

Algorithmic Competition, with Humans

Matthew Leisten¹

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¹Federal Trade Commission. This work represents my views alone and not those of the Commission, its Commissioners, or the United States Government.

Algorithmic (automated) pricing

For many (most?) firms, pricing partially *automated*

Does this increase markups above competitive levels?

- ▶ Yes, no, maybe, depending on model

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Algorithms may:

- ▶ Provide *commitment* to irrational pricing off-path
- ▶ Improve *prediction* so pricing on-path is ex post rational

This paper

Study **algorithmic competition with managerial override**

1. Firms design algorithms, mapping rivals' price to own
2. (or 2+) Algorithms run *or* firms choose prices manually

Choosing manually is *costly* but *useful* if algorithms are “failing”

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I show this is:

- (Tractable): analytic solution exists
- (Instructive): highlights roles of prediction and commitment
- (Falsifiable): significantly refines equilibrium predictions
- (Sufficient): explains patterns in real pricing data...
- (Necessary*): ...in ways existing models cannot

The game, in general

Two-stages, symmetric differentiated duopoly, demand $q(p_i, p_{-i})$

Stage 1: Firms simultaneously set algorithms σ_i

Stage 2: Firms simultaneously defer to algorithm *or* choose price

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- Action set is $\{ \underbrace{\mathbb{R}^+}_{\text{Override, set a price}}, \underbrace{\sigma}_{\text{Defer to algorithm}} \}$

- One-shot pricing
- Overriding may come at cost c ; no marginal costs

Pricing stage details

Algorithms in place, $\sigma = (\sigma_1, \sigma_2)$ define subgame.

If rival chooses a *price* p_{-i} :

$$a_i = \arg \max_{a \in \{\mathbb{R}^+, \sigma\}} p_i q_i(p_i, p_{-i}) - c * 1[a_i \in \mathbb{R}^+]$$

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If rival chooses its *algorithm* σ_{-i} :

$$a_i = \arg \max_{a \in \{\mathbb{R}^+, \sigma\}} p_i q_i(p_i, x_{-i} + z_{-i} p_i) - c * 1[a_i \in \mathbb{R}^+]$$

In Pictures

Pricing Stage Lemma

Informal summary of theoretical results

If $c = \infty$, *any* price between Bertrand and collusive is possible.

Details

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If $c = 0$, \exists equilibrium in which:

- ▶ Algorithms *match* changes in rival price but *undercut* levels
- ▶ Only one firm, chosen at random, overrides
- ▶ Undercutting designed so:

$$\sigma_i(p^{BR}(\sigma_i)) = p^{BR}(p^{BR}(\sigma_i))$$

Prices far from competitive in general

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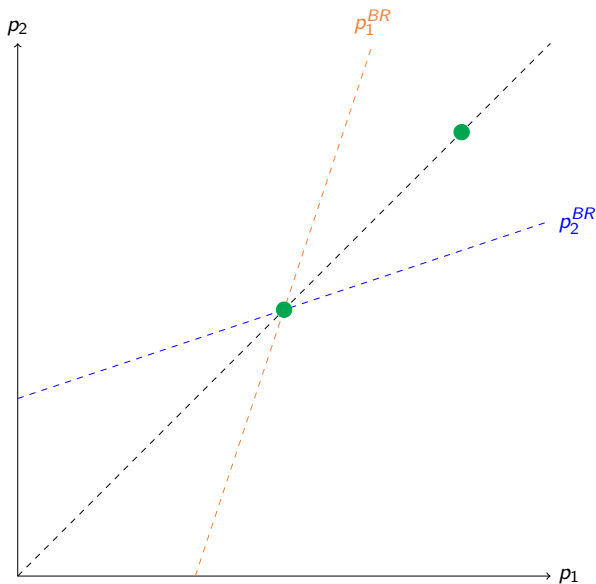
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Prices far from competitive in general

