# A COLECTION OF FAKE PROOFS.

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## Introduction

There are many well-known “proofs” of false statements such as $0 = 1$. They typically contain an obviously incorrect step, for example, many popular proofs of $0 = 1$ amount to starting with the true statement $0 \cdot x = 0 \cdot y$ for arbitrary $x$ and $y$, and then “simplifying” both sides by 0, getting the obviously wrong statement $x = y$ (from which we could infer that any pair of numbers are equal). The division by zero may be somewhat hidden in the critical step, but it is typically easy to spot. The common characteristic of the fake proofs included in this work is that their wrong step or fallacy is intended to be particularly intricate and not so easily spotted. Solutions explaining what is wrong on each fake proof are included at the end.

**Disclaimer:** Part of the text (including mathematical equations) in the fake proofs has been generated using AI. An interesting complementary exercise can be to spot what parts of the text have been automatically generated.

# Fake Proofs

## 1 0=1

### 1.1 Using dominated convergence

**False Theorem 1.** $0 = 1$.

*Fake Proof.* We build a sequence of functions whose integrals are always 1 and converge (almost everywhere) to 0.

1. **Normalized sequence.** For each integer $n \geq 1$ set
$$f_n(x) \;=\; (n+1)\,x^n, \qquad 0 \leq x \leq 1.$$
Integrating we get
$$\int_0^1 f_n(x)\,dx = (n+1)\int_0^1 x^n\,dx = (n+1)\cdot\frac{1}{n+1} = 1 \quad \text{for every } n. \tag{1}$$

2. **Pointwise (a.e.) limit.** For every $x \in [0,1)$ we have $x^n \to 0$ as $n \to \infty$. Since the singleton $\{1\}$ has Lebesgue measure 0, we get
$$f_n(x) \xrightarrow[n\to\infty]{} 0 \quad \text{for a.e. } x \in [0,1]. \tag{2}$$

3. **Uniform control off a tiny set (Egoroff).** Fix $\varepsilon > 0$. Egoroff's theorem provides a set $E_\varepsilon \subset [0,1]$ with $m(E_\varepsilon) < \varepsilon$ such that the convergence in (2) is uniform on $K_\varepsilon := [0,1] \setminus E_\varepsilon$. Hence there exists $N = N(\varepsilon)$ satisfying
$$|f_n(x)| < \varepsilon \quad \text{for all } x \in K_\varepsilon \text{ and } n \geq N. \tag{3}$$

4. **Bounding the "bad" part.** On the small set $E_\varepsilon$ we use the crude bound $|f_n(x)| \leq n+1$, so
$$\int_{E_\varepsilon} |f_n(x)|\,dx \;\leq\; (n+1)\,m(E_\varepsilon) \;<\; \varepsilon\,(n+1). \tag{4}$$
Choosing $n \geq \max\{N(\varepsilon),\, \lceil \varepsilon^{-2} \rceil\}$ gives $\varepsilon(n+1) \leq \varepsilon^{-1}$.
Because $\varepsilon > 0$ was arbitrary, set
$$g(x) \;:=\; \sup_{n\geq 1} |f_n(x)|,$$
which is in $L^1([0,1])$ and dominates $(f_n)$.

5. **Applying dominated convergence.** Assuming the domination, (2) and the dominated-convergence theorem we get
$$\lim_{n\to\infty}\int_0^1 f_n(x)\,dx = \int_0^1 \big(\lim_{n\to\infty} f_n(x)\big)\,dx = \int_0^1 0\,dx = 0. \tag{5}$$

6. **Contradiction.** From (1) the same limit equals 1, therefore $0 = 1$.

$\square$

## 1.2 Using complex integration.

**False Theorem 2.** $0 = 1$.

*Fake Proof.* Consider the unit circle parameterized by $\gamma(t) = e^{it}$ for $t \in [0, 2\pi]$, and the contour integral
$$I = \oint_\gamma \frac{\log z}{z}\,dz.$$

