

**CS 310**  
**Homework Assignment No. 1**  
Due on Tue 1/20/2004

1. Use truth tables to determine whether the following logical equivalences are correct:

- (a)  $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
- (b)  $(p \vee q) \vee r \equiv p \vee (q \vee r)$
- (c)  $(p \rightarrow q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$
- (d)  $(p \vee q) \vee r \equiv (p \leftrightarrow q) \leftrightarrow r$

2. Consider the following statements:

- (a)  $\forall x \forall y (x < y)$ .
- (b)  $\forall x \exists y (x < y)$ .
- (c)  $\exists x \forall y (x < y)$ .
- (d)  $\exists x \exists y (x < y)$ .

Determine their truth value assuming that the universe of discourse is:

- (1) The set of all integers.
- (2) The set of positive integers.
- (3) The set of negative integers.
- (4) The set  $A = \{1, 2, 3, 4, 5\}$ .

3. Consider the following premises:

- 1. If A is red then B is green.
- 2. If C is red then D is green.
- 3. A is red or C is red.
- 4. B is not green.

Use a formal argument to prove that D is green (write in three columns containing respectively a label, a proposition and a reason).

4. Let  $a, b, c$  be integers satisfying  $a^2 + b^2 = c^2$ . Give two different proofs that  $abc$  must be even,

- (a) by considering various parity cases;
- (b) using argument by contradiction.

5. Prove the following statements using mathematical induction:

- (a) If  $n$  is an integer greater than or equal to 4, then  $n! > 2^n$ .
- (b)  $7^n - 1$  is divisible by 6 for all positive integers  $n$ .