

1) 4.3  $G = (V, E)$  Undirected graph  
 whether  $G$  contains simple cycle  
 of length  $\geq 4$   
 run time at most  $O(|V|^3)$

- simple cycle exists if there is a pair  
 of distinct vertices such that they have  
 at least 2 common neighbors

-  $O(|V|^2)$  pair of distinct vertices

-  $(x, y)$  we distinct vertices ~~such that they~~

- check if  $x \neq y$  (which where  $x \neq y$ ) to  
 see if they have 2 common neighbors

- For each vertex  $A$  that is  
 adjacent to  $x$ , check to see if  
 it is also a vertex that is adjacent  
 to  $y$

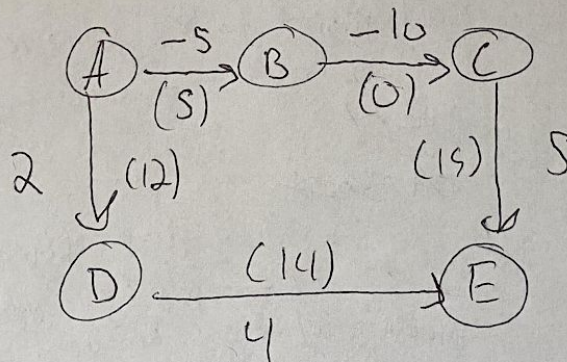
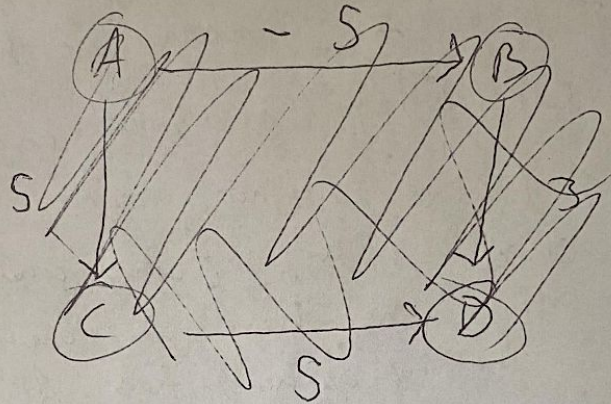
- ~~Abort~~

- Do this through hashing  
 (get's up a hash table ~~for~~  
 populating w/ every adjacent vertex)

- the hashing is done in  $O(1)$  time

2) 4.8

Algorithm for finding shortest path from node S to node T. Directed graph w/ Negative edges. Add a large Constant to each edge weight so that all weights become positive, then run Dijkstra's also at node S & return shortest path found to node T.



Go from A to E, shortest path  
~~Ans~~ Before Constant - A, D, E length = 6  
 Constant value = +10 After Constant - A, B, C, E length = 20  
 (A, D, E) is now 26



3) 4-11

Directed graph w/ positive  
edge lengths  
returns length of shortest cycle  
in the graph  
At most  $O(V^3)$

- Let  $G$  be a graph
- Matrix  $M_{AB}$  w/ shortest path between  $A$  &  $B$
- For any vertice  $C, D$ , there is a shortest paths between  $C$  &  $D$
- $M_{CD} + M_{DC}$  is Length of cycle
- Compute the minimum of  $M_{CD} + M_{DC}$  for any pair of vertices

Run Time

- Any row  $R$  of ~~the~~ the matrix  $M$  ~~can~~ is  $O(V^2)$  by using Dijkstra's algorithm
- ~~time~~ computing  $M_{CD} + M_{DC}$  is  $O(V)$   
 $O(V^2) + O(V) = O(V^2)$