

**CS 310-0**  
**Homework Assignment No. 5**  
Due Tue 2/20/2001

1. There are 2 Mathematics books, 3 Physics books and 5 Computer Science books on a shelf.
  - (a) In how many ways can the 10 books be placed on the shelf?
  - (b) In how many ways can they be placed if the Computer Science books must remain together?
  - (c) If we place the books at random, what is the probability that the Computer Science books will end up all together?
2. A city has streets in the direction E-W and avenues in the direction N-S, making a perfect grid. A taxicab has to go from the intersection of 23rd street and 1st avenue to the intersection of 30th street and 7th avenue following a path of minimum length.
  - (a) In how many ways can it be done?
  - (b) Assume that the taxicab needs gas and there is a gas station at the intersection of 27th street and 5th avenue. How many (minimal) paths go through that intersection?
  - (c) Assume that there is an accident at the intersection of 27th street and 5th avenue and the taxicab wants to avoid it. How many (minimal) paths can the taxicab follow now?
3. Find the number of integer solutions to the following equation

$$x_1 + x_2 + x_3 = 12$$

with each one of the following restrictions:

- (a)  $x_1, x_2, x_3 \geq 0$ .
  - (b)  $x_1, x_2, x_3 > 0$ .
  - (c)  $1 \leq x_1, 2 \leq x_2, 3 \leq x_3$ .
4. Find the number of positive integers not greater than 9450 that are divisible by 3, 5 or 7.
5. (Pigeonhole Principle) Prove that if we select more than half of all subsets of a given set  $X$ , at least two of them consist of a set  $A$  and its complement  $\overline{A} = X - A$ .
6. We define recursively a sequence  $x_n$  in the following way:

$$x_0 = 2; \quad x_{n+1} = \frac{x_n^2 + 1}{2x_n} \quad (n \geq 0).$$

Prove by induction:  $0 < x_n - 1 \leq 1/2^{2^n - 1}$  for every  $n \geq 0$ . (Hint: begin by proving that  $x_{n+1} - 1 = (x_n - 1)^2 / 2x_n$ .)