COMP/BMME 775, Medical Image Analysis Fall 2014, Pizer
Assignment 2: **Segmentation by deformable models**. Due in sections
Part 2, of 4. Shape space for ASM and test images Assigned 30 Sept. 2014, Due 9 Oct. 2014

In the multipart assignment of which this component is a part you are ultimately to compare two deformable model methods for segmenting deformed ellipsoids in 3D images, namely geodesic snakes using SNAP and active shape models (ASM). The assignment is due in sections.

II. This second section involves preparing to use ASM by forming a shape space on a PDM using PCA. It is due on Thursday, 9 October. It consists of three parts. In the first part you will create object descriptions that you will use as training data to form a shape space. In the second part you will create a shape space by PCA, i.e., treating the data as if it were entirely Euclidean. In the third part you will create an alternative shape space by Euclideanizing and commensurating the non-Euclidean components of the data before you apply PCA.

Part 1. Creating the object data
The images that you will be segmenting will contain ellipsoids as objects. There will be up to 9 different variations: of the 3 center coordinates: c_x, c_y, c_z; of the three principal radii: r_x, r_y, r_z; and of the 3 degrees of freedom in the orientation of the ellipsoid's principal directions: these orientations will be represented a rotation of the standard x,y,z coordinate system by two angles giving the latitude and longitude of the axis of rotation and a third angle: the angle of rotation about the axis (note that the quaternion representation of rotation directly encodes this information).

In a 1:128 × 1:128 × 1:128 image let the mean ellipsoid be centered at (64,64,64), have principal radii of 45, 27, and 18 voxels, and have an orientation such that the longest axis is along x, the next longest is along y, and the least long axis along z. The mean will be understood to have a rotation angle of zero.

You will need to create a PDM from each of many of these ellipsoids. This is provided for the base ellipsoid (with the mean values listed previously) in a file named *base_ellipsoid.mat*. It will be a 1D array, called $\mathbf{z}_{\mathrm{base}}$, of 222 features: an *x*, a *y*, and a *z* at 74 different points spaced around the ellipsoid, i.e., $(x_1, y_1, z_1, x_2, y_2, z_2, \ldots, x_{74}, y_{74}, z_{74})$. In that array its center is (0,0,0), its radii are as specified above, and its rotation angle is zero. Every other ellipsoid can be computed by first applying *r* stretchings: r_x/r_{base}, r_y/r_{base}, r_z/r_{base}, respectively to the *x*, *y*, and *z* coordinates of the base ellipsoid, followed by applying the rotation to all of the resulting points to produce x',y',z' for each point, followed by a translation $(c_i - c_{i,\mathrm{base}})$, i=1,2,3, different for each coordinate) on each of the resulting 3 coordinate values (don't forget also to add 64 to each center coordinate to get it to fit correctly into the image).

Produce a random ellipsoid PDM by sampling from many different zero-mean Gaussians, each with their own standard deviations, as follows. The standard deviations you choose should be a parameter of your program and should keep the ellipsoid center within 6 voxels in each direction from the mean, the ellipsoid radii within 10% of the mean, and the rotation angles each within $\pi/12$ radians of the mean. Thus you will need to sample from the following zero-mean Gaussians:

A 3D Gaussian on the center point offset: $c_x - c_{x,base}$, $c_y - c_{y,base}$, $c_z - c_{z,base}$, understood as Euclidean variables.

A 3D Gaussian on $r_x / r_{x,base}$, $r_y / r_{y,base}$, $r_z / r_{z,base}$, understood as positive variables that thus require the log transformation before the Gaussian is applicable.

A 2D Gaussian on the Euclideanized axis direction, understood as point on a 2-sphere. You can create the points by sampling from a 2D Euclidean Gaussian and considering each result as being on the tangent plane to the sphere at the north pole. This is equivalent to using the 2D duples directly as an angle pair.

A 1D Gaussian on the rotation angle. Again you can use a Euclidean Gaussian on the angle variable.

Create n=25 such random ellipsoids centered at (64,64,64) and store their PDM 222-tuples in a data matrix A_{coords}.

