CS 310
Homework Assignment No. 2
Due on Tue 1/28/2003

1. Prove or disprove: If A, B and C are sets such that $A \triangle C = B \triangle C$ then $A = B$.

2. In a university with 300 students enrolled 140 students are taking French, 120 are taking business, 130 are taking music, 30 are taking French and business, 40 are taking business and music, 50 are taking French and music, and 10 are taking French, business and music. How many students:
(a) are taking exactly two of those subjects?
(b) are taking exactly one of those subjects?
(c) are not taking any of the three subjects?

3. Let L be the set of all strings (including the null string) over $A = \{a, b\}$ than can be constructed by repeated application of the following rules:
0) λ (the null string) is in L.
1) If $\alpha \in L$ then $a\alpha b \in L$.
2) If $\alpha \in L$ and $\beta \in L$ then $\alpha\beta \in L$.
Determine which of the following strings are in L, which ones are not, and why: *aaabbb*, *ababab*, *aababb*, *aab*, *ba*, *aabbba*, *abbaab*.

4. Find the properties (reflexive, transitive, symmetric, antisymmetric) verified by the following relations:
(a) The following relation on $\mathbb{R}$: $x \,\mathcal{R}\, y \Leftrightarrow x - y \in \mathbb{Z}$.
(b) The following relation on $\mathbb{R}$: $x \,\mathcal{R}\, y \Leftrightarrow x - y \in \mathbb{N}$.
(c) The following relation on $\mathbb{R}^+$: $x \,\mathcal{R}\, y \Leftrightarrow x/y \in \mathbb{Q}$.
(d) The following relation on $\mathbb{R}^+$: $x \,\mathcal{R}\, y \Leftrightarrow x/y \in \mathbb{Z}$.

5. Determine whether the relation $x \,\mathcal{R}\, y \Leftrightarrow \exists k \in \mathbb{Z}^+,\ x^k = y$, is a partial order on $\mathbb{Z}^+$.

6. Prove that the following is an *equivalence relation* on $\mathbb{N} \times \mathbb{N}$:

$$(a, b) \,\mathcal{R}\, (c, d) \Leftrightarrow a + d = b + c\,.$$

7. Let $f, g : \mathbb{R}^+ \to \mathbb{R}^+$ be the functions $f(x) = 1/x$, $g(x) = x + 3$. Find $g \circ f$, $f \circ g$, f^2, g^2, $g \circ f^2$, $f \circ g \circ f$, $f^2 \circ g$, $f \circ g^2$, $g \circ f \circ g$, $g^2 \circ f$.

8. For each one of the following functions, determine if it is one-to-one (but not onto), onto (but not one-to-one), or a bijection. Justify the answer. If it is a bijection, find its inverse.
(a) $f : \mathbb{N} \to \mathbb{N}$, $f(x) = x^2$.
(b) $f : \mathbb{R}^+ \to \mathbb{R}^+$, $f(x) = x^2$.
(c) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$.
(d) $f : \mathbb{R} \to [0, 1)$, $f(x) = x - \lfloor x \rfloor$.