1.8. Integration using Tables and CAS

The use of tables of integrals and Computer Algebra Systems allow us to find integrals very quickly without having to perform all the steps for their computation. However we often need to modify slightly the original integral and perhaps complete or simplify the answer.

Example: Find the following integral using the tables at the end of Steward's book:

$$\int \frac{\sqrt{x^2-1}}{x}\,dx = \cdots .$$

Answer: In the tables we find the following formula No. 41:

$$\int \frac{\sqrt{a^2-u^2}}{u}\,du = \sqrt{a^2-u^2} - a\cos^{-1}\frac{a}{|u|} + C\,,$$

hence, letting $a = 1$, $u = 1$ we get the answer:

$$\int \frac{\sqrt{1-x^2}}{x}\,dx = \boxed{\sqrt{1-x^2} - \cos^{-1}\frac{1}{|x|} + C}\,.$$

Example: Find the integral:

$$\int \frac{x^2}{\sqrt{9+4x^2}}\,dx = \cdots$$

Answer: In the tables the formula that resembles this integral most is No. 26:

$$\int \frac{u^2\,du}{\sqrt{a^2+u^2}} = \frac{u}{2}\sqrt{a^2+u^2} - \frac{a^2}{2}\ln\left(u+\sqrt{a^2+u^2}\right) + C\,,$$

hence making $a = 3$, $u = 2x$:

$$\begin{aligned}\int \frac{x^2}{\sqrt{9+4x^2}}\,dx &= \frac{1}{8}\int \frac{u^2\,du}{\sqrt{a^2+u^2}} \\ &= \frac{1}{8}\left\{\frac{u}{2}\sqrt{a^2+u^2} - \frac{a^2}{2}\ln\left(u+\sqrt{a^2+u^2}\right)\right\} + C \\ &= \boxed{\frac{x}{8}\sqrt{9+4x^2} - \frac{9}{16}\ln\left(2x+\sqrt{9+4x^2}\right) + C}\,.\end{aligned}$$

Example: Find the same integral using Maple.

Answer: In Maple we enter at the prompt:

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> int(x^2/sqrt(9+4*x^2),x);
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and it returns:

$$\frac{x}{8}\sqrt{9+4x^2} - \frac{9}{16}\,\text{arcsinh}\left(\frac{2}{3}x\right)$$

First we notice that the answer omits the constant C. On the other hand, it involves an inverse hyperbolic function:

$$\text{arcsinh}\, x = \ln\left(x + \sqrt{1+x^2}\right),$$

hence the answer provided by Maple is:

$$\frac{x}{8}\sqrt{9+4x^2} - \frac{9}{16}\ln\left(\frac{2x}{3} + \sqrt{1+\frac{4x^2}{9}}\right) =$$
$$\frac{x}{8}\sqrt{9+4x^2} - \frac{9}{16}\ln\left(2x + \sqrt{9+4x^2}\right) + \frac{9}{32}\ln(3),$$

so it differs from the answer found using the tables in a constant $\frac{9}{32}\ln(3)$ which can be absorbed into the constant of integration.