2000_m}{\text{Total Enrollment in } 2000_m}$$

to proxy for how college-centric a particular market is. I then use this variable, enrollment in 2000, and the interaction of the rooms per student variable with growth in enrollment since 2000, to instrument for rivals' capacity. This interaction instrument is reminiscent of many "shift-share" instruments.[3] It scales growth in enrollment (the "shift" in market size) by the ratio between capacity and enrollment (the "share" of rooms per student). For exogeneity to plausibly hold, I estimate this specification using only hotels constructed since 2000.

A separate approach is to use hotel-specific measures of demand volatility and uncertainty. Re-examining Equation 5, there are two hotel-specific demand shifters, $\lambda_{b(j),w(t)}$ and ξ_{jt}. One of these, $\lambda b(j), w(t)$ is clearly anticipated by the hotel, i.e. the hotel knows demand varies for hotels of its brand between weekdays and weekends. The other, ξ_{jt}, may be unknown to the hotel depending on modeling assumptions. Returning to my discussion in Chapter 1, ξ_{jt} is known under the assumption of unknown draws (UD) and is unknown under the assumption of known draws (KD). If I assume known draws, then I can construct hotel-specific measures

[3] This type of instrument is commonly found in the labor economics literature, originating with Bartik (1991). See Goldsmith-Pinkham *et al.* (2018) for a recent discussion of these instruments' validity.

of volatility and uncertainty as follows:

$$\text{Volatility}_j = \text{var}(\xi_{jt} + \lambda_{b(j),w(t)})$$

$$\text{Uncertainty}_j = \frac{\text{var}(\xi_{jt})}{\text{Volatility}_j}.$$

Note that my primary specification throughout Chapters 1 and 2 of this dissertation assumed unknown draws. There does not seem to be a feasible way at the moment to create credible hotel-specific measures of volatility and uncertainty under this assumption. A note about including market-wide demand shocks: because market-wide demand shocks affect hotel-specific demand differently than hotel-specific shocks of the same size, i.e., market-wide shocks also affect rivals, it would be difficult to interpret any hotel-specific measure of volatility and uncertainty that included both types of shock. For instance, if I specified $\text{Volatility}_j = \text{var}(\xi_{jt} + \lambda_{b(j),w(t)} + x_t + \lambda_t)$ and $\text{Uncertainty}_j = \frac{\text{var}(\xi_{jt}+\lambda_t)}{\text{Volatility}_j}$, I may be attributing to uncertainty what is really the effect of more *hotel-specific volatility* on demand via variance in ξ_{jt}. In light of this, I account for market-wide demand characteristics via a market fixed effect in the hotel-specific regressions. My main specification is

$$\text{Capacity}_j = \beta_{0j} + \beta_1 \text{Volatility}_j + \beta_2 \text{Uncertainty}_j + \sum_g \beta_g \mathbf{1}[g(j) = g] + u_j.$$

3.3 Empirical Strategy: Market Structure

I then turn my attention to the question of how demand volatility and uncertainty on overall market structure, specifically the number of firms and aggregate market capacity. The most straightforward way to do this is to project these market structure variables onto demand uncertainty and volatility for each market:

$$\text{Capacity}_m = \beta_0 + \beta_1 \text{Volatility}_m + \beta_2 \text{Uncertainty}_m + u_m \tag{6}$$

$$\text{Hotel Count}_m = \beta_0 + \beta_1 \text{Volatility}_m + \beta_2 \text{Uncertainty}_m + u_m. \tag{7}$$

While I do use this approach, the clear drawback is omitted variable bias: several other characteristics of the market, including aggregate demand and costs of building capacity, may induce bias in these estimates if they are correlated with volatility and uncertainty. Because aggregate demand is especially important in determining the number of hotels and rooms in a market, even small degrees of correlation with the included regressors may induce large biases in my estimated coefficients. As mentioned before, college enrollment is not an especially good proxy for market size by itself, so it is not useful as a control in this regression. I discuss how to address this more effectively in future work in the concluding section of this chapter.

Variation in enrollment, while not a good proxy for market size *cross-sectionally*, is perhaps a good predictor of growth in market size for a given market over time. This appears to be the case, based on the first stage estimates using the shift-share instruments of Approach 5 (see the F-statistic reported in Table 2). This allows me to examine how market structure evolves in response to growth in market size in the presence of volatility and uncertainty, a related (but not identical) question to how market structure depends on volatility and uncertainty cross-sectionally.

To do this, I use a long-differences approach. I partition the years since 2000 into two sub-periods: 2000 through 2008 and 2009 through 2016. These periods, while also conveniently equal in length, correspond roughly to hotel construction that began before and after the 2008 financial crisis. I use long differences to account for the fact that hotel construction has lumpiness and a certain degree of "time-to-build." A hotel that opens in 2004 may be responding to a demand shock in 2002 or 2003 or in anticipation of demand growth in 2005 onward.

I construct a measure of market expansion during each of these periods using similar logic to the shift-share instruments in the previous subsection. I calculate

$$\%\Delta\text{Market Size}_{m,t,t-8} = \frac{\text{Enrollment}_{mt} - \text{Enrollment}_{m,t-8}}{\text{Enrollment}_{m,t-8}} \times \frac{\text{Enrollment}_{m,t-8}}{\text{Capacity}_{m,t-8}}.$$

If enrollment grows by five percent, we would expect this to mean more in terms of growth in capacity or number of hotels in a market where the university is important for demand. As a result, I rescale enrollment by enrollment per capacity, a measure of the university's importance for demand, to create a predictor of percent growth in capacity or number of hotels that may arise from a given percent growth in enrollment.

