*Assignment 3: Fairness, Maximum Likelihood Estimation, and Text Classification*

*Machine Learning*

*Fall 2019*

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### 💡 Learning Objectives

- Learn about the connection between probabilistic criteria for algorithmic fairness and Bayesian Networks.

- Solidify your understanding of the Naïve Bayes algorithm by applying it to movie review sentiment classification.

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## 1 Bayesian Networks and Algorithmic Fairness

In assignment 1 of this module we discussed how Bayesian methods can be used to reason about algorithmic fairness. We've just had some lengthy discussions about fairness within the context of the COMPAS algorithm. We touched upon some of the limitations of statistically based notions of fairness. Nevertheless, they do have a potential role to play, and you should know what the most common definitions of fairness are and what assumptions they make.

As context for the reading and to help us have common notation, suppose we have the following random variables.

- $R$ is a prediction generated by our algorithm.

- $A$ is a sensitive attribute

- $Y$ is the thing we're trying to predict (we want $R = Y$ if we are predicting accurately)

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### 🔗 External Resource(s) (40 minutes)

Read Fairness and Machine Learning Chapter 2. Start at the section *Formal non-discrimination criteria* and read up to (but not including) the section *Calibration and sufficiency*.

#### ⚠ Notice

- Don't get hung up on the ROC curves. We can certainly discuss this on NB, but it is not required to understand what is going on here. The presentation earlier in the linked reading is also pretty clear.

- The notation they use in this reading for conditional independence is an upside down T with only one line (instead of our notation, $\perp\!\!\!\perp$).

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### Exercise 1 (10 minutes)

Thinking back to the COMPAS example, which definition of fairness given in the reading was Propublica using? Which definition of fairness was Northpointe using?

> ☆ **Solution**
>
> - Northpointe is using sufficiency $Y \perp\!\!\!\perp A \mid R$. You'll notice that in the reading they say that sufficiency is the same thing as matching positive and negative predictive value for all values of the protected attribute.
>
> - Propublica is using separation $R \perp\!\!\!\perp A \mid Y$. This fairness principle requires the false positive and true positive rates to be the same across for all values of the protected attribute.

If you're interested in playing around with the COMPAS data more extensively, we have put together a notebook that reproduces the calculation that are at the heart of the two competing notions of fairness. The notebook is written with a couple of notebook exercises, but these are totally optional. We expect the default will be that folks will just take a look if they want to (or skip this if they don't). If you want to spend some extra time and solidify your understanding of true positive rate, false positive rate, positive predictive value, etc., then it might be worth doing it as a set of exercises.

For the purposes of posting on NB, the conversion of the notebook is at the end of the document.

## 2  Text Classification with Bag of Words

Next we'll be applying Naïve Bayes to the task of classifying text.

> ⤢ **External Resource(s) (60 minutes)**
>
> This will be done in the Assignment 3 companion notebook.

(to make this more readable, we have included the PDF of the companion notebook at the end).

## 3  The Intelligent Design of Jenny Chow

This assignment is fully described on the Intelligent Design of Jenny Chow Canvas page. There is also an alternative described on the assignment page if you can't attend. Make sure to look at the assignment before going to the play since we are asking you to capture some of your reactions / thoughts so that you can bring them to class on Monday for discussion.

# Exploring the ProPublica COMPAS Analysis

In this notebook you'll get a chance to examine the data used in the ProPublica story yourself.

*Disclaimer:* Please don't over interpret what you find in the data. We know from our discussions that methodology is key to being able to properly interpret findings. Our goal here will be to reproduce results from the readings we did for last class.

First, we'll download and parse the data into a data frame.

In [2]:

```python
import pandas as pd

!wget https://raw.githubusercontent.com/propublica/compas-analysis/master/compas-scores-two-years.csv
df = pd.read_csv('compas-scores-two-years.csv')
df
```

```
--2019-10-24 03:07:49--  https://raw.githubusercontent.com/propublica/compas-
analysis/master/compas-scores-two-years.csv
Resolving raw.githubusercontent.com (raw.githubusercontent.com)... 151.101.0.133, 151.101.64.133,
151.101.128.133, ...
Connecting to raw.githubusercontent.com (raw.githubusercontent.com)|151.101.0.133|:443...
connected.
HTTP request sent, awaiting response... 200 OK
Length: 2546489 (2.4M) [text/plain]
Saving to: 'compas-scores-two-years.csv'

compas-scores-two-y 100%[===================>]   2.43M  --.-KB/s    in 0.09s

2019-10-24 03:07:49 (26.4 MB/s) - 'compas-scores-two-years.csv' saved [2546489/2546489]
```