**Argument A (antiderivative).** On any region where a branch of $\log z$ is holomorphic we have

$$\frac{d}{dz}\big(\tfrac{1}{2}(\log z)^2\big) = \frac{\log z}{z}.$$

Hence $\frac{1}{2}(\log z)^2$ serves as an antiderivative of $\frac{\log z}{z}$, so by the fundamental theorem for line integrals around a closed curve,

$$I = 0.$$

**Argument B (direct parameterization).** Along $\gamma(t) = e^{it}$ and taking the (principal) branch formally, $\log(e^{it}) = it$ and $dz = ie^{it}\,dt$. Then

$$I = \int_0^{2\pi} \frac{\log(e^{it})}{e^{it}}\, ie^{it}\,dt = \int_0^{2\pi} (it)\,i\,dt = -\int_0^{2\pi} t\,dt = -\frac{(2\pi)^2}{2} = -2\pi^2.$$

Combining the two evaluations yields $0 = -2\pi^2$, and dividing both sides by $-2\pi^2$ gives $0 = 1$. QED(?)

$\square$

# 2 All triangles are equilateral

## 2.1 Using affine geometry

**False Theorem 3.** *Every nondegenerate Euclidean triangle is equilateral.*

*Fake Proof.* Let $T = \triangle ABC$ be any nondegenerate triangle in the plane.

**Step 1 (Affine normalization via the Steiner inellipse).** Let $E(T)$ be the Steiner inellipse of $T$, i.e. the unique ellipse of maximal area inscribed in $T$. It is tangent to the three sides at their midpoints and is centered at the centroid $G_T$ of $T$. Since every ellipse is an affine image of the unit circle, there exists an affine isomorphism

$$\Phi : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$

such that $\Phi\big(E(T)\big) = \mathbb{S}^1$ (the unit circle) and $\Phi(G_T) = 0$. Define $T^* = \Phi(T)$. Then $T^*$ is a triangle circumscribed about $\mathbb{S}^1$, and $0$ is the image of the centroid of $T$.

**Step 2 (The circle is the incircle of $T^*$ and its center is $0$).** Because $\Phi$ is affine, it preserves lines and tangency. Hence $\mathbb{S}^1$ is tangent to each side of $T^*$. By uniqueness of the inscribed circle in a triangle, $\mathbb{S}^1$ is the incircle of $T^*$, so its center $I_{T^*}$ (the incenter) is 0:

$$I_{T^*} = 0 = \Phi(G_T).$$

Thus the incenter of $T^*$ coincides with the image of the centroid of $T$.

**Step 3 (Centroid is natural under affine maps).** For a triangle, the centroid is the barycenter of the three vertices with equal weights. If $\Phi(x) = Lx + t$ is affine, then

$$G_{T^*} = \frac{\Phi(A) + \Phi(B) + \Phi(C)}{3} = \Phi\left(\frac{A + B + C}{3}\right) = \Phi(G_T) = 0.$$

Hence $G_{T^*} = I_{T^*}$.

**Step 4 (Barycentric characterization).** In barycentric coordinates relative to $T^*$ with side lengths $a, b, c$, the incenter is $(a : b : c)$ while the centroid is $(1 : 1 : 1)$. Therefore $I_{T^*} = G_{T^*}$ if and only if $(a : b : c) = (1 : 1 : 1)$, i.e. $a = b = c$. Thus $T^*$ is equilateral.

**Step 5 (Canonicity via reflection; independence of choices).** The normalization in Step 1 can be justified as "canonical" as follows. Using the complex plane, there exists a Schwarz–Christoffel map from the upper half-plane to $T$, and then an affine post-composition realizing $\Phi$ as above. Reflecting across the straight sides (Schwarz reflection principle) extends the construction across the boundary; the tangency data and straightness of the sides force the extension to be compatible on overlaps. Hence the choice of $\Phi$ is canonical up to a global similarity.

**Step 6 (Descent).** Since $T^*$ is equilateral and the construction is canonical up to a similarity, the equilateral property descends back to $T = \Phi^{-1}(T^*)$. Therefore $T$ is equilateral. $\square$

**Addendum: An "energy" rephrasing.** Define the scale-free functional

$$\mathcal{F}(T) \;=\; \frac{a^2 + b^2 + c^2}{\Delta(T)}\,,$$

where $\Delta(T)$ is the area of $T$. By Weitzenböck's inequality one has $\mathcal{F}(T) \geq 4\sqrt{3}$ with equality if and only if $T$ is equilateral. Under the normalization of Step 1, $T$ is sent to $T^*$ with incircle $\mathbb{S}^1$ of radius 1, and a standard computation using $a = 2R\sin\alpha$, $\Delta = rs$ shows that $\mathcal{F}(T^*) = 4\sqrt{3}$. Hence $T^*$ is equilateral, and by the canonicity of the normalization (Step 5), the original $T$ must be equilateral as well. $\square$

# Solutions

## 1 0=1

### 1.1 Using dominated convergence

The wrong step was in claiming that the "dominating function" is integrable. Actually, it is not.

