

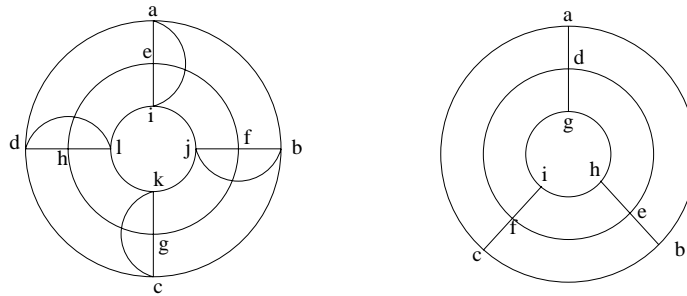
**CS 310-0**  
**Homework Assignment No. 8**  
 Not Due

1. Assume that the 26 (capital) letters of the English alphabet

$\{A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z\}$

are represented as (loop-free) graphs. Classify them by homeomorphism. How many classes of graph-homeomorphic letters are there?

2. For each of the following graphs find an Euler circuit and a Hamiltonian cycle, or prove that there is none.



3. The five Platonic solids are the following:

name	faces	edges	vertices	edges per vertex	edges per face
tetrahedron	4	6	4	3	3
hexahedron	6	12	8	3	4
octahedron	8	12	6	4	3
dodecahedron	12	30	20	3	5
icosahedron	20	30	12	5	3

All Platonic solids have connected loop-free regular<sup>1</sup> planar graphs. Their dual graphs are also loop-free and regular. Prove that the table above describes all possible connected loop-free regular graphs whose dual graphs are also loop-free and regular. Hint: Call  $d$  = the degree of the vertices of the given graph (= number of edges that meet at each vertex),  $d'$  = the degree of the vertices of the dual graph (= number of edges surrounding each region of the given graph). If the graph is loop-free then  $d' \geq 3$ . Since the dual graph must also be loop-free then  $d \geq 3$ . Next, pose three equations: (1) Euler's formula for planar graphs, (2) a relation among  $d, v, e$ , (3) a relation among  $d', r, e$ . Find all positive integer solutions to those equations such that  $d, d' \geq 3$ .

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<sup>1</sup>A regular graph is a graph whose vertices have all the same degree.

4. Consider the following arithmetic expression:

$$a * (b + c \uparrow d \uparrow 2) / (e + f * h \uparrow i - j * (k + 5)).$$

The hierarchy for evaluation of arithmetic expressions (from do first to do last) is: (1) “ $\uparrow$ ”, (2) “ $*$ ” and “ $/$ ”, (3) “ $+$ ” and “ $-$ ”. The inner parenthesis must be evaluated first, and operations with the same priority level are evaluated from left to right, except  $\uparrow$ , which is evaluated from right to left—for instance,  $2/3*5$  means  $(2/3)*5$ , but  $2 \uparrow 3 \uparrow 5$  means  $2 \uparrow (3 \uparrow 5)$ .

- (a) Represent the given expression with a binary rooted tree.
- (b) Write it in Polish notation.
- (c) Write it in reverse Polish notation.