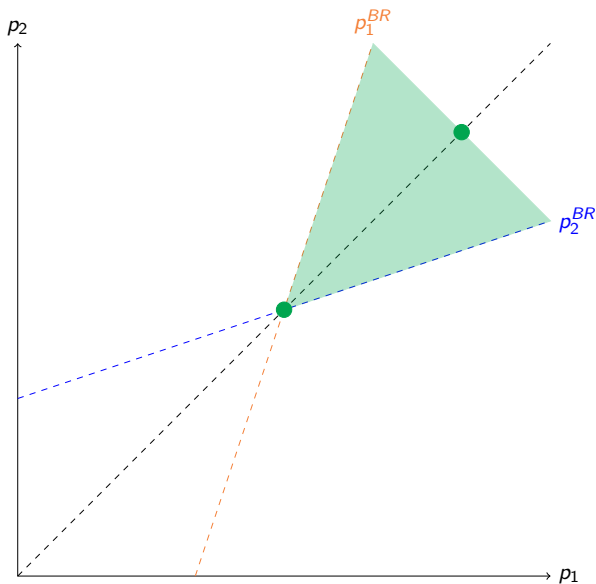
“Collusion by algorithm” sometimes also an equilibrium

Technical details If $c > 0$

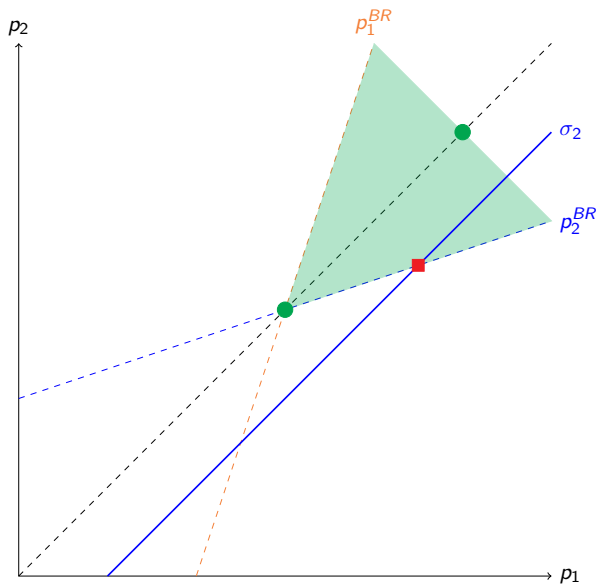
The theory, in pictures



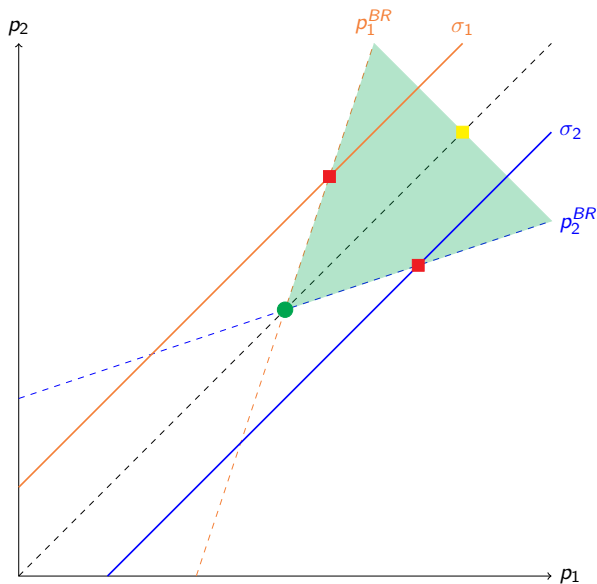
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The theory, in pictures



Commitment vs. Prediction

Suppose state of nature θ drawn between algorithm-setting and pricing stages:

$$q_i = q(p_i, p_{-i}, \theta)$$

or

$$\pi_i = q(p_i, p_{-i})(p - \theta)$$

If $c = 0$ and algorithms cannot condition on θ , then prices generically Bertrand.

