Autoencoders

ALS User Meeting

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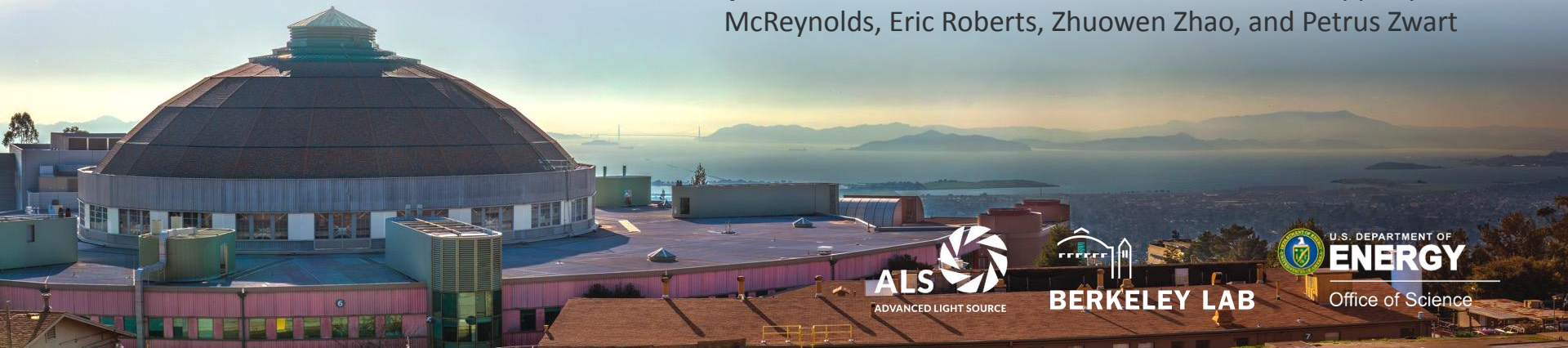
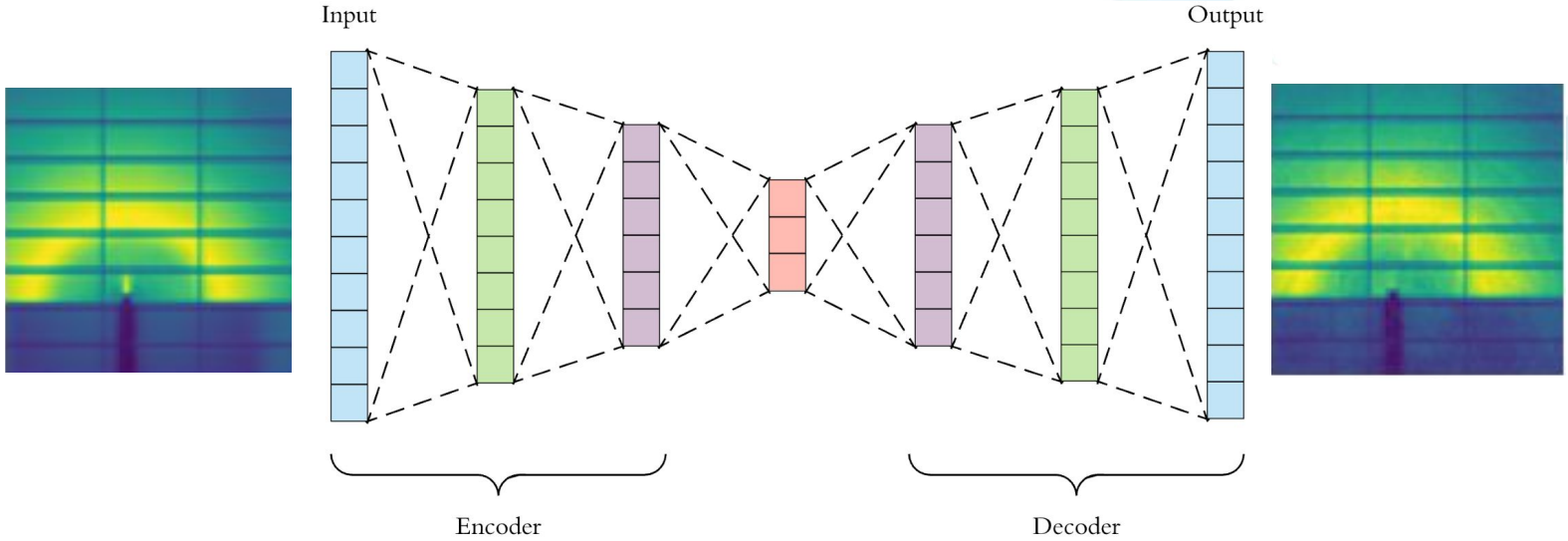


