Lecture 6 - Recitation class

Today's class is a recitation class: you will solve problems in groups, and I will providing guidance as needed.

This lecture will **not** be recorded.

- If at the end of class you have not completed all problems, you can finish them after class
- When you are all done, you can `push` your solutions for ungraded feedback. Follow instructions here: [https://ufl.instructure.com/courses/404371/assignments/4422035 (https://ufl.instructure.com/courses/404371/assignments/4422035)](https://ufl.instructure.com/courses/404371/assignments/4422035)
  - Submission is not required

Analytical Problems

These problems are typically solved with paper and pencil.

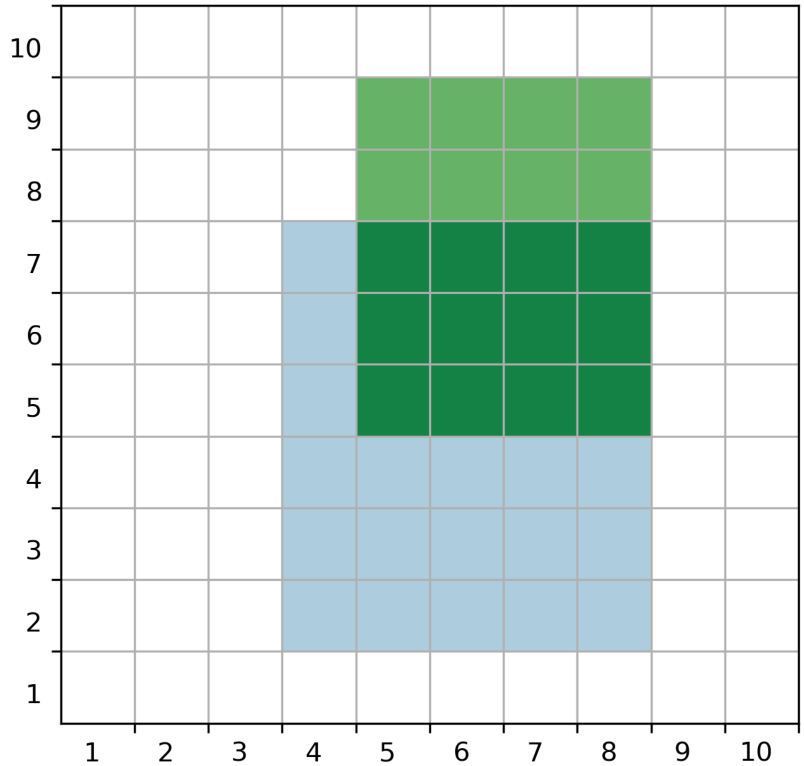
In [15]:
```
from IPython.display import Image # library module for embedding images that a
re kept when converting Notebook to PDF!
```

Exercise 1

The figure below shows a sample space and two events. Each box in the grid represents one outcome, and outcomes are equally likely. Let A be the event shown in blue (the one with more probability), and let B be the event shown in green (the one with less probability). Answer the questions and give the numerical values for the probabilities below (no need to show any work on this problem):

In [36]:
```
Image('figures/Twoevents.png', width=400, height=500) # note that you can control the size of the image
```

Out[36]:



1. $P(A) =$
2. $P(B) =$
3. $PA \cap B) =$
4. $P(A \cup B) =$
5. $P(A|B) =$
6. Are A and B mutually exclusive?

Exercise 2

Let $(S, \mathcal{F}, P)$ be a probability space with events A, B, C such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(C) = 0.1$. Find the following probabilities under the specified conditions. Using Venn diagrams will help in solving some of these problems.

1. Either A or B occurs if A and B are mutually exclusive.
2. A and B occur if A and B are mutually exclusive.
3. Consider $P(A \cap B) = 0.1$. What is $P(A^c \cap B)$?
4. Consider $P((A \cup B)^c) = 0.42$. Are A and B mutually exclusive?

Exercise 3

Using Venn diagram show that:

- $(A \cap B) \subset A$ and $(A \cap B) \subset B$
- $\overline{\left(\overline{A}\right)} = A$
- $A \subset B \Rightarrow \overline{B} \subset \overline{A}$
- $A \cup \overline{A} = \Omega$
- $\overline{A \cap B} = \overline{A} \cup \overline{B}$ (DeMorgan's Law 1)
- $\overline{A \cup B} = \overline{A} \cap \overline{B}$ (DeMorgan's Law 2)

To solve some of these problems, it is sufficient to *color-code* the different events to show that a statement is true.

Exercise 4

Consider the experiment where a fair 4-sided die is rolled twice and we assume that all sixteen possible outcomes are equally likely. Let X and Y be the result of the 1st and the 2nd roll, respectively.

We want to compute the conditional probability $P(A|B)$, where

$$A = \{\max(X, Y) = m\}$$

and

$$B = \{\min(X, Y) = 2\}$$

and m takes each of the values $1, 2, 3, 4$.

For each value of m, compute the probability $P(A|B)$.

To start solving this problem, draw the table of all possible pair of outcomes.

Exercise 5

Consider the experiment where you pick 3 cards at random from a deck of 52 playing cards (13 cards per suit) **without replacement**, i.e., at each card selection you will not put it back in the deck, and so the number of possible outcomes will change for each new draw.

Let D_i denote the event the card is diamonds in the ith draw.

Use the tree-based sequential diagram to compute the following probabilities:

1. $P(D_1 \cap D_2)$
2. $P(D_1 \cap D_2 \cap \overline{D_3})$
3. $P(D_3|D_1 \cap D_2)$

Simulation Problems

These problems are solved using Python simulations.

Make sure you load the necessary libraries, modules and magics.

Exercise 6

A magician has in her pocket a fair coin and a two-headed coin. She chooses one at random and flips it. Let H_i denote the event that the outcome of flip i is heads.

Use a Python simulation to find $P(H_3|H_1 \cap H_2)$.

Exercise 7

Challenge: A magician has in her pocket a fair coin and a two-headed coin. She chooses one at random and flips it. How many times must the coin up heads consecutively for the probability that the next flip is heads to be 99.9%?

The trick to make this implementation simple and easy is to consider "heads" 1 and "tails" 0.

In [ ]:

```

```