Prediction vs. commitment, a synthesis

	Full Commitment	No Commitment
Full Prediction	"Anything goes"	One-sided override or collusion
No Prediction	"Anything goes"	Generically Bertrand

In sustaining supracompetitive prices, prediction and commitment are *substitutes*: one or the other is good enough

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Most settings w/ algorithmic pricing involve *repeated* interaction

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- ▶ If i overrides, choose best response to rival's algorithm:

$$p_{it} = \arg \max_p \pi(p, p - x_{-i}, \theta_t)$$

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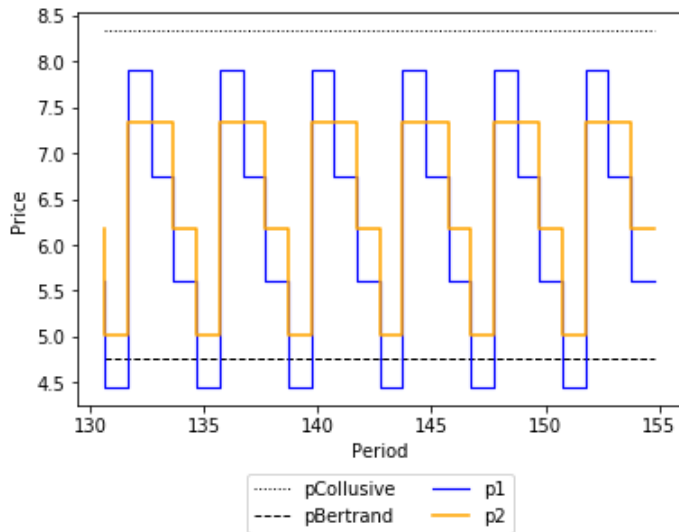
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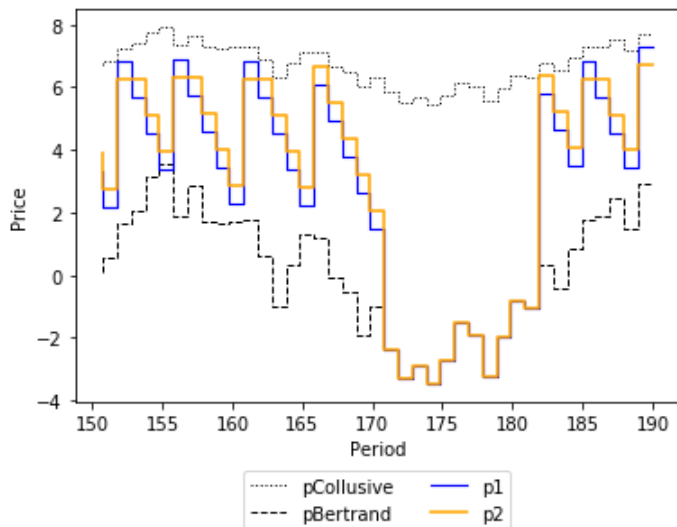
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Edgeworth Cycles



Bertrand Reversion



From theory to data

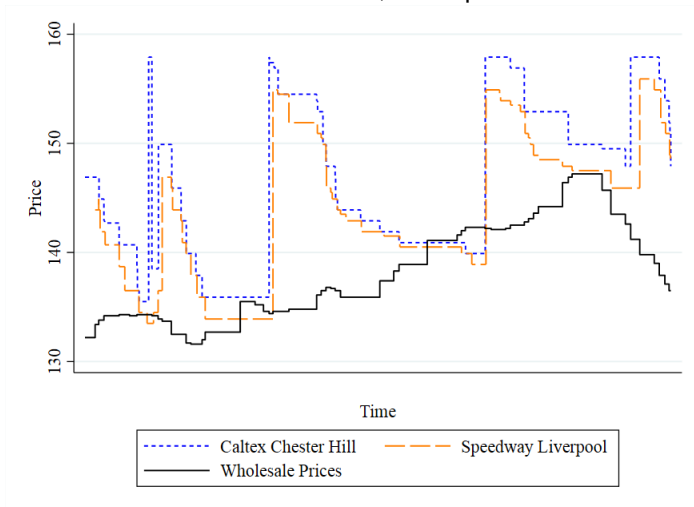
Key model predictions:

- ▶ Existence of Edgeworth cycles [Here](#)
- ▶ Price decreases should seem automated
 - ▶ Decreases should be uniform in size
 - ▶ Price “matching” should happen quickly [Here](#)
- ▶ Price increases should resemble override
 - ▶ Prices should better reflect marginal costs after increases [Here](#)
 - ▶ Increases should be timed for when opportunity cost of undercutting low [Here](#)
- ▶ Cost volatility may lead to “freefall” pricing
 - ▶ Unusually large price decreases only when marginal costs volatile [Here](#)
 - ▶ Less so for increases [Here](#)