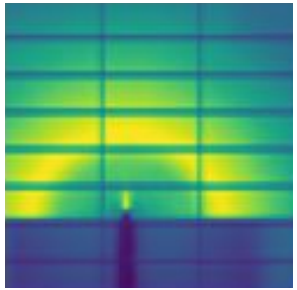
What is an autoencoder?

Neural network that learns efficient data embeddings. It consists of 2 parts: an encoder and a decoder.

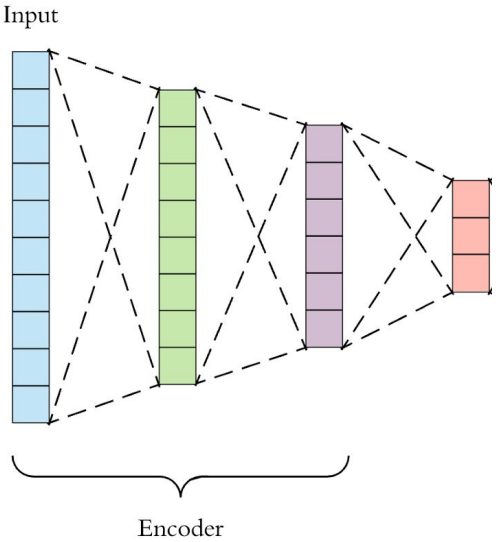


Encoder