Part 2. Analyzing the object data via PCA

Each geometric variation of an ellipsoid will be a compound of all 9 parameter variations, so it lives in a 9-dimensional space. Your job is to select the top 9 eigenmodes from a principal shape analysis applied to the PDM and to compute the fraction of the total variance captured by these eigenmodes. You should compute the properties of 9 modes of variation and display them by showing the ellipsoid at -2 times its principal standard deviation from the mean, 0 times its principal standard deviation from the mean, and +2 times its principal standard deviation from the mean. You can visualize these ellipsoids best using the matlab program provided as *disp3DMesh.m* on the sakai site for this assignment. That program requires 4 *.mat* files that are respectively called *north*, *south*, *east*, and *west*, which are also provided on the sakai site for this assignment. Note that by pushing the rotate (circular arrow) button when using *disp3DMesh.m*, you are in a mode of changing the viewpoint interactively using your mouse. You can also zoom in and out by switching to zoom in / zoom out mode by pushing the corresponding button.

With regard to forming the shape space, you should write a program that

1. takes in the *n* tuples of N/3 (x_i,y_i,z_i) points (here n will be 25 and N/3 will be 74). Your program should compute the mean of the *n* PDMs and subtract that mean from each PDM. Save the resulting mean for use in later sections of this assignment.

2. from the results of step 1, forms the matrix AA^T. with $A = A_{coords}$. Then compute *(N-1)* times the total variance as trace $(AA)^T$. The total variance is given by the trace of the matrix 25×25 $(1/(N-1))\ AA^T$, and the variance captured by the 9 eigenmodes is the sum of their 9 principal variances. Turn is this value.

3. does principal component analysis of the 222-tuples. That is, do eigenanalysis of the 25×25 AA^T, and generate 9 eigenvalues, sorted in decreasing order,. After multiplication by A^T (and normalization, if you will need unit eigenvectors) these eigenvectors will form 9 principal directions (eigenmodes of the estimated covariance matrix $(1/(n-1))\ A^TA$). The eigenvalues of AA^T will be principal variances σ_i^2 times

(n-1). Save the 9 unit eigenvectors and their 9 principal variances for use in later parts of this assignment.

4. For each of i = 1, 2, …, 9, display the following triple of PDMs: the mean PDM – $2\sigma_i$ × the i^{th} eigenvector, the mean PDM, and the mean PDM + 2 σ_i × the i^{th} eigenvector. Answer the question: How are these related to the 9 known modes of variation?

Part 3. Analyzing the object data via CPNS, i.e., Euclideanization, commensuration, and then PCA

Transform each ellipsoid PDM to its center point offset, by computing and subtracting its center of mass; its scale variable γ, by computing the square root of sum of squares of the individual point offsets from the center of mass; as well as the scaled, centered PDM, which lives on a unit sphere. Use the PNS program called *PNSmain.m* that you can find on the sakai site for the assignment to transform each of the *n* training ellipsoids into a tuple of *n-1*=24 Euclideanized values. Multiply each ellipsoid's Euclideanized values by its scale value γ to produce the values commensurate with the center position values, and also compute the commensurated γ value, namely $\bar{\gamma}\, ln\left(\frac{\gamma}{\bar{\gamma}}\right)$. Each training ellipsoid's result will be a 28-tuple of commensurated values: 24 from the sphere, 3 from the center, and the scale. The PNS will also produce a polar system, which you can use to invert a commensurated Euclideanized tuple into a boundary PDM, using the provided program *PNSe2s.m*. *PNSmain.m* produces two arrays as output. The first is the Euclideanized values (called *resmat*), and the second is the polar values (called *PNS*). This polar values array provide the second input (also called *PNS*) to *PNSe2s.m*. The other (first) input to *PNSe2s.m* is a 1D array of Euclideanized values (also called *resmat*).

Create an *n* × 28 data matrix A, and use your PCA program to produce 9 eigenmodes and eigenvalues and the fraction of the total variance captured by these eigenmodes. Compare this to the fraction you obtained via direct use of PCA without Euclideanization, in part 2. With the use of the inversion to PDM program and the display program, display the 9 modes of variation as you did in part 2. Answer the question: How are these related to the 9 known modes of variation?