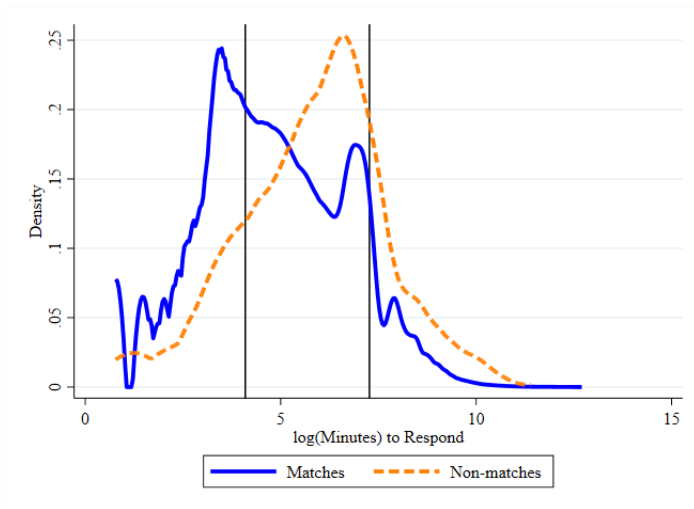
Conclusion

Edgeworth cycles in real life

FuelWatch: timestamped price changes for every gas station in New South Wales, 2018-present



Speed of responses



Note: vertical lines at 1 hour and 1 day

[Details](#)

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Only resets are strategic

What predicts p_{it} ?

Define jump as

1. $p_{it} > p_{i,t-dt}$, i.e., i increases price
2. $p_{i,t-dt} \leq p_{i,t-2dt} \forall i$, i.e., not a match of a rival's increase

Variable	Partial Correlation	
	Non-Jumps	Jumps
Rival Price	0.73	0.41
Wholesale Price	0.18	0.36
Traffic Volume	0.02	0.04

Back

Resets timed strategically

Probit regressions to predict 1[Jump occurs at t]:

Variable	(1) Coef (Std. Err)	(2) Coef (Std. Err)	(3) Coef (Std. Err)
Wholesale Cost	0.280*** (0.017)	0.281*** (0.017)	0.301*** (0.016)
Rival Price	0.023** (0.010)	0.023** (0.010)	0.009 (0.011)
Lagged Own Price	-0.235*** (0.012)	-0.236*** (0.012)	-0.238*** (0.01)
Traffic Volume	- -	- -	-0.056*** (0.005)
N	728,032	728,032	475,190
Pr[Y=1]	0.071	0.071	0.072

Back

Freefall pricing

Construct volatility measures using (1) historical rack price volatility and (2) OVX index

Variable	(1) Coef (Std. Err)	(2) Coef (Std. Err)	(3) Coef (Std. Err)	(4) Coef (Std. Err)	(5) Coef (Std. Err)
Δ Wholesale	-	-0.030*** (0.010)	-0.043*** (0.010)	-0.008 (0.010)	-0.016 (0.011)
Volatility	0.078*** (0.013)	0.071*** (0.013)	- (0.013)	0.073*** (0.015)	- (0.015)
OVX	-	-	0.054*** (0.013)	-	0.069*** (0.022)
N	412,622	412,538	412,538	412,538	412,538
Fixed Effects	-	-	-	Monthly	Monthly

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Conclusions

Algorithmic competition with managerial override is:

- (Tractable): analytic solution exists
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Implications:

1. Algorithms can do damage! But must consider (1) extent and ease of human involvement, (2) predictive abilities of algorithms
2. Instead of puzzling over Edgeworth cycles, back out an algorithm/human combo that generates it

Thank you!

