CS 310
Homework Assignment No. 7
Not due

1. In this problem the *universal set* is the set of natural numbers $\mathbb{N}$. Let S be the collection of subsets X of $\mathbb{N}$ such that either X of its complement $\overline{X}$ is finite. Show that $(S, \cup, \cap, \overline{}, \emptyset, \mathbb{N})$ is a Boolean algebra.

2. Prove that the implication operator $\rightarrow$ given by the table

x	y	$x \rightarrow y$
1	1	1
1	0	0
0	1	1
0	0	1

   is functionally complete.

3. Design a combinatorial circuit that multiplies the binary numbers $x_2 \, x_1$ and $y_2 \, y_1$. The output will be of the form $z_4 \, z_3 \, z_2 \, z_1$.

4. Design a combinatorial circuit that works as a 2-to-4-line demultiplexer, i.e. it has two inputs x_0, x_1 and four outputs y_0, y_1, y_2, y_3, so that output $y_k = 1$ and the other outputs are 0, precisely when $x_1 \, x_0$ is the binary number k.

5. Design a combinatorial circuit with three inputs x_1, x_2, x_3 and one output y that computes $y = x_1 + x_2 + x_3 \pmod 2$, i.e., $y = 0$ if $x_1 + x_2 + x_3$ is even and $y = 1$ if it is odd.

6. Design a finite-state machine that inputs a string of a's and b's (read from left to right) and outputs the number of a's plus twice the number of b's modulo 4. For instance the input "*abaababbaaaba*" would produce the output "1301302012312"

7. In a computer system users must choose a password verifying the following requirements:
   (a) It must start with a letter.
   (b) It must end with a letter or digit.
   (c) It must contain at least a one small letter, a capital letter and a digit.
   In order to simplify the problem assume that the set of input symbols is $\{s, c, d\}$ for "small letter", "capital letter" and "digit" respectively. Design a finite-state automaton (as simple as possible) that accepts strings (of whatever length) verifying exactly the given criteria.