Out[2]:

| | id | name | first | last | compas_screening_date | sex | dob | age | age_cat | race | juv_fel_count | dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | miguel hernandez | miguel | hernandez | 2013-08-14 | Male | 1947-04-18 | 69 | Greater than 45 | Other | 0 | |
| 1 | 3 | kevon dixon | kevon | dixon | 2013-01-27 | Male | 1982-01-22 | 34 | 25 - 45 | African-American | 0 | |
| 2 | 4 | ed philo | ed | philo | 2013-04-14 | Male | 1991-05-14 | 24 | Less than 25 | African-American | 0 | |
| 3 | 5 | marcu brown | marcu | brown | 2013-01-13 | Male | 1993-01-21 | 23 | Less than 25 | African-American | 0 | |
| 4 | 6 | bouthy pierrelouis | bouthy | pierrelouis | 2013-03-26 | Male | 1973-01-22 | 43 | 25 - 45 | Other | 0 | |
| 5 | 7 | marsha miles | marsha | miles | 2013-11-30 | Male | 1971-08-22 | 44 | 25 - 45 | Other | 0 | |
| 6 | 8 | edward riddle | edward | riddle | 2014-02-19 | Male | 1974-07-23 | 41 | 25 - 45 | Caucasian | 0 | |
| 7 | 9 | steven stewart | steven | stewart | 2013-08-30 | Male | 1973-02-25 | 43 | 25 - 45 | Other | 0 | |
| 8 | 10 | elizabeth thieme | elizabeth | thieme | 2014-03-16 | Female | 1976-06-03 | 39 | 25 - 45 | Caucasian | 0 | |
| 9 | 13 | bo bradac | bo | bradac | 2013-11-04 | Male | 1994-06-10 | 21 | Less than 25 | Caucasian | 0 | |

| | id | name | first | last | compas_screening_date | sex | dob | age | age_cat | race | juv_fel_count | dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 14 | benjamin franc | benjamin | franc | 2013-11-26 | Male | 1986-06-01 | 27 | 25 - 45 | Caucasian | 0 | |
| 11 | 15 | ellyaher lanza | ellyaher | lanza | 2013-10-03 | Male | 1992-08-18 | 23 | Less than 25 | African-American | 0 | |
| 12 | 16 | kortney coleman | kortney | coleman | 2013-01-01 | Female | 1978-08-22 | 37 | 25 - 45 | Caucasian | 0 | |
| 13 | 18 | jarrod turbe | jarrod | turbe | 2013-10-09 | Male | 1974-12-02 | 41 | 25 - 45 | African-American | 0 | |
| 14 | 19 | craig gilbert | craig | gilbert | 2013-10-30 | Female | 1968-06-14 | 47 | Greater than 45 | Caucasian | 0 | |
| 15 | 20 | samuel seraphin | samuel | seraphin | 2014-06-03 | Male | 1985-03-25 | 31 | 25 - 45 | African-American | 0 | |
| 16 | 21 | mario hernandez | mario | hernandez | 2014-03-24 | Male | 1979-01-25 | 37 | 25 - 45 | Hispanic | 0 | |
| 17 | 22 | darrious davis | darrious | davis | 2013-12-22 | Male | 1990-06-22 | 25 | 25 - 45 | African-American | 0 | |
| 18 | 23 | neil heckart | neil | heckart | 2013-11-17 | Male | 1984-12-24 | 31 | 25 - 45 | Caucasian | 0 | |
| 19 | 24 | michael lux | michael | lux | 2014-11-15 | Male | 1985-01-08 | 31 | 25 - 45 | Caucasian | 0 | |
| 20 | 25 | columbus wilson | columbus | wilson | 2014-05-02 | Male | 1951-06-28 | 64 | Greater than 45 | African-American | 0 | |
| 21 | 26 | vandivuiet williams | vandivuiet | williams | 2013-04-18 | Male | 1994-11-29 | 21 | Less than 25 | African-American | 0 | |
| 22 | 27 | nelson avalo | nelson | avalo | 2014-10-16 | Male | 1988-08-06 | 27 | 25 - 45 | Caucasian | 0 | |
| 23 | 28 | janel denicola | janel | denicola | 2013-11-22 | Female | 1995-03-22 | 21 | Less than 25 | Caucasian | 0 | |
| 24 | 30 | dominic pabon | dominic | pabon | 2013-02-08 | Male | 1992-01-23 | 24 | Less than 25 | Hispanic | 0 | |
| 25 | 32 | russell sottile | russell | sottile | 2013-01-25 | Male | 1973-01-10 | 43 | 25 - 45 | Caucasian | 0 | |
| 26 | 33 | andre ashley | andre | ashley | 2013-05-11 | Male | 1983-08-24 | 32 | 25 - 45 | Other | 0 | |
| 27 | 37 | deandrae counts | deandrae | counts | 2013-05-06 | Male | 1989-02-08 | 27 | 25 - 45 | African-American | 0 | |
| 28 | 38 | victoria soltau | victoria | soltau | 2013-03-18 | Female | 1979-09-03 | 36 | 25 - 45 | Caucasian | 0 | |
| 29 | 39 | najee sapp | najee | sapp | 2013-02-20 | Male | 1989-04-27 | 26 | 25 - 45 | African-American | 0 | |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| 7184 | 10962 | yolani moratz | yolani | moratz | 2013-08-08 | Female | 1979-11-05 | 36 | 25 - 45 | Caucasian | 0 | |
| 7185 | 10963 | paul wyatt | paul | wyatt | 2013-11-06 | Male | 1973-09-19 | 42 | 25 - 45 | Caucasian | 0 | |
| 7186 | 10964 | terrence brown | terrence | brown | 2013-02-23 | Male | 1979-06-08 | 36 | 25 - 45 | African-American | 0 | |
| 7187 | 10965 | anthony fields | anthony | fields | 2013-03-13 | Male | 1959-02-20 | 57 | Greater than 45 | Caucasian | 0 | |