Questions or comments?
mattleisten@gmail.com
<https://mleisten.github.io>

Algorithms as commitment: Brown and Mackay (2021), Salcedo (2015)

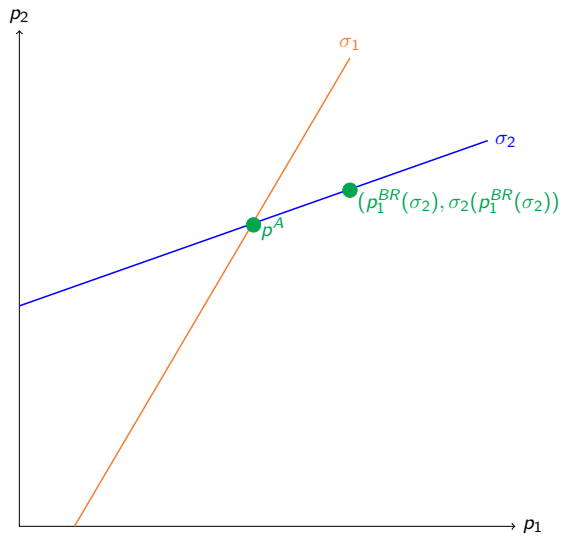
Algorithms as prediction: Miklos-Thal and Tucker (2019), O'Connor and Wilson (2019)

Algorithms as learning: Asker et al. (2021), Assad et al. (2020), Johnson et al. (2020), Calvano et al. (2020), Klein (2019)

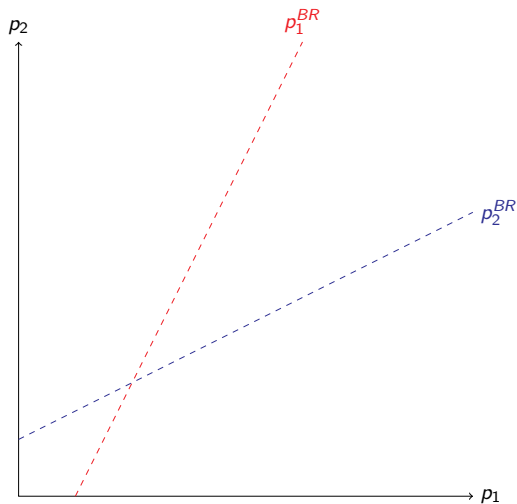
Forebears in conduct: Klemperer and Meyer (1989), Rubinstein and Abreu (1988), Maskin and Tirole (1988), Salop (1986)

Gasoline: Assad et al. (2020), Byrne and DeRoos (2019), Clark and Houde (2013), Wang (2009), Hosken et al. (2008), Noel (2007)

Pricing, Illustrated

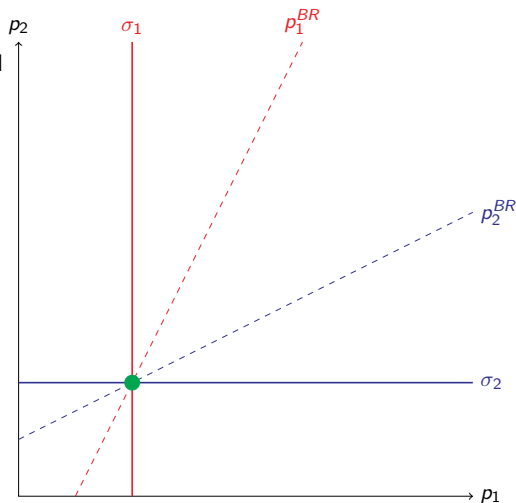


No override ($c = \infty$, Brown and Mackay, 2021)



