

CS 310
Homework Assignment No. 3
Due on Tue 2/3/2004

1. Give a Θ notation of the form $\Theta(n^k)$ ($k \in \mathbb{Z}$) for the number of times the statement 'x := x+1' is executed in the following algorithm:

```
1: procedure silly_loop(n)
2:   x := 0
3:   for i:=1 to n do
4:     for j:=1 to 10*i*i do
5:       for k:=1 to 2*j do
6:         x := x+1
7:   end silly_loop
```

2. Consider the following procedure:

```
1: procedure ackerman(m,n)
2:   if m = 0 then
3:     return(n+1)
4:   elsif n = 0 then
5:     return(ackerman(m-1,1))
6:   else
7:     return(ackerman(m-1,ackerman(m,n-1)))
8: end ackerman
```

- (a) Prove that $\text{ackerman}(0,n)$, $\text{ackerman}(1,n)$ and $\text{ackerman}(2,n)$ are $\Theta(n)$.
(b) Prove that $\text{ackerman}(3,n) = \Theta(2^n)$.
(c) Prove that $\text{ackerman}(4,n) = \Omega(2^{2^n})$.
(d) Find all values of $\text{ackerman}(m,0)$ ($m \geq 0$) that can be written in our universe as a base 10 number (assume that our universe contains 10^{80} atoms).
3. We have a box with n balls, two of them are colored red and the rest are colored blue. We extract the balls one by one (without returning them to the box) at random until finding a red ball. Measuring the time by the number of extractions, find the *best-case time*, *worst-case time* and *average-case time* needed to find a red ball.

4. You need to pour exactly 1 floz of water into a pot, but you only have two containers with capacity for 36 floz and 49 floz respectively. You are allowed to transfer water among the containers as you wish, but you cannot measure directly any amount of water that is a fraction of one of the containers. Pose the problem as a Diophantine equation, solve it, and use the solution to find a way of measuring exactly 1 floz with the two containers.
5. We have a number of stamps of various denominations and want to mail a package that requires \$3.25 postage. In each of the following cases determine if we have the appropriate stamps to get exactly the required postage. Justify the answers.
- (a) 1000 8¢-stamps, 500 10¢-stamps and 300 22¢-stamps.
 - (b) 20 15¢-stamps and 5 50¢-stamps.
 - (c) 100 50¢-stamps and 100 33¢-stamps.
 - (d) 4 50¢-stamps and 4 35¢-stamps.