| | id | name | first | last | compas_screening_date | sex | dob | age | age_cat | race | juv_fel_count | dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **7188** | 10966 | shameel koya | shameel | koya | 2013-03-04 | Male | 1979-10-12 | 36 | 25 - 45 | Caucasian | 0 | |
| **7189** | 10967 | george bedward | george | bedward | 2014-05-31 | Male | 1981-05-26 | 34 | 25 - 45 | African-American | 0 | |
| **7190** | 10969 | eric sparks | eric | sparks | 2013-01-11 | Male | 1991-07-13 | 24 | Less than 25 | African-American | 0 | |
| **7191** | 10971 | eugene drogus | eugene | drogus | 2014-01-07 | Male | 1948-10-17 | 67 | Greater than 45 | Caucasian | 0 | |
| **7192** | 10972 | matt munoz | matt | munoz | 2013-09-12 | Male | 1984-03-22 | 32 | 25 - 45 | Caucasian | 0 | |
| **7193** | 10975 | warren aiken | warren | aiken | 2013-09-05 | Male | 1990-09-30 | 25 | 25 - 45 | African-American | 0 | |
| **7194** | 10976 | arleen martin | arleen | martin | 2014-12-19 | Female | 1985-08-14 | 30 | 25 - 45 | Caucasian | 0 | |
| **7195** | 10977 | adrian williams | adrian | williams | 2013-09-18 | Male | 1981-03-09 | 35 | 25 - 45 | African-American | 0 | |
| **7196** | 10979 | angelita diaz | angelita | diaz | 2013-09-06 | Male | 1972-07-19 | 43 | 25 - 45 | African-American | 0 | |
| **7197** | 10980 | jarvis yates | jarvis | yates | 2014-02-27 | Male | 1987-08-27 | 28 | 25 - 45 | African-American | 0 | |
| **7198** | 10981 | orett harrison | orett | harrison | 2013-12-25 | Male | 1984-03-31 | 32 | 25 - 45 | African-American | 0 | |
| **7199** | 10982 | austin harris | austin | harris | 2013-10-01 | Male | 1992-07-07 | 23 | Less than 25 | Caucasian | 0 | |
| **7200** | 10984 | shantrina stfort | shantrina | stfort | 2013-11-05 | Female | 1995-06-06 | 20 | Less than 25 | African-American | 0 | |
| **7201** | 10985 | kyle miller | kyle | miller | 2014-01-22 | Male | 1986-04-08 | 30 | 25 - 45 | African-American | 0 | |
| **7202** | 10987 | ceasar gomez | ceasar | gomez | 2013-03-31 | Male | 1990-02-07 | 26 | 25 - 45 | Hispanic | 0 | |
| **7203** | 10988 | luis fernandez | luis | fernandez | 2013-10-27 | Male | 1971-09-19 | 44 | 25 - 45 | Hispanic | 0 | |
| **7204** | 10989 | rodney montgomery | rodney | montgomery | 2013-12-28 | Male | 1985-09-28 | 30 | 25 - 45 | African-American | 0 | |
| **7205** | 10990 | christopher tun | christopher | tun | 2013-05-28 | Male | 1992-04-28 | 23 | Less than 25 | Caucasian | 0 | |
| **7206** | 10992 | alexander vega | alexander | vega | 2013-05-10 | Male | 1994-07-15 | 21 | Less than 25 | Caucasian | 0 | |
| **7207** | 10994 | jarred payne | jarred | payne | 2014-05-10 | Male | 1985-07-31 | 30 | 25 - 45 | African-American | 0 | |
| **7208** | 10995 | raheem smith | raheem | smith | 2013-10-20 | Male | 1995-06-28 | 20 | Less than 25 | African-American | 0 | |
| **7209** | 10996 | steven butler | steven | butler | 2013-11-23 | Male | 1992-07-17 | 23 | Less than 25 | African-American | 0 | |
| **7210** | 10997 | malcolm simmons | malcolm | simmons | 2014-02-01 | Male | 1993-03-25 | 23 | Less than 25 | African-American | 0 | |
| **7211** | 10999 | winston gregory | winston | gregory | 2014-01-14 | Male | 1958-10-01 | 57 | Greater than 45 | Other | 0 | |

| | id | name | first | last | compas_screening_date | sex | dob | age | age_cat | race | juv_fel_count | dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7212 | 11000 | farrah jean | farrah | jean | 2014-03-09 | Female | 1982-11-17 | 33 | 25 - 45 | African American | 0 | 0 |
| 7213 | 11001 | florencia sanmartin | florencia | sanmartin | 2014-06-30 | Female | 1992-12-18 | 23 | Less than 25 | Hispanic | 0 | |

7214 rows × 53 columns

In the ProPublica dataset they only used data where the "days_b_screening_arrest" feature was in the range [-30, 30].

In [0]:

```
# filter these based on propublica analysis (not sure why this doesn't match
https://fairmlbook.org/classification.html)
df = df[(df['days_b_screening_arrest'] <= 30) | (df['days_b_screening_arrest'] >= -30)]
```

## Reproducing Calculations of False Positive Rate and Positive Predictive Value

From a statistical point of view (but not necessarily a social justice point of view) the debate between Propublica and NorthPointe boiled down to what is the rate way to measure bias in an algorithm. As you saw in the first part of the assignmnet, ProPublica used the evidence that the false positive rate differed between blacks and whites as evidence of bias. Northpointe argued used the fact that the positive predictive values across the two groups were the same as evidence *against* bias.

