TMAT/AMAT 118 Practice Midterm 2

Name: _____

1. Given a function $f : \mathbb{R} \to \mathbb{R}$ and $a \in \mathbb{R}$, what does it mean for f to be continuous at a?

2. Let

$$f(x) = \begin{cases} 1/x & \text{if } x < 3, \\ 3x & \text{if } x \geq 3. \end{cases}$$

At which points is f not continuous?

3. Suppose f is continuous at a, and

$$\lim_{x \to a^+} f(x) = c.$$

What is $f(a)$?

4. Give the definition of the derivative of a function f at a point a, in two different ways.

5. Using the definition of the derivative and basic properties of limits, prove that if f and g are both differentiable at a, then

$$(f + g)'(a) = f'(a) + g'(a).$$

6. Compute the derivative of $f(x) = \sqrt{x}$ directly from the definition of the derivative. Do not use the power rule.

7. Complete the following to give the product rule for derivatives: If f and g are both functions which are differentiable everywhere, then $(fg)' =$ _____.

8. Find the first and second derivatives of $f(x) = 3x^3 + 2x^2 + x$.

9. For $f(x)$ as in the previous problem, find the slope of the tangent line to the curve $y = f(x)$ at $(1, 6)$.

10. What is the second derivative of $f(x) = \cos x$?

11. What is the tenth derivative of $f(x) = \sin x$?

12. Complete the following to give the chain rule: If g is differentiable at a and f is differentiable at $g(a)$, then $(f \circ g)'(a) =$ _____.

13. What are the first and second derivatives of $f(x) = \tan x$?

14. What is the derivative of $f(x) = e^{\sin 3x}$?

15. What is the derivative of $f(x) = (\ln x)^3$?

16. What is the derivative of $f(x) = \log_3 x$?

17. What is the derivative of $2^{(x^5)}$?

18. Find the derivative of a function $y = f(x)$ defined implicitly by $x^2 + 4y^2 = 4$.

19. What is the derivative of $x^{\sin 3x}$? HINT: Use logarithmic differentiation.

20. What is the derivative of $(\sin x)^{\cos x}$?

21. Suppose the distance in miles traveled by a car after t hours is given by $r(t) = 3t + \sqrt{t}$. Find expressions for the velocity $v(t)$ and acceleration $a(t)$ of the car at time t.

22. Suppose I drop a ball from the top of a 200m tower. What is the velocity of the ball just before it hits the ground? What is the acceleration? You may assume that the distance $r(t)$ in meters that the ball has dropped after t seconds is given by $r(t) = 4.9t^2$.