CHAPTER 8

# Trees

## 8.1. Trees

**8.1.1. Terminology.** A *tree* is a connected undirected graph with no simple circuits.

A *rooted tree* is a tree in which a particular vertex is designated as the root and every edge is directed away from the root.

We draw rooted trees with the root at the top. The arrows indicating the directions of the edges can be omitted.

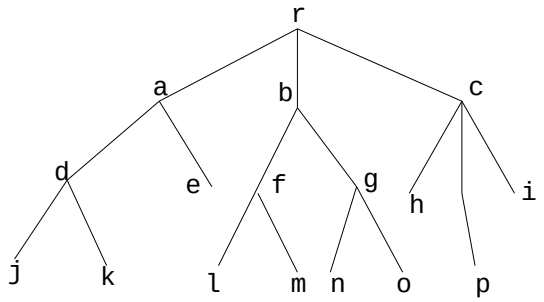


FIGURE 8.1. A rooted tree.

The *level* of a vertex $v$ is the length of the simple path from the root to $v$. The *height* of a rooted tree is the maximum level of its vertices.

Let $T$ be a tree with root $v_0$. Suppose that $x$, $y$ and $z$ are vertices in $T$ and that $(v_0, v_1, \ldots, v_n)$ is a simple path in $T$. Then:

1. $v_{n-1}$ is the *parent* of $v_n$.
2. $v_0, v_1, \ldots, v_{n-1}$ are *ancestors* of $v_n$.
3. $v_n$ is a *child* of $v_{n-1}$.
4. If $x$ is an ancestor of $y$, $y$ is a *descendant* of $x$.
5. If $x$ and $y$ are children of $z$, $x$ and $y$ are *siblings*.
6. If $x$ has no children, it is called a *terminal vertex* or *leaf*.

7. If $x$ is not a terminal vertex, it is an *internal* or *branch vertex*.
8. The *subtree of $T$ rooted at $x$* is the graph $(V, E)$, where $V$ is $x$ together with its descendants and $E$ = edges of simple paths from $x$ to some vertex in $E$.