

Design Summary

Data Structures

- **Ring Buffers (Lists, max 30 elements):**
 - Used for storing recent competitor prices, own prices, demands, and profit history.
 - Why? Keeps memory usage predictable and low, avoids unbounded growth.
 - Size: 30 periods (enough for short-term trends, fits well within RAM limits).

Online Update Equations

- **Mean & Variance (Welford's Algorithm):**
 - Updates mean and variance in $O(1)$ time, no need to recalculate from scratch.
 - Example (for demand):

```
n += 1
delta = new_value - mean
mean += delta / n
m2 += delta * (new_value - mean)
```
 - Used for both demand and competitor price statistics.
- **Profit Momentum (Exponential Moving Average):**
 - Tracks changes in profit to adapt base price.
 - Formula:

```
momentum = alpha * profit_change + (1 - alpha) * previous_momentum
```
 - Alpha is set to 0.1 for smooth adaptation.

Cold Start & Edge Case Handling

- **Cold Start:**
 - If no previous state, initialize all buffers and stats to default values.
- **Corrupted State:**
 - If state is missing or invalid, fall back to cold start.
- **Price Bounds:**
 - All prices are clipped to $[1.0, 20.0]$ for safety.
 - If price calculation fails (NaN/Inf), revert to base price.

Policy & Trade-offs

- **Base Price:**
 - Starts at 10.0, nudged by profit momentum and demand trend.
- **Competitor Response:**
 - Follows competitor price partially (30% gap), plus a trend-following signal.
- **Demand Response:**
 - Raises price if demand is strong and stable, lowers if weak.
 - If demand is volatile, price is more conservative.
- **Why this approach?**
 - Simple, fast, and robust.
 - No heavy ML, just smart use of streaming stats and bounded history.
 - Easy to debug and explain.

Rationale

- **Efficiency:**
 - All updates are $O(1)$, so the function is fast and meets timing constraints.
- **Memory:**
 - Bounded buffers ensure memory stays low, even for long runs.
- **Robustness:**
 - Handles missing/corrupted data gracefully.
- **Transparency:**
 - Logic is clear, easy to audit, and tweak.

For details, see comments in `duopoly.py`.