I then compute percentage growth in the number of properties and number of rooms during these time periods as well. I estimate the following regressions:

$$\%\Delta\text{Total Rooms}_m = \beta_{0,pd} + \beta_1\text{Volatility}_m + \beta_2\text{Uncertainty}_m + \beta_3\%\Delta\text{Market Size}_m$$

Table 1: Demand estimates

Parameter	Estimate	Std. Err.
Price sensitivity, α	-1.475	0.032
Nesting parameter, γ	0.284	0.004
	Mean Across all Hotels	
	Market-Level	Hotel-Level
Volatility	0.318	0.098
Uncertainty	0.323	0.878

Data sources: STR, Inc., Expedia, and *Travel Weekly*

$$\%\Delta\text{Number of Properties}_m = \beta_{0,pd} + \beta_1\text{Volatility}_m + \beta_2\text{Uncertainty}_m + \beta_3\%\Delta\text{Market Size}_m$$

where there are different intercepts based on the period considered (pre-financial crisis and post-financial crisis). I also perform regressions that replace volatility and uncertainty with the interaction of volatility and uncertainty with the adjusted percentage growth in market size. Note that these specifications embed several rather strong assumptions that may be addressed with more data. I discuss these assumptions in the conclusion of this chapter.

4 Results

Demand estimates are similar to those in Chapter 1 of this dissertation. This is expected, given the similarity of both, the specification and the data used, to demand in that chapter. These estimates are reported in Table 1. The key variables that arise out of these demand estimates are those of volatility and uncertainty for each market. I report those, for each market, in Figure 4.

There is considerable variation in both measures; notably, markets vary between 20% of market-wide demand variation being explained by non-predictable events (South Bend, IN) to over 40% being explained by non-predictable events (College Station, TX). There doe not appear to be much correlation between volatility and uncertainty. Perhaps not surprisingly, the most volatile markets are heavily college-centric, and the most predictable markets are college-centric and have much tourism that revolves around football games.

4.1 Cross-Market Differences

I report estimates from several regressions based on the discussion in Section 1 in Table 2. Volatility consistently has a statistically insignificant effect on hotel size, but uncertainty consistently predicts larger hotels. This is true, even if I control for differences in market size and building costs by including rival capacity and account for endogeneity in rival capacity by using lagged rival capacity and/or instrumenting for rival capacity using shift-share instruments, though I lose statistical power, in part on account of using fewer observations. These estimates ultimately show (1) that demand fluctuations affect the optimal choice of capacity, but only if prices are unable to respond to these fluctuations, and (2) that uncertainty increases hotel capacity, suggesting hotels are on the "slack markets" side of the divide in Figure 1. This suggests that hotel capacity is sufficiently inexpensive to build and empty rooms sufficiently inexpensive to maintain to keep as insurance for failing to adjust prices during high demand events.

Table 2: Market demand volatility, uncertainty, and capacity

	(1)	(2)	(3)	(4)	(5)
Dep. Var.	Capacity	Capacity	Capacity	Capacity	Capacity
	Estimate (Std. Err.)	Estimate (Std. Err.)	Estimate (Std. Err.)	Estimate (Std. Err.)	Estimate (Std. Err.)
Volatility	−11.032	−6.257	−2.667		−11.622
	(11.318)	(12.567)	(13.705)		(35.873)
Uncertainty	77.913	84.564	59.459	34.853	49.788
	(26.418)	(27.073)	(27.160)	(44.055)	(32.728)
Rival Capacity (000s)		0.566			
		(0.488)			
2000 Rival Capacity (000s)			0.358	−0.327	0.169
			(0.818)	(1.047)	(2.797)
2000 Enrollment (000s)				0.080	
				(0.216)	
Quality FE	Y	Y	Y	Y	Y
IV					Shift-Share
First Stage F					22.49
N	1,051	1,051	452	333	333

Data source: STR, Expedia, *Travel Weekly*, and university websites. All hotels used in first two specifications. Only hotels opened since 2000 used in specification (3), and only hotels opened since 2000 in college towns used in specifications (4) and (5).

The results for within-market capacity differences in Table 3 complement the cross-market results. Because the hotel-level measures of volatility and uncertainty involve a different set of demand shifters, their magnitudes are not comparable to those of the analogous coefficients in Table 2. However, the coefficient on hotel-level uncertainty remains positive and nearly statistically significant at the 10% confidence level, consistent with the general evidence that

Table 3: Hotel demand volatility, uncertainty, and capacity

Dep. Var.	Capacity
	Estimate (Std. Err.)
Hotel Volatility	17.564 (18.058)
Hotel Uncertainty	12.636 (8.134)
Quality FE	Y
Market FE	Y
N	1,053

Data sources: STR, Expedia, and *Travel Weekly*.

hotels choose to be larger if they expect to be unable to sufficiently adjust prices to reflect demand shifts. Note that there may be simultaneity bias (or, depending on the desired interpretation, measurement error) in the hotel-specific estimates: large hotels may face less demand uncertainty over time because they attract many visitors. Small hotels are more likely to have idiosyncratic nights in which they sell out of their (limited) number of rooms for reasons difficult to predict (and difficult for the econometrician to observe), which would increase their uncertainty faced. Note that this would bias my coefficient on uncertainty upwards (i.e., towards zero), so the effect of uncertainty on capacity choice may in fact be even larger than reported.

