CS 310-0
Homework Assignment No. 1
Due Tue 1/16/2001

1. We define the connective *nor* by:

$$p \downarrow q \Leftrightarrow \neg(p \vee q)$$

   Make its truth table. Write the following statements using $\downarrow$ only:
   (a) $\neg p$
   (b) $p \wedge q$
   (c) $p \vee q$
   (d) $p \rightarrow q$
   (e) $p \leftrightarrow q$
   (For instance: $\neg p \Leftrightarrow p \downarrow p$.)

2. Use truth tables to determine if the following logical equivalences are correct:
   (a) $p \wedge (q \veebar r) \Leftrightarrow (p \wedge q) \veebar (p \wedge r)$
   (b) $(p \veebar q) \veebar r \Leftrightarrow p \veebar (q \veebar r)$
   (c) $(p \rightarrow q) \rightarrow r \Leftrightarrow p \rightarrow (q \rightarrow r)$
   (d) $(p \veebar q) \veebar r \Leftrightarrow (p \leftrightarrow q) \leftrightarrow r$
   (e) $p \wedge q \Leftrightarrow (p \rightarrow (q \rightarrow \mathrm{F}_0)) \rightarrow \mathrm{F}_0$

3. Prove the following logical equivalences by using laws of logic:
   (a) $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$
   (b) $(p \rightarrow q) \rightarrow r \Leftrightarrow (p \vee r) \wedge (q \rightarrow r)$

4. Consider the following statements:
   (a) $\forall x \forall y \exists z \, [x < y \rightarrow (x < z) \wedge (z < y)]$.
   (b) $\forall x \, (x^2 \neq 2)$.
   (c) $\exists x \forall y \, (x^2 < y \leftrightarrow 1 < 2\,y)$.
   Determine their truth value assuming that the universe of discourse is:
   (1) The set of all integers.
   (2) The set of all rational numbers.
   (3) The set of all real numbers.

5. Find a model and a countermodel for each of the following statements:
   (a) $\forall x \exists y \, (x < y)$.
   (b) $\exists x \forall y \, (x \leq y)$.
   (c) $\exists x \exists y \forall z \, (z = x \vee z = y)$.
   (d) $\forall x \exists y \, (x + y = z)$.

6. Write the negation of the following quantified statement in prenex normal form, leaving the statement inside in conjuctive normal form:

$$\forall x \exists y \forall z \, (y < z \rightarrow x < z)$$