CS 310-0
Homework Assignment No. 3
Due Fri 4/21/2000

1. Let $f, g : \mathbb{R}^+ \to \mathbb{R}^+$ be the functions $f(x) = x + 1$, $g(x) = 1/x$.
   (a) For $n \in \mathbb{Z}^+$ find f^n, g^n, $f^n \circ g$ and $(f^n \circ g)^{-1}$ (adjust the codomain of $f^n \circ g$ if necessary so that the inverse can be defined).
   (b) Write 43/10 as a suitable composition of f and g applied to 1, i.e.: $43/10 = f^{n_1} \circ g \circ f^{n_2} \circ g \circ \cdots \circ f^{n_k}(1)$, where $n_1, n_2, \ldots, n_k$ are positive integers.

2. The hyperbolic functions are defined in the following way:
   (a) Hyperbolic sine: $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$.
   (b) Hyperbolic cosine: $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$.
   (c) Hyperbolic tangent: $\tanh(x) = \dfrac{\sinh(x)}{\cosh(x)}$.
   Prove that $\tanh : \mathbb{R} \to (-1, 1)$ is a one-to-one correspondence and find its inverse.

3. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = x\,(x^2 - 6x + 11)$. Find $f^{-1}([6, \infty))$.[1]

4. Let $f : A \to A$ a function from a set A to itself. An element $x \in A$ is called a *fix point* of f if $f(x) = x$. Assume that $A = \mathbb{R}^*$ and $f(x) = \dfrac{x^2 + 9}{2x}$. Find all fix points of f.

5. For each one of the following functions, determine if it is one-to-one (but not onto), onto (but not one-to-one), or a one-to-one correspondence. If it is a one-to-one correspondence, find its inverse.
   (a) $f : \mathbb{N} \to \mathbb{N}$, $f(x) = x^3$.
   (b) $f : \mathbb{R}^* \to \mathbb{R}^+$, $f(x) = 1/|x|$, where $|x|$ = absolute value of x.
   (c) $f : \mathbb{Q} \to \mathbb{Q}$, defined by cases in the following way:
   $$f(x) = \begin{cases} -x & \text{if } x \in \mathbb{Z} \\ x + 1 & \text{if } x \in \mathbb{Q} - \mathbb{Z} \end{cases}$$
   (d) $f : \mathbb{N} \times \{0, 1, 2\} \to \mathbb{N}$, $f(a, b) = 3a + b$.

6. Let A be a set, $\mathcal{P}(A)$ the family of subsets of A, and $\{0,1\}^A = \{f \mid f : A \to \{0,1\}\}$ the set of all functions from A to $\{0,1\}$. Prove that $\mathcal{P}(A)$ and $\{0,1\}^A$ have the same cardinality. (Hint for each subset $B \subseteq A$ consider the function $f_B : A \to \{0,1\}$ defined by $f_B(x) = 1$ if $x \in B$, and $f_B(x) = 0$ otherwise.)

[1] Here "f^{-1}" means preimage set, not inverse function.