CS 310
Homework Assignment No. 1
Due on Wed 4/6/2005

1. Use truth tables to determine whether the following logical equivalences are correct:
(a) $p \wedge (q \oplus r) \equiv (p \wedge q) \oplus (p \wedge r)$
(b) $(p \oplus q) \oplus r \equiv p \oplus (q \oplus r)$
(c) $(p \rightarrow q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$
(d) $(p \oplus q) \oplus r \equiv (p \leftrightarrow q) \leftrightarrow r$
(e) $p \rightarrow (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

2. Consider the following statements:
(a) $\forall x \forall y\, (x < y)$.
(b) $\forall x \exists y\, (x < y)$.
(c) $\exists x \forall y\, (x < y)$.
(d) $\exists x \exists y\, (x < y)$.
Determine their truth value assuming that the universe of discourse is:
(1) The set of all integers.
(2) The set of positive integers.
(3) The set of negative integers.
(4) The set $A = \{1, 2, 3, 4, 5\}$.

3. Consider the following premises:
1. If A is red then B is green.
2. If C is red then D is green.
3. A is red or C is red.
4. B is not green.

Use a formal argument to prove that D is green (write it in three columns containing respectively a label, a proposition and a reason).

4. Let a, b, c be integers satisfying $a^2 + b^2 = c^2$. Give two different proofs that abc must be even,
(a) by considering various parity cases;
(b) using argument by contradiction.

5. Prove the following:

(a) There exists a pair of consecutive integers such that one is a perfect square and the other one is a perfect cube.

(b) The product of two of the numbers $65^{1000} - 8^{2001} + 3^{177}$, $79^{1212} - 9^{2399} + 2^{2001}$, and $24^{4493} - 5^{8192} + 7^{1777}$ is nonnegative. (Do not try to evaluate these numbers!).