In Northpointe's report, they have a table which lists various statistics for blacks versus whites using the COMPAS risk scores as the predictor. Here is the relevant information from their report.

| Race | $\hat{y} = 1$ | True positive rate | False Positive Rate | Positive Predictive Value | Negative Predictive Value |
|---|---|---|---|---|---|
| white | $decile \geq 1$ | 1.00 | 1.00 | 0.39 | 1.00 | |
| | $decile \geq 2$ | 0.85 | 0.64 | 0.46 | 0.79 |
| | $decile \geq 3$ | 0.74 | 0.47 | 0.5 | 0.76 |
| | $decile \geq 4$ | 0.64 | 0.35 | 0.54 | 0.74 |
| | $decile \geq 5$ | 0.52 | 0.23 | 0.59 | 0.71 |
| | $decile \geq 6$ | 0.41 | 0.15 | 0.64 | 0.69 |
| | $decile \geq 7$ | 0.29 | 0.09 | 0.68 | 0.66 |
| | $decile \geq 8$ | 0.20 | 0.05 | 0.71 | 0.65 |
| | $decile \geq 9$ | 0.12 | 0.03 | 0.70 | 0.63 |
| | $decile \geq 10$ | 0.05 | 0.01 | 0.70 | 0.61 |
| black | $decile \geq 1$ | 1.00 | 1.00 | 0.51 | 1.00 | |
| | $decile \geq 2$ | 0.95 | 0.83 | 0.55 | 0.77 |
| | $decile \geq 3$ | 0.89 | 0.68 | 0.58 | 0.73 |
| | $decile \geq 4$ | 0.81 | 0.56 | 0.60 | 0.69 |
| | $decile \geq 5$ | 0.72 | 0.45 | 0.63 | 0.65 |
| | $decile \geq 6$ | 0.63 | 0.34 | 0.66 | 0.62 |
| | $decile \geq 7$ | 0.51 | 0.25 | 0.69 | 0.59 |
| | $decile \geq 8$ | 0.39 | 0.16 | 0.72 | 0.57 |
| | $decile \geq 9$ | 0.26 | 0.09 | 0.74 | 0.54 |
| | $decile \geq 10$ | 0.12 | 0.03 | 0.79 | 0.51 |

### *Notebook Exercise 1*

Write code to reproduce the table above. Here are some hints to help you.

- If you're fuzzy on what each of these statistic means (false positive rate, true positive rate, etc.), consider checking out Binary

- If you're fuzzy on what each of these statistic means (false positive rate, true positive rate, etc.), consider checking out Binary Diagnostic Tests.
- You'll want to use the column `df['two_year_recid']` as your indicator of true positive versus true negative (positive means that the person recidivated).
- To select narrow the data frame to just contain people of a particular race you can use the following snippet ( `race` would be a string that is either "Caucasian" or "African-American").

```
df_for_race = df[df['race'] == race]
```

- To generate a particular row of the table, you'll want to loop over all possible thresholds where the model would predict recidivate ($\hat{y} = 1$).
- You can count the number of elements in a Pandas series that satisfy some criterion using the following technique. For instance, if we wanted to calculate the number of elements in "some_column" that are greater than 0 and less than 30, we could use the following code.

```
((df['some column'] > 0).sum() & (df['some column'] < 30).sum()
```

- It's up to you how you want to generate the table. You can simply print out the values within a loop as you compute them, or you could populate a data frame with your calculations and then plot them (this is what we did in the solution).

In [23]:

```
# ***Solution***

# we're going to create a data frame to hold all of the results.   Think of this
# as a represntation of the
results = pd.DataFrame(columns=['race', 'decile >=', 'true_positive_rate', 'false_positive_rate', '
positive_predictive_value', 'negative_predictive_value'])

for race in ['Caucasian', 'African-American']:
    df_for_race = df[df['race'] == race]
    y = df_for_race['two_year_recid']
    for thresh in range(1, 11):
        yhat = df_for_race['decile_score'] >= thresh
        true_positive_rate = ((y == 1) & (yhat == 1)).sum() / (y == 1).sum()
        false_positive_rate = ((y == 0) & (yhat == 1)).sum() / (y == 0).sum()
        if (yhat == 1).sum() == 0:
            positive_predictive_value = float('nan')
        else:
            positive_predictive_value = ((y == 1) & (yhat == 1)).sum() / (yhat == 1).sum()

        if (yhat == 0).sum() == 1:
            negative_predictive_value = float('nan')
        else:
            negative_predictive_value = ((y == 0) & (yhat == 0)).sum() / (yhat == 0).sum()

        results = results.append({'race': race,
                                  'decile >=': str(thresh),
                                  'true_positive_rate': true_positive_rate,
                                  'false_positive_rate': false_positive_rate,
                                  'positive_predictive_value': positive_predictive_value,
                                  'negative_predictive_value': negative_predictive_value}, ignore_i
dex=True)
results
```

/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:18: RuntimeWarning: invalid value encountered in long_scalars

