CS 310
Homework Assignment No. 1
Due on Tue 1/21/2003

1. Use truth tables to determine whether the following logical equivalences are correct:
   (a) $p \wedge (q \veebar r) \equiv (p \wedge q) \veebar (p \wedge r)$
   (b) $(p \veebar q) \veebar r \equiv p \veebar (q \veebar r)$
   (c) $(p \rightarrow q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$
   (d) $(p \veebar q) \veebar r \equiv (p \leftrightarrow q) \leftrightarrow r$

2. Let p and q be primitive statements such that $p \rightarrow q$ is false. Find the truth value of the following:
   (a) $q \rightarrow p$
   (b) $\overline{p} \rightarrow \overline{q}$
   (c) $\overline{q} \rightarrow \overline{p}$

3. Consider the following statements:
   (a) $\forall x \forall y \, (x < y)$.
   (b) $\forall x \exists y \, (x < y)$.
   (c) $\exists x \forall y \, (x < y)$.
   (d) $\exists x \exists y \, (x < y)$.
   Determine their truth value assuming that the universe of discourse is:
   (1) The set of all integers.
   (2) The set of positive integers.
   (3) The set of negative integers.
   (4) The set $A = \{1, 2, 3, 4, 5\}$.

4. Consider the following premises:
   1. If A is large then B is small.
   2. If C is large then D is small.
   3. A is large or C is large.
   4. B is not small.
   Use a formal argument to prove that D is small (write in three columns containing respectively a label, a proposition and a reason).

5. Prove the following statements using mathematical induction:

   (a) $1 \cdot 2^0 + 2 \cdot 2^1 + 3 \cdot 2^2 + 4 \cdot 2^3 + \cdots + n \cdot 2^{n-1} = (n-1)2^n + 1$ for $n \geq 1$.

   (b) $10^n < 2^{2^n}$ for $n \geq 4$.