Compresses the input data into latent space vector.



1024x1024=1,048,576



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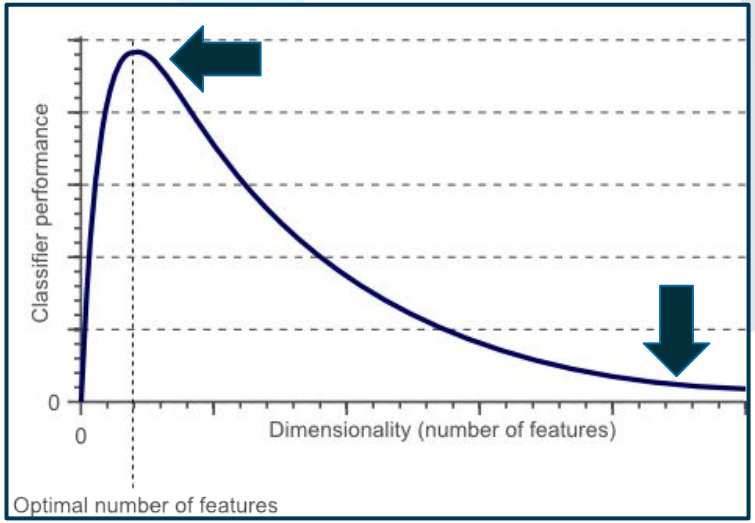
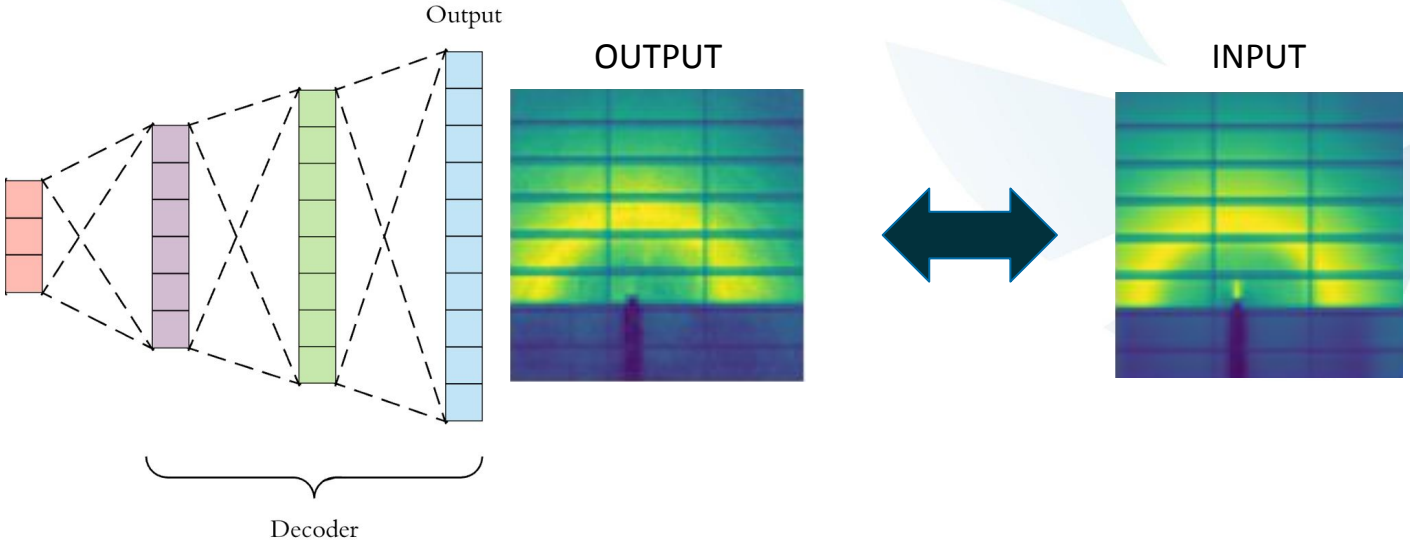


