CS 310
Homework Assignment No. 7
Due on Tue 5/25/2005

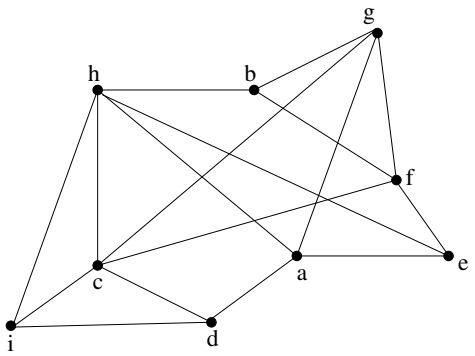
1. Consider the following arithmetic expression:

$$a * (b + c \uparrow d \uparrow 2)/(e + f * h \uparrow i - j * (k + 5)) .$$

The hierarchy for evaluation of arithmetic expressions (from do first to do last) is: (1) "$\uparrow$", (2) "$*$" and "$/$", (3) "$+$" and "$-$". The inner parenthesis must be evaluated first, and operations with the same priority level are evaluated from left to right, except $\uparrow$, which is evaluated from right to left—for instance, $2/3 * 5$ means $(2/3) * 5$, but $2 \uparrow 3 \uparrow 5$ means $2 \uparrow (3 \uparrow 5)$.

(a) Represent the given expression with a binary rooted tree.
(b) Write it in Polish notation.
(c) Write it in reverse Polish notation.

2. Use (1) the Breadth-First Search Algorithm and (2) the Depth-First Search Algorithm to find two spanning trees of the following graph with its edges ordered in alphabetic order:



3. In this problem the *universal set* is the set of natural numbers $\mathbb{N}$. Let S be the collection of subsets X of $\mathbb{N}$ such that either X of its complement $\overline{X}$ is finite. Show that $(S, \cup, \cap, \overline{\ }, \emptyset, \mathbb{N})$ is a Boolean algebra.

4. Prove that the implication operator $\rightarrow$ given by the table

x	y	$x \rightarrow y$
1	1	1
1	0	0
0	1	1
0	0	1

is functionally complete.

(next page $\rightarrow$)

5. Let '$x_1\, x_0$' and '$y_1\, y_0$' be two 2-bit binary numbers, and '$z_3\, z_2\, z_1\, z_0$' its 4-bit product; e.g. if '$x_1\, x_0$' $= 10_{(2} = 2_{(10}$ and '$y_1\, y_0$' $= 11_{(2} = 3_{(10}$, then '$z_3\, z_2\, z_1\, z_0$' $= 0110_{(2} = 6_{(10}$. Find a Boolean expression for the function $f(x_0, x_1, y_0, y_1) = z_1$, and design a combinatorial circuit that computes it.