CS 310-0
Homework Assignment No. 5
Due Fri 2/18/2000

1. Find the number of diagonals in a regular n-side polygon.

2. Find the number of integer solutions to the following equation
$$x_1 + x_2 + x_3 + x_4 = 16$$
with each one of the following restrictions:
(a) $x_1, x_2, x_3, x_4 \geq 0$.
(b) $x_1, x_2, x_3, x_4 > 0$.
(c) $1 \leq x_1$, $2 \leq x_2$, $3 \leq x_3$, $4 \leq x_4$.

3. Find the number of integer solutions to the following equation
$$x_1 + x_2 + x_3 = 12$$
with the restrictions: $0 \leq x_1$, $0 \leq x_2 < 6$, $0 \leq x_3 < 10$.

4. A group of people are in a meeting. Of this group, 26 people are married, 29 are from Illinois, 30 are male, 9 are married and from Illinois, 7 are married and male, and 8 are from Illinois and male. What is the minimum possible number of people in that meeting?

5. In a class the students must choose 3 out of 4 subjects A, B, C, D to write an essay about. Subject A is chosen by 21 students, subject B by 18, subject C by 15 and subject D by 12. How many students are there in the class?

6. Let A be the set of all 8-digit numbers in base 3 (so they are written with the digits $0, 1, 2$ only), including those with leading zeroes such as 00120010. The *Hamming distance* between two elements of A is the number of places where they differ, for instance the Hamming distance between 11201001 and 11020020 is 5, because they differ in the 3rd, 4th, 5th, 7th and 8th places.
(a) Find the number of elements in A.
(b) Given an element $a \in A$, find the number of elements in A whose Hamming distance to a is exactly 3.
(c) Given an element $a \in A$, find the number of elements in A whose Hamming distance to a is 3 or less.
(d) Prove that given 12 elements from A, two of them coincide in at least 2 places.

7. Prove the following by induction:

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{2n^3 + 3n^2 + n}{6}\,.$$

8. We define recursively a sequence x_n in the following way:

$$x_0 = 0\,; \qquad x_{n+1} = \sqrt{2 + x_n} \quad (n \geq 0)\,,$$

i.e.: $x_1 = \sqrt{2}$, $x_2 = \sqrt{2 + \sqrt{2}}$, $x_3 = \sqrt{2 + \sqrt{2 + \sqrt{2}}}$, etc.

Prove the following by induction:

(a) x_n is nonnegative, increasing and less than 2, i.e.: $x_n < x_{n+1} < 2$ for every $n \geq 0$.

(b) $0 < 2 - x_n \leq 2^{-n+1}$ for every $n \geq 0$.

What is the value of the following infinite nested radical?:

$$\lim_{n\to\infty} x_n = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots}}}} =$$