Decoder

Reconstructs the input data by using the latent space vector.



Hidden Layers

x: input, y: output

Linear transformation

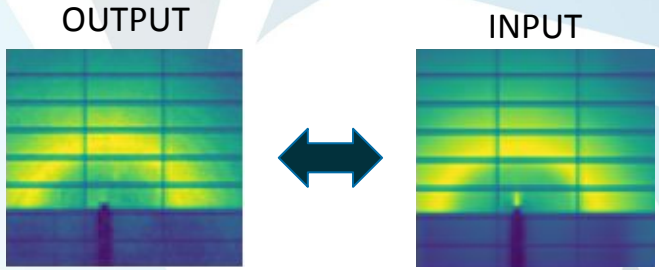
$$y = xW^T + b$$

Weights Bias

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} w_{11} & w_{21} & w_{31} & w_{41} \\ w_{12} & w_{22} & w_{32} & w_{42} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

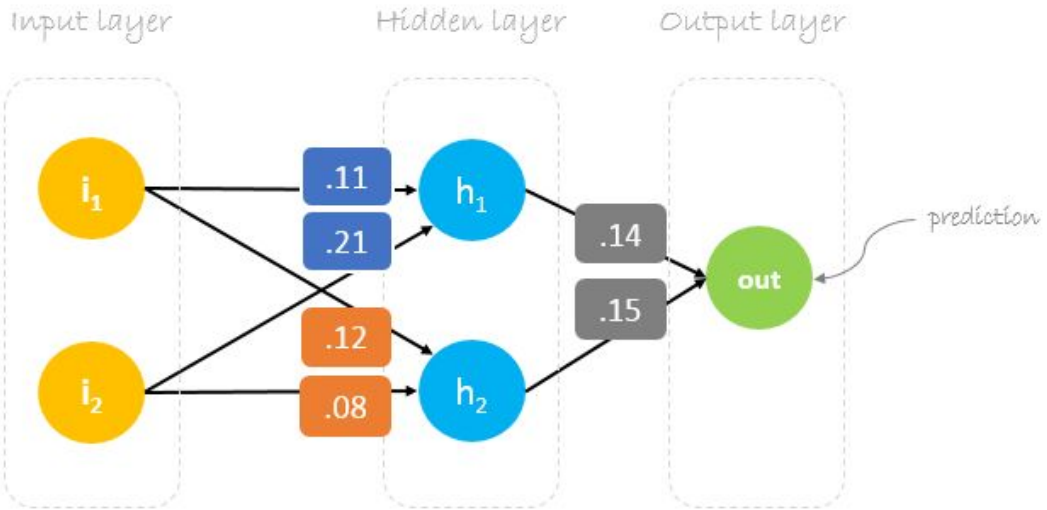
Backpropagation

$$Error = \frac{1}{2}(\hat{Y} - Y)^2$$



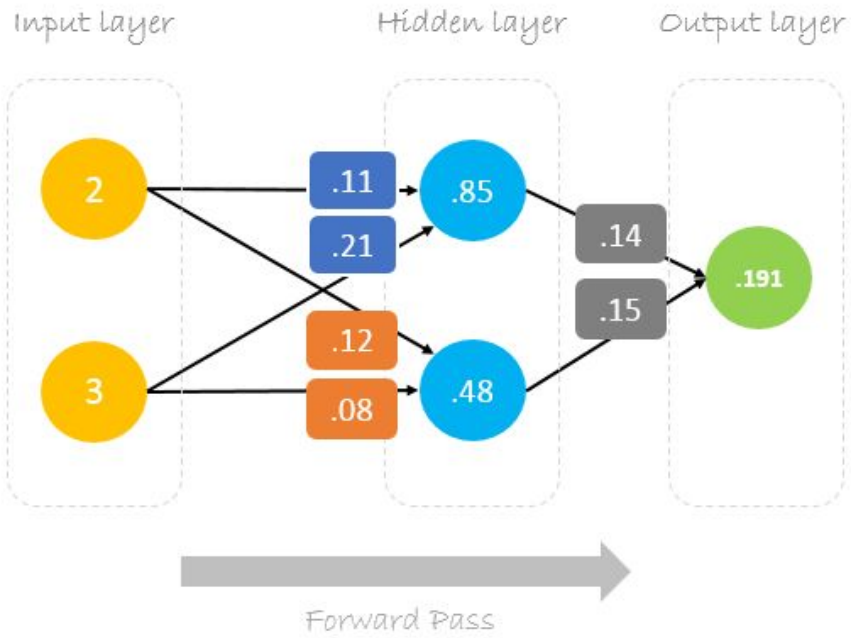
Gradient descent: $$w_i' = w_i - lr\left(\frac{\partial Error}{\partial w_i}\right)$$

Backpropagation example



This example was adapted from: https://hmkcode.com/ai/backpropagation-step-by-step/