Recall that the allegedly dominating function was defined by

$$g(x) \;=\; \sup_{n\geq 1}(n+1)x^n, \qquad 0 \leq x \leq 1.$$

#### Estimating $g(x)$ near $x = 1$

Fix $x \in (0,1)$ and let $t := -\log x > 0$. Treating $n$ as a real variable and maximizing

$$h(n) = \log\bigl((n+1)x^n\bigr) = \log(n+1) - nt,$$

we find

$$h'(n) = \frac{1}{n+1} - t = 0 \quad\Longrightarrow\quad n_* = \frac{1}{t} - 1.$$

Rounding $n_*$ to the nearest integer yields the discrete maximize, so

$$g(x) \approx (n_* + 1)x^{n_*} = \frac{1}{t}\, e^{-(\frac{1}{t}-1)t} = \frac{e^{-1}}{t}\, e^t \sim \frac{e^{-1}}{t} \quad \text{as } x \to 1^-.$$

Consequently

$$g(x) \;\gtrsim\; \frac{C}{-\log x} \quad (x \to 1^-), \qquad \text{with } C = e^{-1}.$$

A simpler explicit bound, valid for all $0 < x < 1$, is

$$g(x) \;\geq\; \frac{1}{-e\log x}. \tag{†}$$

#### Failure of integrability

Integrate the bound (†) near the endpoint $x = 1$. For $0 < \delta < 1$,

$$\int_{1-\delta}^{1} g(x)\,dx \;\geq\; \frac{1}{e}\int_{1-\delta}^{1} \frac{dx}{-\log x}.$$

Substituting $u = -\log x$ ($x = e^{-u}$, $dx = -e^{-u}\,du$) gives

$$\int_{1-\delta}^{1} \frac{dx}{-\log x} = \int_{0}^{-\log(1-\delta)} \frac{e^{-u}}{u}\,du \;\geq\; \int_{0}^{\varepsilon} \frac{du}{u} = \infty \quad (\varepsilon > 0).$$

Hence

$$\int_{1-\delta}^{1} g(x)\,dx = \infty \quad \Longrightarrow \quad g \notin L^1([0,1]).$$

### Why Egoroff's theorem cannot solve the problem

Egoroff's theorem ensures that for each $\varepsilon > 0$ there exists a set $K_\varepsilon \subset [0,1]$ with $m(K_\varepsilon) > 1-\varepsilon$ on which $f_n \to 0$ *uniformly.* However, the exceptional set $E_\varepsilon = [0,1] \setminus K_\varepsilon$ depends on $\varepsilon$. Taking the *supremum* over all $n$ at every point "remembers" the worst individual $n$ and recreates the spike $g(x) \sim 1/(-\log x)$ as $x \to 1^-$. No single integrable envelope works for the entire sequence, so the dominated–convergence theorem is inapplicable. Step 5 of the fake proof therefore collapses, and with it the conclusion $0 = 1$.

## 1.2 Using complex integration

There is *no* single-valued holomorphic branch of $\log z$ on any domain that winds once around 0. The unit circle encloses 0, so you cannot pick a holomorphic antiderivative $\frac{1}{2}(\log z)^2$ on a domain containing $\gamma$ as a closed curve. Argument A implicitly assumes such a branch exists.

In Argument B, writing $\log(e^{it}) = it$ continuously for $t \in [0, 2\pi]$ sneaks in the same issue: the principal branch of log jumps by $2\pi i$ after a full loop around 0. Treating log as continuous along the entire loop is illegitimate. Once the branch-cut/monodromy problem is handled correctly, the contradiction disappears.

# 2 All triangles are equilateral

## 2.1 Using affine geometry

This note pinpoints the illegitimate moves in the slick (but wrong) argument that "all triangles are equilateral." Up to the point where the normalized triangle $T^*$ is shown to satisfy $I_{T^*} = G_{T^*}$, the reasoning can be made correct. The collapse happens when the conclusion is pulled back to the original triangle $T$.

