Response Letter – Contrastive Learning as Optimal Homophilic Graph Structure Learning

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We thank the reviewers for their constructive feedback on our work-in-progress submission. Following the review process, we continued developing our theoretical framework, motivated both by the valuable insights from reviewers and our ongoing research efforts. Rather than expanding the scope of our investigation, we focused on strengthening the theoretical foundations within the established framework, leading to substantially more rigorous mathematical results.

Enhanced Per-Node Formulation. Following reviewer suggestions for clearer mathematical definitions and motivated by the definition of node homophily, we refined our per-node formulation to significantly improve both theoretical rigor and practical applicability. While per-node homophily likelihood was already defined in our original submission, we removed the problematic aggregation over all nodes (with its assumption of conditional independence) and eliminated the artificial query/reference distinction while resolving technical issues with isolated nodes discovered during revision. This reformulation was essential for enabling the theoretical framework development described below, as the original definitions were insufficient for establishing rigorous mathematical connections.

Developing Novel Theoretical Framework. The enhanced per-node formulation enabled us to develop a probabilistic framework based on influence matrices that bridges discrete graph structures and continuous representations—a non-trivial theoretical contribution requiring careful mathematical construction. We established two fundamental theorems: Theorem 1 proving that our probabilistic homophily likelihood properly generalizes traditional node homophily, and Theorem 2 establishing the exact mathematical equivalence $\mathcal{L}^{(i)}_{\text{contrastive}} = -\log L_i$ between contrastive loss and negative log-likelihood of homophily. The key insight of our work lies not in mathematical complexity, but in revealing a fundamental connection that was previously hidden through the proper theoretical lens provided by the influence matrix formulation.

Strengthened Theoretical Claims. The reviewer-identified need for rigorous mathematical definitions enabled us to prove exact mathematical equivalences rather than the gradient alignment arguments in our original submission. This represents a substantially stronger theoretical result that provides a solid foundation for understanding why contrastive learning is effective for similarity-based tasks. The influence matrix framework provides a general foundation extensible to other similarity measures beyond our current focus on softmax-normalized similarities.

Comprehensive Framework Analysis. The stronger theoretical results enabled detailed analysis of practical considerations, including the ε-imperfectness parameter for understanding discrete-to-continuous transitions and how representation learning captures both explicit relationships and latent homophily potential. We developed insights about how standard generalization practices ensure that learned homophily reflects meaningful similarities rather than spurious label correlations, providing both the optimization target and quality assurance mechanism for learning meaningful graph structures. We also enhanced the presentation with comprehensive illustrations showing the connection between representation learning and graph homophily.

The reviewers' insights were instrumental in identifying theoretical gaps that, when addressed, transformed what initially appeared as empirical observations into rigorous theoretical equivalences. While we maintained the same scope regarding empirical validation, the mathematical foundations are substantially strengthened, providing a solid basis for future empirical investigations and practical applications in graph neural networks and adaptive graph construction.