4.2 Growth and Market Structure

If uncertainty tends to increase capacity investment, it stands to reason that markets that face more demand uncertainty may also have *fewer* hotels. This is for two reasons. First is the obvious reason that, if hotels are larger, given some aggregate demand for hotels in a market, fewer hotels are needed to satisfy that demand. The second is that the inability to adjust prices to demand fluctuations is, overall, detrimental to an individual hotel's performance (see Chapter 1 for evidence of this; there is the caveat that the inability to adjust prices to demand fluctuations may have either positive or negative effects on market-wide profitability, as alluded to in Chapter 2). As a result, for some fixed cost of entry that does not vary with capacity, there may be fewer hotels that find it profitable to enter, all else equal. For a given fixed cost structure, this second effect may also yield less capacity overall in the market, all

Table 4: Demand volatility, uncertainty, and market structure

Dep. Var.	(1Num) No. of Properties	(2Num) No. of Properties	(1Cap) Market Capacity (000s)	(2Cap) Market Capacity (000s)
	Estimate (Std. Err)	Estimate (Std. Err)	Estimate (Std. Err)	Estimate (Std. Err)
Volatility	−88.095	−120.848	−8.561	−12.207
	(39.012)	(49.799)	(4.083)	(5.211)
Uncertainty	−129.098	−153.711	−9.096	−14.556
	(105.982)	(147.383)	(11.090)	(15.421)
2016 Enrollment (000s)		0.270		0.041
		(0.580)		(0.061)
N	25	18	25	18

Data sources: STR, Expedia, *Travelers Weekly*, and university websites.

else equal.

I examine these effects by regressing the number of hotels and the number of rooms on market volatility and uncertainty. The results are in Table 4 along with a regression that uses college enrollment as an additional control. These regressions are difficult to interpret as causal, because market size is likely the most important determinant of supply in horizontally-differentiated markets like hotels (see, e.g. Bresnahan & Reiss (1991) and Ellickson (2007)) and is omitted from this specification. I use enrollment in college town to proxy market demand characteristics in specification (2) Table 4. As mentioned earlier, however, college enrollment is a poor proxy for aggregate market demand, as many of the largest colleges are located in remote towns with little else. Further work is needed to better account for cross-market heterogeneity in market size, and more markets are needed for more power and to credibly include more market characteristics as controls without overfitting.

Given the insufficient data to study cross-market heterogeneity in *static* or *equilibrium* structure, I instead analyze how demand volatility and uncertainty affect how the supply of rooms responds to *growth* in market demand. This is more feasible because the response to *changes* in market demand is less likely to depend on market size— a doubling of demand should yield a doubling of the supply of rooms, all else equal, in both, large markets and small. I report the estimates of the coefficients in Equations 6 and 7 in Table 5.

Specifications (1Num) and (1Cap) both show that growth in enrollment is associated with supply expansion, while the presence of volatility and uncertainty has little bearing on how a market expands. However, specifications (2Num) and (2Cap) suggest that the effect of de-

Table 5: Market growth, uncertainty, and volatility

Dep. Var.	(1Num) %ΔProperties	(2Num) %ΔProperties	(1Cap) %ΔCapacity	(2Cap) %ΔCapacity
	Estimate (Std. Err)	Estimate (Std. Err)	Estimate (Std. Err)	Estimate (Std. Err)
Volatility	0.228		−0.051	
	(0.217)		(0.257)	
Uncertainty	0.641		0.464	
	(0.495)		(0.586)	
%ΔMarket Size$_m$	0.027	−0.268	0.045	−0.139
	(0.007)	(0.075)	(0.009)	(0.100)
× Volatility		0.185		0.073
		(0.049)		(0.065)
× Uncertainty		0.618		0.439
		(0.162)		(0.218)
$\mathbf{1}[2009-2016]$	−0.243	−0.256	−0.243	−0.248
	(0.051)	(0.043)	(0.060)	(0.057)
N	36	36	36	36

Data sources: STR, Inc, Expedia, *Travelers Weekly*, and university websites.

mand growth on supply expansion comes mainly through the presence of demand uncertainty. Markets that face no demand uncertainty— that is, markets in which prices can adequately respond to demand shifts even when capacity cannot— do not seem to respond to long-term demand growth by expanding capacity. A pithy way of saying this is that hotels seem to substitute capacity response to long-term demand shifts for price responses to short-term demand fluctuations when the latter are difficult to achieve.

5 Conclusion

In this paper, I demonstrated that demand uncertainty affects market structure in meaningful ways. The most robust evidence is in support of market-level uncertainty yielding larger hotels. I also provide some evidence that within-market, hotel-specific increases in demand uncertainty also yield increases in capacity. I also showed evidence that suggests the presence of uncertainty may reduce the number of hotels, even if each hotel is, on average, larger in size. This ultimately means the effect of demand uncertainty on total room capacity, in equilibrium, in indeterminate. It should be noted that these results should not be taken too seriously, in light of my relatively small number of markets and the aforementioned issues with accounting for differences in market size. Finally, I showed that the presence of demand uncertainty exacerbates the response of hotel supply to long-term changes in demand.

My theoretical exercise shows that, in other markets in which capacity is more costly to build, the sign of the various effects of demand uncertainty on market structure may be different. However, the broad takeaway remains the same: the effects of demand fluctuations on capacity choice and market structure seem to be magnified when prices cannot adequately respond to these fluctuations. As firms become better at predicting demand, price discriminating, and adjusting prices to changes in conditions in real-time, old lessons governing market structure may no longer be valid.

More work is needed. Most closely to the present analysis, a broader cross-section of markets, as well as better controls to proxy for market size and demand characteristics other than volatility and uncertainty that may affect market structure, would make my estimates more credible. The key omitted supply variable from this analysis is the cost of building capacity, though the fixed cost of entry also matters. Obviously it matters for my discussion of the number of properties and market structure more generally, but it matters for a single hotel's capacity choice as well— if fixed costs are high, entry will likely be less frequent, but hotels will be larger conditional on entry. As a result, high fixed costs, if they are correlated with my measure of uncertainty, would yield similar patterns of larger and fewer hotels in certain markets in my data. While I do not have a reason to believe fixed costs and demand uncertainty would be correlated, cost data or structural fixed cost estimates may lend further credence to my conclusions.

More broadly, there is more work to be done to understand why differences in uncertainty, and not volatility, primarily affects market structure in the hotel industry. Though my model provides some insights into why demand uncertainty relative to volatility may matter for capacity choice, further insights may come from a more sophisticated analysis of how prices are set— hotel pricing occurs dynamically over time as uncertainty is resolved, and so pricing exists on a continuum between volatility and uncertainty paradigms— and from a broader model that also incorporates the entry decision before a capacity choice is made.

