CS 310
Homework Assignment No. 2
Due on Tue 1/27/2004

1. In a university with 300 students enrolled 140 students are taking French, 120 are taking business, 130 are taking music, 30 are taking French and business, 40 are taking business and music, 50 are taking French and music, and 10 are taking French, business and music. How many students:
(a) are taking exactly two of those subjects?
(b) are taking exactly one of those subjects?
(c) are not taking any of the three subjects?

2. Assume A, B, C are finite sets.
(a) Show that $|A \cup B| = |A| + |B| - |A \cap B|$.
(b) Find a similar formula for $|A \cup B \cup C|$. Show that your formula holds for all finite sets A, B, C.

3. Find the properties (reflexive, transitive, symmetric, antisymmetric) verified by the following relations:
(a) Strict inequality of integers: $x \,\mathcal{R}\, y \Leftrightarrow x < y$.
(b) Set disjointness: $A \,\mathcal{R}\, B \Leftrightarrow A \cap B = \emptyset$.
(c) The following relation on $\mathbb{Q}$: $x \,\mathcal{R}\, y \Leftrightarrow x - y \in \mathbb{Z}$.
(d) The following relation on $\mathbb{Q}$: $x \,\mathcal{R}\, y \Leftrightarrow x - y \in \mathbb{N}$.

4. A string s is a *substring* of t if there are strings u and v with $t = usv$. Let $S = \{a, b\}^*$ be the set of all strings over $\{a, b\}$. Assume that $\preccurlyeq$ represents the relation $s \preccurlyeq t \equiv$ "s is a substring of t".
(a) Prove that $\preccurlyeq$ is a partial order on S.
(b) Draw the Hasse diagram of $\preccurlyeq$ on the set $S' = \{s \in S \,:\, |s| \leq 2\}$.

5. Prove that the following is an *equivalence relation* on $\mathbb{Z} \times \mathbb{Z}^*$:

$$(a, b)\,\mathcal{R}\,(c, d) \Leftrightarrow a\,d = b\,c\,.$$

Is it an equivalence relation on $\mathbb{Z} \times \mathbb{Z}$?

6. Let f and g be the functions from $\mathbb{Z}^+$ to $\mathbb{Z}^+$ ($\mathbb{Z}^+$= positive integers) defined by $f(n) = n^2$, $g(n) = 2^n$. Find the compositions $f \circ f$, $g \circ g$, $f \circ g$, and $g \circ f$.