Out[23]:

| | race | decile >= | true_positive_rate | false_positive_rate | positive_predictive_value | negative_predictive_value |
|---|---|---|---|---|---|---|
| 0 | Caucasian | 1 | 1.000000 | 1.000000 | 0.402439 | NaN |
| 1 | Caucasian | 2 | 0.852665 | 0.638987 | 0.473318 | 0.784404 |
| 2 | Caucasian | 3 | 0.736677 | 0.469388 | 0.513848 | 0.749503 |
| 3 | Caucasian | 4 | 0.642633 | 0.348346 | 0.554054 | 0.730284 |
| 4 | Caucasian | 5 | 0.526646 | 0.231527 | 0.605042 | 0.706796 |
| 5 | Caucasian | 6 | 0.410658 | 0.144265 | 0.657191 | 0.683146 |
| 6 | Caucasian | 7 | 0.294671 | 0.090077 | 0.687805 | 0.657012 |

| | race | decile | true_positive_rate | false_positive_rate | positive_predictive_value | negative_predictive_value |
|---|---|---|---|---|---|---|
| 7 | Caucasian | 8 | 0.203762 | 0.052780 | 0.722222 | 0.638520 |
| 8 | Caucasian | 9 | 0.118077 | 0.032372 | 0.710692 | 0.619648 |
| 9 | Caucasian | 10 | 0.047022 | 0.013371 | 0.703125 | 0.605877 |
| 10 | African-American | 1 | 1.000000 | 1.000000 | 0.528414 | NaN |
| 11 | African-American | 2 | 0.952381 | 0.823141 | 0.564542 | 0.768229 |
| 12 | African-American | 3 | 0.890316 | 0.667266 | 0.599208 | 0.730263 |
| 13 | African-American | 4 | 0.815409 | 0.553357 | 0.622803 | 0.683486 |
| 14 | African-American | 5 | 0.721776 | 0.434053 | 0.650748 | 0.644809 |
| 15 | African-American | 6 | 0.629749 | 0.327338 | 0.683111 | 0.618523 |
| 16 | African-American | 7 | 0.515784 | 0.236811 | 0.709345 | 0.584481 |
| 17 | African-American | 8 | 0.390048 | 0.147482 | 0.747692 | 0.555035 |
| 18 | African-American | 9 | 0.260567 | 0.086930 | 0.770570 | 0.524269 |
| 19 | African-American | 10 | 0.119315 | 0.028777 | 0.822878 | 0.496020 |

In [25]:

```python
import matplotlib.pyplot as plt

fig, ax = plt.subplots()
legend_strings = []
for race, df_by_race in results.groupby('race'):
    plt.plot(df_by_race['false_positive_rate'], df_by_race['true_positive_rate'])
    for _, row in df_by_race.iterrows():
        ax.annotate(str(row[1]), (row[3], row[2]))
    legend_strings.append(race)

plt.legend(legend_strings)
plt.xlabel('false positive rate')
plt.ylabel('true positive rate')
plt.show()
```
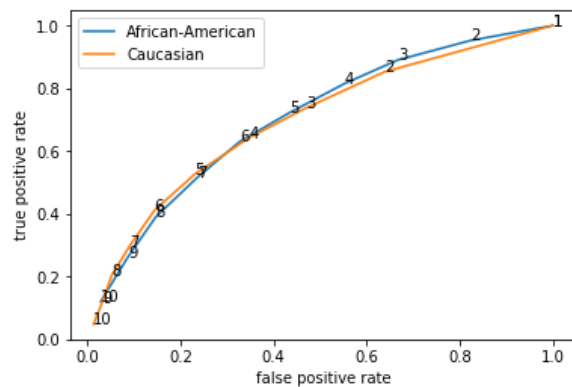


Notice how for a given threshold (annotated as text), the curves have vastly different false positive rates.

In [26]:

```python
fig, ax = plt.subplots()
legend_strings = []
for race, df_by_race in results.groupby('race'):
    plt.plot(df_by_race['negative_predictive_value'], df_by_race['positive_predictive_value'])
    for _, row in df_by_race.iterrows():
        ax.annotate(str(row[1]), (row[5], row[4]))
    legend_strings.append(race)

plt.legend(legend_strings)
plt.xlim([0, 1])
plt.ylim([0, 1])
plt.xlabel('negative predictive value')
plt.ylabel('positive predictive value')
plt.show()
```
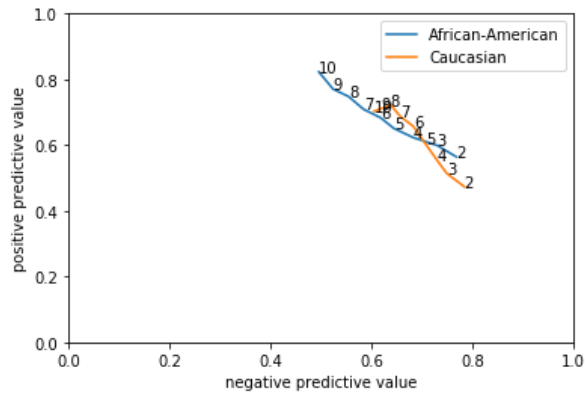
Notice how for a threshold of 4 or 5, the positive predictive values are almost identical.

## Sanity Check Using Sklearn

We can compare our calculations to the ROC curve (false positive rate versus true postive rate).

