CS 310-0

Homework Assignment No. 1

Due Fri 1/14/2000

1. Write the truth table for the following statements.
   (a) $p \to (q \to r)$
   (b) $(p \to q) \to r$

2. Let p and q be primitive statements such that $p \to q$ is false. Find the truth value of the following:
   (a) $q \to p$
   (b) $\neg p \to \neg q$
   (c) $\neg q \to \neg p$

3. Write the following statement in *conjunctive normal form*:
$$p \leftrightarrow (q \to r)$$

4. We define the connective *nand* by:
$$p \uparrow q \Leftrightarrow \neg(p \wedge q)$$
   Make its truth table. Write the following statements using $\uparrow$ only
   (a) $\neg p$
   (b) $p \wedge q$
   (c) $p \vee q$
   (d) $p \to q$
   (e) $p \leftrightarrow q$
   (For instance: $\neg p \Leftrightarrow p \uparrow p$.)

5. Use truth tables to determine if the following logical equivalences are correct (in exercise 1 you already made the truth table for the left hand sides):
   (a) $p \to (q \to r) \Leftrightarrow (p \wedge q) \to r$
   (b) $(p \to q) \to r \Leftrightarrow (\neg p \to r) \wedge (q \to r)$

6. Solve problem 5 by using the laws of logic instead of truth tables.

7. Check the following logical implications:
   (a) $(\neg p \to p) \Rightarrow p$
   (b) $p \Rightarrow (\neg p \to q)$
   (c) $(p \to q) \Rightarrow ((q \to r) \to (p \to r))$

8. Given the following set of premises:
   1. $(\neg p \to p) \to p$
   2. $p \to (\neg p \to q)$
   3. $(p \to q) \to ((q \to r) \to (p \to r))$

   and using only *Modus Ponens* as rule of inference:
   $$\begin{array}{rl} & p \\ & p \to q \\ \hline \therefore & q \end{array}$$
   prove: $p \to p$.

9. What truth values of p, q and r are compatible with the following set of premises?
   1. $p \leftrightarrow \neg q$
   2. $q \leftrightarrow \neg r$
   3. $r \leftrightarrow (\neg p \wedge \neg q)$

10. Consider the following statements:
    (a) $\forall x \forall y\,(x \leq y)$
    (b) $\forall x \exists y\,(x \leq y)$
    (c) $\exists x \forall y\,(x \leq y)$
    (d) $\exists x \exists y\,(x \leq y)$
    Determine their truth value assuming that the universe of discourse is:
    (1) The set of all integers.
    (2) The set of positive integers.
    (3) The set of negative integers.
    (4) The set $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

11. Assume that the universe of discourse is the set of integers. Prove the following, stating the method or principle being used:
    (a) $\exists x\,(x^2 = x)$.
    (b) $\forall x\,\forall y\,(x + y < 10 \to x < 2 \vee y < 8)$.

12. Write the negation of the following quantified statement in prenex normal form, leaving the statement inside in disjunctive normal form:
    $$\forall \varepsilon[\varepsilon > 0 \to \exists N\,\forall n\,(n > N \to |a_n| < \varepsilon)]\,.$$