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A Proofs

I restate and prove the three propositions in the main text, in order, below.

Proposition 3.1. Let $p_V(\lambda, \infty; z)$ be the optimal volatility price of a firm that faced no capacity constraint but marginal cost z (i.e., the monopoly price), and define $z(\lambda, c)$ by

$$p_V(\lambda, \infty; z(\lambda, c)) = p_V(\lambda, c).$$

Then

$$\frac{\partial \Pi_V(c)}{\partial c} = [1 - \bar{\lambda}_V(c)]\mathbb{E}(z(\lambda, c)|\lambda > \bar{\lambda}(c))$$

and

$$\frac{\partial \Pi_U(c)}{\partial c} = [1 - \bar{\lambda}_U(c)]p_U(c).$$

Proof: I begin with $\frac{\partial \Pi_V(c)}{\partial c}$. Assume that $\bar{\lambda}_V(c)$ is between 0 and 1. Then *ex ante* profits for the volatility-facing firm are

$$\Pi_V(c) = \int_0^{\bar{\lambda}_V(c)} p_V(\lambda, c) q(p_V(\lambda, c), \lambda)\, d\lambda + \int_{\bar{\lambda}_V(c)}^1 p_V(\lambda, c) c\, d\lambda.$$

Differentiate using Leibniz rule to obtain

$$\begin{aligned}\frac{\partial \Pi_V(c)}{\partial c} = \int_0^{\bar{\lambda}_V(c)} \frac{\partial}{\partial c}[p_V(\lambda, c) q(p_V(\lambda, c), \lambda)]\, d\lambda + \frac{\partial \bar{\lambda}_V(c)}{\partial c}[p_V(\bar{\lambda}_V(c), c) q(p_V(\bar{\lambda}_V(c), c), \lambda)] \\ + \int_{\bar{\lambda}_V(c)}^1 \frac{\partial}{\partial c}[p_V(\lambda, c)c]\, d\lambda - \frac{\partial \bar{\lambda}_V(c)}{\partial c}[p_V(\bar{\lambda}_V(c), c)c].\end{aligned}$$

Note that $q(p_V(\bar{\lambda}_V(c), c), \lambda) = c$, so the second and fouth terms on the right hand side of this equation cancel. This yields

$$\frac{\partial \Pi_V(c)}{\partial c} = \int_0^{\bar{\lambda}_V(c)} \frac{\partial}{\partial c}[p_V(\lambda, c) q(p_V(\lambda, c), \lambda)]\, d\lambda + \int_{\bar{\lambda}_V(c)}^1 \frac{\partial}{\partial c}[p_V(\lambda, c)c]\, d\lambda.$$

Next, note that, so long as the capacity constraint is nonbinding (i.e., $\lambda < \bar{\lambda}_V(c)$), optimal price and quantity for the volatility-facing firm do not change. Thus, the first term is also zero. All that remains is

$$\frac{\partial \Pi_V(c)}{\partial c} = \int_{\bar{\lambda}_V(c)}^{1} \frac{\partial}{\partial c}[p_V(\lambda, c)c] \; d\lambda.$$

Differentiating the integrand yields

$$\frac{\partial \Pi_V(c)}{\partial c} = \int_{\bar{\lambda}_V(c)}^{1} \frac{\partial p_V(\lambda, c)}{\partial c} c + p_V(\lambda, c) \; d\lambda. \tag{8}$$

Next, note that, when capacity is binding and revenues are concave, $p_V(\lambda, c)$ solves

$$q(p_V(\lambda, c), \lambda) = c$$

and downward-sloping (i.e., invertible) demand implies

$$p_V(\lambda, c) = q^{-1}(c, \lambda).$$

This means

$$\frac{\partial p_V(\lambda, c)}{\partial c} = 1 \Bigg/ \left. \frac{\partial q(p, \lambda)}{\partial p} \right|_{p = p_V(\lambda, c)}.$$

Inserting this into 8, we obtain

$$\frac{\partial \Pi_V(c)}{\partial c} = \int_{\bar{\lambda}_V(c)}^{1} \frac{c}{\partial q(p, \lambda)/\partial p\big|_{p = p_V(\lambda, c)}} + p_V(\lambda, c) \; d\lambda. \tag{9}$$

Now, consider a monopolist facing marginal cost z and no capacity constraint. The monopolist chooses price according to

$$p_V(\lambda, \infty; z) = \operatorname{argmax}_p (p - v) q(p, \lambda)$$

yielding first order condition

$$0 = (p_V(\lambda, \infty; z) - z) \left. \frac{\partial q(p, \lambda)}{\partial p} \right|_{p = p_V(\lambda, \infty; z)} + q(p_V(\lambda, \infty; z), \lambda)$$

Choose marginal cost $z(\lambda, c)$ so that this first order condition is satisfied at a quantity of c:

$$0 = (p_V(\lambda, \infty; z(\lambda, c)) - z(\lambda, c)) \frac{\partial q(p, \lambda)}{\partial p}\bigg|_{p = p_V(\lambda, \infty; z(\lambda, c))} + c.$$

Rearranging, we obtain

$$v(\lambda, c) = \frac{c}{\partial q(p, \lambda)/\partial p\big|_{p = p_V(\lambda, \infty; z(\lambda, c)}} + p_V(\lambda, \infty; z(\lambda, c)).$$

Because quantity must equal c at this price, $p_V(\lambda, \infty; z(\lambda, c)) = p_V(\lambda, c)$. This immediately implies that the above expression is the integrand of 9.