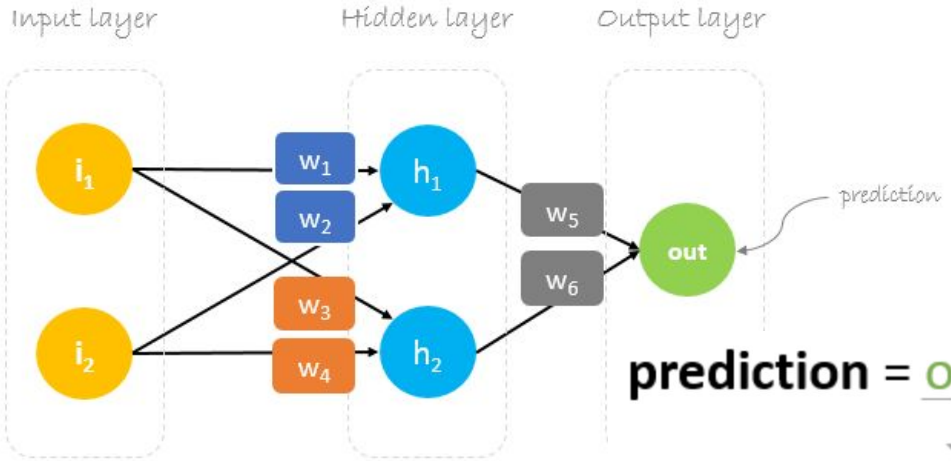
Backpropagation example



Actual output = 1

Error = 0.327

Backpropagation example



$$\text{prediction} = \text{out}$$

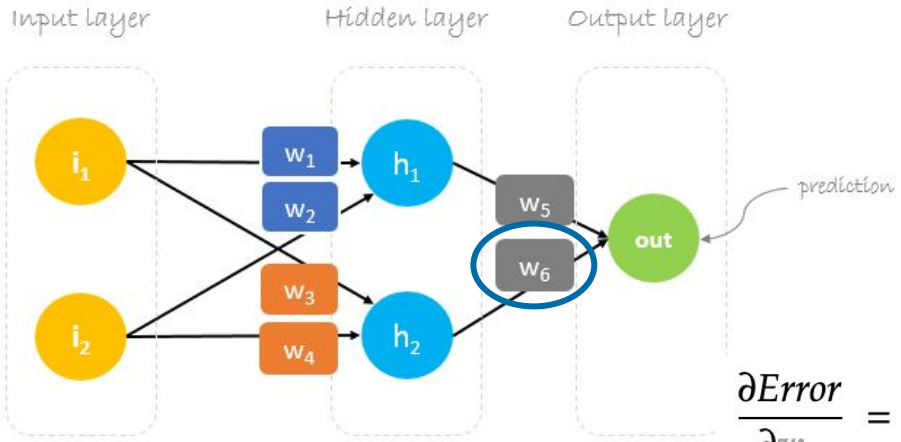
$$\text{prediction} = (h_1)\, w_5 + (h_2)\, w_6$$

$$h_1 = i_1 w_1 + i_2 w_2$$

$$h_2 = i_1 w_3 + i_2 w_4$$

$$\text{prediction} = (i_1 w_1 + i_2 w_2)\, w_5 + (i_1 w_3 + i_2 w_4)\, w_6$$

Backpropagation example



$$w_6' = w_6 - lr\left(\frac{\partial Error}{\partial w_6}\right)$$

$$\frac{\partial Error}{\partial w_6} = \frac{\partial Error}{\partial \hat{Y}} \times \frac{\partial \hat{Y}}{\partial w_6}$$

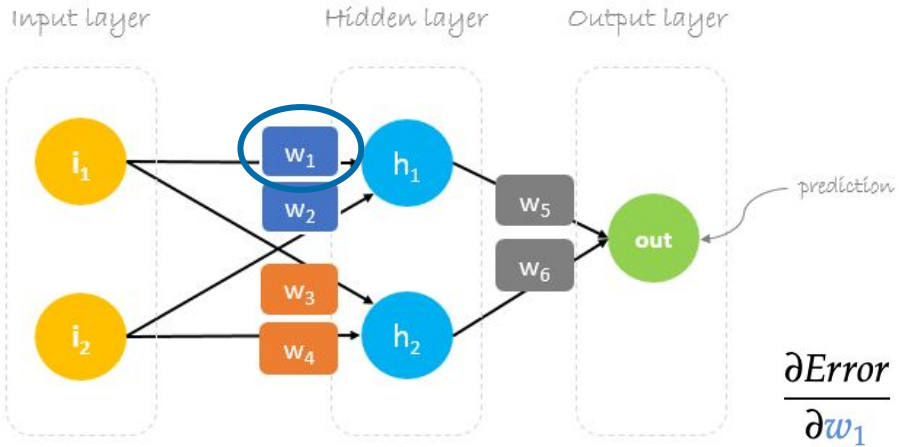
$$\frac{\partial Error}{\partial w_6} = \frac{\partial 0.5(\hat{Y} - Y)^2}{\partial \hat{Y}} \times \frac{\partial[(i_1 w_1 + i_2 w_2)w_5 + (i_1 w_3 + i_2 w_4)w_6]}{\partial w_6}$$

