

Tidal Forces and Apparent Size: Application to the Earth–Moon System

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(AI assisted)

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Human's Original Prompt:

Assume planet A has a moon B, and it is hence subject to tidal forces from B. Also assume that with time moon B changes its distance to planet A. What is the relation between said tidal forces and the apparent size of B as seen from A? If an exact computation is too complex feel free to make simplification assumptions such as the change in distance is relatively small enough to replace exact formulas with asymptotic simpler ones.

AI's Answer:

1. Introduction

Consider a planet A with a moon B . The moon exerts a tidal force on the planet, whose magnitude depends on the moon's mass, its distance, and the planet's size. If the moon's orbital distance changes slowly, its apparent angular size in the planet's sky also changes.

The purpose of this note is to relate the moon's apparent size to the strength of its tidal effect, and to evaluate the relationship in the concrete case of the Earth–Moon system.

2. Scaling law

Let M_B and R_B be the mass and physical radius of the moon, r its distance from the planet, and θ its apparent angular diameter as seen from the planet. For small angular sizes, we approximate

$$\theta \approx \frac{2R_B}{r}. \quad (1)$$

The characteristic tidal acceleration at the planet's surface scales as

$$a_{\text{tidal}} \propto \frac{M_B}{r^3}. \quad (2)$$

Eliminating r using the relation for θ yields

$$a_{\text{tidal}} \propto \left(\frac{M_B}{R_B^3} \right) \theta^3. \quad (3)$$

Up to an overall constant, the tidal force is proportional to the moon's mean density multiplied by the cube of its apparent angular size. For a given moon whose density does not change, this simplifies to

$$a_{\text{tidal}} \propto \theta^3. \quad (4)$$

Human's Prompt:

Can you can use the Earth–Moon system as an application example?

AI's Answer:

3. Application to the Earth–Moon system

The Moon's average radius is

$$R_M \approx 1.74 \times 10^6 \text{ m},$$

and the average Earth–Moon distance is

$$r_{\text{mean}} \approx 3.84 \times 10^8 \text{ m.}$$

The Moon's orbit is elliptical, with approximate perigee and apogee distances

$$r_{\text{per}} \approx 3.63 \times 10^8 \text{ m}, \quad r_{\text{apo}} \approx 4.06 \times 10^8 \text{ m.}$$

Using the small-angle approximation, the apparent angular diameter at each distance is

$$\theta \approx \frac{2R_M}{r}.$$

Numerically, this gives:

$$\begin{aligned} \theta_{\text{mean}} &\approx 0.52^\circ, \\ \theta_{\text{per}} &\approx 0.55^\circ, \\ \theta_{\text{apo}} &\approx 0.49^\circ. \end{aligned}$$

Thus, the Moon appears roughly 11%–12% larger at perigee than at apogee.

4. Tidal variation estimate

From the scaling relation $a_{\text{tidal}} \propto \theta^3$, the ratio of tidal strength at perigee to that at apogee is

$$\frac{a_{\text{per}}}{a_{\text{apo}}} = \left(\frac{\theta_{\text{per}}}{\theta_{\text{apo}}} \right)^3 \approx (1.116)^3 \approx 1.39. \quad (5)$$

Equivalently, using directly the ratio of distances:

$$\frac{a_{\text{per}}}{a_{\text{apo}}} = \left(\frac{r_{\text{apo}}}{r_{\text{per}}} \right)^3 \approx 1.39.$$

Therefore, tidal forces due to the Moon are approximately 39% stronger at perigee than at apogee.

5. Conclusion

The Earth–Moon system provides a concrete illustration of the general principle that, for a fixed moon, the tidal force experienced by a planet scales with the cube of the moon’s apparent angular size. In practical terms, the observed difference between “supermoon” and “micromoon” aligns with roughly a 40% variation in tidal strength over the lunar orbit.

Human’s Objection

If the Moon and the Sun have approximately the same apparent size from Earth, why Moon tides are stronger than Sun tides?

AI’s Answer:

Apendix: Dialogue on Why Lunar Tides Dominate Solar Tides

The following informal dialogue summarizes a conceptual refinement that arose while discussing why the Moon exerts stronger tidal forces on Earth than the Sun, despite their similar apparent sizes.

Claim (initial explanation).

Tidal forces scale like M/r^3 , while apparent size scales like $1/r$. Thus distance affects tides much more strongly than appearance. Even though the Sun is more massive, its great distance reduces its tide-generating effect to less than half that of the Moon.

Human's Objection

Somehow I am not convinced of that explanation. Assume that the Moon grows to be the size of the Sun and we place it at the same distance from Earth as the Sun (without changing its density). The angular size of the enlarged Moon would not change, but how about its tidal force on Earth?

AI's Answer:

Objection (the user's thought experiment).

Suppose we enlarge the Moon until its physical radius matches the Sun's, and place it at the same distance as the Sun, without changing its density. Its apparent size in the sky would then be unchanged. Would its tidal effect be the same as the present Moon's, or would it match (or exceed) the Sun's? This thought experiment suggests that the original explanation based purely on $1/r$ versus $1/r^3$ may be incomplete.

Refinement (resolution of the issue).

Let ρ be the mean density of the tide-generating body and θ its apparent angular diameter. For small angles one has

$$\theta \approx \frac{2R}{r}, \quad M = \frac{4}{3}\pi R^3 \rho.$$

Since tidal acceleration scales like M/r^3 , eliminating R and r yields

$$a_{\text{tidal}} \propto \rho \theta^3.$$

Thus, if two bodies have the same density and same apparent size, they generate the same tide, regardless of their actual mass or distance. Density matters as much as appearance.

Application to the Sun and the Moon.

The Sun and Moon subtend nearly equal angular diameters as seen from Earth. However, their mean densities differ substantially:

$$\rho_{\text{Moon}} \approx 3.34 \text{ g/cm}^3, \quad \rho_{\odot} \approx 1.41 \text{ g/cm}^3.$$

Since

$$\frac{a_{\text{Moon}}}{a_{\odot}} \approx \frac{\rho_{\text{Moon}}}{\rho_{\odot}} \approx 2.4,$$

lunar tides are naturally stronger than solar tides. The earlier appeal to distance effects was accurate but incomplete: the decisive factor is that the Moon is much denser than the Sun, while sharing its angular size.

Conclusion.

The dialogue illustrates that a complete understanding of tidal dominance requires recognizing the dependence

$$a_{\text{tidal}} \propto \rho \theta^3,$$

which cleanly explains why the Moon exerts stronger tides than the Sun. This relation also validates the user's thought experiment: enlarging the Moon and moving it outward while preserving its density and angular size leaves its tidal effect essentially unchanged.