Now, examine the case for the uncertainty-facing firm. First, examine the problem of choosing the optimal $p_U(c)$ given capacity constraint c:

$$p_U(c) = \text{argmax}_p \int_0^{\bar{\lambda}(p,c)} pq(p, \lambda)\ d\lambda + \int_{\bar{\lambda}(p,c)}^1 pc\ d\lambda \tag{10}$$

where $\bar{\lambda}(p, c)$ is the value of λ above which the capacity constraint binds, given price p. Because $pq(p, \lambda)$ is strictly concave in p and $\bar{\lambda}(p, c)$ is continuous in p, this can be solved using a first-order condition, and the derivative of *ex ante* profits with respect to c can be found using the envelope theorem.

Write the *ex ante* profits:

$$\Pi_U(c) = \int_0^{\bar{\lambda}_U(c)} p_U(c) q(p_U(c), \lambda)\ d\lambda + \int_{\bar{\lambda}_V(c)}^1 p_U(\lambda, c) c\ d\lambda.$$

Using the envelope theorem, we can write the derivative with respect to capacity as

$$\frac{\partial \Pi_U(c)}{\partial c} = \frac{\partial \bar{\lambda}(p, c)}{\partial c}\bigg|_{p = p_U(c)} [p_U(c) q(p_U(c), \bar{\lambda}_U(c))] - \frac{\partial \bar{\lambda}(p, c)}{\partial c}\bigg|_{p = p_U(c)} [p_U(c) c] + \int_{\bar{\lambda}_U(c)}^1 \frac{\partial}{\partial c}\Big[pc\Big]_{p = p_U(c)} d\lambda.$$

Because $q(p_U(c), \bar{\lambda}_U(c)) = c$, the first two terms on the right hand side of the above equation cancel. This yields

$$\begin{aligned}\frac{\partial \Pi_U(c)}{\partial c} &= \int_{\bar{\lambda}_U(c)}^{1} \frac{\partial}{\partial c}\Big[pc\Big]_{p=p_U(c)} \, d\lambda \\ &= \int_{\bar{\lambda}_U(c)}^{1} p_U(c) \, d\lambda \\ &= Pr(\lambda > \bar{\lambda}_U(c))p_U(c)\end{aligned}$$

where the last term simply re-expresses the integral in terms of the probability of a sellout. □

> **Proposition 3.2.** Suppose there exists a choke price $p_{choke}(\lambda)$ and that $qq_{p\lambda} - q_p q_\lambda > 0$, where subscripts denote derivatives. Let $\bar{c}$ be the level of capacity above which sellouts never occur in either setting, U or V, and let $0 < c^* < c^{**} < 1$. Then
>
> $$\lambda_V(c) - \lambda_U(c) < 0 \text{ if } c \in [0, c^*)$$
>
> and
>
> $$\lambda_V(c) - \lambda_U(c) > 0 \text{ if } c \in (c^{**}, \bar{c}].$$

Proof: Define the following object: $p_V(c) = p_V(c, \bar{\lambda}_V(c))$. This is the volatility price at the threshold λ above which sellouts occur. $p_V(\lambda, c)$ is the monopoly price below this threshold and the price such that $q(p_V(\lambda, c), \lambda) = c$ above this threshold. At $\bar{\lambda}_V(c)$ these thresholds equal each other. Thus, $p_V(c, \bar{\lambda}(c))$ follows the monopoly price $\bar{\lambda}_V(c)$ as c increases. That is,

$$p_V(c) = -\frac{q(p_V(c), \bar{\lambda}_V(c))}{\frac{\partial q(p_V(c), \bar{\lambda}_V(c))}{\partial p}}.$$

Now examine $p_U(c)$. Differentiating the objective in Equation 10 yields the following first order condition:

$$\int_0^{\bar{\lambda}_U(c)} p_U(c) \frac{\partial q(p_U(c), \lambda)}{\partial p} + q(p_U(c), \lambda) \, d\lambda + \int_{\bar{\lambda}_U(c)}^{1} c d\lambda = 0.$$

which, after some maniuplation, can be conveniently expressed as

$$p_U(c) = \frac{\mathbb{E}(\tilde{q}(p_U(c),\lambda,c))}{\bar{\lambda}_U(c)\mathbb{E}\left[\frac{\partial q(p_U(c),\lambda)}{\partial p}\middle|\lambda < \bar{\lambda}_U(c)\right]} \tag{11}$$

$$= -\left(\frac{\mathbb{E}(q(p_U(c),\lambda)|\lambda < \bar{\lambda}_U(c))}{\mathbb{E}\left[\frac{\partial q(p_U(c),\lambda)}{\partial p}\middle|\lambda < \bar{\lambda}_U(c)\right]} + \frac{(1-\bar{\lambda}_U(c))c}{\bar{\lambda}_U(c)\mathbb{E}\left[\frac{\partial q(p_U(c),\lambda)}{\partial p}\middle|\lambda < \bar{\lambda}_U(c)\right]}\right). \tag{12}$$

where $\tilde{q}(p,\lambda,c) = \min\{q(p,\lambda),c\}$. Both $p_V(c)$ and $p_U(c)$ are continuous in c by continuity of q and the distribution of λ. For sufficiently small values of c, I claim $p_U(c) > p_V(c)$. If c is vanishingly small, $\lambda_U(c)$ approaches zero, $p_V(c)$ and $p_U(c)$ approach $p_{choke}(0)$, and the first term on the right hand side of 12 approaches the value at $\lambda = \lambda_U(c)$:

$$\frac{\mathbb{E}(q(p_U(c),\lambda,c)|\lambda < \bar{\lambda}_U(c))}{\mathbb{E}\left[\frac{\partial q(p_U(c),\lambda)}{\partial p}\middle|\lambda < \bar{\lambda}_U(c)\right]} \to \frac{\mathbb{E}(q(p_U(c),\lambda)|\lambda = \bar{\lambda}_U(c))}{\mathbb{E}\left[\frac{\partial q(p_U(c),\lambda)}{\partial p}\middle|\lambda = \bar{\lambda}_U(c)\right]}.$$

If $p_V(c) = p_U(c)$, the term on the right hand side of A equals $p_V(c)$. Because, as $c \to 0$, $p_U(c), p_V(c) \to p_{choke}(0)$, the first term in in 12 approachs $p_V(c)$. The second term in 12 is weakly positive, implying that $p_U(c) > p_V(c)$ for sufficiently small values of c.

