MATLAB Assignment 1

Due Thursday, September 21 at 11:59 PM (Maryland time) on Gradescope

Instructions:

On Canvas, see the file MATLAB_basics.pdf to learn how to get MATLAB and do some basic commands first. You may work with up to two other people (**groups of three maximum**). If you choose to work together, you may simply submit **one copy, and everyone will be receiving the same grade**. **Make sure to include all names when submitting to Gradescope!**

Submitting: To get an idea of what you should be submitting, you can first download the file `Example_Matlab_File.m` in the Files section. Open it in Matlab. Then at the top of the program, click on the PUBLISH tab. Click on the Publish button, and it should output an html file with all the code/output. This format is what your Matlab assignment should look like. When you are done with the actual Matlab project, click the Publish button, save this as a PDF, and **upload this to Gradescope.** There is a tab on ELMS that links you to Gradescope. **Remember to separate each problem by a section using the double percent signs.** Even if you have the correct code, **if there is no output, you will NOT receive full credit!**

(separate problems by using double percent signs as shown in the example file!!!!)

1. For this problem, we will keep everything as rational numbers. Copy the following first

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format rat
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Consider the following system of equations

$$\begin{aligned} x_1 - 5x_2 - 3x_3 - 8x_4 &= -5 \\ x_1 + 6x_2 + 3x_3 + 25x_4 &= 14 \\ -x_1 + 4x_2 - 3x_3 + 5x_4 &= -3 \\ 3x_1 - 2x_2 - 3x_3 + 15x_4 &= 6 \end{aligned}$$

   (a) Define the augmented matrix for the system in Matlab, and denote it by A.

   (b) Apply elementary row operations (see item (10) in MATLAB_basics.pdf) to put the matrix in **ROW REDUCED ECHELON FORM**. Do NOT suppress the output for each operation you do, and do NOT use the `rref` command for this question.

   (c) Redefine your matrix A as in part (a), and now simply use the `rref` command on the matrix A. **From hereon, we will assume the usage of the command without showing the row operations in Matlab**.

(d) Use the `disp` or `fprintf` command to state what the solution of the system is. If there is no solution, explain why. If there is a unique solution, state what it is. If there are infinite solutions, find the general solution.

You may need to add LINE BREAKS if your explanation is long. If the grader cannot read what you wrote because the text was chopped off, you won't get full credit!

2. (Make sure to use double percent signs to make problem 2 a separate section!)

Let a and b be any two real numbers. We will show that the vector $\begin{pmatrix} a \\ b \end{pmatrix}$ lies in the span of $S = \left\{ \begin{pmatrix} -60 \\ 275 \end{pmatrix}, \begin{pmatrix} 350 \\ 400 \end{pmatrix} \right\}$.

(a) We need to tell MATLAB to treat a and b as symbolic variables. Therefore begin by including the following: `syms a b`. If it gives you an error, you may need to install an add-on for symbolic computation (click on the hyperlink it gives and follow the instructions).

(b) If $\begin{pmatrix} a \\ b \end{pmatrix}$ is written as a linear combination $c_1 \begin{pmatrix} -60 \\ 275 \end{pmatrix} + c_2 \begin{pmatrix} 350 \\ 400 \end{pmatrix}$, find an appropriate matrix and apply the `rref` command to find the coefficients of the linear combination.

(c) Use the `disp` or `fprintf` command to explicitly state what the weights/coefficients c_1 and c_2 are in the linear combination in part (b).

(d) Consider the set of two vectors $T = \left\{ \begin{pmatrix} -60 \\ 275 \end{pmatrix}, \mathbf{w} \right\}$. Find an explicit vector $\mathbf{w}$ (with actual numbers!) that is NOT $\begin{pmatrix} -60 \\ 275 \end{pmatrix}$ or the zero vector, so that NOT every vector $\begin{pmatrix} a \\ b \end{pmatrix}$ lies in the span of T. **Explain why your choice of w works, and provide an explicit vector that is *not* in the span of T.**

You may need to add LINE BREAKS if your explanation is long. If the grader cannot read what you wrote because the text was chopped off, you won't get full credit! This holds for all parts that you must explain. This is the last warning!

3. (a) Define a matrix A that would help determine if the set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\} = \left\{ \begin{pmatrix} 1 \\ 5 \\ 6 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ -2 \\ 5 \\ -4 \\ 1 \end{pmatrix}, \begin{pmatrix} -4 \\ 5 \\ 1 \\ 5 \\ -1 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ -7 \\ 0 \\ 2 \end{pmatrix} \right\}$ is linearly independent or linearly dependent.

(b) Use the `rref` command and then use `disp` or `fprintf` to **CLEARLY EXPLAIN** if the set is linearly independent or dependent. If you say something like "there's infinite solutions" you will get very little credit! Be very clear what you are saying! (Look back at how linear independence is defined!)
If the set is linearly dependent, use `disp` or `fprintf` to express $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, and $\mathbf{v}_4$ as a non-trivial linear combination of the zero vector.

4. Let

$$S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix} \right\}.$$

(a) Define an appropriate matrix and apply the `rref` command to determine if $\mathbf{w} = \begin{pmatrix} 25 \\ 55 \\ 10 \end{pmatrix}$ can be expressed as a linear combination of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

(b) If no such combination exists, briefly explain why. Otherwise, **if there is a unique** linear combination, find it and use `disp` or `fprintf` to state what it is. **If there is more than one way** to write the linear combination, use `disp` or `fprintf` to **show TWO different linear combinations that make** w.

5. Let

$$S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\} = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 12 \\ 14 \end{pmatrix} \right\}.$$

(a) Define an appropriate matrix and applying the `rref` command to determine if the vector $\mathbf{v}_1$ can be expressed as a linear combination of $\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$. Make sure to show the output of the RREF. **Then simply state yes or no.**

(b) Define an appropriate matrix and applying the `rref` command to determine if the vector $\mathbf{v}_2$ can be expressed as a linear combination of $\mathbf{v}_1, \mathbf{v}_3, \mathbf{v}_4$. Make sure to show the output of the RREF. **Then simply state yes or no.**

(c) Define an appropriate matrix and applying the `rref` command to determine if the vector $\mathbf{v}_3$ can be expressed as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4$. Make sure to show the output of the RREF. **Then simply state yes or no.**

(d) Define an appropriate matrix and applying the `rref` command to determine if the vector $\mathbf{v}_4$ can be expressed as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$. Make sure to show the output of the RREF. **Then simply state yes or no.**

(e) From the previous parts, **cite a result from class/the textbook** to decide whether the set of vectors is linearly independent or linearly dependent. You can use `disp` or `fprintf` to explain your reasoning.

(f) Suppose we only tested if $\mathbf{v}_2$ can be expressed as a linear combination of $\mathbf{v}_1, \mathbf{v}_3, \mathbf{v}_4$. Can we conclude that S is linear independent or dependent? From your answer in part (e), does this contradict the theorem from class? You can use `disp` or `fprintf` to explain your reasoning.

There is a short 1 minute video (Gradescope_tutorial.mp4) in the Files section on how to upload to Gradescope. If you are working with other people, MAKE SURE TO ADD THE GROUP MEMBERS WHEN YOU UPLOAD THE FILE. There should be click on a Group Members button for you to list the people in your group that will count as 1 submission. Also make sure to MARK YOUR PAGES AS SHOWN IN THE VIDEO.