$$\frac{\partial Error}{\partial w_6} = (\hat{Y} - Y)(i_1 w_3 + i_2 w_4)$$

$$\frac{\partial Error}{\partial w_6} = (\hat{Y} - Y)h_2$$

$$\boxed{w_6' = w_6 - lr\,(\hat{Y} - Y)h_2}$$

Backpropagation example



$$w_1' = w_1 - lr\left(\frac{\partial Error}{\partial w_1}\right)$$

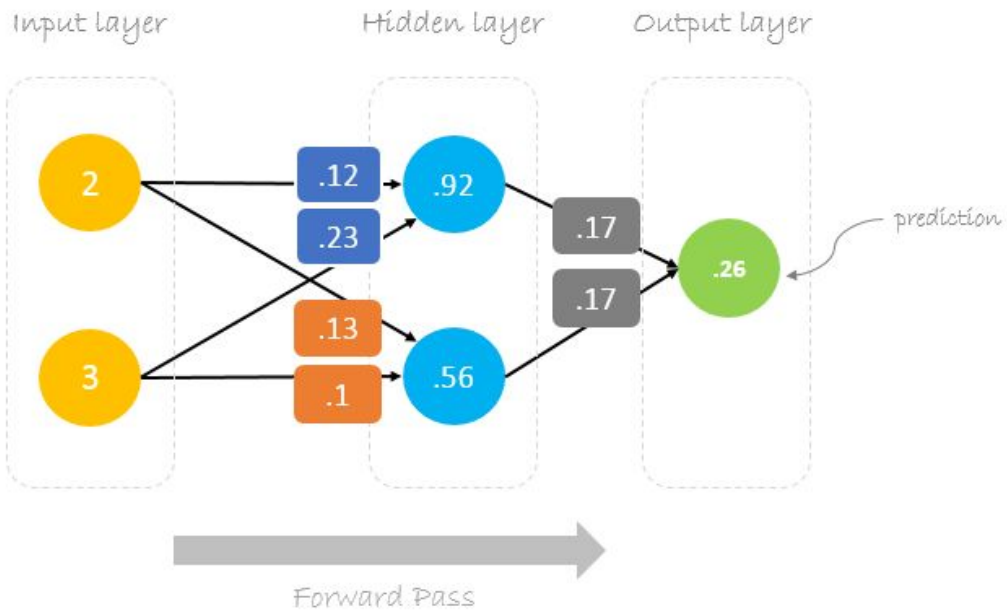
$$\frac{\partial Error}{\partial w_1} = \frac{\partial Error}{\partial \hat{Y}} \times \frac{\partial \hat{Y}}{\partial h_1} \times \frac{\partial h_1}{\partial w_1}$$

$$\frac{\partial Error}{\partial w_1} = \frac{\partial 0.5(\hat{Y} - Y)^2}{\partial \hat{Y}} \times \frac{\partial [h_1 w_5 + h_2 w_6]}{\partial h_1} \times \frac{\partial [i_1 w_1 + i_2 w_2]}{\partial w_1}$$

$$\frac{\partial Error}{\partial w_1} = (\hat{Y} - Y) w_5 i_1$$

$$\boxed{w_1' = w_1 - lr\,(\hat{Y} - Y) w_5 i_1}$$

Backpropagation example



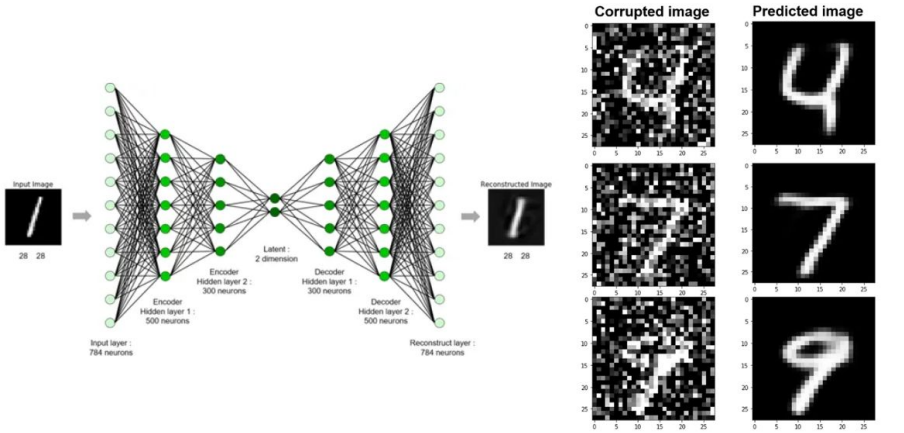
Before = 0.191

Actual output = 1

Error = 0.274

Common applications

- Image denoising
- Data compression
- Feature extraction
- Recommendation systems
- Image Generation
- Many others



https://omdena.com/blog/denoising-autoencoders/