If $p_U(c) > p_V(c)$, the $q(p_U(c),\lambda) < q(p_V(c),\lambda)$ for any value of λ. In particular, $q(p_U(c),\bar{\lambda}_V(c)) < q(p_V(c),\bar{\lambda}_V(c)) = c$, implying that $\bar{\lambda}_U(c) > \bar{\lambda}_V(c)$ for sufficiently small c.

Now consider what happens as $c \to \bar{c}$. When $c \geq \bar{c}$, note that $\bar{\lambda}_U(c) = \bar{\lambda}_V(c) = 1$. By continuity of $\bar{\lambda}$, this implies that $\bar{\lambda}_U(c), \bar{\lambda}_V(c) \to 1$ as $c \to \bar{c}$. Examining 12, this implies that the second term on the right hand side approaches zero. Thus,

$$p_U(c) - p_V(c) \to \frac{q(p_V(c),\bar{\lambda}_V(c))}{\frac{\partial q(p_V(c),\bar{\lambda}_V(c))}{\partial p}} - \frac{\mathbb{E}(q(p_U(c),\lambda)|\lambda < \bar{\lambda}_U(c))}{\mathbb{E}\left[\frac{\partial q(p_U(c),\lambda)}{\partial p}\middle|\lambda < \bar{\lambda}_U(c)\right]}.$$

Because $\bar{\lambda}_V(c)$ and $\bar{\lambda}_U(c)$ approach 1, this can instead be written as

$$p_U(c) - p_V(c) \to \frac{q(p_V(c),1)}{\frac{\partial q(p_V(c),1)}{\partial p}} - \frac{\mathbb{E}(q(p_U(c),\lambda))}{\mathbb{E}\left[\frac{\partial q(p_U(c),\lambda)}{\partial p}\right]}.$$

Suppose $p_U(c) = p_V(c)$. The assumption that $q_{p\lambda} q - q_p q_\lambda > 0$ implies that

$$\frac{\partial}{\partial \lambda} \frac{q(p, \lambda)}{\frac{\partial q(p,\lambda)}{\partial p}} < 0.$$

This means that the second term on the right hand side of A is larger than the first. As a result, $p_U(c) - p_V(c)$ is negative in a neighborhood of $\bar{c}$, a contradiction. Furthermore, by concavity, if $p_U(c) > p_V(c)$, by concavity, $p_U(c) - p_V(c)$ is even more negative. Thus, $p_U(c) < p_V(c)$ in a neighborhood of $\bar{c}$.

If $p_U(c) < p_V(c)$, then $\bar{\lambda}_U(c) > \bar{\lambda}_V(c)$ at this particular value of c. Thus, $\bar{\lambda}_V(c) - \bar{\lambda}_U(c)$ in a neighborhood of $\bar{c}$. □

Theorem 3.3 Suppose there exists a choke price $p_{choke}(\lambda)$ and that $q q_{p\lambda} - q_p q_\lambda > 0$, where subscripts denote derivatives. Then there exists some c^* and c^{**} such that

$$\frac{\partial \Pi_V(c)}{\partial c} > \frac{\partial \Pi_U(c)}{\partial c}$$

if $c < c^*$, and

$$\frac{\partial \Pi_V(c)}{\partial c} < \frac{\partial \Pi_U(c)}{\partial c}$$

if $c > c^{**}$ and $c < \bar{c}$.

Proof: First, consider a small capacity c. By Proposition 3.2, if c is sufficiently small, $\bar{\lambda}_V(c) < \bar{\lambda}_U(c)$. That is, the uncertainty-facing firm is less likely to sell out than the volatility facing firm. Furthermore, $p_U(c) < p_{choke}(0)$; otherwise it would earn zero revenues.

Examining Equation 9, it is clear that for c sufficiently small, the first term in the integrand disappears, because $\frac{\partial q(p,\lambda)}{\partial p} \neq 0$. This means we can write

$$\frac{\partial \Pi_V(c)}{\partial c} \to \int_{\bar{\lambda}_V(c)}^{1} p_V(\lambda, c)\, d\lambda, \quad c \to 0$$

which is the same as

$$\frac{\partial \Pi_V(c)}{\partial c} \to (1 - \bar{\lambda}_V(c)) \mathbb{E}(p_V(\lambda, c) | \lambda \geq \bar{\lambda}_V c), \quad c \to 0. \tag{13}$$

Note that $p_V(0, c) \to p_{choke}(0)$ as $c \to 0$, which immediately implies that as c becomes very small, $p_V(c,\lambda) \to p_V(0,\lambda) \geq p_{choke}(0)$ for all λ. This means that the expectation term 13 is greater than or equal to $p_{choke}(0)$ in the limit.

To summarize, for c sufficiently small:

$$p_U(c) \leq p_{choke}(0) \leq \mathbb{E}(p_V(c,\lambda)|\lambda > \bar{\lambda}_V(c))$$

and, by Propoistion 3.2, for c sufficiently small:

$$(1-\bar{\lambda}_V(c)) > (1-\bar{\lambda}_U(c)).$$

These two inequalities imply

$$(1-\bar{\lambda}_U(c))p_U(c) < (1-\bar{\lambda}_V(c))\mathbb{E}(p_V(c,\lambda)|\lambda > \bar{\lambda}_V(c)),$$

so $\frac{\partial \Pi_U(c)}{\partial c} < \frac{\partial \Pi_V(c)}{\partial c}$.

