MATLAB Assignment 3

Due Thursday, October 26 at 11:59 PM EDT (Maryland time) on Gradescope

Instructions:

On ELMS, see the file MATLAB_basics.pdf to learn how to get MATLAB and do some basic commands first. You may work with up to two other people (**groups of three total**). If you choose to work together, you may simply submit **one copy, and everyone will be receiving the same grade**. **Make sure to include all names when submitting to Gradescope!**

Submitting: To get an idea of what you should be submitting, you can first download the file `Example_Matlab_File.m` in the Files section. Open it in Matlab. Then at the top of the program, click on the PUBLISH tab. Click on the Publish button, and it should output an html file with all the code/output. This format is what your Matlab assignment should look like. When you are done with the actual Matlab project, click the Publish button, save this as a PDF, and **upload this to Gradescope.** There is a tab on ELMS that links you to Gradescope. **Remember to separate each problem by a section using the double percent signs.** Even if you have the correct code, **if there is no output, you will NOT receive full credit!**

(separate problems by using double percent signs as shown in the example file!!!!)

1. Use `format rat`. Define

$$A = \begin{pmatrix} 3 & -8 & 22 & -8 & 2 \\ 6 & -8 & -2 & -8 & 44 \\ -3 & 3 & 15 & 3 & -27 \\ -1 & 1 & 6 & 1 & -9 \\ 0 & 5 & -7 & 5 & 25 \\ 4 & 2 & -3 & 2 & 66 \end{pmatrix}.$$

   (a) Use the `rref` command and then determine a basis for the column space and for the kernel of matrix A. You can use `disp` or `fprintf` to show your answer.

   (b) Suppose A is now the **augmented matrix** of a system of equations. What is the solution to the system? If it is unique, state it. If there are infinite solutions, describe it in parametric vector form. If there are no solutions, explain why.

2. Use `format rat`. Define

$$A = \begin{pmatrix} 5 & -3 & 6 & 7 \\ 0 & 0 & 0 & 1 \\ 7 & -5 & 3 & -7 \\ 4 & 0 & 3 & -7 \end{pmatrix}.$$

   (a) Find a basis for the row space of A. Use `disp` or `fprintf`, and **express the vectors as row vectors using brackets**, like $\{[3\ \ 4\ \ 6\ \ 1], [1\ \ 3\ \ 5\ \ 7]\}$.

(b) Find a basis for the column space of A. Use `disp` or `fprintf` to show your answer.

(c) Does $\dim(\text{row}(A)) = \dim(\text{col}(A))$? Simply state yes or no.

(d) Are $\text{row}(A)$ and $\text{col}(A)$ equal as sets? Use `disp` or `fprintf` to **briefly explain.**

3. Use `format rat`. Consider the set

$$\mathcal{B} = \{(4, 1, -5, 5), (2, 6, 4, -7), (4, 3, 0, 1), (-8, 6, 7, 7))\}$$

Let $\mathbf{v} = (2, -30, 13, -10)$.

(a) Use appropriate Matlab commands to find $\mathbf{u}$ where $[\mathbf{u}]_{\mathcal{B}} = \mathbf{v}$.

(b) Use appropriate Matlab commands to find $\mathbf{w}$ where $\mathbf{w} = [\mathbf{v}]_{\mathcal{B}}$.

Make sure to show your computations, and use `disp` or `fprintf` to briefly explain what you are doing.

4. Use `format rat`. Consider the polynomials

$$f_1(x) = 7 - 3x + x^2 + 7x^3 + 2x^4$$

$$f_2(x) = 9 - 3x - 9x^2 - 5x^3 - 6x^4$$

$$f_3(x) = 1 - x + 3x^2 + 4x^3 + 3x^4$$

$$f_4(x) = 5 - 3x - x^2 + x^4$$

that lie in $\mathbb{P}_4$ (the vector space of polynomials of degree at most 4). Let

$$W = \text{Span}\{f_1(x), f_2(x), f_3(x), f_4(x)\}.$$

(a) Denote each of the 4 vectors $\mathbf{v}_i = [f_i(x)]_{\mathcal{B}}$ to be the coordinate vector of $f_i(x)$ relative to the basis $\mathcal{B} = \{1, x, x^2, x^3, x^4\}$ in $\mathbb{P}_4$. Define these as **column** vectors in Matlab as `v1`, `v2`, ...

(b) Define $A =$ [`v1 v2 v3 v4`] to be the matrix whose i^{th} column is the vector $\mathbf{v}_i$.

(c) Use appropriate commands with the matrix above to help find a basis $\mathcal{S}$ for W. Use `disp` or `fprintf` to explicitly show what the basis is **when written as polynomials (not vectors)**.

(d) Is the set $\{f_1(x), f_2(x), f_3(x), f_4(x)\}$ linearly independent or linearly dependent? If it is linearly dependent, find an explicit dependent relationship. If it is linearly independent, use `disp` or `fprintf` to justify your answer.

5. Use `format short`. We first show the set of vectors

$$\mathcal{B} = \{1, \cos(x), \cos^2(x), \cos^3(x), \cos^4(x)\}$$

is linearly independent in the vector space of real-valued functions. That is, we want to show the equation

$$a_0 \cdot 1 + a_1 \cos(x) + a_2 \cos^2(x) + a_3 \cos^3(x) + a_4 \cos^4(x) = 0 \tag{1}$$

is true only when $a_0 = a_1 = a_2 = a_3 = a_4 = 0$. Note that (1) must hold for all values of x. If the set of functions was linearly dependent, then for any dependent relation, say $3 - 4\cos(x) + 12\cos^2(x) - 2\cos^3(x) + 2\cos^4(x) = 0$ **(this is not a true equation)**, we should still observe linear dependency for explicit values of x — *e.g.* if we plugged in $x = \pi/23$.

(a) By subbing in a value for x in (1), we get a linear equation in terms of variables a_0, a_1, a_2, a_3, and a_4. Sub in $x = 0.1, 0.2, 0.3, 0.4, 0.5$ to create a linear system of 5 equations and 5 unknowns. Define its coefficient matrix by A in Matlab.

(b) Observe the system has a non-trivial solution if and only if (1) would have a non-trivial solution. Use appropriate Matlab commands to conclude the system only has a trivial solution, thus implying $\mathcal{B}$ is a linearly independent set of vectors.

(c) Let $\mathcal{C} = \{1, \cos(x), \cos(2x), \cos(3x), \cos(4x)\}$. We have the following identities:

$$\cos(2x) = -1 + 2\cos^2(x)$$

$$\cos(3x) = -3\cos(x) + 4\cos^3(x)$$

$$\cos(4x) = 1 - 8\cos^2(x) + 8\cos^4(x).$$

With the help of the identities, determine the $\mathcal{B}$-coordinate vectors for the 5 vectors in $\mathcal{C}$. Define these as **column** vectors in Matlab as `u1, u2,`...

(d) Using part (c), define an appropriate matrix B and use Matlab commands to determine if $\mathcal{C}$ is a linearly independent set or not. Use `disp` or `fprintf` to briefly explain your answer.

(e) If $D = \text{Span}(\mathcal{B})$, explain why $\mathcal{C}$ forms a basis for D.