CS 310-0
Homework Assignment No. 6
Due Fri 5/19/2000

1. A computer virus works in such a way that it creates two copies of itself every second for two seconds and then dies. At time 0 sec one copy of the virus enters the computer. How many copies of the virus will be created at time n sec? How many copies of the virus will there be in the computer at time n sec? (Note: the sequence of copies created at time n sec starts like this: $x_0 = 1, x_1 = 3, x_2 = 8, x_3 = 22, \ldots$)

2. The following operation is called *iterated power*: $a \uparrow\uparrow n = a^{a^{\cdot^{\cdot^{a}}}}$, with n a's piled in the tower of exponents—for instance, $2 \uparrow\uparrow 4 = 2^{2^{2^2}} = 2^{2^4} = 2^{16} = 65536$.[1] Give a recursive definition for the iterated power.

3. Ackerman's function $A : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ is defined for every $m, n \geq 0$ by the following double recurrence:
   (a) $A(0, n) = n + 1$ for every $n \geq 0$.
   (b) $A(m, 0) = A(m - 1, 1)$ if $m \geq 1$.
   (c) $A(m, n) = A(m - 1, A(m, n - 1))$ if $m, n \geq 1$.
   Find close-form formulae for $A(1, n)$, $A(2, n)$, $A(3, n)$ and $A(4, n)$.[2] Find all other values of $A(m, n)$ $(m \geq 5)$ whose decimal representation can be written in our universe (assume that our universe has 10^{80} atoms.)

4. You need to pour exactly 1 tsp of water into a pot, but you only have two containers with capacity for 36 tsp and 49 tsp respectively. You are allowed to transfer water among the containers as you wish, but you cannot measure directly any amount of water that is a fraction of one of the containers. Pose the problem as a Diophantine equation, solve it, and use the solution to find a way of measuring exactly 1 tsp with the two containers.

5. We have a number of stamps of various denominations and want to mail a package that requires \$3.25 postage. In each of the following cases determine if we have the appropriate stamps to get exactly the required postage. Justify the answers.
   (a) 1000 8¢-stamps, 500 10¢-stamps and 300 22¢-stamps.
   (b) 20 15¢-stamps and 5 50¢-stamps.
   (c) 100 50¢-stamps and 100 33¢-stamps.
   (d) 4 50¢-stamps and 4 35¢-stamps.

6. Four comets visit the Earth every 560, 400, 675 and 588 years respectively. They coincide by chance in year Y. In what year will they coincide again?

[1] This notation is due to Donald Knuth: $a \uparrow n = a^n$ represents the usual power; $a \uparrow\uparrow n$ is the iterated power; $a \uparrow\uparrow\uparrow n =$ the iterated-iterated power, and so on.

[2] The formulae may include any arithmetic operation, powers and iterated powers.