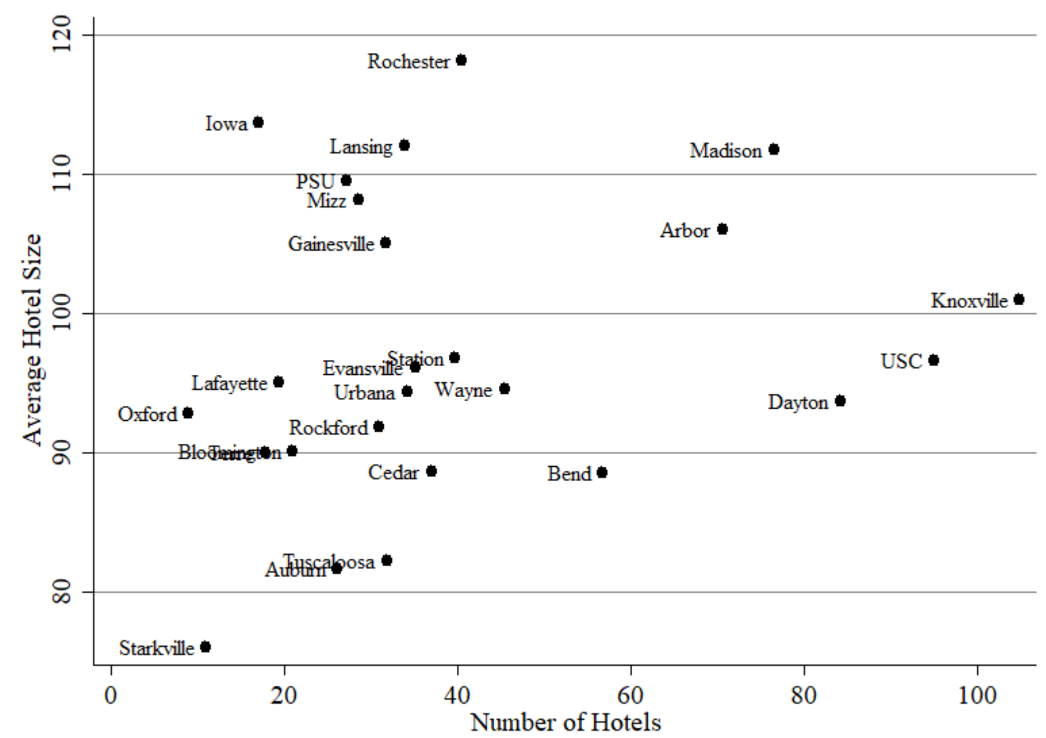
Now, consider high levels of capacity. By Proposition 3.2, for some c close enough to $\bar{c}$, the volatility facing firm will never sell out, but the uncertainty facing firm will sometimes sell out. By Proposition 3.1, this implies that $\frac{\partial \Pi_V(c)}{\partial c} = 0$ but $\frac{\partial \Pi_U(c)}{\partial c} > 0$. This immediately gives us our result that