In [7]:

```python
from sklearn import metrics

for race in ['African-American', 'Caucasian']:
    df_filtered = df[df['race'] == race]
    y = df_filtered['two_year_recid']
    scores = df_filtered['decile_score']
    fpr, tpr, thresholds = metrics.roc_curve(y, scores)
    plt.plot(fpr, tpr)
plt.legend(['Caucasian', 'African-American'])
plt.xlabel('false positive rate')
plt.ylabel('true positive rate')
plt.show()
```
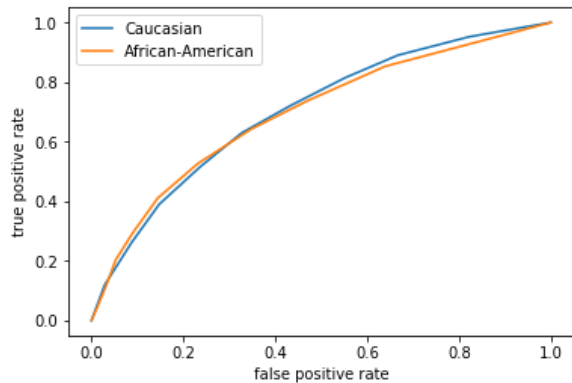
# Sentiment Classification Using Naïve Bayes

*Abstract*

In this notebook you'll be implementing the Naïve Bayes algorithm and applying it to the task of classifying the sentiment of a movie review. We're going to take a mostly bottom-up approach here where we directly connect the math we've been learning with the algorithm. At the end we'll compare our results to sklearn's built-in implementation of the algorithm to both sanity check our own implementation and potentially allow those who want to to run more experiments with an implementation that has more options.

## Sentiment Analysis

The [Wikipedia Article on Sentiment Analysis](#) provides the following definition for sentiment analysis.

> Sentiment analysis (also known as opinion mining or emotion AI) refers to the use of natural language processing, text analysis, computational linguistics, and biometrics to systematically identify, extract, quantify, and study affective states and subjective information. Sentiment analysis is widely applied to voice of the customer materials such as reviews and survey responses, online and social media, and healthcare materials for applications that range from marketing to customer service to clinical medicine.

In this notebook we'll be focusing on predicting the sentiment of a movie review from IMDB based on the text of the movie review. This dataset is one that was originally used in a Kaggle competition called [Bag of Words meets Bag of Popcorn](#) (you'll understand that joke by the end of this notebook!)

The [data](#) consists of the following.

> The labeled data set consists of 50,000 IMDB movie reviews, specially selected for sentiment analysis. The sentiment of reviews is binary, meaning the IMDB rating < 5 results in a sentiment score of 0, and rating >=7 have a sentiment score of 1. No individual movie has more than 30 reviews. The 25,000 review labeled training set does not include any of the same movies as the 25,000 review test set. In addition, there are another 50,000 IMDB reviews provided without any rating labels.

Our goal will be to see if we can learn a model, using Naïve Bayes on a training set to accurately estimate sentiment of new reviews.

Without further ado, let's download and parse the data into a data frame.

In [0]:

```python
import gdown
import pandas as pd

gdown.download('https://drive.google.com/uc?authuser=0&id=1Z8bwIBa_0gFe9-
C2W0goZ72lQfFMbxjS&export=download',
              'labeledTrainData.tsv',
              quiet=False)
df = pd.read_csv('labeledTrainData.tsv', header=0, delimiter='\t')
df
```

Let's look at the average sentiment to see what we are dealing with (1 is positive sentiment and 0 is negative)

In [0]:

```python
df['sentiment'].mean()
```

Looks like we're dealing with a balanced set of positives and negatives.

Next, let's look at a particular review. To make the output look nicer, we'll create a [new Pandas series with line wrapping](#).

In [0]:

```python
# this takes a little while to run
```

```
reviews_wrapped = df['review'].str.wrap(80)
```

In [0]:
```
print(reviews_wrapped.iloc[20])
```

## The Bag of Words Model

We know that in order to apply Naïve Bayes we need to convert each of our reviews into a vector of features. There are lots of different methods to convert text into vectors. In this notebook we'll be using a pretty basic (bugt suprisingly powerful) form of vectorization where we construct a feature vector with $k$ entries (where $k$ is the total number of unique words in the dataset) and for any particular review we set the corresponding entry to $1$ if that word appears in the dataset and $0$ otherwise. This representation is called the **bag of words** since the encoding of the text into features is independent of where the words occur in the text (you could shuffle the words in the review and still have the same feature vector). The Wikipedia article on Bag of Words has more information.

Instead of writing our own code to convert from text to a bag of words representation we're going to use scikit learn's built-in count vectorizer. Before we apply it to the data, let's apply it to toy dataset to help you better understand the bag of words model.

In [0]:
```
from sklearn.feature_extraction.text import CountVectorizer
import numpy as np

# the binary feature makes it so only the presence or absence of the word is
# returned (rather than the count)
vectorizer = CountVectorizer(binary=True)
toy_data = ['This is review one.  It has some words.', 'This is review two.  It also has some words.']
vectorizer.fit(toy_data)
X_toy = vectorizer.transform(toy_data)
X_toy
```

The first thing to notice about the data is that it's stored in a sparse matrix format. A sparse matrix is useful when the elements of your matrix are mostly zeros. If you have a lot of unique words, but each piece of text only contains a small fraction of those words you will end up with a sparse matrix. In this notebook we'll be limiting the size of our vocabulary and converting the matrix to dense. This is to ease implementation. If you want to leave things as a sparse matrix, please feel free to do so.