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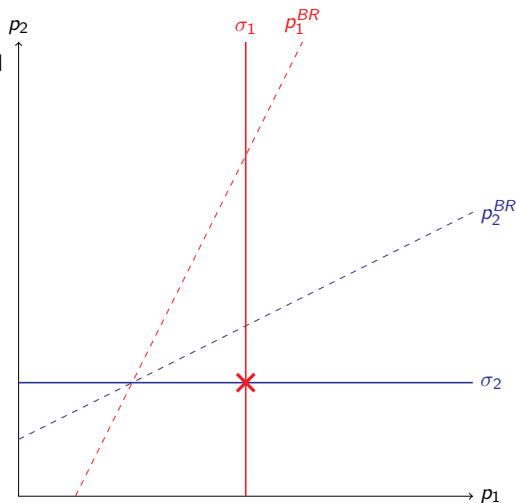
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...but cannot sustain
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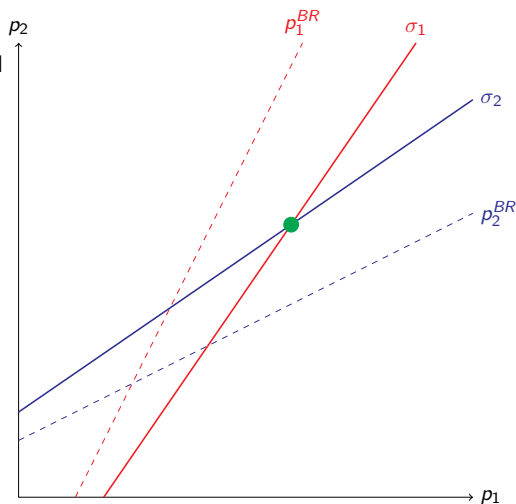


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Upward sloping algorithms
soften competition, sustain
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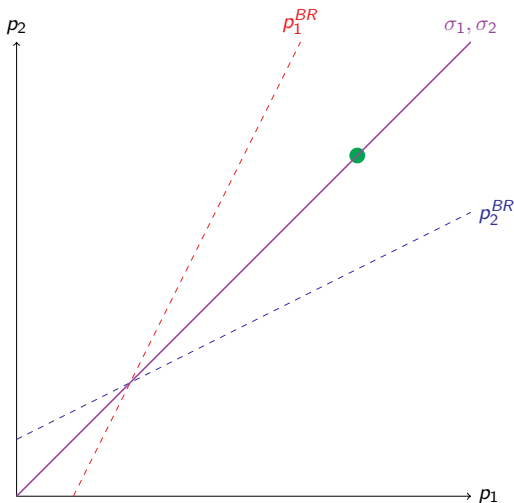
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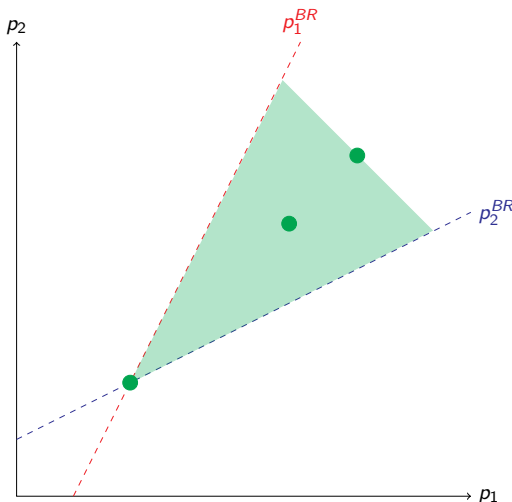
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Really, “anything goes”



Pricing stage game

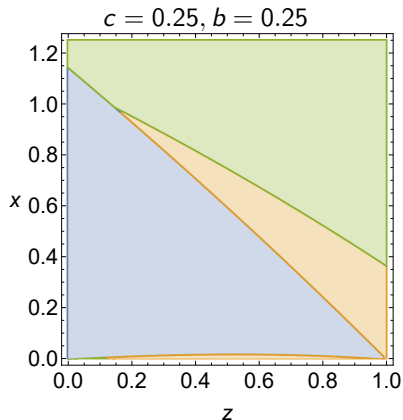
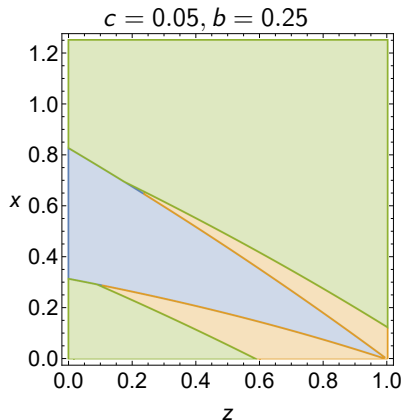
Proposition: One of these must be true:

1. Algorithms are sustained in an equilibrium. Prices are p^A .
2. One firm overrides its algorithm in an equilibrium. Prices are $(p^*(\sigma_{-i}), \sigma_{-i}(p_i^*(\sigma_{-i})))$.
3. The unique equilibrium is Bertrand pricing.

