**CS 310**
**Homework Assignment No. 4**
Due on Tue 2/11/2003

**1.** Ten people go to a movie.
(a) In how many ways can they be placed in the queue of the movie theater?
(b) Assume that those people are divided into three groups, one with 2 people, another one with 3 people and another one with 5 people. In how many ways can they be placed in the queue if the members of the same group must remain together?

**2.** In a class the students must choose 3 out of 4 subjects $A, B, C, D$ to write an essay about. Subject $A$ is chosen by 21 students, subject $B$ by 18, subject $C$ by 15 and subject $D$ by 12. How many students are there in the class?

**3.** Find the number of integer solutions to the following equation

$$x_1 + x_2 + x_3 = 12$$

with each one of the following restrictions:
(a) $x_1, x_2, x_3 \geq 0$.
(b) $x_1, x_2, x_3 > 0$.
(c) $1 \leq x_1$, $2 \leq x_2$, $3 \leq x_3$.

**4.** Let $A$ be the set of all 8-digit numbers in base 3 (so they are written with the digits $0, 1, 2$ only), including those with leading zeroes such as 00120010. The *Hamming distance* between two elements of $A$ is the number of places where they differ, for instance the Hamming distance between 11201001 and 11020020 is 5, because they differ in the 3rd, 4th, 5th, 7th and 8th places.
(a) Find the number of elements in $A$.
(b) Given an element $a \in A$, find the number of elements in $A$ whose Hamming distance to $a$ is exactly 3.
(c) Given an element $a \in A$, find the number of elements in $A$ whose Hamming distance to $a$ is 3 or less.
(d) Prove that given 12 elements from $A$, two of them must coincide in at least 2 places. (Hint: given $a_1, \ldots, a_{12} \in A$ look at the sets $B_k = \{x \in A \mid H(a_k, x) \leq 3\}$ $(k = 1, \ldots, 12)$, where $H(a, x) =$ Hamming distance between $a$ and $x$. Use a cardinality argument to show that two of them must have some common element $a \in B_i \cap B_j$. What can we say about $H(a_i, a_j)$?)

**5.** We have three boxes with balls. The first one has 9 white balls and 1 red ball. The second one has 5 white balls and 5 red balls. The third one has 1 white ball and 9 red balls. We choose one of the boxes at random (with the same probability) and take a ball from it, which turns out to be red. What is the probability that the box chosen is the one with 9 red balls?