In [0]:
```
print("feature vectors", X_toy.todense())
print(vectorizer.get_feature_names())
```

### Notebook Exercise 1

Referencing back to what you know about the bag of words model and our toy data, explain why the vectors look the way they do. Make up your own toy data (or add to ours) and see if the results make sense.

***Solution***

The first entry of the vector corresponds to the word 'also', which occurs only in the second review (thus the first line after feature vectors has a 0 for the first entry and the second line has a 1). The second entry of the vector corresponds to the word 'has', which is in both reviews (so both vectors have a 1 there). The pattern continues as you go through the vector.

## Vectorizing the Whole Dataset

Now that you have a general idea what bag of words is all about, let's apply it to our movie reviews. To make our lives easier we're only going to include words in our feature vector if they occur in at least 100 reviews. Doing this will help with overfitting (although next assignment we will be learning another technique to deal with this). While we're at it we'll also convert the sentiment labels to a numpy array.

In [0]:
```
vectorizer = CountVectorizer(binary=True, min_df=100)
vectorizer.fit(df['review'])
X = vectorizer.transform(df['review']).todense()
```

```python
y = np.array(df['sentiment'])
print("X.shape", X.shape)
print("y.shape", y.shape)
```

As a quick intuition builder, let's look at a word we think would probably differ across sentiment values.

In [0]:

```python
terrible_index = vectorizer.get_feature_names().index('terrible')
print("terrible occurs in", X[y==1, terrible_index].mean(), "for Y=1")
print("terrible occurs in", X[y==0, terrible_index].mean(), "for Y=0")
```

Sorta makes sense (but please someone do some looking into what those reviews are where it terrible appears and it is Y=1)

## The General Form of Naïve Bayes

Last Assignment we showed how the Titanic problem could be solved using the Naïve Bayes Model. Specifically we computed what's known as the odds ratio.

$$\frac{p(Y \mid S)p(C = 1 \mid S)p(M \mid S)p(S)}{p(Y \mid \neg S)p(C = 1 \mid \neg S)p(M \mid \neg S)p(\neg S)}$$

Recall that $Y$ meant young passenger, $C$ represented fare class, $M$ represented "is male", and $S$ represented survival.

Hopefully it is pretty clear that while we derived the formula in the specific case of the Titanic model, the logic we applied is completely general. If we assume that we are doing binary classification provided values $x_1, x_2, ..., x_d$, then the odds ratio can be written in the following way.

$$\frac{p(Y = 1 \mid X_1 = x_1, X_2 = x_2, ..., X_d = x_d)}{p(Y = 0 \mid X_1 = x_1, X_2 = x_2, ..., X_d = x_d)} = \frac{p(Y = 1) \times p(X_1 = x_1 \mid Y = 1) \times ... \times p(X_d = x_d \mid Y = 1)}{p(Y = 0) \times p(X_1 = x_1 \mid Y = 0) \times ... \times p(X_d = x_d \mid Y = 0)}$$

We also learned last assignment that if this odds ratio is greater than 1, we should predict positive. While the odds ratio is a totally valid way to attack the problem, it can be numerically unstable. Therefore, most people use the log odds ratio instead (you just hit both sides with a log).

### Notebook Exercise 2

Compute the log odds ratio for the Naïve Bayes algorithm by taking the log of both sides of the preceding equation. Simplify as much as possible using properties of logarithms. What must be true of the log odds ratio in order to predict $Y = 1$?

*Solution*

The answer is pretty straight forward. We use the fact that log of a ratio is the difference in the log of the numerator and denominator. We also use the fact that log of a product is the sum of the logs of each of the terms.

$$\log \left( \frac{p(Y = 1 \mid X_1 = x_1, X_2 = x_2, ..., X_d = x_d)}{p(Y = 0 \mid X_1 = x_1, X_2 = x_2, ..., X_d = x_d)} \right) = \log p(Y = 1) - \log p(Y = 0) + \sum_{i=1}^{d} \left( \log p(X_i = x_i \mid Y = 1) - \log p(X_i = x_i \mid Y = 0) \right)$$

If the log odds ratio is greater than or equal to 0, we should predict $Y = 1$ (at least our probability will be at least 0.5).

## Fitting the Parameters of the Model

What we see from looking at the log odds ratio equation is that in order to apply the model we must have an estimate of the following probabilities.

- $p(Y = 0)$
- $p(Y = 1)$
- $p(X_i = x_i \mid Y = 0)$ (for $i$ from 1 to $d$)
- $p(X_i = x_i \mid Y = 1)$ (for $i$ from 1 to $d$)

The strategy for fitting these parameters will be the same as was described in the previous assignment (we'll see a more formal justification of why this works in the next assignment). In order to estimate a probability, we'll just count the number of times the

For instance, if we want to estimate $p(Y = 0)$ we would count the number of instances in the dataset where $Y = 0$ and divide that by the total number of instances in the dataset. If we wanted to esitmated $p(X_i = 1 \mid Y = 0)$ (suppose X_i represents the word "terrible") we would count the number of reviews that included the word terrible and had sentiment 0 and divide that by the number of reviews that were sentiment 0 (hopefully that makes sense given the definition of $p(A \mid B) = \frac{p(A,B)}{p(B)}$.