Note: Bertrand may still exist if (1) or (2) is true.

Pricing stage: example

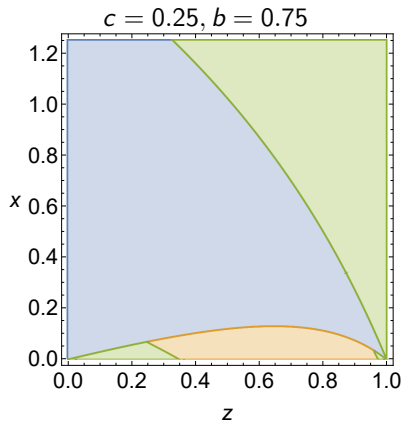
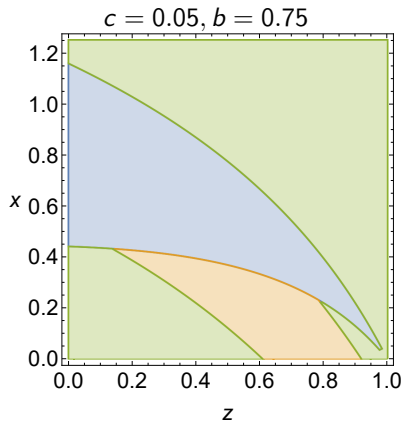
Linear demand: $q_i = 1 - p_i + bp_{-i}$



Algorithmic, One-sided override, Bertrand

Pricing stage: example

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Algorithmic, One-sided override, Bertrand

Public randomization

Four types of equilibrium: “Algorithmic”, “Only Firm 1 Overrides”, “Only Firm 2 Overrides”, “Bertrand”

Existence profile $r(\sigma) \in \{0, 1\}^4$, with entry $r_j = 1$ if equilibrium of type j exists when algorithms are σ

Assumptions:

- ▶ Probability of equilibrium type j is measurable w.r.t. $r(\sigma)$.
- ▶ If \exists a non-Bertrand equilibrium, Bertrand is never played.
- ▶ All non-Bertrand equilibria are played with positive probability.

What if $c > 0$?

Generally difficult to characterize. Less ambitious question: is collusion by algorithm possible?

Equivalent to: Can firm i do better than collusion by inducing rival to override their algorithm, as in the $c = 0$ case?

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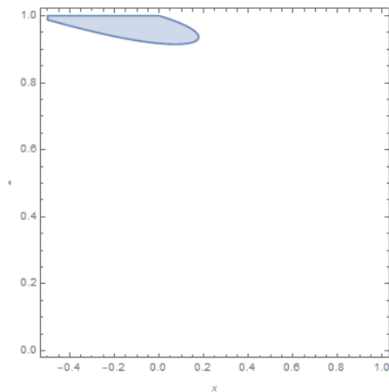
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Rule of thumb / conjecture: Scope for collusion by algorithm decreasing, then increasing, in c .

Example: $q_i = 1 - p_i + .5p_{-i}$

Feasibility

Rationality



c sufficiently small so $-i$ overrides

c sufficiently large so i does not override

Example: $q_i = 1 - p_i + .5p_{-i}$

Rationality, Feasibility

Searching for “equilibrium”

Simulate to find equilibrium in algorithms $\{x_i\}$:

1. Start with $x_{-i} = 0$, set grid X
2. For each gridpoint $x \in X$:
 - ▶ Simulate i average payoffs over time, setting $x_i = x$.
 - ▶ Set x_{-i} equal to i 's best gridpoint
 - ▶ Iterate to convergence

Speed of responses

i changes price at t , previous rival price change at $t - dt$.

Compute distance between i 's latest price change and last rival's price change:

$$M_{it} = |(p_{it} - p_{i,t-dt}) - (p_{-i,t-dt} - p_{-i,t-2dt})|$$

Plot distribution of dt when $M_{it} = 0$ versus not

Placebo test: “freerise”

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Volatility	-0.022 (0.021)	-0.013 (0.021)	- (0.022)	0.032 (0.025)	- (0.040)
OVX	-	-	-0.046** (0.022)	-	0.049 (0.040)
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