$$\frac{\partial \Pi_V(c)}{\partial c} < \frac{\partial \Pi_U(c)}{\partial c}$$

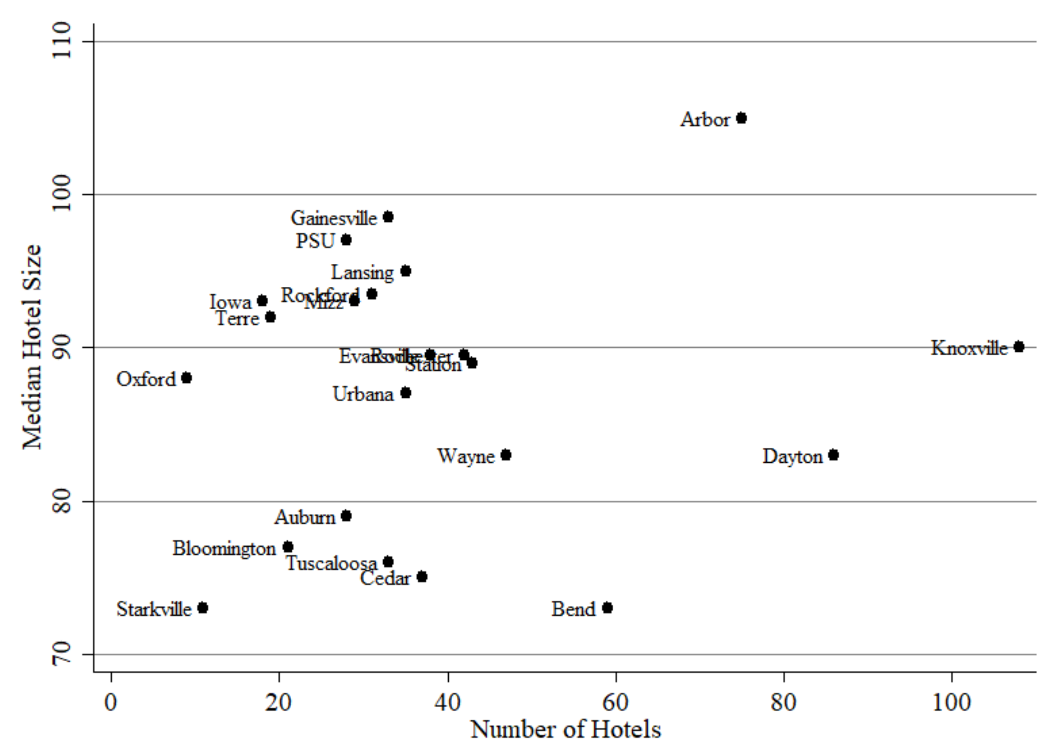
for sufficiently large c. $\square$

Figure 2: Market structure heterogeneity

(a) Mean hotel size

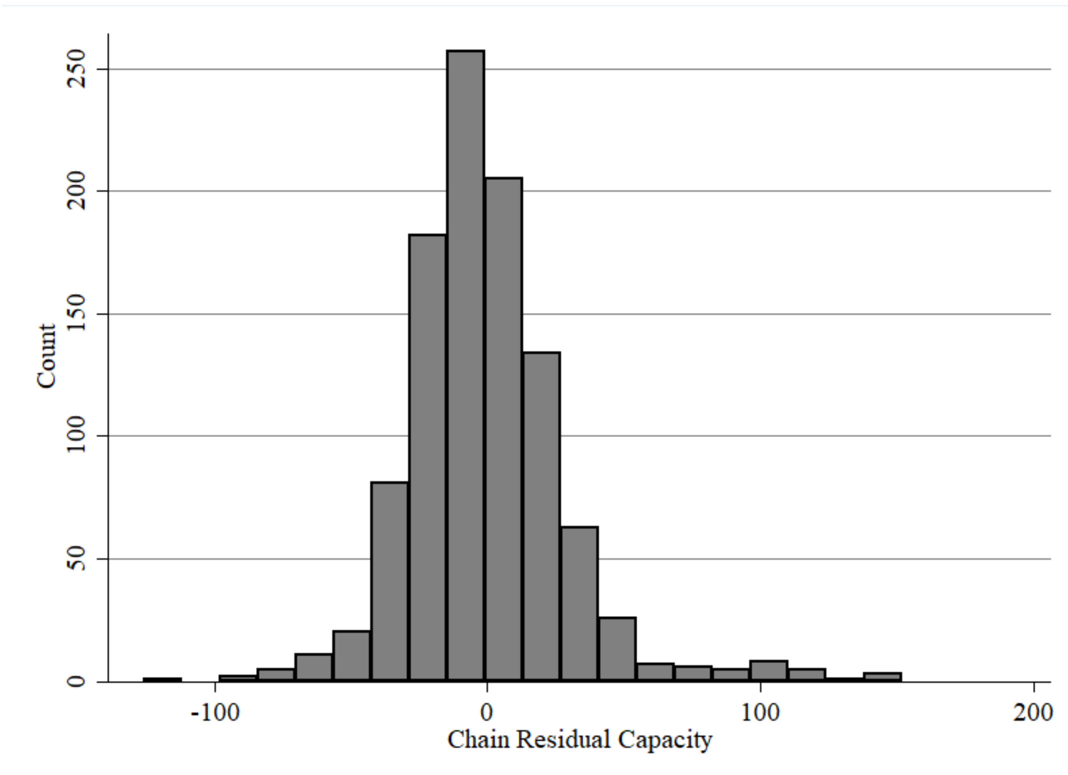


(b) Median hotel size



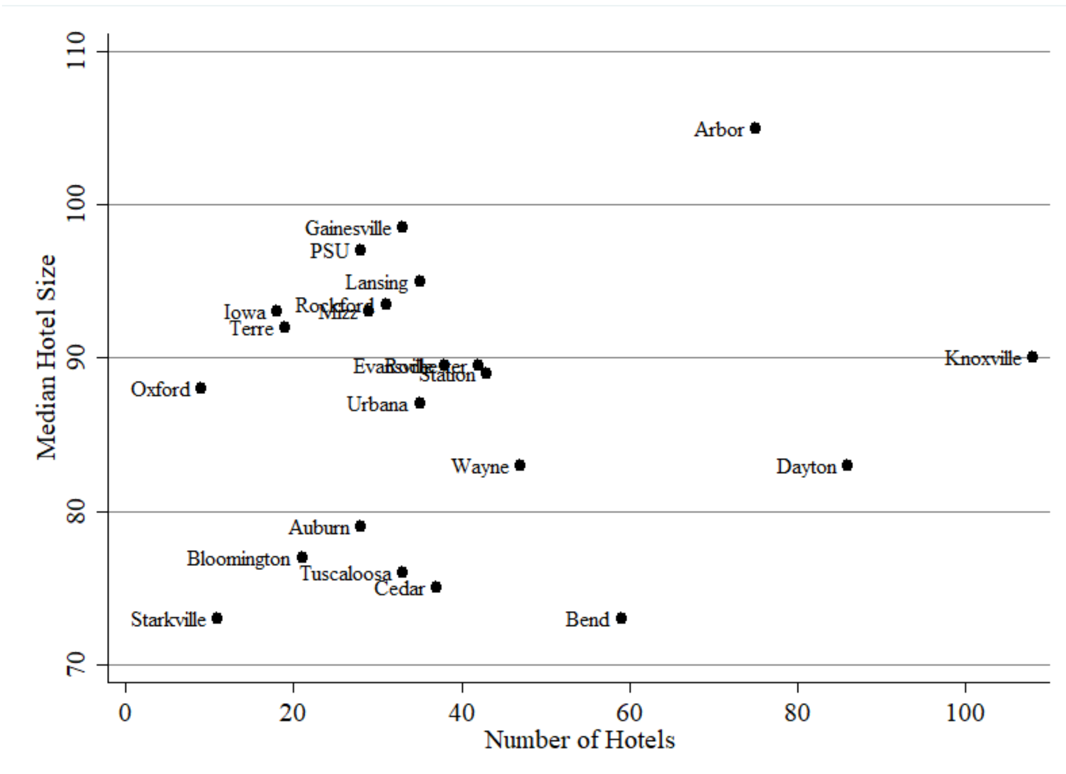
Data source: STR, Inc.

Figure 3: Residual hotel size, conditional on brand



Data source: STR, Inc. Plots frequency of residual from regression of capacity on fixed effects for each brand represented in dataset. Capacity measured in total number of rooms. Does not include independent hotels.

Figure 4: Volatility and uncertainty estimates, by market



Data sources: STR, Inc., Expedia, and *Travel Weekly*