## Notebook Exercise 3

Write a function that takes as input $X$ and $y$ and returns a tuple containing the following elements (in this order)

1. The probability of Y=1
2. The probability of Y=0
3. A vector where the $i$th entry represents $p(X_i = 1 \mid Y = 1)$
4. A vector where the $i$th entry represents $p(X_i = 1 \mid Y = 0)$

To help you out we've included a unit test and a function stub.

In [0]:

```
def fit_nb_model(X, y):
    """ Fit the parameters of a Naive Bayes model given a bag of words model
        with binary counts (X) and class labels (y).

        Returns: a tuple with the following components in order
                The probability of Y=1,
                The probability of Y=0,
                A vector where the $i$th entry represents $p(X_i=1|Y=1)$
                A vector where the $i$th entry represents $p(X_i=1|Y=0)$

    >>> X = np.array([[1, 0, 1], [1, 0, 0], [0, 0, 1], [1, 1, 1]])
    >>> y = np.array([1, 0, 1, 0])
    >>> fit_nb_model(X, y)
    (0.5, 0.5, array([0.5, 0. , 1. ]), array([1. , 0.5, 0.5]))
    """
    return None

import doctest
doctest.testmod()
```

In [0]:

```
# ***Solution***
def fit_nb_model(X, y):
    """ Fit the parameters of a Naive Bayes model given a bag of words model
        with binary counts (X) and class labels (y).

        Returns: a tuple with the following components in order
                The probability of Y=1,
                The probability of Y=0,
                A vector where the $i$th entry represents $p(X_i=1|Y=1)$
                A vector where the $i$th entry represents $p(X_i=1|Y=0)$

    >>> X = np.array([[1, 0, 1], [1, 0, 0], [0, 0, 1], [1, 1, 1]])
    >>> y = np.array([1, 0, 1, 0])
    >>> fit_nb_model(X, y)
    (0.5, 0.5, array([0.5, 0. , 1. ]), array([1. , 0.5, 0.5]))
    """
    X_1 = X[y == 1, :]
    X_0 = X[y == 0, :]
    return y.mean(), 1 - y.mean(), X_1.mean(axis=0), X_0.mean(axis=0)

import doctest
doctest.testmod()
```

## Inference

Once we have the model parameters fitted, we have to be able to do inference on new data. This will amount to computing our log odds ratio and seeing if it is greater than 0. If the log odds ratio is greater than 0, we return a predicted sentiment of 1, otherwise we return a sentiment of 0.

Before having you implement the inference code, let's split into a train and test set and use your code to fit the model.

In [0]:

```
from sklearn.model_selection import train_test_split

X_train, X_test, y_train, y_test = train_test_split(X, y)
p_y_1, p_y_0, p_x_y_1, p_x_y_0 = fit_nb_model(X_train, y_train)
```

## Notebook Exercise 4

Implement a function called `get_nb_prediction` that takes as input the NB model ( `p_y_1, p_y_0, p_x_y_1, p_x_y_0` ) and a dataset (X), computes the log odds ratio and returns a vector of predictions for that data.

To get you started we have a function stub.

Hint: you can write a nested loop with one over the data points and one over the features and compute log probabilities as you go. (you can speed this up using numpy features, but don't worry about computational speed)

In [0]:

```
def get_nb_predictions(p_y_1, p_y_0, p_x_y_1, p_x_y_0, X):
    """ Predict the labels for the data X given the Naive Bayes model """
    return None

# here is some test code that will call your model on the first 100 test points
# (for speed of development).
y_pred = get_nb_predictions(p_y_1, p_y_0, p_x_y_1, p_x_y_0, X_test[:100,:])
```

In [0]:

```
# ***Solution***
# This will be really slow, but we hope it is more readable for folks that are
# less familiar with numpy
def get_nb_predictions(p_y_1, p_y_0, p_x_y_1, p_x_y_0, X):
    """ Predict the labels for the data X given the Naive Bayes model """
    log_odds_ratios = np.zeros(X.shape[0])
    for i in range(X.shape[0]): # loop over data points
        print("progress", i/X.shape[0])
        log_odds_ratios[i] += np.log(p_y_1) - np.log(p_y_0)
        for j in range(X.shape[1]): #loop over words
            if X[i, j] == 1:
                log_odds_ratios[i] += np.log(p_x_y_1[0, j]) - np.log(p_x_y_0[0, j])
            else:
                log_odds_ratios[i] += np.log(1 - p_x_y_1[0, j]) - np.log(1 - p_x_y_0[0, j])
    return (log_odds_ratios >= 0).astype(np.float)

y_pred = get_nb_predictions(p_y_1, p_y_0, p_x_y_1, p_x_y_0, X_test[:100,:])
```

## Calculating Accuracy

Now we'll test our model on all of the test data and see how accurate it is.

In [0]:

```
y_pred = get_nb_predictions(p_y_1, p_y_0, p_x_y_1, p_x_y_0, X_test)
print("accuracy is", (y_pred == y_test).mean())
```

## Sanity Check

Just to see if we're in the ball park, let's try scikit learn's built-in implementation of Naïve Bayes.

In [0]:

```
from sklearn.naive_bayes import MultinomialNB

model = MultinomialNB()
```

```
model.fit(X_train, y_train)
y_pred = model.predict(X_test)
np.mean(y_pred == y_test)
```