## What is actually correct

Let $T$ be any nondegenerate triangle and $E(T)$ its Steiner inellipse, centered at the centroid $G_T$ and tangent to the three sides at their midpoints. There exists an affine isomorphism $\Phi$ with $\Phi(E(T)) = \mathbb{S}^1$ and $\Phi(G_T) = 0$. Write $T^* = \Phi(T)$.

- Tangency is preserved by affine maps, so $\mathbb{S}^1$ is tangent to each side of $T^*$. Hence $\mathbb{S}^1$ is the *incircle* of $T^*$ and its center (the incenter) is $I_{T^*} = 0$.
- The centroid is affine-natural: $G_{T^*} = \Phi(G_T) = 0$.
- In barycentric coordinates relative to $T^*$ with side lengths $(a, b, c)$, one has $I_{T^*} = (a : b : c)$ and $G_{T^*} = (1 : 1 : 1)$, so $I_{T^*} = G_{T^*}$ iff $a = b = c$. Thus $T^*$ *is equilateral.*

All of the above can be justified without trickery.

## The fatal mistake: treating equilateralness as affine-invariant

[Key failure] The property "equilateral" is preserved by similarities but *not* by general affine maps. In particular, from "$T^*$ is equilateral" one cannot conclude that $T = \Phi^{-1}(T^*)$ is equilateral unless $\Phi$ is a similarity.

*Proof.* Similarities preserve all angles and scale all lengths by a common factor, hence they preserve equilateralness. For a counterexample with a non-similarity, start from the equilateral triangle with vertices

$$A = (0, 0), \quad B = (1, 0), \quad C = \left(\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}\right),$$

and apply the shear $S(x, y) = (x + y, y)$. The images are

$$A' = (0, 0), \quad B' = (1, 0), \quad C' = \left(\tfrac{1}{2} + \tfrac{\sqrt{3}}{2}, \tfrac{\sqrt{3}}{2}\right).$$

A direct computation gives the squared side lengths

$$|A'B'|^2 = 1, \qquad |A'C'|^2 = \frac{7}{4} + \frac{\sqrt{3}}{2}, \qquad |B'C'|^2 = \frac{7}{4} - \frac{\sqrt{3}}{2},$$

which are unequal; $S$ turns an equilateral triangle into a scalene one. $\square$

In the fake proof, Step 6 says: "Since $T^*$ is equilateral, $T$ is equilateral." That is exactly the non sequitur above.

## The “conformal reflection” red herring

Another seductive gloss claims canonicity via Schwarz–Christoffel maps and the Schwarz reflection principle. This insinuates that the affine normalization is “conformal enough” to transport equilateralness back to $T$.

**Lemma 1.** *A real-affine map $\Phi(x) = Lx + t$ is conformal (angle-preserving) iff $L^T L = \lambda I$ for some $\lambda > 0$, i.e. $L$ is a similarity. In particular, the generic ellipse→circle map is* not *conformal.*

Thus, appealing to holomorphic reflection across the sides after post-composing with a non-similarity affine $L$ does not make the construction conformal. The paragraph is merely decorative: it does not validate pulling the equilateral property back to $T$.

## The “energy addendum” sleight of hand

The addendum considers the scale-free functional

$$\mathcal{F}(T) = \frac{a^2 + b^2 + c^2}{\Delta(T)}.$$

Weitzenböck's inequality says $a^2 + b^2 + c^2 \geq 4\sqrt{3}\,\Delta(T)$ with equality iff $T$ is equilateral. The fake proof claims that under the normalization one “computes” $\mathcal{F}(T^*) = 4\sqrt{3}$, which would force $T^*$ equilateral. But for a general triangle normalized to have inradius $r = 1$, there is no reason for equality in Weitzenböck to hold. The “computation” is circular: it reasserts what must be proved.

## How to repair (and why it then becomes trivial)

If one *insists* that $\Phi$ be a similarity (so that equilateralness is preserved), then $\Phi$ can map $E(T)$ to a circle only when $E(T)$ was already a circle, i.e. $T$ was already equilateral. The argument becomes correct but tautological.

**Bottom line.** The sophisticated machinery up to the equilateral shape of $T^*$ is fine. The collapse occurs precisely when one tries to pull that conclusion back through a *non-similarity* affine map, and this is not saved by the conformal-reflection gloss nor by the “energy” rephrasing.