MATLAB Assignment 4

Due Thursday, November 30 at 11:59 PM EDT (Maryland time) on Gradescope.

Instructions:

On ELMS, see the file MATLAB_basics.pdf to learn how to get MATLAB and do some basic commands first. You may work with up to two other people (**groups of three total**). If you choose to work together, you may simply submit **one copy, and everyone will be receiving the same grade**. **Make sure to include all names when submitting to Gradescope!**

Submitting: To get an idea of what you should be submitting, you can first download the file `Example_Matlab_File.m` in the Files section. Open it in Matlab. Then at the top of the program, click on the PUBLISH tab. Click on the Publish button, and it should output an html file with all the code/output. This format is what your Matlab assignment should look like. When you are done with the actual Matlab project, click the Publish button, save this as a PDF, and **upload this to Gradescope.** There is a tab on ELMS that links you to Gradescope. **Remember to separate each problem by a section using the double percent signs.** Even if you have the correct code, **if there is no output, you will NOT receive full credit!**

(separate problems by using double percent signs as shown in the example file!!!!)

1. **Use the `short` format for this problem.**

   (a) Input the following matrix in Matlab: $A = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 1 & 5 & 1 & 3 \\ 0 & 2 & -3 & 5 \\ 1 & 2 & 1 & 0 \end{pmatrix}$.

   (b) Compute `[P,D]=eig(A)`, which creates a diagonal matrix D whose diagonal entries are the eigenvalues of A, and P is a matrix whose columns correspond to eigenvectors of the eigenvalues.

   (c) Suppose you are only given the information of the matrix D above. Use **a result from class** to determine whether or not A is diagonalizable. Use `disp` or `fprintf` to briefly explain your answer.

2. **Use the `rat` format for this problem.** We look at an example of an inner product that is not the dot product. Consider the inner product in $\mathbb{P}_n$ where

$$\langle f(x), g(x) \rangle = \int_0^1 f(x)g(x)\,dx.$$

   (a) Look up the `int` (integral) command, and compute the inner product of $f(x) = 2x^3 - 4x^2 + x - 2$ and $g(x) = x^5 - 3x + 1$. (make sure to put an asterisk when multiplying with x).

(b) Are the two functions $f(x)$ and $g(x)$ orthogonal? Show your work, and use `disp` or `fprintf` to briefly explain your answer.

(c) Create 2 polynomials explicitly $h_1(x)$, $h_2(x)$, **NEITHER OF WHICH IS A CONSTANT POLYNOMIAL** (*i.e.* they must be degree 1 or higher) that ARE orthogonal. Show/justify that they are orthogonal using Matlab.

Hint: Don't overcomplicate the polynomials. Try to find one simple polynomial that satisfies the condition, and then try to write it as a product.

3. **Use the `short` format for this problem.**

(a) Define the vectors $(2, 3, -3, -6)$, $(6, -1, 4, 1)$, $(0, 5, -3, 6)$, $(-4, 5, -2, 4)$ as **column vectors**. Label them `u1`, `u2`, `u3`, `u4`.

(b) Define `A=[u1 u2 u3 u4]`

(c) We now apply the Gram–Schmidt process to the vectors. Input

```
v1=u1
v2=u2-dot(u2,v1)/dot(v1,v1)*v1
```

Continue the process and define `v3` and `v4` by the Gram–Schmidt process.

(d) From the vectors in the previous part, create an **orthonormal basis** for $\mathbb{R}^4$ by dividing each vector by its magnitude. You can use the `norm` command to help. Denote these vectors as `w1`, `w2`, ...

(e) Define `Q=[w1 w2 w3 w4]`.

(f) Define $R = Q^T A$, and check that $A = QR$.

(g) Input

```
[Q1,R1] = qr(A,0)
```

to find a QR-factorization immediately. Observe your matrices are slightly different from our computation! This is because an orthonormal basis is not unique (think about how the first vector you defined could have been any of the 4).

4. **Use the `rat` format for this problem.**

(a) Define $A = \begin{pmatrix} 3 & 6 & -7 \\ 4 & -4 & 1 \\ 7 & -6 & 3 \end{pmatrix}$, and assume that it's the standard matrix of a linear transformation.

(b) Suppose $\mathcal{B} = \left\{ \begin{pmatrix} 3 \\ 4 \\ -3 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} \right\}$. Define the three vectors in $\mathcal{B}$ by `v1`, `v2`, `v3` in Matlab (as column vectors).

(c) Use the previous parts to help you find the $\mathcal{B}$-matrix for the transformation. Denote this matrix by `C`.

(d) Compute $C \begin{pmatrix} 17 \\ 17 \\ 17 \end{pmatrix}$. Denote this by `values`. Use `disp` or `fprintf` to briefly explain what these numbers mean in terms of the mapping, linear combinations, and basis.

(e) Verify your explanation in the previous part makes sense by computing out the linear combination and mapped vector.

5. **Use the `rat` format for this problem.** Let

$$S_1 = \{(5, -2, 1, -5, 0), (-3, 1, 6, 4, 2), (-6, 1, 0, 4, 2)\}$$

$$S_2 = \{(2, 3, -4, 3, 1), (-14, -8, 10, -8, -8), (-3, 2, -3, 2, -2)\}.$$

(a) Use appropriate Matlab commands (show your work!) to determine whether S_1 or S_2 is linearly independent. Use `disp` or `fprintf` to state which set is linearly independent. You do not need to explain why.

(b) Let W denote the span of the set chosen in the previous part. Thus, W is a subspace of $\mathbb{R}^5$. Denote the vectors in your chosen set as `u1`, `u2`, `u3`.

(c) Use the Gram–Schmidt process to find an orthogonal basis for W. Denote the vectors by `v1`, `v2`, `v3`.

(d) Define $\mathbf{y} = (7, -9, 0, 3, 2)$ (as a column vector).

(e) Express $\mathbf{y}$ as a sum of two vectors, one in W, denoted by `z1`, and the other in $W^\perp$, denoted by `z2`.

(f) What point in W is closest to $\mathbf{y}$?